

For the Journal of the Franklin Institute.

General Problem of Trussed Girders. By DE VOLSON WOOD,
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(Continued from page 311.)

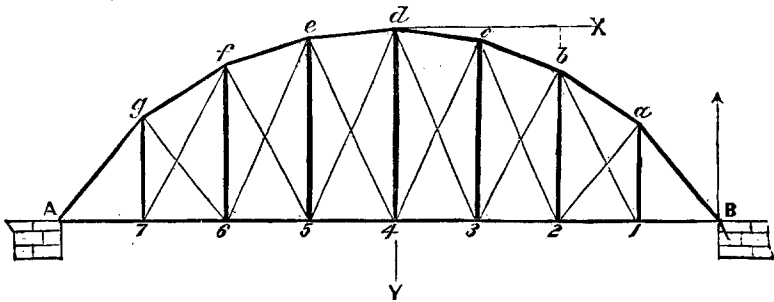
The formulas which express the strains upon the several parts of trusses which are in common use, should be as simple as possible, and when possible should be continuous functions of the strain and distance from some fixed point, as one end for instance. This can generally be done when the curve of the chord follows an algebraic law.

We will suppose that the bays on the lower chord are all equal to each other, and that the load consists of equal weights placed at the upper or lower joints. If these hypotheses do not correspond with any practical case resort may be had to the equations already given.

14°. *Let the lower chord be horizontal and the upper one parabolic, or, if polygonal, let the vertices of the polygon be in a parabolic arc.*

Some parts of this problem were very fully discussed in my article on the "*Trussed Arch*," found in the April number of this *Journal* for this year, on page 223. I shall therefore notice it very briefly here, merely showing how to apply equations (20) to this case.

FIG. 11.



Let Fig. 11, represent the truss.

Let N = total number of bays in the horizontal tie,

n = the number of the bay considered, counting from either end,

D = $d4$ = greatest depth of the truss,

l = length of a bay = $1 - 2 = 2 - 3$, &c.,

p = one of the equal weights which constitute the load,

w = the total load when uniformly distributed over the span ;
and the other notation the same as before given.

$$\text{Then } p = \frac{W}{N}.$$

We first suppose that the bridge is uniformly loaded throughout

which is the form taken by the equation when the bars act as braces. (See eq. 15, p. 228 in the April No.)

Equation (31) shows that the strains vary as the product of the segments into which the span is divided by the vertical bar which passes through the upper end of the inclined tie.

It is easy to show that the strain on the horizontal chord is greatest when the bridge is completely loaded; hence, for an uniform load we have for the greatest stress on this chord

$$H_1 = \frac{p l N^2}{8D}, \text{ as given in equation (27).}$$

Similarly, the greatest strain on the arch is for a full load; and if the load be uniform, we have

$$F \cos i = H_1 = \frac{p l N^2}{8D} \quad . \quad . \quad . \quad (32)$$

as given above.

From (30) we may find

$$\cos i = \frac{l N^2}{\sqrt{l^2 N^2 + 16 D^2 (N - 2n + 1)^2}}, \quad . \quad . \quad . \quad (33)$$

which reduces (32) to

$$F = \frac{p}{8D} \sqrt{l^2 N^2 + 16 D^2 (N - 2n + 1)^2} \quad . \quad . \quad . \quad (34)$$

From (30) we may also find that

$$\frac{1}{\cos \theta} = \frac{\sqrt{16 D^2 N^2 (N - n)^2 + l^2 N^2}}{4 D n (N - n)}$$

which reduces (31) to

$$F_2 = \frac{(N - n + 1)(n - 1) \sqrt{16 D^2 N^2 (N - n)^2 + l^2 N^2}}{8 D n (N - n)} p \quad (35)$$

Equations (27), (34) and (35) are necessary and sufficient for solving the problem.

Before leaving this problem it will be well to observe that for a load uniformly distributed over the whole length $v = \frac{1}{2} w = \frac{1}{2} n p$. But in this case only one-half the load on each of the end panels causes strains on the truss, because said half is sustained directly by the abutments, or supports. Making this deduction and we have for the re-action which causes strains $v = \frac{1}{2} (N - 1) p$ as given in equation (26).

