Original Communications

ON THE POSSIBILITY OF CONSTRUCTING AN EINTHOVEN TRIANGLE FOR A GIVEN SUBJECT

Frank N. Wilson, M.D., J. Marion Bryant, M.D., and Franklin D. Johnston, M.D.
Ann Arbor, Mich.

INTRODUCTION

For more than a third of a century students of electrocardiography have struggled with the concepts that underlie the equilateral triangle of Einthoven, Fahr, and De Waart, and have continuously debated the validity of the assumptions it involves. During this long period numerous investigators have attempted to settle these disputes by generating an electric field of the appropriate kind in a human cadaver, or in some sort of model, and comparing the experimental results with the theoretical predictions. The more important of these studies have been reviewed in a recent article from this laboratory,[1] and the results of a cadaver experiment, of the kind in question, performed here a number of years ago have now been published[2] in connection with a discussion of the possibility of converting the Einthoven triangle into an equilateral tetrahedron.

It is our present purpose to present and discuss the results of some experiments of this same general sort in which the cadaver, or model, has been replaced by the body of a normal human subject. This substitution offers some very obvious advantages and it involves no very great flight of the imagination. We have wondered why we did not attempt it long ago and from what source came the suggestion that led us finally to make it. The train of thought concerned cannot now be traced to its origin with any certainty, but it may well have been the result of a discussion, in the autumn of 1944, when one of us appealed to Dr. Kenneth S. Cole and Dr. Alvin M. Weinberg for advice as to the best way of measuring the magnitude of the potential variations of a central terminal connected to the limb electrodes through equal resistances. Dr. Cole expressed, on this occasion, some dissatisfaction with the means previously employed for this purpose and intimated that a more direct approach to the problem was preferable and ought to be feasible. Apparently, his first reaction was that it might be

From the Department of Internal Medicine, University of Michigan Medical School.
The observations upon which this article is based were made with the aid of a grant from the S. S. Kresge Foundation.
Presented before the Third Inter-American Cardiological Congress, Chicago, Ill., June 13-17, 1948.

possible to obtain the desired information by mapping the isopotential lines of the cardiac field on the surface of the trunk. After further consideration, however, he remarked that the problem "might be reversed" by introducing a three-phase alternating current into the body by way of the limb electrodes and finding on the anterior and posterior surfaces of the chest the points at the same potential as the node formed by the three phases in Y or star connection. Some months later, preparations for carrying out this experiment were made but they were never completed, partly because it was found that when two limb electrodes were connected to a source of low frequency alternating current and also to a central terminal, through equal resistances, the location of the line on the body surface corresponding in potential to this terminal was determined chiefly by the relative magnitude of the two "contact" or "skin" resistances involved. The question then arose as to whether the measurement and equalization of these resistances (by the method of equalizing the resistances in the limb leads described by Einthoven, Bergansius, and Bijtel[3]) would not defeat the main purpose of the investigation. Dr. Cole suggested later that we connect the current source to two of the limb electrodes, using each of the three pairs in turn, and plot the points at the same potential as the third. The results of a few experiments of this kind are described in a later section of this paper.

The experiments with which we are at present mainly concerned were begun in September, 1946. We have frequently interrupted them for the purpose of analyzing the results of those already done and of studying the theoretical questions and the technical problems encountered. To a very large extent they have been of an unsystematic, preliminary, and exploratory kind. We are far from satisfied with the data collected thus far and are in the process of trying to perfect the technical procedures required to the point where the variation of the results on repetition of a given experiment will be negligible for our purpose. For this reason, the conclusions suggested by the observations available at this time must be regarded as subject to future revision.

When we began our investigations we were not aware of the work of Burger and Van Milaan[4] of Utrecht, who for several years have been studying electric fields generated in a model of the human body which they have built. Since we have become familiar with their publications we have been considerably influenced by their ideas, and particularly by their elegant method of constructing a triangle which summarizes all the information, concerning the nature of the electrical field in their model, obtainable by measuring the potential differences between electrodes corresponding in location to the limb electrodes used in clinical electrocardiography.

In our earliest experiments, one set of three electrodes was placed on the extremities in the usual way. Three electrodes were also placed on the anterior surface of the body, one just above the symphysis pubis and the other two near the junctions of the arms with the trunk. Three additional electrodes were put on the back and these were located as nearly as possible directly behind the corresponding electrodes of the anterior set. From these nine electrodes the following leads were taken: (1) the usual limb leads (I, II, and III); (2) corresponding leads from the anterior electrodes (IA, IIA, and IIIA); (3) the same leads from the posterior electrodes (IP, IIP, and IIIP); and (4) a lead from each of the anterior electrodes to the corresponding posterior electrode (RR, LL, and FF).

It should be pointed out that whereas the exact locations of the limb electrodes are a matter of no importance, the positions of the electrodes on the trunk must be determined with considerable precision if consistent results are to be obtained in experiments on different subjects. With this in mind the following plan for placing these electrodes was adopted. Each of the upper anterior electrodes was put in the area bounded laterally by the palpable coracoid process of the scapula and above by the inferior margin of the clavicle. The corresponding posterior electrode was then placed directly behind it and immediately below the scapular spine. The inferior posterior electrode was put in the midline on the lower part of the coccyx, and the corresponding anterior electrode straight in front of it, on the midline and just above the pubic symphysis. Even though considerable care was used in the placement of these electrodes, it seems not unlikely, when we consider that human chests vary greatly in size and shape, that the personal equation involved in deciding upon their exact positions sometimes had an effect upon the relative magnitudes of the deflections in the leads requiring their use.

The electrocardiograms of normal subjects which have been taken by this system of nine leads are all much like that reproduced in Fig. 1. It will be noted that the deflections in Leads RR, LL, and FF are strikingly similar in general outline but very different in magnitude. Those of Lead LL are largest, and those of Lead FF smallest. This is true of all of our records of this kind. In taking these sagittal leads the galvanometer connections were made in such a way that relative positivity of the posterior electrode produced an upward deflection in the completed record. The relations between the nine leads from the electrodes on the front and back of the trunk are expressed by the equations that follow. In these equations the symbols IA, IIP, RR, etc., may be regarded as representing the leads for which they stand, considered as vectors which are to be added vectorially according to the parallelogram law, or as representing the deflections in these leads, which are to be added algebraically.

$$(1) \quad IP + RR = IA + LL$$
$$(2) \quad IIP + RR = IIA + FF$$
$$(3) \quad IIIP + LL = IIIA + FF$$
$$(4) \quad IP + IIIP = IIP$$
$$(5) \quad IA + IIIA = IIA$$

In the experiments in which an artificial field was established in the trunk we employed a vacuum-tube beat-frequency oscillator to generate a current alternating 25 times per second. This current was introduced into the chest by way of two small circular electrodes and was maintained constant by means of a sensitive rectifier-type milliammeter. The diameter of the electrodes, the distance between their centers, and the magnitude of the current passed into the body have varied in different experiments. No systematic study of the effects of these variations has been made, but in general, the potential differences between

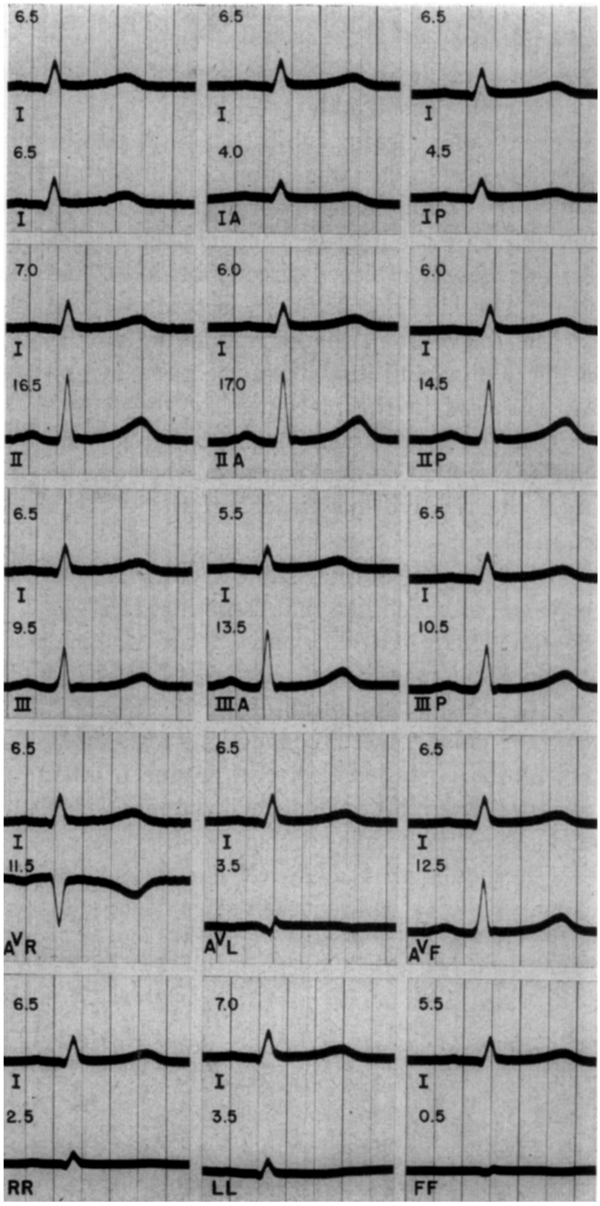


Fig. 1.—Experiment 5, *a*. Electrocardiogram of a normal subject showing the standard limb leads (I, II, and III), the corresponding anterior (IA, IIA, and IIIA), and posterior leads (IP, IIP, and IIIP), the augmented unipolar leads (aV_R, aV_L, and aV_F), and the sagittal leads (RR, LL, and FF).

the extremities have been roughly proportional to the product of the current and the distance between the input electrodes. It will be convenient to refer to this product as the electrical moment.

At the beginning of an experiment a point on the chest was selected and the centers of the input electrodes were placed on a line drawn through this point in such a way as to make the distances of their centers from it equal. In the case of a given subject the electric field established in the trunk by connecting these electrodes to the oscillator is then adequately defined by giving the location of the point mentioned, the distance between the centers of the electrodes, the magnitude and character of the current, and the angle between the line specified and the horizontal. After the first two or three experiments the location and manipulation of the input electrodes was greatly simplified by fastening several pairs of these electrodes to a small circular protractor. This made it possible, after the center of the protractor had been placed over the chosen point, to vary the position of the current axis in the manner desired by selecting the proper pair of connections instead of by removing and replacing the electrodes. We shall speak of the input as vertical when the current axis was parallel to the midsternal line, and as horizontal when this axis was perpendicular to this line.

