rods coupling them in pairs. On the lower ends of the shafts are keyed four horizontal driving wheels, 1 foot 4 inches in diameter, which, as arranged, grip between them the mid-rail. This rail is the ordinary double-headed variety, bolted to suitable sleepers. The eccentrics for working the valve of the leading cylinder are keyed one on each of the forward grip wheel shafts. Those for working the valve of the after cylinder are in like manner keyed one on each of the trailing shafts. A flat iron table is bolted to the frame to support each link. The links are provided with suitable bosses to slide on these tables. Reversing is simply effected in the usual way from the foot-plate. The eccentrics for the outside cylinders are keyed on the leading horizontal axle, as they could not be provided for elsewhere.

(To be continued.)

For the Journal of the Franklin Institute.

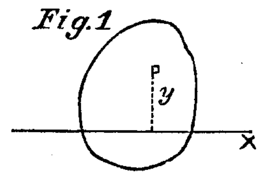
Moment of Inertia of Surfaces. By De Volson Wood, Prof. C.E., University of Michigan.

In the investigation of the resistance of beams or columns, the moment of inertia of the transverse sections plays a very important part. We may always deduce the moment of inertia of a surface from the well known expression $\iint y^2\, dy\, dx$; but we may often facilitate operations by deducing from it some general rules. It is my purpose in the following article to give a general discussion of the subject, and apply each principle in the development to the solution of a problem.

General Expression.

The moment of inertia of a surface is the sum of the products found by multiplying each elementary area by the square of its distance from any assumed axis.

Fig. 1



Let dA = an elementary area.
y = its distance from the axis.
and I = the moment of inertia of the surface.

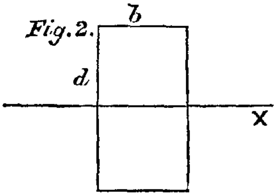
Then, according to the definition,
$y^2 dA$ = the moment of inertia of an element,

$$I = \int y^2 dA, \quad . \quad . \quad . \quad (1.)$$

Let the surface be referred to rectangular co-ordinates. Then $dA = dy\, dx$;

$$\therefore I = \iint y^2\, dy\, dx, \quad . \quad . \quad . \quad . \quad (2.)$$

Fig. 2.



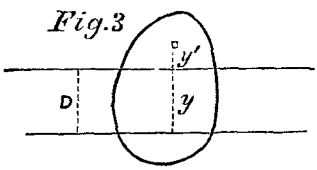
EXAMPLE.—To find the moment of inertia of a rectangle about an axis passing through the centre and parallel to the ends.

Let b = the breadth, d = the depth; then we have

$$I = \int_0^b \int_{-\frac{1}{2}d}^{+\frac{1}{2}d} y^2\, dy\, dx = \frac{1}{12} d^3 \int_0^b dx = \frac{1}{12} bd^3 \quad (3.)$$

FORMULA OF REDUCTION.

Fig. 3



To find the relation between the moments of inertia about two parallel axes, one of which passes through the centre of gravity.

Let D = the distance between them.
y^1 = the ordinate from the axis which passes through the centre.

Let y = the ordinate from the other axis.
I^1 = the moment of inertia about the axis through the centre.

We have $y = D + y^1$; hence (1) becomes

$$\int y^2\, dA = \int y^{1^2} dA + \int 2D\, y^1\, dA + D^2 \int dA.$$

But, because x^1 passes through the centre, we have $2D\int y^1\, dA = 0$, when it is integrated so as to include the whole area, and $\int dA = A$; hence, we find

$$\int y^2\, dA = \int y^{1^2}\, dA + D^2 A.$$

$$\text{or } I = I^1 + D\, A, \quad . \quad . \quad . \quad (4.)$$

This is called the *formula of reduction*, and may thus be enunciated: The moment of inertia of a surface about any axis, equals the moment of inertia about an axis parallel to it which passes through the centre, *plus* the area into the square of the distance between them.