In order to bring together, in this connexion, all the equations for computing the strains on this truss, I copy the following from the article above referred to.

For maximum shearing in the panel system when the load is on the upper chord, we have

$$F_2 \cos i = \frac{(N - n) n}{2N} p \quad . \quad . \quad . \quad (36)$$

For maximum strain on the upper and lower chords use equations (27) and (32).

In the *Triangular System*, for maximum shearing, when the load is on the upper chord, we have

$$F_2 \cos \theta = \frac{(N-n)^2}{2N} \left[\frac{4n^2-1}{4nN-4n^2-1} \right] p, \quad (37)$$

with the load on the lower chord, we have

$$F_2 \cos \theta = \frac{(N-n)(N-n+1)}{2N} \left[\frac{4n^2-1}{4nN-4n^2-1} \right] p \quad (38)$$

$$\tan \theta = \frac{lN^2}{2D[N^2-(N-2n-1)^2]} \quad (39)$$

Equations (27) and (32) may also be used for this system, for finding the strains on the chords.

15°. *Let both chords be horizontal.*

For this make $i=0$ in equations (20) and they become,

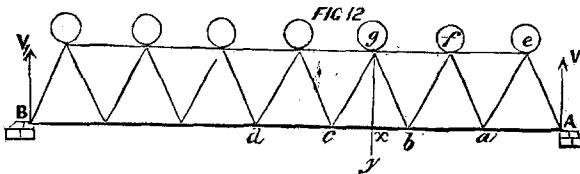
$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= V - \sum_o^x P \\ HD &= Vx_1 - \sum_o^x Px \end{aligned} \right\} \quad (40)$$

in which D is the total depth.

The second of these equations shows that the ties or braces, as the case may be, resist all the shearing stress.

For an uniform load $v = \frac{1}{2}wL$, and at the middle section $\sum_o^x P = \frac{1}{2}wL$, which in (40) gives $F_2=0$, which shows that no ties or braces are needed at the middle. But it will be shown hereafter, that if the uniform load consists of equal weights placed on the joints, there will, in some cases, be a strain on the middle braces.

LET US FIRST CONSIDER A SYSTEM OF TRIANGULAR TRUSSING—see Fig. 12.



Let the bays be of equal length, and the bars be equally inclined.

Let the equal weights be on the joints of the upper chord. Then the total load is np . $\therefore V = \frac{1}{2}np$.

Let a vertical section be made just at the left of g , and call bc the n^{th} bay counting from A ; or to be more specific, let n be the number of bays between the end and the foot of the brace considered. Then we readily find for the section gx ,

$$\begin{aligned} \sum_o^x P &= np \\ x_1 &= (n - \frac{1}{2}) l \\ \sum_o^x Px &= \frac{1}{2}n(n-1)pl. \end{aligned}$$

These in (40) give,

$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= \frac{1}{2} p (N - 2n) \\ H_1 D &= \frac{1}{4} [N(2n - 1) - 2n(n - 1)] pl \end{aligned} \right\} \quad (41)$$

We also find $\tan \theta = \frac{l}{2D}$; $\sin \theta = \frac{l}{\sqrt{4D^2 + l^2}}$; $\cos \theta = \frac{2D}{\sqrt{4D^2 + l^2}}$.

For the braces which incline the same way as g , c , $\sin \theta$ will be negative; for all others it is positive.

If N is even, we have $n = \frac{1}{2} N$ for the middle braces, which in the second of (41) gives $F_2 = 0$; or there is no strain on the middle braces; but if N is odd, we shall have $n = \frac{1}{2} (N + 1)$ or $= \frac{1}{2} (N - 1)$ for the middle braces. These values in the second of (41) will give $F_2 \sin \theta = + \frac{1}{2} p$. For the end braces $n = 0$, which gives $F_2 \sin \theta = \frac{1}{2} Np = v$.