Since alternating current was employed, the current axis had one direction during one-half of the cycle and exactly the opposite direction during the other half. The deflections recorded by the various leads used are, therefore, sine waves (considerably distorted in some instances). The measurements given represent the full amplitude of these waves minus the width of the light beam, which is about 2.0 millimeters. Plus and minus signs have been assigned to these measurements in such a way as to fulfill Einthoven's law whenever this law could be applied. In some instances each lead was taken simultaneously with Lead I, and when this was done deflections which were in phase with those of Lead I were considered positive and deflections which were 180° out of phase with those of Lead I were considered negative, or vice versa. For reasons which will appear later, the deflections of Leads RR, LL, and FF were made opposite in sign to those of unipolar Leads V_R, V_L, and V_F, respectively, except when the current axis was parallel to the sagittal plane, in which case they were considered all positive or all negative.

AN ILLUSTRATIVE EXPERIMENT

The experiment illustrated in Figs. 2,*A* and 2,*B* and summarized in Table I was performed on Sept. 24, 1946. A current of 0.5 milliampere was passed through the trunk by way of electrodes 1.1 cm. in diameter, which were equidistant from a point 4.0 cm. to the left of the midsternal line and at the level of the fourth intercostal space. The distance between them was 5.0 centimeters. The various leads were taken with the current axis horizontal and repeated after this axis had been rotated clockwise through 30, 60, 90, 120, and finally 150 degrees. The deflections in each of the records were measured and Einthoven's manifest potential difference, E, and the angle made by the electrical axis with the horizontal were computed in the customary way for each set of standard limb

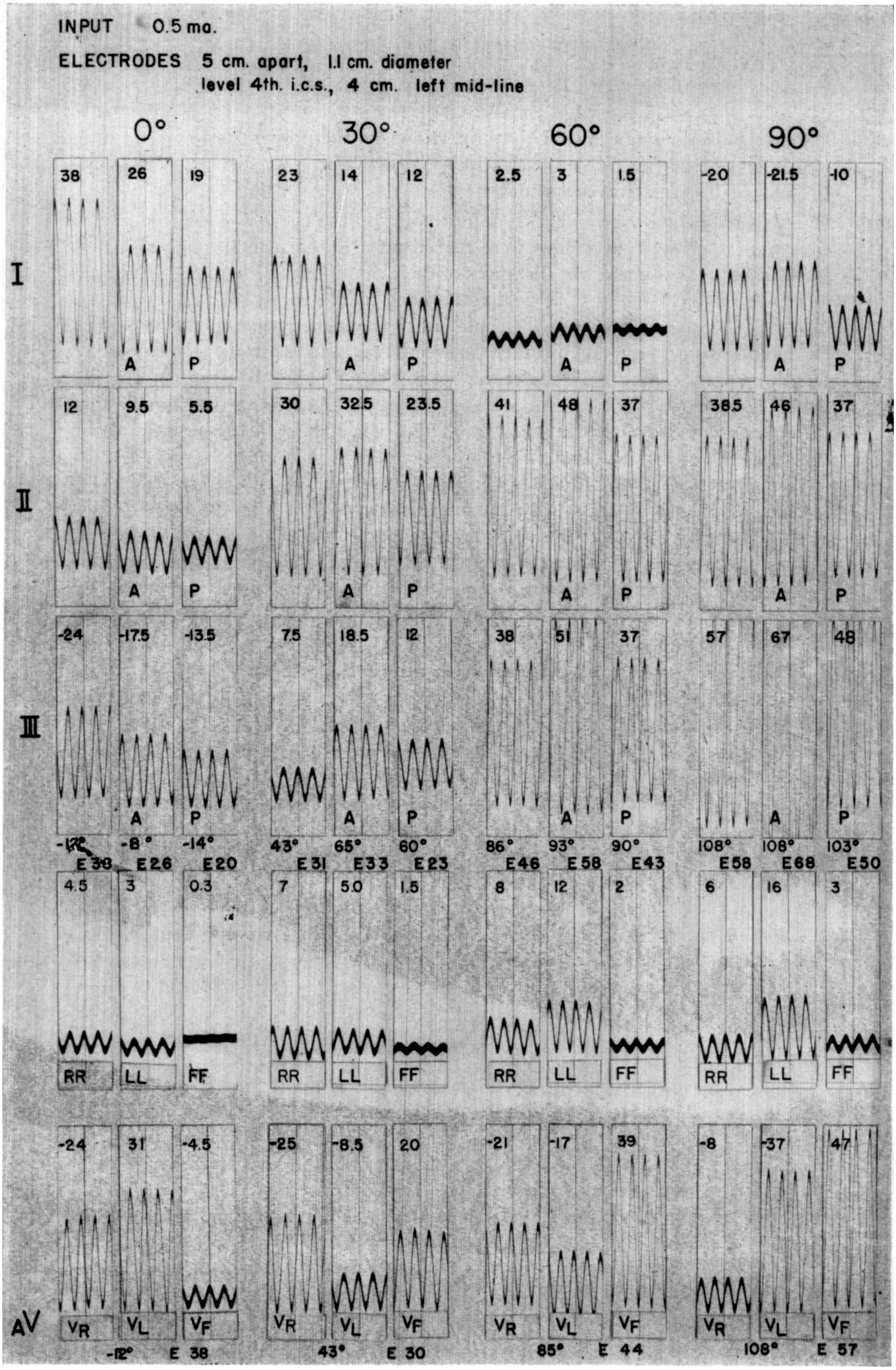


Fig. 2.—*A*, Experiment 3, *b*. Input electrodes to the left of the midline. Deflectons in the various leads when the angle made by the current axis with the horizontal was 0, 30, 60, and 90 degrees, respectively.

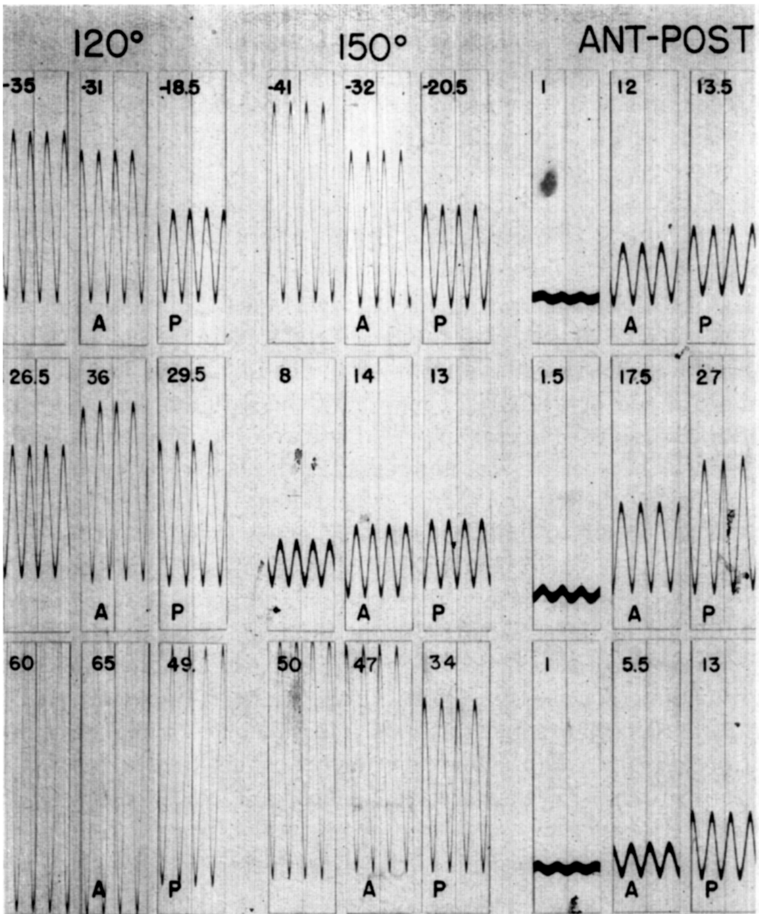


Fig. 2.—*B*, Experiment 3, *b*. Continuation of Fig. 2, *A*. Deflections in the various leads when the angle between the current axis and the horizontal was 120 and 150 degrees, and anteroposterior when one input electrode was on the back and the other on the front of the chest.

leads, unipolar limb leads, anterior leads, and posterior leads. The values for these quantities which appear in Figs. 2,*A* and 2,*B* were obtained graphically. Those which are given in Table I were computed. In the case of those measurements which did not fulfill Einthoven's law, the measurement for Lead III was revised, for the purpose of these computations, to the extent required to bring it into accord with those for Leads I and II.

The last set of leads shown in Fig. 2,*B* was taken at the end of the experiment for the purpose of studying the field produced by a sagittal current axis. One electrode was placed on the front of the chest, at the level of the fourth intercostal space and over the right margin of the sternum, and the other on the back, close to the seventh dorsal spine. The two input electrodes were then far apart and, since the current was not reduced, the electrical moment was much greater than when the other records were taken. It will be noted that the deflections in the standard limb leads are extremely small; in other experiments it was found that they could be abolished altogether by moving one of the input electrodes a short distance in one direction or another. It was never possible, however, to bring the deflections to zero in all of the standard limb leads and in all of the anterior or posterior leads at the same time. When the current axis has a sagittal direction the deflections in the sagittal leads, RR, LL, and FF, are naturally larger than when this axis is parallel to, or makes a small angle with, the frontal plane. In the few experiments of this kind performed, however, the deflections in Lead FF were always smaller, and usually much smaller, than those of Leads RR and LL.