EXAMPLE.—To find the moment of inertia of a rectangle about an axis which coincides with one end. The distance between the axes will be $\frac{1}{2}d = D$, and $A = bd$; hence, (4) and (3) will give

$$I = \frac{1}{12} bd^3 + \tfrac{1}{4}d^2.\ \ bd = \tfrac{1}{3}bd^3, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5.)$$

From (4) we find

$$I^1 = I - D^2A, \quad . \quad . \quad . \quad . \quad . \quad (6,)$$

which is sometimes very convenient.

For instance, it is easier to find the moment of inertia of a triangle, about an axis which passes through its apex, and parallel to the base, than about any other axis; but having found this, we may easily find it about an axis parallel to it which passes through the centre.

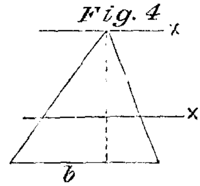
To illustrate,

Let b = the base, and
d = the altitude of the triangle.

Then, taking the origin at the vertex, and we have

$$x : y :: b : d \therefore x = \frac{b}{d} y,$$

Fig. 4



and (2) becomes

$$\text{I} = \int_0^d y^2 x \, dy = \frac{b}{d}\int_0^d y^3 \, dy = \tfrac{1}{4} bd^3, \qquad (7.)$$

The area $= \frac{1}{2} bd = \text{A}$.
$\text{D} = \frac{2}{3} d$ = the distance from the vertex to the centre; hence, (6) becomes

$$\text{I}^1 = \tfrac{1}{4}{}^3 bd - \tfrac{1}{2} bd \times \frac{4}{9} d^2 = \frac{1}{36} bd^3, \qquad (8.)$$

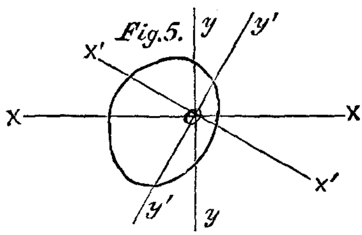
If the base be the axis, (8) and (4) give

$$\text{I} = \frac{1}{36} bd^3 + \tfrac{1}{2} bd \times \frac{1}{9} d^3 = \frac{1}{12} bd^3, \qquad (9.)$$

Relation between Rectangular Axes.

We will now find the relation between the moments of inertia about the axes of two systems of rectangular co-ordinates, having the same origin, but inclined to each other.

Fig. 5.



Let O, Fig. 5, be the origin, and
$y\text{O}y^1 = x\text{O}x^1 = \beta$.
Also, I_x = the moment of inertia about $x\text{O}x$.
I_y = the moment of inertia about $y\text{O}y$.
I_{x^1} = the moment of inertia about $x^1\text{O}x^1$.
I_{y^1} = the moment of inertia about $y^1\text{O}y^1$.

Then, by the definition, we have

$$\text{I}_x = \int y^2 d\text{A}, \qquad \text{I}_y = \int x^2 d\text{A}, \qquad (10,)$$

$$\text{I}_{x^1} = \int {y^1}^2 d\text{A}, \qquad \text{I}_{y^1} = \int {x^1}^2 d\text{A}, \qquad (11.)$$

For the transformation of rectangular co-ordinates, we have

$$\left.\begin{aligned} x^1 &= x \cos\beta - y \sin\beta \\ y^1 &= x \sin\beta + y \cos\beta \end{aligned}\right\} \qquad (12).$$

Hence, we have

$$x^{12} + y^{12} = x^2 + y^2,$$

which is called an *isotropic function*, (from the Greek ισος, equal, and τροπω, turning, *i.e.* the distance from the origin to any point is the same for all inclinations of the systems of co-ordinate axes.)

$$\text{Let B} = \int xy \, d\text{A} \text{ and B}^1 = \int x^1 y^1 d\text{A}, \qquad (13.)$$

Then, by combining (10), (11), (12), and (13), we readily find the following equations:

$$\left.\begin{array}{l} I_{x1} = I_x \cos^2\beta + I_y \sin^2\beta - 2B\cos\beta\sin\beta \\ I_{y1} = I_y \sin^2\beta + I_y \cos^2\beta + 2B\cos\beta\sin\beta \\ B_1 = (I_x - I_y)\cos\beta\sin\beta + B(\cos^2\beta - \sin^2\beta) \end{array}\right\} \quad (14).$$