Before discussing other aspects of this experiment we shall explain the figures enclosed in parentheses in Table I. In general, any number of forces acting at the same point can be added vectorially according to the parallelogram law, or can be regarded as equivalent to three forces each parallel to one axis of a system of mutually perpendicular axes equal in number to the dimensions of the space under consideration. It has always been held that the electromotive force of the heart at a given instant is a directed quantity of this kind, and this view is certainly correct if the electrical field of the heart is equivalent to that of a mathematical dipole, as Einthoven and his associates assumed. In the experiment under consideration, the distance between the positive and negative poles was not, as in the case of such a dipole, extremely small in comparison with the distance of the lead electrodes from them. It seemed desirable, therefore, to ascertain whether, for example, a current input of 0.3 milliamperes at 30 degrees was equivalent, under these circumstances, to a current input of 0.3 cos 60° milliamperes at 0 degrees plus a current input of 0.3 sin 60° milliamperes at 90 degrees. The cosine of 60 degrees is 0.5 and the sine 0.866. If we multiply the observed deflection in Lead I listed under 0 degrees in the table by the former, we obtain the figure 19.0. The deflection for the same lead listed under 90 degrees is −20.0, and this multiplied by 0.866 gives −17.32. The expected deflection in Lead I for a current axis of 60 degrees is then the algebraic sum of these two figures, or 1.68. The observed value is 2.5. For a current axis of 30 degrees the deflection in Lead I measured 23.0 millimeters. The calculated amplitude of this deflection is 22.9 millimeters. If the other figures given in the

TABLE I. MAGNITUDE OF DEFLECTIONS, MANIFEST POTENTIAL DIFFERENCE (E), AND THE ANGLE MADE BY THE ELECTRICAL AXIS WITH THE HORIZONTAL AS DETERMINED IN EXPERIMENT 3b.

CURRENT AXIS / LEAD	0°		90°		30°		60°		120°		150°		AP
I	38.0		−20.0		23.0 (22.9)		2.5 (1.7)		−35.0 (−36.3)		−41.0 (42.9)		1.0
II	12.0	−12° 38.9	38.5	110° 59.4	30.0 (29.6)	42° 31.1	41.0 (39.3)	82° 42.4	26.5 (27.3)	125° 63.8	8.0 (8.8)	141° 55.3	1.5
III	−24.0		57.0		7.5 (6.7)		38.0 (37.6)		60.0 (63.6)		50.0 (51.8)		1.0
IA	26.0		−21.5		14.0 (11.8)		−3.0 (−5.6)		−31.0 (−31.6)		−32.0 (−33.2)		12.0
IIA	9.5	−9° 26.3	46.0	108° 69.0	32.5 (31.2)	68° 31.5	48.0 (44.5)	96° 54.1	36.0 (35.1)	118° 67.3	14.0 (14.8)	132° 49.1	17.5
IIIA	−17.5		67.0		18.5 (19.6)		51.0 (50.2)		65.0 (66.7)		47.0 (48.0)		5.5
IP	19.0		−10.0		12.0 (11.5)		1.5 (0.8)		−18.5 (−18.2)		−20.5 (−21.5)		13.5
IIP	5.5	−14° 19.2	37.0	102° 49.5	23.5 (23.3)	60° 23.4	37.0 (34.8)	89° 39.2	29.5 (29.3)	112° 48.2	13.0 (13.7)	127° 35.4	27.0
IIIP	−13.5		48.0		12.0 (11.8)		37.0 (34.0)		49.5 (47.4)		34.0 (35.2)		13.0
AV_R	−24.0		−8.0		−25.0		−21.0		5.0		17.0		1.5
AV_L	31.0		−37.0		8.5		−17.0		−47.0		−45.0		0
AV_F	−4.5		47.0		20.0		40.0		42.0		28.0		1.5
RR	4.5		6.0		7.0 (6.9)		8.0 (7.5)		−2.5 (−2.9)		−1.0 (0.9)		57.5
LL	−3.0		16.0		−5.0 (−5.4)		12.0 (12.3)		15.0 (15.3)		9.0 (10.6)		31.0
FF	0.3		−3.0		−1.5		−2.0		−3.0		−2.0		10.0
$(V_T)_P-(V_T)_A$	0.6		6.3		0.2		6.0		4.8		2.0		

The theoretical magnitudes of the deflections are shown in parentheses directly below the observed magnitudes.

For each set of standard limb leads, anterior leads, and posterior leads, the manifest potential difference E and the angle which gives the position of the electrical axis, found by the customary method, are given following the figure for the deflection in Lead II, Lead IIA, and Lead IIP.

table are examined it will be seen that, considering all the possibilities of error involved, the agreement between the observed and calculated deflections is surprisingly good. Hereafter, when the distance between the input electrodes was not greater than 5 cm., we shall confine the discussion chiefly to the tracings obtained when the current axis was horizontal and those obtained when it was vertical. It will be convenient to use the letter H to designate the former and the letter V to designate the latter. Such symbols as $(\mathrm{I})_{\mathrm{H}}$, $(\mathrm{II})_{\mathrm{H}}$, $(\mathrm{II})_{\mathrm{V}}$, and $(\mathrm{III})_{\mathrm{V}}$ will be used to represent the deflections in the various leads under these two different conditions.

The sagittal leads RR, LL, and FF measure the differences in potential between the apices of the triangle formed by the posterior leads and the corresponding apices of the triangle formed by the anterior leads. These relations are expressed by the following equations:

(6) $(V_R)_P - (V_R)_A = RR$

(7) $(V_L)_P - (V_L)_A = LL$

(8) $(V_F)_P - (V_F)_A = FF$

Here the symbol $(V_R)_P$ represents the true potential of the electrode on the posterior aspect of the right shoulder and the other symbols have a like significance. If these equations are added and the sum is divided by three, we obtain the expression

(9) $1/3\ [(V_R)_P + (V_L)_P + (V_F)_P)] - 1/3\ [(V_R)_A + (V_L)_A + (V_F)_A] = 1/3\ (RR + LL + FF)$

or, what amounts to the same thing,

(10) $(V_T)_P - (V_T)_A = 1/3\ (RR + LL + FF)$

where $(V_T)_P$ is the potential of a central terminal connected through equal resistances to the three posterior electrodes, and $(V_T)_A$, the potential of central terminal connected through equal resistances to the three anterior electrodes. When the sum in the second member of this equation is positive, the anterior terminal is negative with respect to the posterior.

In Table I the figure obtained by adding the deflections in Leads RR, LL, and FF algebraically and dividing the sum by 3 is given for each position of the current axis. Observe that the calculated difference in potential between the two central terminals was small when the current axis was horizontal or nearly so, and relatively large when this axis was vertical or nearly vertical. The anterior terminal is negative with respect to the posterior for all of the positions of the current axis listed.

There is a rough correspondence between the relative size of the deflections in the unipolar leads from the two arms and the relative size of the deflections in Leads RR and LL. In the three instances in which the deflections in Lead aV_L are very much larger than the deflections in Lead aV_R, the deflections in Lead LL are much larger than those in Lead RR (Table I, columns headed 90, 120, and 150). In one instance (Column 30) the deflections in Lead aV_R are much larger than those of Lead aV_L and the deflections of Lead RR are somewhat arger than those of Lead LL. In the other two cases this relation does not hold;

in one of them (Column 0) the deflections of Lead aV_L are considerably larger than those of Lead aV_R, but the deflections of Lead LL are smaller than those of Lead RR; in the other (Column 60) the opposite is true. The deflections in Lead FF are very small for every position of the current axis and it is difficult to demonstrate any relation of the kind in question between this lead and Lead aV_F.

Compared to the apices of the anterior triangle, the corresponding apices of the posterior triangle are more distant from the input electrodes but otherwise similarly situated with respect to them. One would, therefore, expect that when the potential of a given apex of the former became positive or negative the potential of the corresponding apex of the latter must show a fluctuation of the same sign but of smaller magnitude. The coccygeal and pubic electrodes are farther apart than the two electrodes of the other two pairs, but they are also much farther from the precordium. Does this account for the smallness of the deflections in Lead FF? Are the potential variations of the leg electrode regularly smaller than those of the arm electrodes even when the current axis is vertical? These questions cannot be answered now, but we are confident that they can be answered in the near future.

CONSTRUCTION AND USE OF THE BURGER TRIANGLE

Referring once more to the data shown in Table I, we may point out that the magnitude of Einthoven's manifest electromotive force (E) shows striking variations with the position of the current axis. It is much larger for a vertical than for a horizontal axis and is greatest when the current flow is parallel to Lead III (120 degrees). There are also pronounced discrepancies between the position of the axis defined by the input electrodes and the position of the electrical axis calculated in the customary manner. When the former axis made an angle of 60 degrees with the horizontal, the latter was separated from it by an angle of 22 degrees when the calculations were based on the standard limb leads; 36 degrees when they were based on the anterior leads; and 29 degrees when they were based on the posterior leads. For this position of the current axis the deflections in Leads I, IA, and IP were very small so that the calculated axis was nearly perpendicular to these leads.

For both horizontal and vertical current flow, the deflections in Lead III were larger than those in Lead II and the deflections in Lead aV_L were larger than those in Lead aV_R. It is apparent, therefore, that the differences between the true situation existing in this experiment and that which Einthoven and his associates postulated should be attributed to the eccentric position of the input electrodes. These were placed to the left of the midsternal line, and the left shoulder, being closer to the source of the field than the right, displayed larger potential variations than its fellow.

The methods devised by Burger and Van Milaan[4] make it possible to present practically all of the significant data pertaining to Experiment 3,*b* in a single diagram (Fig. 3). They regard the deflection in a given lead as the scalar product of two vectors, one of which represents that lead and the other the electromotive force responsible for the field. In the Gibb's notation the scalar, or dot, product of two vectors, **A** and **B**, is written **A·B** in clarendon type. The value of this expression is obtained by multiplying the product of the lengths of the two vectors by the cosine of the angle between the positive direction of the one and the positive direction of the other. This amounts to the same thing as multiplying the length of one vector by the projection of the other upon it. When one vector is of unit length the scalar product is equal to the length of the other multiplied by the cosine of the angle between the directions which the two vectors define.

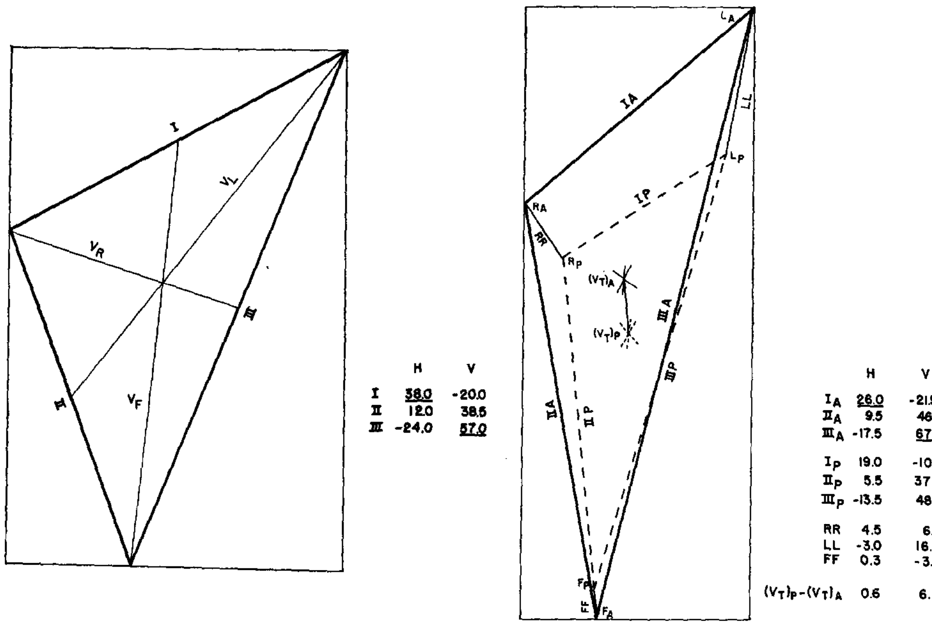


Fig. 3.—Experiment 3, *b*. Diagrams, constructed by the method of Burger and Van Milaan,[4] which depict in graphic form the material shown in Figs. 2, *A* and 2, *B* and in Table I.

In the present discussion we need not be concerned with the absolute, but only with the relative lengths of the vectors with which we shall have to deal. We may, therefore, consider that one which represents the electromotive force as of unit length. When this is done the deflection in any lead is the product of the length of that lead and the cosine of the angle which its positive direction makes with that of the current axis. The problem of finding the vectors which depict the leads used in our experiment then becomes a relatively simple one, for we know for each of these leads the size of the deflections both when the current axis was horizontal and when it was vertical. In the one case the unit vector points from right to left (0 degrees) and in the other it points straight down (90 degrees). The direction of Lead I, for example, with respect to the first of these unit vectors, is defined by the angle of which the tangent is equal to $(I)_V$ divided by $(I)_H$, or $-20/38$ (Table I). Performing the indicated operation, we obtain the figure -0.5263, and on consulting a table of the natural trigonometric functions find that the angle sought is approximately -28 degrees. The length of Lead I, which gives the size of the deflection in this lead, when the current axis is parallel to it and may, therefore, be represented by the symbol $(E)_I$, can be found by adding the squares of $(I)_V$ and $(I)_H$ and extracting the square root of the sum thus obtained. In the present instance this calculation gives the figure 42.94. Table II gives the length and direction with respect to the horizontal of each of the twelve leads used in the experiment under consideration.

TABLE II. LENGTH AND DIRECTION, WITH RESPECT TO THE HORIZONTAL, OF THE TWELVE LEADS USED

LEAD	LENGTH	ANGLE (DEGREES)
I	42.9	−28
II	40.3	73
III	64.0	114
IA	33.7	−40
IIA	47.0	78
IIIA	70.0	103
IP	21.5	−28
IIP	37.0	81
IIIP	50.0	109
RR	7.5	53
LL	16.3	101
FF	3.0	−84

The rather tedious computations involved in this method can be avoided by the use of a straightedge, square, and ruler. To construct a given lead, proceed as follows: On a horizontal line lay off a segment of a length equal to the amplitude of the deflection for a horizontal input. If this deflection is positive, draw a vertical line through the right, and if it is negative, through the left end of this segment. On this vertical line measure from the horizontal line a length equal to the amplitude of the deflection for a vertical input. This second segment should extend downward when this deflection is positive, and upward when it is negative. Draw the hypotenuse of the right triangle of which these two segments are the perpendicular sides. This line represents the lead vector and its positive direction is from the beginning of its horizontal to the end of its vertical component.

In constructing a diagram such as that shown in Fig. 3, it is, of course, necessary to find only those lead vectors that are independent. When the lead vectors already drawn are sufficient to locate all the electrodes, the others can be found by drawing lines between the points representing the electrodes which they connect.

The augmented unipolar limb leads are represented by the medians of the triangle corresponding to the standard limb leads. The unaugmented unipolar limb leads are those segments of these medians lying between their intersection and the apices of the triangle. The medians of the anterior and the posterior triangle have a similar significance. The vector drawn from the intersection of the medians of the former to the intersection of the medians of the latter depicts a lead from the central terminal of the first to that of the second.

To avoid a possible misunderstanding it should be emphasized that the nine lead diagram of Fig. 3 is not an attempt to reproduce a three dimensional figure upon a flat surface. The sagittal leads RR, LL, and FF appear in this diagram only because they showed deflections when the current axis was parallel to the frontal plane. In other words, these leads had components in the frontal plane, and it is these that appear in the diagram. The sagittal components of these leads, by virtue of which they displayed deflections when the current axis was parallel to the sagittal plane, are not shown.

When the deflections in two of the three leads of an oblique triangle are known, the direction and length of the vector which represents the electromotive force acting can be found. The method of doing this is somewhat different from that employed in the case of an equilateral triangle and requires some words of explanation.

The scalar product of two vectors, **A** and **B**, is equal to the sum of the scalar products obtained by multiplying each of the mutually perpendicular components of the one by the corresponding component of the other; that is,

$\mathbf{A}\cdot\mathbf{B} = a_1b_1 + a_2b_2$, where a_1 is the length of the horizontal component of **A**, b_1 is the length of the horizontal component of **B**, and a_2 and b_2 are the lengths of the vertical components of **A** and **B**, respectively.

Let us assume that Leads I and II are those for which the deflections are known. Let D_I and D_{II} stand for these known deflections, and X and Y for the horizontal and vertical components of the unknown electromotive force. We may then form the following equations:

$$(11)\quad D_I = (I)_H X + (I)_V Y$$

$$(12)\quad D_{II} = (II)_H X + (II)_V Y$$

The second member of each of these equations is the sum of the product of the horizontal components of the lead vector and the electromotive force and the product of their vertical components. These equations can easily be solved for the unknowns, X and Y. If the first is divided by the length of Lead I and the second by the length E_{II} of Lead II, we obtain:

$$(13)\quad \frac{D_I}{E_I} = \frac{(I)_H X}{E_I} + \frac{(I)_V Y}{E_I}$$

$$(14)\quad \frac{D_{II}}{E_{II}} = \frac{(II)_H X}{E_{II}} - \frac{(II)_V Y}{E_{II}}$$

These last equations show that the deflection in a given lead divided by its length is equal to the sum of the projections of the horizontal and vertical components of the electromotive force, and, therefore, to the projection of this force as a whole upon that lead. In obtaining the magnitude and direction of the electromotive force graphically, the procedure in the case of the Burger triangle differs from that employed in the case of the Einthoven triangle in only one respect. The former does not have sides that are equal in length and each of the known deflections must be divided by the length of the lead to which it belongs before operations are begun.

EFFECT OF THE POSITION OF THE INPUT ELECTRODES WITH REFERENCE TO THE MIDLINE UPON THE FORM OF THE BURGER TRIANGLE

Five experiments were performed upon Subject 4 and these are designated by the symbols 4,*b*, 4,*c*, 4,*d*, 4,*e*, and 4,*f*. In all these experiments the input electrodes (1.1 cm. in diameter) were at the same vertical level (fourth intercostal space at the sternal margin) and the current passed through them was 0.5 milliamperes. In three instances they were 5.0 cm. apart (4,*b*, 4,*d*, and 4,*f*); in the first of these, the point midway between them was 5.0 cm. to the right of the mid-

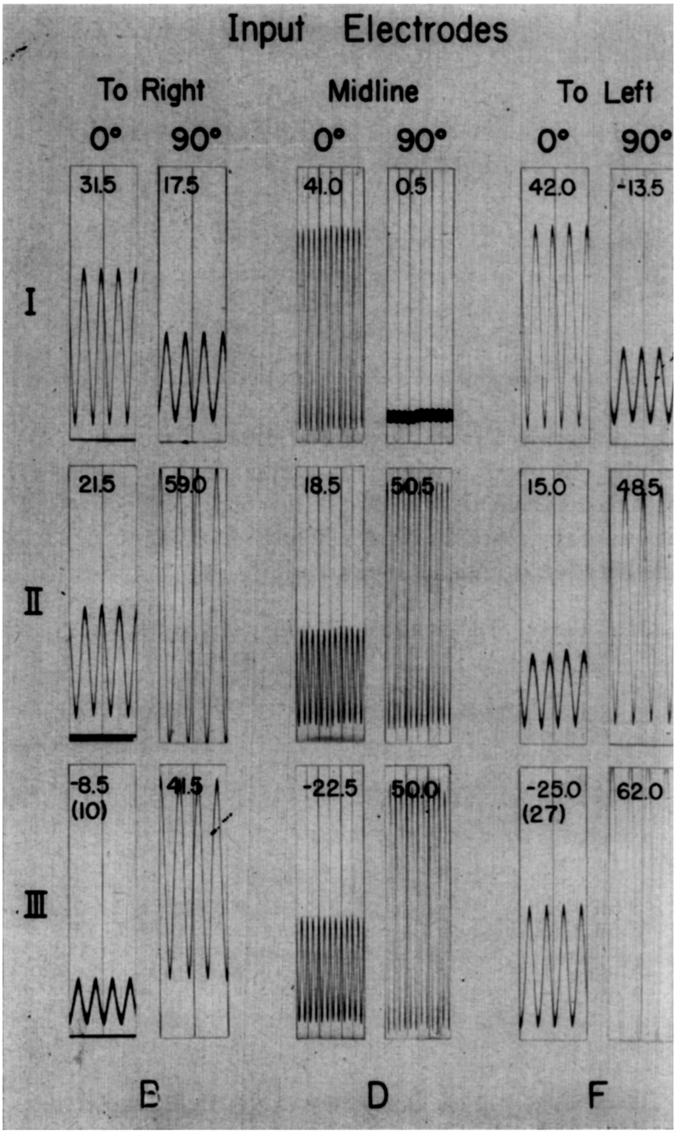


Fig. 4.—Experiments 4, *b*, 4, *d*, and 4, *f*. Input electrodes to the right of the midline in 4, *b*, in the **midline in 4, *d*, and to the left of the midline in 4, *f*.** The deflections in the standard limb leads when **the current axis was horizontal (0°) and when it was vertical** (90°).

sternal line, in the second it was on this line, and in the third it was 5.0 cm. to the left of it. The tracings obtained in these three experiments when the current axis was horizontal and when it was vertical are reproduced in Fig. 4, and the corresponding triangles are shown in Fig. 5. Experiments 4,*c* and 4,*e* were identical with Experiments 4,*b* and 4,*d*, respectively, except that the distance between