By adding the 1st and 2d of (14), we find

$$I_{x1} + I_{y1} = I_x + I_y, \quad . \quad . \quad . \quad (15,)$$

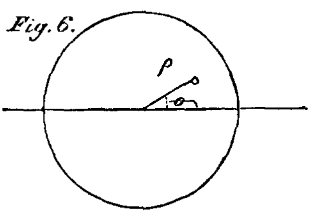
which is *isotropic;* and from which we see that *the sum of the moments of inertia about pairs of rectangular axes having the same origin is constant.*

From (10) and (15) we readily find that

$$I_x + I_y = I_{x1} + I_{y1} = \int (x^2 + y^2)\, dA = \int \rho^2 dA, \quad . \quad (16,)$$

in which $x^2 + y^2 = \rho^2$ = a variable distance; and hence, from the definition, equation (16) is the moment of inertia about an axis perpendicular to the plane of the surface. This is called the *polar moment of inertia.* We see that it equals the sum of the moments about two rectangular axes which lie in the surface, and whose origin is on the polar axis.

Fig. 6.



Let θ = the variable angle; then will $\rho d\rho d\theta$ = an elementary polar area = dA; hence, (16) becomes

$$\iint \rho^3\, d\rho\, d\theta = I_p = \text{the } \textit{polar moment of inertia}, \quad . \quad . \quad (17.)$$

EXAMPLES.—1. To find the polar moment of inertia of a circle, about an axis passing through its centre, we have only to integrate (17) between the limits 0 and r for ρ, and 0 and 2π for θ; hence, we have

$$I_p = \int_0^{2\pi}\int_0^r \rho^3 d\rho d\theta = \tfrac{1}{2}\pi r^4, \quad . \quad . \quad . \quad . \quad (18.)$$

2. We may find in the same way that we found equation (3) that the moment of inertia of a rectangle about an axis passing through its centre and parallel to its sides, is $\frac{1}{12} b^3 d$; hence, the polar moment of a rectangle about an axis passing through its centre is, by equation (16),

$$I_p = \frac{1}{12} bd\,(b^2 + d^2) = \frac{1}{12} A\,(\text{diagonal})^2, \quad . \quad . \quad . \quad (19,)$$

that is, it equals one-twelfth the area multiplied by the square of the diagonal.

Knowing the polar moment, it is easy, in some cases, to find the moment about an axis in the surface.

For instance, if the moments about the rectangular axes are equal, equation (16) will give

$$2\int x^2\, dA = \int \rho^2\, dA;$$

$$\therefore I_x = \tfrac{1}{2} I_p, \quad . \quad . \quad . \quad . \quad . \quad (20.)$$

Hence, equations (20) and (18) give, for the moment of inertia of a circle, about an axis in the surface, and passing through the centre,

$$I = \tfrac{1}{4}\pi r^4, \quad . \quad . \quad . \quad . \quad . \quad (21,)$$

and, for the moment of a square about an axis in the surface, parallel to one side and passing through the centre, (20) and (19) will give

$$I = \frac{1}{24} b^2 (b^2 + b^2) = \frac{1}{12} b^4.$$

If the polar moment and one of the rectangular moments are known, the other rectangular moment may be easily found; for equation (16) gives—

$$I_y = I_p - I_x, \quad . \quad . \quad . \quad . \quad (21.)$$

MAXIMUM AND MINIMUM MOMENTS.

To find the position of the axes (for any assumed origin) which shall give a maximum moment about one axis and minimum about the other, we will suppose that I_x, I_y, and B, have been found for any assumed position of the axis.

By differentiating the first of (14), and placing it equal zero, we find

$$D\beta = -2(I_x - I_y)\cos\beta\sin\beta + 2B(\sin^2\beta - \cos^2\beta) = 0;$$

$$\therefore \frac{-2B}{I_x - I_y} = \frac{2\cos\beta\sin\beta}{\sin^2\beta - \cos^2\beta} = \tan 2\beta, \quad . \quad (22.)$$

The second derivative, placed equal zero, gives $\tan 2\beta = \pm 1$; hence, of the two values of β found from (22), one will give the position of I_{x1} for a maximum, and the other the position for a minimum.