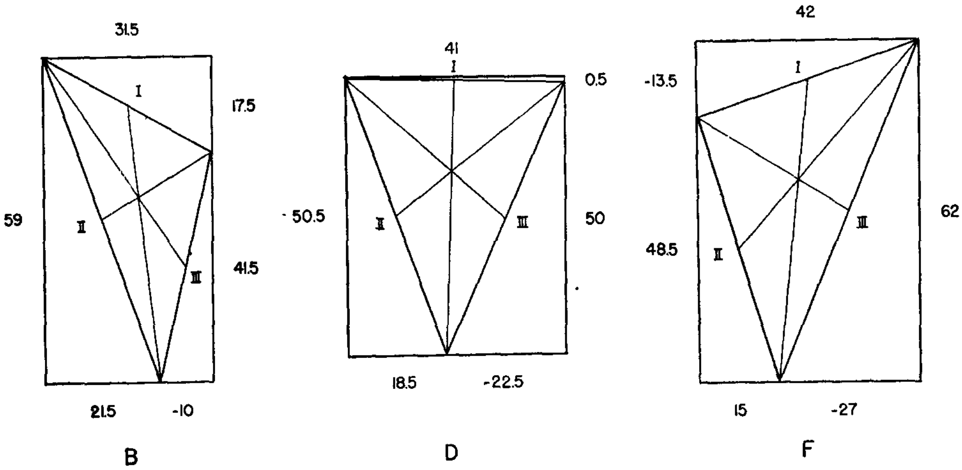


Fig. 5.—The Burger triangles corresponding to the deflections shown in Fig. 4.

the input electrodes was 2.0 cm. instead of 5.0 centimeters. The effects of this reduction in the electrical moment are summarized in Table III, which gives for all of the five experiments the amplitudes of the deflections in the limb leads for two positions of the current axis, the length (E_I, E_{II}, etc.) of each lead vector, and the angle between it and the horizontal.

TABLE III. EFFECT OF REDUCTION OF DISTANCE BETWEEN INPUT ELECTRODES (ELECTRICAL MOMENT)

EXPERIMENT	INPUT ELECTRODES		DEFLECTIONS					
	POSITION	DISTANCE APART	LEAD I		LEAD II		LEAD III	
			0 H	90 V	0 H	90 V	0 H	90 V
4b	5 cm. right	5 cm.	31.5	17.5	21.5	59.0	−8.5	42.0
4c	5 cm. right	2 cm.	15.5	8.0	10.5	25.0	−4.5	15.5
4d	Midline	5 cm.	41.0	0.5	18.5	50.5	−21.5	50.0
4e	Midline	2 cm.	22.5	0.5	10.0	21.0	−12.5	20.5
4f	5 cm. left	5 cm.	42.0	−13.5	15.0	48.5	−25.0	62.0
			E_I	ANGLE (DEGREES)	E_{II}	ANGLE (DEGREES)	E_{III}	ANGLE (DEGREES)
4b	5 cm. right	5 cm.	36.0	29	62.8	70	43.2	103
4c	5 cm. right	2 cm.	17.6	27	27.1	67	16.1	106
4d	Midline	5 cm.	41.0	0	53.3	70	54.4	114
4e	Midline	2 cm.	22.5	0	23.2	65	23.9	121
4f	5 cm. left	5 cm.	44.1	−18	50.8	73	67.3	113

It is of interest that when we reduced the distance between the input electrodes from 5.0 to 2.0 cm., without altering the size of the current flowing, the amplitude of the deflections fell somewhat more than 50 per cent, on the average, but the shape of the Burger triangle did not materially change.

When the point midway between the centers of the electrodes was on the midsternal line, the triangle was isosceles, or very nearly so, with the side corresponding to Lead I shorter than those corresponding to Leads II and III. When this point was to the right of the midline, Lead II was longer than Lead III and the angle which defines the direction of Lead I was positive; when it was to the left of the midline, Lead III was larger than Lead II and the angle between Lead I and the horizontal was negative. There can be no doubt that in the first of these three cases the potential variations of the two arms were of approximately the same magnitude. In the second the potential variations of the right arm were greater than those of the left for both positions of the current axis; in the third the reverse was true.

ON SEPARATING THE HORIZONTAL AND VERTICAL COMPONENTS OF THE FIELD AND BRINGING THE POTENTIAL OF THE CENTRAL TERMINAL TO ZERO

In Experiment 4,*d* the Burger triangle was very close to isosceles (Fig. 5). Let us make it exactly so by equalizing the absolute magnitudes of the deflections in Leads II and III; this process gives for the amplitudes of the deflections in the various leads the following figures:

$(\mathrm{I})_H$	$(\mathrm{II})_H$	$(\mathrm{III})_H$	$(V_R)_H$	$(V_L)_H$	$(V_F)_H$
41.0	20.5	−20.5	−20.5	20.5	0
$(\mathrm{I})_V$	$(\mathrm{II})_V$	$(\mathrm{III})_V$	$(V_R)_V$	$(V_L)_V$	$(V_F)_V$
0	50.3	50.3	−16.7	−16.7	33.4

In this case, it is clearly possible to separate the horizontal and the vertical components of the electromotive force, for the purpose, for example, of obtaining an accurate vectorcardiogram. Lead I yields a deflection of 41 mm. when the current axis is horizontal and no deflection when this axis is vertical. Lead V_F gives a deflection of 33.4 mm. when the current axis is vertical and no deflection when it is horizontal. By recording Lead I with the "horizontal channel" of the vectorcardiograph operating at its "normal" sensitivity, and Lead V_F simultaneously with the "vertical channel" of the vectorcardiograph operating at a sensitivity 41/33.4 times as great, we should obtain an accurate record of the changes in the position of the electrical axis.

When the current axis was horizontal the deflection in Lead V_F was zero; that is, the central terminal and the left leg were at the same potential. The spatial relations between the input electrodes and the junctions of the limbs with the trunk were such that there can be no doubt that this potential was zero in the sense that it was midway between that of the region where the current entered and that of the region where the current left the body. On the other hand, the shape of the Burger triangle indicates that when the current axis was vertical the potential of the central terminal was not zero in this sense. It may seem probable that the shape and dimensions of the triangle, which are given, supply the data necessary for the computation of the potential of this terminal, but this is not the case, unless it be assumed that the point midway between the input electrodes was "electrically" equidistant from the limb electrodes. Actually, this point was much farther, in the geometric sense, from the junction of the left leg with the trunk than from the junctions of the arms with the trunk. The assumption mentioned amounts to the supposition that the field in this experiment was equivalent to that of a centric dipole in a homogeneous conducting sphere. In the case of a model of this kind, and leads from electrodes that are equidistant from the center of the homogeneous spherical volume conductor, the Burger triangle and the geometric triangle defined by the three lead electrodes are similar in the technical Euclidean sense; each side of the one is parallel to the corresponding side of the other. Under the circumstances specified, the same statement holds for other figures, including the Burger tetrahedron and the corresponding geometric tetrahedron defined by four lead electrodes not all in the same plane. Under all other circumstances, the Burger triangle or tetrahedron, as the case may be, differs in shape from the geometric triangle or tetrahedron of which the lead electrodes are the apices.

If we make the assumption in question, the potential of the central terminal when the current axis was vertical can be found in the following way: The tangent of half the angle at that apex of the isosceles triangle corresponding to the leg electrode is given by the ratio $\frac{1/2(I)_H}{(II)_V}$, or $\frac{20.5}{50.3}$, which is equal to 0.4075.

This is the tangent of 22 degrees. Knowing this angle, we can find the other angles of the triangle. The angle made with the horizontal by the radius vector from the center of the circle defined by the apices of the triangle to that apex which corresponds to the electrode on the left arm is 90 degrees minus 44 degrees, or 46 degrees. The true potential of this electrode when the current axis was vertical was, therefore, R sin −46 degrees where R is the radius of the circle mentioned and therefore the length of the leads from the center of this circle to the apices of the triangle. Since the true potential of the leg electrode was consequently R cos 0 degrees when the current axis was vertical, we have the equation

$$R \cos 0° - R \sin -46° = (II)_V$$

or

$$R(1 + 0.695) = 50.3, \text{ and } R = 29.7$$

The true potential of the left leg was then 29.7 tenths of a millivolt and the true potential of the central terminal 29.7 minus 33.4 or −3.7 tenths of a millivolt. We have carried out this computation for the sake of introducing the problem of reducing the potential variations of the central terminal to zero. We do not believe that the assumption on which it is founded is valid.

In a preceding section of this paper, it was pointed out that the vectors drawn from the intersection of the medians of the Burger triangle to its apices represent the three unaugmented unipolar limb leads. Since these leads measure the potential of the limb electrodes with respect to that of the central terminal, we may consider each apex of this triangle at the same potential as the corresponding extremity. We may likewise regard the intersection of the medians as at the same potential as a central terminal connected to the limb electrodes through equal resistances. With the midpoint of each side of the triangle we may associate a potential half-way between the potentials of the apices it connects. We may suppose that along each side and each median of the triangle the potential falls uniformly from the end of that side or median where it is higher to the end where it is lower. Every point inside the triangle is then at a potential which may be regarded as a weighted mean of the potentials of its apices. When the dipole responsible for the field and the lead electrodes lie in the same plane and the dipole is inside the geometric triangle which these electrodes define, one of the points of the corresponding Burger triangle is at zero potential for every position of the dipole axis. When the dipole is outside the electrode triangle it is clear that all of its apices, and, consequently, all points inside the Burger triangle, must be positive for some positions of the electrical axis and negative for others.

When the resistances in the arms of the central terminal are equal, its potential is the mean of the potentials of the apices of the triangle. By making these resistances unequal and giving them the proper relative magnitude, it is, however, possible to make the potential of the central terminal equal to that of any point on the perimeter of the triangle or inside it. We may regard CR, CL, and CF leads as leads from a central terminal connected to two apices of the triangle by infinite resistances. In this case, the potential of the terminal is the same as that of the third apex. Augmented unipolar limb leads are leads from a central terminal with equal resistances in two of its branches and an infinite resistance in the third. In this case, the reference potential is that of the midpoint of one of the sides of the triangle. By varying one of the three resistances without altering the other two, we can make this potential equal to that of any point lying on the median which runs from the apex corresponding to the altered resistance to the midpoint of the opposite side. The same result is obtained by leaving this resistance unchanged and altering the other two in equal measure. By making one resistance infinite and varying one of the others, we can give the central terminal a potential equal to that of any point on the side of the triangle opposite to the apex that has been disconnected. By making unequal changes in two of the three resistances, we can shift the potential of the terminal to that of any point lying inside any of the circumscribed areas into which the medians divide the triangle.

The change in the potential of the central terminal effected by a specified alteration of the relative magnitudes of the resistances in its three branches may be computed in the following way: Consider the equations:

$$(15)\quad 1/r_a\,(V_R - V_T^1) = i_a$$
$$(16)\quad 1/r_b\,(V_L - V_T^1) = i_b$$
$$(17)\quad 1/r_c\,(V_F - V_T^1) = i_c$$

in which r_a, r_b, and r_c are the resistances between the central terminal and the junctions with the trunk of the right arm, left arm, and left leg, respectively; V_T^1 is the potential of this terminal when these resistances are unequal; V_R, V_L, and V_F are the open circuit potentials of the three extremities; and i_a, i_b, and i_c

are the currents flowing toward the central terminal through the corresponding resistances. By Kirchhoff's current law, the sum of these three currents is zero, and if we add Equations 15, 16, and 17 and multiply by $r_a r_b r_c$, we obtain the expressions:

$$(18)\quad r_b r_c (V_R - V_T^1) + r_a r_c (V_L - V_T^1) + r_a r_b (V_F - V_T^1) = 0$$

and
$$(19)\quad V_T^1 = \frac{r_b r_c V_R + r_a r_c V_L + r_a r_b V_F}{r_b r_c + r_a r_c + r_a r_b}$$

When the three resistances are equal the potential of the central terminal, (V_T) is given by:

$$(20)\quad V_T = \frac{V_R + V_L + V_F}{3}$$

and
$$(21)\quad V_T^1 - V_T = \frac{r_b r_c (V_R - V_T) + r_a r_c (V_L - V_T) + r_a r_b (V_F - V_T)}{r_b r_c + r_a r_c + r_a r_b}$$

Since

$$(22)\quad r_a r_b (V_F - V_T) = -r_a r_b (V_R - V_T) - r_a r_b (V_L - V_T)$$

it follows that

$$(23)\quad V_T^1 - V_T = \frac{r_b (r_c - r_a)(V_R - V_T) + r_a (r_c - r_b)(V_L - V_T)}{r_b r_c + r_a r_c + r_a r_b}$$

$$(24)\quad V_T^1 - V_T = \frac{\left(\frac{r_c}{r_a} - 1\right)(V_R - V_T) + \left(\frac{r_c}{r_b} - 1\right)(V_L - V_T)}{\frac{r_c}{r_a} + \frac{r_c}{r_b} + 1}$$

It is convenient to have also expressions for the currents i_a, i_b, and i_c in terms of the open circuit voltages (I), (II), and (III) in the three limb leads and the resistances r_a, r_b, and r_c. For certain purposes the internal resistances between the junctions of the extremities with the trunk must be considered. We shall here regard these as equal and small in comparison with the external resistances (r_a, r_b, and r_c) and represent them by the symbol r_1. The expressions referred to are given in a previous paper from this laboratory.[5] The currents mentioned appear in them as fractions with the denominator

$$(25)\quad r_1^2 + 2r_1 (r_a + r_b + r_c) + 3 (r_a r_b + r_a r_c + r_b r_c)$$

Representing this denominator by k we have:

$$(26)\quad ki_a = -(r_1 + 3r_c)(I) - (r_1 + 3r_b)(III)$$
$$(27)\quad ki_b = (r_1 + 3r_c)(I) - (r_1 + 3r_a)(II)$$
$$(28)\quad ki_c = (r_1 + 3r_b)(II) + (r_1 + 3r_a)(III)$$

Equation 19 is true for all positions of the current axis and all values of V_F', including the value zero. Consequently,

$$(29) \quad r_b r_c \, (V_R)_H + r_a r_c \, (V_L)_H + r_a r_b \, (V_F)_H = 0$$

$$(30) \quad r_b r_c \, (V_R)_V + r_a r_c \, (V_L)_V + r_a r_b \, (V_F)_V = 0$$

These equations may be solved for the ratios r_a/r_c, r_b/r_c, and r_b/r_a, the last of which is the second divided by the first.

$$(31) \quad \frac{r_a}{r_c} = \frac{(V_R)_V(V_L)_H - (V_R)_H(V_L)_V}{(V_F)_H(V_L)_V - (V_F)_V(V_L)_H}$$

$$(32) \quad \frac{r_b}{r_c} = \frac{(V_L)_V(V_R)_H - (V_L)_H(V_R)_V}{(V_F)_H(V_R)_V - (V_F)_V(V_R)_H}$$

$$(33) \quad \frac{r_b}{r_a} = \frac{(V_F)_V(V_L)_H - (V_F)_H(V_L)_V}{(V_F)_H(V_R)_V - (V_F)_V(V_R)_H}$$

These are the relative magnitudes of the resistances required to bring the potential of the central terminal to zero when the true potentials of the limb electrodes are known for two positions of the current axis, provided that the dipole is inside the triangle.

When this triangle is isosceles $(V_F)_H$ is zero and Equation 33 gives

$$(34) \quad \frac{r_b}{r_a} = \frac{(V_L)_H}{-(V_R)_H} = \frac{-(III)_H}{(II)_H} = 1$$

and, since $(V_R)_V$ and $(V_L)_V$ are equal and $-(V_R)_H$ and $(V_L)_H$ are equal, Equations 31 and 32 give

$$(35) \quad \frac{r_a}{r_c} = \frac{2(V_R)_V}{(V_F)_V}$$

and

$$(36) \quad \frac{r_b}{r_c} = \frac{2(V_L)_V}{(V_F)_V}$$

These last equations are true not only when the triangle is isosceles, but also whenever $(V_F)_H$ is zero and $\frac{(V_R)_V}{(V_L)_V}$ is equal to $\frac{-(V_R)_H}{(V_L)_H}$; that is, whenever the lead vector V_F is parallel to the vertical axis and the angles made with this axis by the lead vectors V_R and V_L are equal.

We may now turn to a consideration of the triangles (Fig. 5) corresponding to Experiments 4,*b*, 4,*d*, and 4,*f*. For the last, we have for the deflections in the standard and unaugmented limb leads the following figures:

$(I)_H$	$(II)_H$	$(III)_H$	$(V_R)_H$	$(V_L)_H$	$(V_F)_H$
42.0	15.0	−27.0	−19.0	23.0	−4.0
$(I)_V$	$(II)_V$	$(III)_V$	$(V_R)_V$	$(V_L)_H$	$(V_F)_V$
−13.5	48.5	62.0	−11.7	−25.2	36.8

In this experiment, there were deflections in all of the leads used, both when the current axis was horizontal and when it was vertical. In cases of this kind, it is possible to alter the ratio r_b/r_a in such a way as to make the potential of the central terminal and that of the leg electrode equal when the current axis is horizontal. When these two potentials are equal the current i_c is zero.

According to Equation 28 we have, when ic is zero,

$$(37) \quad \frac{r_I + 3r_b}{r_I + 3r_a} = \frac{-(III)_H}{(II)_H}$$

or, when r_I is neglected as negligibly small in comparison with $3r_a$ and $3r_b$,

$$(38) \quad \frac{r_b}{r_a} = \frac{-(III)_H}{(II)_H}$$

In the present instance, this equation gives for the magnitude of the ratio sought the figure 27/15, or 1.8. The result of the indicated alteration of the relative magnitude of the resistances involved would be to make the potential of the central terminal 4 units less positive and to reduce $(V_F)_H$ to zero. The horizontal component of the dipole would then produce no deflection in Lead V_F, and the deflection in this lead would be proportional to the vertical component. This procedure can be carried out whenever the deflection in Lead I, when the current axis is horizontal, is larger than the deflection in either of the other two limb leads. Under certain circumstances, it is possible to obtain a second lead which will yield a deflection proportional to the horizontal component of the dipole and no deflection for the vertical component by altering the ratios r_a/r_c and r_b/r_c in the manner required to make the currents i_a and i_b equal when the current axis is vertical. When the ratio i_b/i_a equals 1 we have from Equations 26 and 27 when the resistance r_I is neglected

$$(39) \quad r_c\,(I)_V - r_a\,(III)_V = -r_c\,(I)_V - r_b\,(II)_V$$

$$(40) \quad \frac{r_a}{r_c} = \frac{-2(I)_V}{\frac{r_b}{r_a}\,(II)_V - (III)_V}$$

By substituting in this equation the values -13.5, 48.5, and 62 given for the deflections in Leads I, II, and III, respectively, when the current axis was vertical and the value 1.8 found for the ratio r_b/r_a, we obtain for r_a/r_c the value 1.067, and for the ratio r_b/r_c the value 1.92. A central terminal connected to the limb electrodes by resistances having these relative magnitudes would be 4 units less positive when the current axis was horizontal and 5.2 units less negative when this axis was vertical than a central terminal connected to these electrodes through equal resistances. Whenever i_a and i_b are equal, when the current axis is vertical, a lead from any point on the right-arm resistor to a point on the left-arm resistor, which is separated from the central terminal by the same resistance, will yield a deflection proportional to the horizontal component of the electromotive force; its vertical component will give rise to no potential difference between such points. In the present instance, the potentials with respect to the central terminal of the three points on its arms separated from it by a resistance equal to r_c have been computed for the two positions of the current axis. For a horizontal current axis, the figures are -15.0, 15.0, and 0, and for a vertical current axis they are -16.9, -16.9, and 33.8 for the points on the right arm, left arm, and leg resistor, respectively.

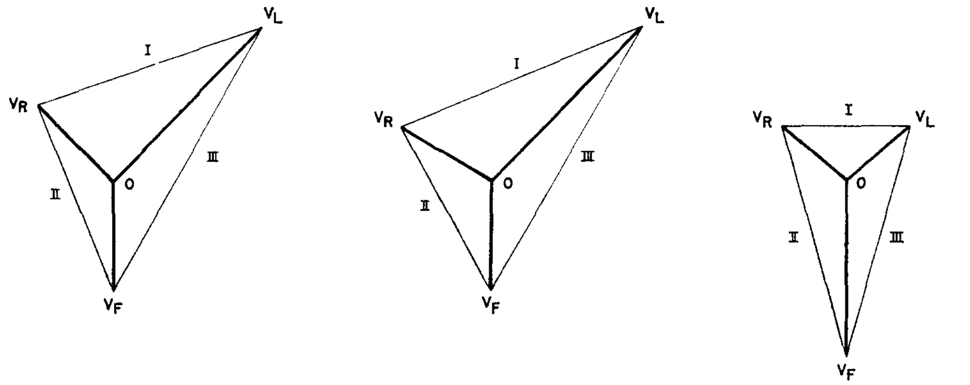
For the other experiments that have been referred to in this paper, the values of the ratios of the resistances in the arms of the central terminal required to make i_c zero when the current axis was horizontal, and i_a equal to i_b when this axis was vertical are as follows: for Experiment 4,*b*, r_a/r_b equals 2.2, r_a/r_c equals 2.5, and r_b/r_c equals 1.2; for Experiment 4,*c* the corresponding figures are 2.1, 3.1, and 1.5, respectively. For Experiment 3,*b* the values are r_b/r_a equals 2.2, r_b/r_c equals 3.5, and r_a/r_c equals 1.6. In all of these experiments, it would, therefore, have been possible to separate the horizontal and vertical components of the electromotive force for any position of the electrical axis by the methods proposed.