Proceeding in a similar way with the second of (14), and we find that when I_{x1} is a maximum, I_{y1} is a minimum, and *vice versa*. Equation (22) in the third of (14) gives $B^1 = 0$.

Let β_1 be the value of β found by equation (22), and $x_1\, y_1$ the corresponding axes, called *principal axes*.

$B^1 = B_1 = 0$, and (14) becomes

$$\left.\begin{aligned} I_{x_1} &= I_x\cos^2\beta_1 + I_y\sin^2\beta_1 - 2B\cos\beta_1\sin\beta_1 \\ I_{y_1} &= I_x\sin^2\beta_1 + I_y\cos^2\beta_1 + 2B\cos\beta_1\sin\beta_1 \\ 0 &= (I_x - I_y)\cos\beta_1\sin\beta_1 + B(\cos^2\beta_1 - \sin^2\beta_1) \end{aligned}\right\} \quad (23.)$$

By adding the 1st and 2d of (23), and then multiplying them together, using the 3d in the reduction, we find

$$I_{x_1} + I_{y_1} = I_x + I_y,$$

$$I_{x_1} I_{y_1} = I_x I_y - B^2,$$

$$\therefore I_{x_1} = \tfrac{1}{2}(I_x + I_y) + \sqrt{\tfrac{1}{4}(I_x - I_y)^2 + B^2};$$

$$I_{y_1} = \tfrac{1}{2}(I_x + I_y) - \sqrt{\tfrac{1}{4}(I_x - I_y)^2 + B^2},$$

by means of which the principal moments may be found without knowing β_1.

EXAMPLE.—To find the principal axes of a rectangle, origin at the centre. We have already found, equation (3), that

$$I_x = \frac{1}{12} bd^3, \text{ and } I_y = \frac{1}{12} b^3 d; \text{ we also have}$$

$$B = \int_{-\frac{1}{2}d}^{\frac{1}{2}d} \int_{\frac{1}{2}b}^{\frac{1}{2}b} xy\, dy\, dx = (\tfrac{1}{4}d^2 - \tfrac{1}{4}d^2)\int_{-\frac{1}{2}b}^{\frac{1}{2}b} x\, dx = 0,$$

which in (22) gives $\beta = 0$, and these in (23) give

$$I_{x_1} = I_x = \tfrac{1}{12} bd^3$$
$$I_{y_1} = I_y = \tfrac{1}{12} b^3 d;$$

hence, those axes which are parallel to the sides and ends of the rectangle are *principal axes.*

Moments in Reference to any System of Rectangular Axes, the Principal Moments being known.

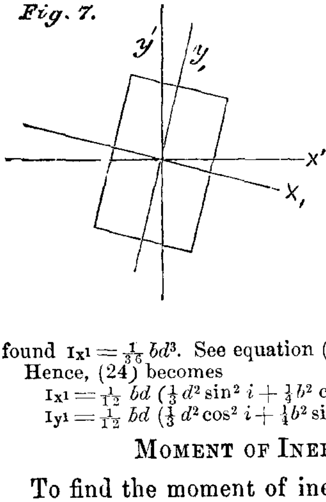
Suppose that the principal moments I_{x_1} and I_{y_1} are known, then we may easily find the moments about axes inclined at any angle; for we have only to make $I_x = I_{x_1}$, $I_y = I_{y_1}$, and $B = 0$ in (14). This done and we have

$$\left.\begin{aligned} I_{x^1} &= I_{x_1} \cos^2\beta + I_{y_1} \sin^2\beta \\ I_{y^1} &= I_{x_1} \sin^2\beta + I_{y_1} \cos^2\beta \\ B^1 &= (I_{x_1} - I_{y_1}) \cos\beta \sin\beta \end{aligned}\right\} \quad . \quad . \quad (24.)$$

If $I_{x_1} = I_{y_1}$, then $I_{x^1} = I_{y^1}$, and the figure may be said to have its moments of inertia perfectly *isotropic.* This is the case with the circle, regular polygons, and many other symmetrical figures.

Examples.—1. Take the case of a rectangle whose sides are inclined at a angle i with the axis of x^1. Then

Fig. 7.



$$\beta = 90° - i, \text{ and (24) gives}$$

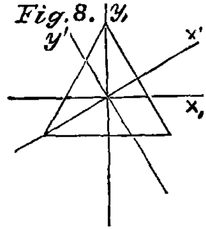
$$I_{x^1} = I_{x_1} \sin^2 i + I_{y_1} \cos^2 i = \tfrac{1}{12} bd\,(d^2 \sin^2 i + b^2 \cos^2 i),$$

$$I_{y^1} = I_{x_1} \cos^2 i + I_{y_1} \sin^2 i = \tfrac{1}{12} bd\,(d^2 \cos^2 i + b^2 \sin^2 i).$$

If $d = b$, then $I_{x^1} = I_{y_1} = \tfrac{1}{12} d^4$, the same as before found.

2. Let the section be an equilateral triangle, with the axis of symmetry inclined at an angle i with the axis of x^1. Then $\beta = 90° - i$. We easily find that $I_{y_1} = \tfrac{1}{48} b^3 d$, and we have found $I_{x^1} = \tfrac{1}{36} bd^3$. See equation (8).

Fig. 8.



Hence, (24) becomes

$$I_{x^1} = \tfrac{1}{12} bd\,(\tfrac{1}{3} d^2 \sin^2 i + \tfrac{1}{4} b^2 \cos^2 i), \quad . . \quad . \quad (25.)$$
$$I_{y^1} = \tfrac{1}{12} bd\,(\tfrac{1}{3} d^2 \cos^2 i + \tfrac{1}{4} b^2 \sin^2 i), \quad . \quad . \quad (26.)$$

Moment of Inertia of a Regular Polygon.

To find the moment of inertia of a regular polygon, let the axis of x pass through a vertex and the centre.

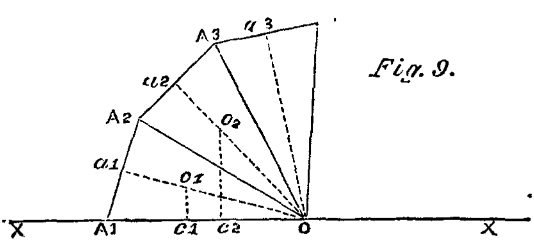


Fig. 9.

Let A_1, A_2, &c., Fig. 9, be the polygon; O its centre; o_1, o_2, &c., the centres of the triangles; oa_1, oa_2, &c., the altitudes.

Let $b = A_1 A_2 = A_2 A_3 = \&c.$

$d = oa_1 = oa_2 =$ the altitude.

$h_1 = o_1 c_1,\ h_2 = o_2 c_2,\ \&c.$

$a = A_1 o A_2 = A_2 o A_3 = \&c. =$ angle at the vortex of each triangle.

$i_1 = a_1 o A_1,\ i_2 = a_2 o A_1,\ \&c., =$ the inclination of the axis of symmetry of each triangle to the axis of x.

$k_1, k_2, k_3, \&c., =$ the moment of inertia of the successive triangles about an axis passing through their centres and parallel to XOX.

$I_1, I_2, I_3, \&c., =$ the moments of inertia of the corresponding triangles about XOX.

$n =$ the number of sides in the polygon.

$a = \frac{1}{2} bd =$ the area of each triangle.

$A = na =$ the area of the polygon.

We have

$$a = \frac{360}{n} \qquad i_1 = \tfrac{1}{2}a \qquad h_1 = \tfrac{2}{3}d \sin i_1$$

$$b = 2a \tan \tfrac{1}{2}a \qquad i_2 = \tfrac{3}{2}a \qquad h_2 = \tfrac{2}{3}d \sin i_2$$

$$\cdot \quad \cdot \quad \cdot \qquad\qquad \cdot \quad \cdot \quad \cdot$$

$$i_n = \frac{2n-1}{2}\, a \qquad h_n = \tfrac{2}{3}d \sin i^n$$

By (25) we have

$$k_1 = \tfrac{1}{6}\, a\, (\tfrac{1}{3}\, d^2 \sin^2 i_1 + \tfrac{1}{4}\, b^2 \cos^2 i_1)$$
$$k_2 = \tfrac{1}{6}\, a\, (\tfrac{1}{3}\, d^2 \sin^2 i_2 + \tfrac{1}{4}\, b^2 \cos^2 i_2)$$
$$k_3 = \tfrac{1}{6}\, a\, (\tfrac{1}{3}\, d^2 \sin^2 i_3 + \tfrac{1}{4}\, b^2 \cos^2 i_3)$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$i_n = \tfrac{1}{6}\, a\, (\tfrac{1}{3}\, d^2 \sin^2 i_n + \tfrac{1}{4}\, b^2 \cos^2 i_n)$$