In order to make clear the limitations of these methods and the kind of cases in which they are applicable, we may consider three hypothetical triangles (Fig. 6). The values assumed for the true potentials of the three electrodes are for each of these triangles, as follows:

Triangle	$(V_R)_H$	$(V_L)_H$	$(V_F)_H$	$(V_R)_V$	$(V_L)_V$	$(V_F)_V$
(A)	$-2.12k$	$4.24k$	0	$-2.12k$	$-4.24k$	$3k$
(B)	$-2.60k$	$4.24k$	0	$-1.5k$	$-4.24k$	$3k$
(C)	$-1.865k$	$1.865k$	0	$-1.506k$	$-1.506k$	$4.888k$

The letter *k* is an arbitrary constant.

In the first case (A) the field is that of a centric dipole in a homogeneous medium. The angles between the radius vectors to the R and L apices of the triangle and the negative one-half of the vertical axis are both 45 degrees. The electrode at the first of these apices is on the surface of the medium, as is also the electrode at the F apex. The other electrode is enough closer to the centric dipole to make its potential variations twice as large as they would be if it too were on this surface. The radius vector to the F electrode makes an angle of 0 degrees with the positive direction of the vertical axis. The methods described give for the ratios of the resistances in the arms of the central terminal the values r_b/r_a equals 2, r_a/r_c equals 1.414, r_b/r_c equals 2.828. These are the values required to reduce the potential of the central terminal to zero.



	H	V		H	V		H	V
V_R	-2.12	-2.12	V_R	-2.60	-1.50	V_R	-1.87	-1.51
V_L	4.24	-4.24	V_L	4.24	-4.24	V_L	1.87	-1.51
V_F	0	3.00	V_F	0	3.00	V_F	0	4.88

A B C

Fig. 6.—Three hypothetical Burger triangles. (See text.)

In the second case (B), the field is again that of a centric dipole in a homogeneous spherical medium and the L electrode is in the same position as in Case (A); it is not at the surface of this medium as are the other two. The radius vector to the R apex makes an angle of 60 and that to the L apex an angle of 45 degrees with the negative direction of the vertical axis. The radius vector to the F apex again makes an angle of zero degrees with the positive direction of this axis. In this instance, the value of r_b/r_a required, when the current axis is horizontal, to make the potential of the central terminal and the current i_c zero is 1.633, and this is equal to $\frac{-(III)_H}{(II)_H}$, or $\frac{4.24}{2.60}$. On the other hand, the values of r_a/r_c and r_b/r_c required to make i_a and i_b equal, when the current axis is vertical, are 51.8 and 84.6, respectively. It is clearly not feasible to equalize these currents in cases of this kind. The values of r_a/r_c and r_b/r_c required to make the potential of the central terminal zero for a vertical current axis are 1.366 and 2.231, respectively. These are the values which make the ratio i_b/i_a equal to the ratio of minus tangent Θ_2 to tangent Θ_1, where Θ_2 is the angle between the positive direction of the horizontal axis and the radius vector to the L apex, and Θ_1 the corresponding angle between this axis and the radius vector to the R apex of the triangle. It is, nevertheless, possible to find in this instance a lead from a point on the right arm resistor to a point on the left arm resistor which will give a deflection proportional to the horizontal component of the dipole and no deflection in response to its vertical component. It is clear that this can be done whenever the arm electrodes are both strongly negative with respect to the central terminal, for the currents i_b and i_a then have the same sign. If the point on the right arm resistor is separated from the central terminal by the resistance A and that on the left arm resistor is separated from this terminal by the resistance B, the two points will be at the same potential provided Ai_a is equal to Bi_b and this condition can always be fulfilled by choosing the two points properly. In the third case (C), all three electrodes are on the surface of the spherical medium and at the apices of an equilateral triangle. The dipole is at a point on the radius vector to the F electrode at a distance from the center of the sphere equal to one-fourth of its radius. In computing the potentials of the apices of the equilateral triangle in this hypothetical case, we have utilized equations for the field of an eccentric dipole in a spherical medium which were developed by Wilson and Bayley.[8] The Burger triangle is isosceles and of the type in which the side corresponding to Lead I is shorter than those corresponding to Leads II and III. Its shape is, therefore, similar to that of the isosceles triangle of Fig. 5. It does not tell us whether the dipole is centric and the geometric triangle defined by the electrodes therefore identical with it in form, or whether the dipole is eccentric and this geometric triangle equilateral, or isosceles but different from it in shape. The assumption that the dipole is centric leads to the conclusion that the potential of the F electrode for a vertical current axis is 3.47k, whereas its true potential is 4.88k, and that the potential of the central terminal is −0.79k instead of 0.63k which is correct. In summing up this section, we may say that in experiments of the kind under consideration, it is always possible to find one lead which will record only the variations of the horizontal, and another which will record only the variations of the vertical component of the electrical field. It is always theoretically possible to modify the relative magnitude of the resistances in the arms of a central terminal connected to the three limb electrodes in such a way as to insure that its potential will be zero for all positions of the electrical axis, provided that the position of the input electrodes is not one that will make all three lead electrodes simultaneously positive or negative. On the other hand, it is not practically possible to do this unless the true potentials of the limb electrodes are known for two positions of the current axis. It does not appear that this necessary information can be obtained in any way other than by measuring the potentials of these electrodes with reference to that of some point possessing, in respect to the circumscribed region where the current enters the body, "electric" and spatial relations identical with those which it bears to the circumscribed region where the current leaves the body. For some positions of the input electrodes it may be that no point which precisely fulfills the prescribed conditions exists. In that case, it may even be difficult to define in a manner satisfactory to everyone exactly what is meant by zero potential in that particular instance. The concept is one that is derived from the consideration of hypothetical situations for which exact mathematical solutions are available. In practice, the solution of the problem of finding a point at zero potential represents an approximation to an ideal based on plausible assumptions of one kind or another. We have as yet made no serious attempt in our experiments to measure the potentials of the limb electrodes with respect to that of a point of the kind mentioned, and shall not now discuss this problem further.

In the title of this article, we have referred to the possibility of constructing an Einthoven triangle for a given subject. In order to do this, it would be necessary to be able to generate in the trunk of the given subject an electrical field closely resembling that associated with the heart beat, and to measure the induced potential differences between the limb electrodes for two different positions of the electrical axis. By the method of Burger and Van Milaan, a triangle corresponding to the data thus obtained could then be constructed.

The extent to which the electrical field set up in the body by connecting two electrodes on the precordium to a source of low frequency alternating current resembles the electrical field of the heart is as yet unknown. It may be possible to generate an artificial field in the trunk more like that of the heart by placing small input electrodes in the esophagus or by introducing them into the right ventricle by the catheterization technique. Since the stimulating effect of the current increases rapidly with the current density, the size of the current that could be safely passed into the body by way of the largest input electrodes permissible in such experiments might be inconveniently small, but with a sufficiently sensitive recording system the field produced could undoubtedly be studied satisfactorily. Much information bearing on the problem of generating an artificial field similar to the cardiac field will certainly come from a comparison of the results of experiments on living subjects with those obtained in like experiments on cadavers and on models.

Human chests differ in size and shape and human hearts in location, but we do not as yet have much information concerning the effects of these variations upon the character of the heart's electrical field. In the majority of our experiments in which the point midway between the input electrodes was on the midsternal line and at the level of the fourth intercostal space, the Burger triangle was very nearly isosceles and of the kind in which the side corresponding to Lead I was shorter than the sides corresponding to Leads II and III. In two instances, however, this triangle was of the opposite type. We do not know whether the shape of the triangle obtained was dependent upon the size and shape of the chest or upon some other factor. A larger series of experiments should decide this question.

THREE ISOPOTENTIAL LINES AND THE RECIPROCITY THEOREM OF HELMHOLTZ

In a preceding section of this paper, it was mentioned that we have carried out a very few experiments of a kind suggested by Dr. Kenneth S. Cole, in which two of the limb electrodes are connected to a current source and the isopotential line corresponding to the potential of the third limb electrode is plotted on the body surface. The three lines obtained in this way may be called the right arm isopotential, the left arm isopotential, and the leg isopotential. They intersect at two points, one on the anterior and the other on the posterior surface of the chest. In the experiments performed thus far, the anterior point has been very close to the midsternal line and usually at the level of the sternal attachment of the fourth costal cartilage or the fourth intercostal space. In the case of one subject, however, it was at the level of the second intercostal space. In the single experiment in which it was located, the posterior point was directly behind the anterior. The isopotential lines found in this instance are shown in Fig. 7.