By equation (4) we have

$$I_1 = k_1 + a h_1^{\,2} = \tfrac{1}{2}\, a\, (d^2 \sin^2 i_1 + \tfrac{1}{12}\, b^2 \cos^2 i_1)$$
$$I_2 = k_2 + a h_2^{\,2} = \tfrac{1}{2}\, a\, (d^2 \sin^2 i_2 + \tfrac{1}{12}\, b^2 \cos^2 i_2)$$
$$I_3 = k_3 + a h_3^{\,2} = \tfrac{1}{2}\, a\, (d^2 \sin^2 i_3 + \tfrac{1}{12}\, b^2 \cos^2 i_3)$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$I_n = k_n + a h_n^{\,2} = \tfrac{1}{2}\, a\, (d^2 \sin^2 i_n + \tfrac{1}{12}\, b^2 \cos^2 i_n).$$

For any portion of the polygon we have

$$\Sigma I = \tfrac{1}{2}\, a\, (d^2\, \Sigma \sin^2 i + \tfrac{1}{12}\, b^2\, \Sigma \cos^2 i), \qquad . \quad . \quad (27,)$$

in which $\Sigma \sin^2 i = \sin^2 \frac{1}{2}\, a + \sin^2 \frac{3}{2}\, a + \sin^2 \frac{5}{2}\, a\ .\ .\ .\ + \sin^2 \frac{2n-1}{2} a$

$$\Sigma \cos^2 i = \cos^2 \tfrac{1}{2}\, a + \cos^2 \tfrac{3}{2}\, a + \cos^2 \tfrac{5}{2}\, a\ .\ .\ .\ + \cos^2 \frac{2n-1}{2} a$$

For the whole polygon $\Sigma \sin^2 i = \Sigma \cos^2 i$; hence, by adding the preceding expressions, we find

$$\Sigma \sin^2 i = \Sigma \cos^2 i = \tfrac{1}{2}\, n;$$

hence, for the whole polygon, equation (27) becomes

$$I = \tfrac{1}{4}\, A\, (d^2 + \tfrac{1}{12}\, b^2), \qquad . \quad . \quad . \quad (28.)$$

For the square, $d = \frac{1}{2}b$, $A = 2b^2$; $\therefore\ I = \frac{1}{12}\, b^4$.

For the hexagon, $d = \frac{1}{2}b \cot 30^\circ$, $A = 3\,bd$; $\therefore I = 0{\cdot}96214\, d^4$.
For the octagon, $d = \frac{1}{2}b \cot 22\frac{1}{2}^\circ$, $A = 4\,bd$; $\therefore I = 0{\cdot}875776\, d^4$.
Let R = the radius of the circumscribing circle.
r = the radius of the inscribed circle.
Then

$$\tfrac{1}{4}b^2 = R^2 - d^2;\ r = d;$$

$$\therefore (28) \text{ becomes } I = \tfrac{1}{12}A(R + 2r).$$

If $r = R$, then $A = \pi r^2$; $\therefore I = \frac{1}{4}\pi r^4$, which is the moment of inertia of a circle as before found.

For the Journal of the Franklin Institute.

A Mode of Determining Graphically the Correction for the Reduction to Centre of Station, and also for Oblique Illumination on Signals in a Geodesic Survey. By John R. Mayer, C.E.

Reduction to the Centre of Station.

C being the centre of a trigonometrical station, O the angle observed between two objects A and B, y the angle between C and B, the *left* hand object, r the distance O C, D the distance A C, and G the distance B C.

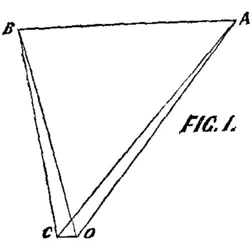


FIG. 1.

Correction expressed in seconds

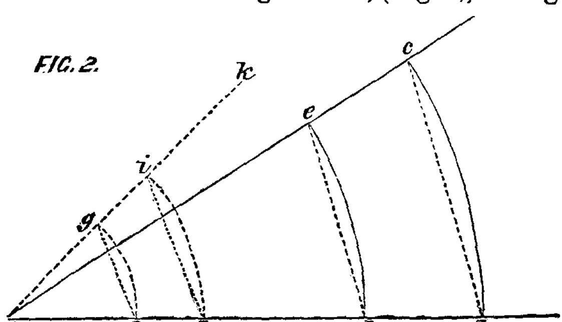
$$= \frac{r \sin(O + y)}{D} R'' - \frac{r \sin y}{G} R''.$$

In the first term call $\sin(O + y)$, a, and we have for this term $D : R'' :: \frac{D}{1000} : \frac{R''}{1000} :: (r\,a)$: correction for the first term.

The quantity $r\,a$ is obtained by a fourth proportional between 90° or radius, r and a the value of this last quantity being taken from the table of natural sines.

Thus we construct a sectoral figure $a\,b\,c$, (Fig. 2,) having $a\,b$ for radius, and the transverse $b\,c = r$, on which the natural sine of a is marked on the lines $a\,b$ and $a\,c$ at d and e from the centre a, which gives $d\,e$ for the quantity $r\,a$.

FIG. 2.



Then, on the same figure $a\ b\ c$, describe an arc $f\,g$, with a radius equal to $\frac{D}{1000}$, and intersect this arc at g with the quantity $\frac{R''}{1000}$, and draw the line $a\,g\,k$. Then, from a as a centre, describe an arc with $d\,e$ as radius which intersects the lines $a\,b$ and $a\,k$ at h and i, the distance $h\,i$ will represent the number of seconds for the correction of the first term.

The same operation is required for the second term,

$$\frac{\frac{r \sin y}{G}}{1000}\ \frac{R''}{100}.$$

The rule of signs is to make the first term *positive* when $(o + y)$ is less than 180°, and apply the negative sign with sin y.

The same scale of equal parts is used for the quantities r, $\frac{D}{1000}$, $\frac{G}{1000}$, and $\frac{R''}{1000}$. $R'' = 206264''{\cdot}8$.

EXAMPLE.—Suppose $o = 51°\ 46'\ 38''$, $y = 79°\ 24'$, $r = 5{\cdot}50$ feet, $D = 27659{\cdot}6$ feet, and $G = 22245{\cdot}4$ feet.

The graphical computation gives:

First term, $+\ 30''{\cdot}75$; second term, $-\ 50''{\cdot}05$. Correction $= -\ 19''{\cdot}30$.

The computation with the table of logarithms gives:

First term, $+\ 30''{\cdot}87$; second term, $-\ 50''{\cdot}12$. Correction $= -\ 19''{\cdot}25$.

Correction for Phase when a Tin Cone or Cylinder is used as Signal.

Z being the angle observed at the station between the sun and the signal, r the radius of the signal, and D the distance.

$$\text{Correction} = \pm \frac{r \cos^2 \frac{1}{2} Z}{D}\ R''.$$

Substituting $\frac{1}{2}$ (cos Z + 1) to $\cos^2 \frac{1}{2}$ Z and calling β the value of $\frac{1}{2}$ (cos Z + 1) from the table of natural cosines, the quantity $r\ \frac{1}{2}$ (cos Z + 1) or $r\,\beta$, will be the fourth proportional between 90° or radius, r and β. Then, dividing D and R″ by 1000, we have

$$\frac{D}{1000} : \frac{R''}{1000} :: r\,\beta : \text{correction}.$$

The construction of the sectoral figure to solve this problem is like the above-mentioned, (Fig. 2.)

With a radius of *about six inches*, this mode of graphical computation gives results which can approximate to less than *a five-hundredth of an unit*.