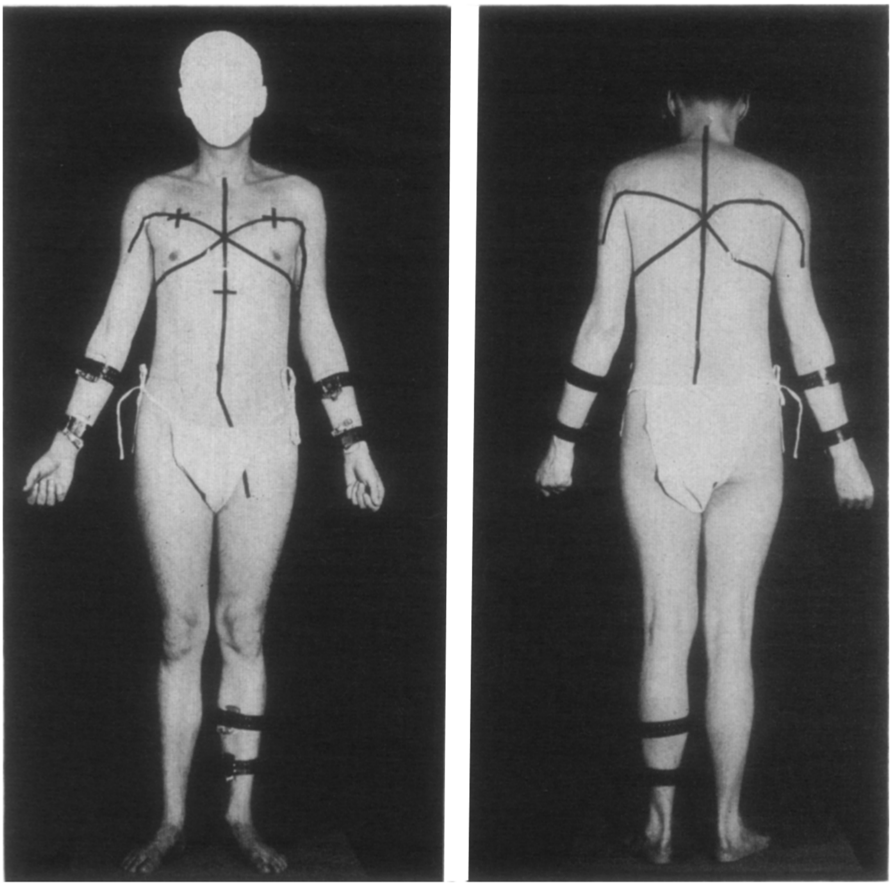


Fig. 7.—Experiment 12. The location of the anterior and posterior intersections of the right arm, left arm, and leg isopotentials.

In Experiment 12, the input electrodes were placed on the leg isopotential at the level required to make the point midway between them and the anterior intersection of the three isopotential lines coincide. The resulting deflection in Lead I was very small and a slight rotation of the input electrodes made it disappear altogether. The wires from the oscillator were then transferred to the electrodes on the arms and the lead wires to the electrodes on the chest. The record obtained again showed no deflection. This procedure was repeated after rotating the current axis through an angle of 90 degrees. This axis was then perpendicular to the leg isopotential and the electrodes were equidistant from it. Under these circumstances, the deflections recorded were of the same size when the oscillator was connected to the chest electrodes and the lead wires to the limb electrodes as when the reverse was the case (Fig. 8). These observations are in accord with the reciprocity theorem. In 1853, Helmholtz,[7] then a young man 32 years of age, proved this theorem theoretically and experimentally for both homogeneous and heterogeneous volume conductors. We believe that the location of the intersections of the three isopotential lines specified will prove to be a very useful procedure. It promises to disclose significant differences between subjects, and since it requires very little time, a large number can be examined in a relatively short period. It will also make it possible in experiments on different subjects to place the input electrodes in such a manner that their positions, from the electrical point of view, will always be the same with respect to the limb electrodes. What is more important, the principles underlying this procedure and the reciprocity theorem suggest a great variety of experiments which may increase our knowledge of the properties of the body considered as a volume conductor of electrical currents of the kind associated with the heart beat.

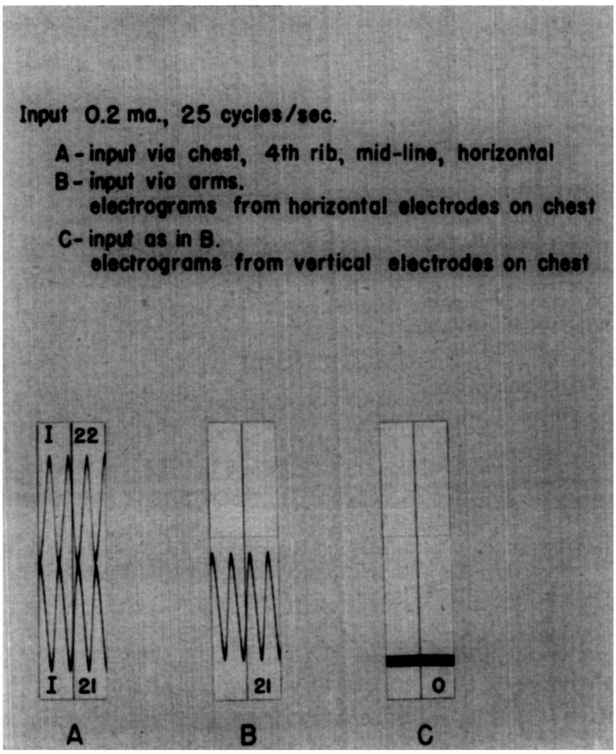


Fig. 8.—Experiment 12. Tracings which illustrate the reciprocity theorem. When there are two electrodes on the limbs and two on the chest, the potential difference between the chest electrodes when the current source was connected to the limb electrodes is equal to the potential difference between the limb electrodes when the current source was connected to the chest electrodes.

THE IMPORTANCE OF THE EFFECTS OF THE POSITION OF THE HEART UPON THE FORM OF THE ELECTROCARDIOGRAM

The ultimate purpose of the work with which this article is concerned is the same as that which led Einthoven and his associates[8] to propose a method of finding the position of the electrical axis of the heart. The first two paragraphs of their famous paper in which this method is described run as follows:

"Die Herzlage beeinflusst die Form des E.K.G. Es ist uns jedoch bei der elektrokardiographischen Untersuchung hauptsächlich darum zu tun, die *Tätigkeit* des Herzens besser zu ermittlen, und man sieht leicht ein, dass, wenn schon durch eine Lageabweichung dieses Organs eine Veränderung in die Form der Kurve hervorgerufen wird, eine Schwierigkeit enstehen muss, um mittels dieser Form auch über die Tätigkeit des Herzens zu urteilen.

Diese Schwierkeit kann am besten gelöst werden, wenn man den Einfluss der Lage vorher genau kennen gelernt hat."

Waller, and many others interested in the electrical aspects of the heart beat, was conscious of this problem long before Einthoven, Fahr, and De Waart attempted to solve it. Today, the differentiation of phenomena produced by displacement or rotation of the heart from those originating within the myocardium itself is still one of the most troublesome of the problems that confront those who attempt to interpret the human electrocardiogram.

There may be little value in computing the exact position of the electrical axis of the heart by Einthoven's method, but there can be no question that the Einthoven triangle has made it possible to recognize with considerable facility peculiarities in the form of the electrocardiogram that result from rotation of the heart about a sagittal axis. The recognition of those peculiarities that result from rotation of the heart about an axis that is not perpendicular to the plane of the limb leads is still extremely difficult. At the same time, our experience with precordial leads has led us to believe that cardiac rotations of this kind are much more often responsible for erroneous interpretations of the electrocardiogram than was formerly suspected. Many changes in the position of the heart that have a profound effect upon the shape of the electrocardiographic deflections cannot at present be recognized by fluoroscopy or by roentgenographic methods. It seems likely, however, that a sound method of taking simultaneously two accurate vectorcardiograms which represent the projections of the cardiac vector upon two different planes and which can be combined to form a spatial curve will contribute heavily to the eventual solution of this important problem. Several methods of this kind have been proposed and some of these have been used to a limited extent. Nevertheless, it has seemed to us that it is desirable to place all methods concerned with the study of the electrical axis of the heart upon a foundation more secure than that upon which they now rest by a thorough experimental study of the distribution in the body of currents similar to those associated with the heart beat. It was with this end in mind that experiments of the kind here reported were undertaken.

SUMMARY

In experiments on normal subjects, two small electrodes on the chest were connected to a source of low frequency current. The resulting differences in potential between the extremities and between other points on the body were measured. By the method described by Burger and Van Milaan, triangles and other figures, which present in graphic form the data obtained in this way, have been constructed.

When the point midway between the input electrodes was in the midsternal line, the triangle corresponding to the standard limb leads was nearly isosceles, and usually, though not always, of the type in which the side corresponding to Lead I was shorter than the other two. When the input electrodes were to the left of the midline, the side of the triangle corresponding to Lead III, and when they were to the right of the midline, the side corresponding to Lead II, was the longest.

When the Burger triangle is oblique, none of the standard or unipolar limb leads yield deflections proportional to either the horizontal or the vertical component of the electrical field. A method of finding two leads, one of which will record the variations of the first of these components, and one which will record the variations of the second, is described. The effect of varying the resistances in the arms of the central terminal upon its potential, and the possibility of reducing the potential variations of this terminal to zero, when the Burger triangle is not equilateral, are discussed.

In a few experiments, the isopotential lines corresponding to the potential of one of the limb electrodes when the other two were connected to a source of low frequency current were plotted on the body surface. The three lines obtained in this way intersect at two points, one on the front and the other on the back of the chest.

REFERENCES

1. Wilson, F. N., Johnston, F. D., Rosenbaum, F. F., and Barker, P. S.: On Einthoven's Triangle, the Theory of Unipolar Electrocardiographic Leads, and the Interpretation of the Precordial Electrocardiogram, AM. HEART J. **32**:277, 1946.
2. Wilson, F. N., Johnston, F. D., and Kossmann, C. E.: The Substitution of a Tetrahedron for the Einthoven Triangle, AM. HEART J. **33**:594, 1947.
3. Einthoven, W., Bergansius, F. L., and Bijtel, J.: Die gleichzeitige Registrierung elektrischer Erscheinungen mittels zwei oder mehr Galvanometer und ihre Anwendung auf die Elektrokardiographie, Arch. f. d. ges. Physiol. **164**:167, 1916.
4. Burger, H. C., and van Milaan, J. B.: (a) Heart Vector and Leads, Brit. Heart J. **8**:157, 1946.
(b) Heart Vector and Leads, Part II. Brit. Heart J. **9**:154, 1947.
5. Bryant, J. M., Johnston, F. D., and Wilson, F. N.: Unipolar Electrocardiographic Leads. Effects Produced by Eliminating the Resistors Between the Limb Electrodes and the Central Terminal, AM. HEART J. (In press).
6. Wilson, F. N., and Bayley, R. H.: The Electrical Field of an Eccentric Dipole in a Homogeneous Spherical Conductor. Unpublished paper.
7. Helmholtz, H.: Ueber einige Gesetze der Vertheilung elektrische Ströme im körperlichen Leitern mit Anwendung auf die thierisch elektrischer Versuche, Ann. d. Physik u. Chem. (Ser. 2) **89**:211, 1853.
8. Einthoven, W., Fahr, G., and de Waart, A.: Ueber die Richtung und die manifeste Grösse der Potenialschwankungen im menschlichen Herzen, und über den Einfluss der Herzlage auf die Form des Elektrokardiogramm, Arch. f. d. ges. Physiol. **150**:308, 1913.