For maximum shearing let all the joints on the left of g be loaded and those on the right be unloaded. Then we readily find:

$$\begin{aligned} \text{the total load} &= (N - n) p, \\ v &= \frac{(N - n)^2}{2N} p, \\ \sum_0^x P &= 0, \\ \sum_0^x P x &= 0, \\ x &= (n - \frac{1}{2}) l. \end{aligned}$$

These substituted in (40) give,

$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= \frac{(N - n)^2}{2N} p \\ H D &= \frac{(N - n)^2}{4N} (2n - 1) l \end{aligned} \right\} \quad (42)$$

But to combine the cases of uniform load throughout, and an uniform load to produce maximum shearing, we will suppose that the uniform load is the weight of the frame, and let $w_1 = w \div N =$ the weight of a bay in length of the frame. Then if $p = w_1$, in (41) those equations will give the strains, and by the aid of (42) we have for the strains,

$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= \frac{1}{2} w_1 (N - 2n) + \frac{(N - n)^2}{2N} p \\ H_1 D &= \frac{1}{4} l \left[\left\{ N(2n - 1) - 2n(n - 1) \right\} w_1 + \frac{(N - n)^2}{N} (2n - 1) p \right] \end{aligned} \right\} \quad (43)$$

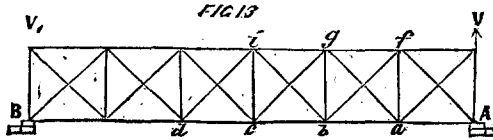
For an uniform load throughout including the weight of the frame we readily have from (41)

$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= \frac{1}{2} (N - 2n) (p + w_1) \\ H_1 D &= \frac{l}{4} [N(2n - 1) - 2n(n - 1)] (p + w_1) \end{aligned} \right\} \quad (44)$$

The third of (44) is greater than the third of (43); hence, for the strains upon the chords we use equations (44). For n less than $\frac{1}{2} N$, the second of (43) is greater than the second of (44); hence, we use the former to find the strains upon the braces.

Similar expressions might be found when the joints of the lower chord are loaded, but we will omit them and pass immediately to the

PANEL SYSTEM.



For a definition of this system see number 8° of this article. Let figure 13 represent the case. Many bridges have been made in this country upon this principle; among which I will mention the following:

Howe's Truss, in which the chords and inclined bars are made of wood, and the vertical ties are of iron, (See this *Journal* vol. iii. 3d series, p. 289, also, *Silliman's Journal* vol. xviii. p. 123): Pratt's Truss, in which the chords and vertical struts are of wood, and the inclined bars are iron ties, (see Vose's *Hand-book of Engineering*, p. 154): Long's Truss, which is composed entirely of wood, and the inclined bars are braces, (see this *Journal* vol. v, 2d series, p. 231.): Whipple's Truss, which is composed entirely of iron, and the inclined bars are ties, (see *Appleton's Dictionary of Mechanics*—the last edition—article Bridges,) and Jone's Truss, which is also made entirely of iron, (see *Scientific American* for 1863, vol. ix, p. 193.) These Trusses differ in the *details* of their construction and not in their mathematical properties, and as it is only the latter that we propose to discuss we shall not dwell upon their points of distinction. Many other trusses partake of some of the properties of those above mentioned.

We now proceed to discuss the theory. The strains upon the diagonal bars will be the same whether they act as ties or braces; also the strain upon them will be the same whether the load be on the upper or lower chord. But the strain on the vertical ties (or struts, as the case may be) is not the same for the load on either chord.

It may be seen from the second of (40) that the *vertical components* of the strains on the several braces which are between two consecutive weights, are equal to each other. For, v being constant, the first member will remain constant as long as $\sum_{\circ}^x P$ is constant, and this is constant between two consecutive weights.

From this we see that when the load is on the lower chord, the *vertical components* of $A f$ and $f a$ Fig. 13, will equal each other; so of $a g$ and $g b$. But if the load be on the upper chord, the vertical components of $f a$ and $a g$ equal each other; so of $g b$ and $b i$. If ties be used instead of braces, and the load be on the upper chord, then will the vertical components on $f b$ and $b g$ equal each other; but if the load

be on the lower chord, the vertical components on af and fb equal each other. If there be no load between c and A , the vertical components will be the same on all the bars between c and A .

Let n be the number of the panel counting from A , in which the strains are considered, which will be the same as the number of a brace (or tie) counting from the same end, and let the other notation be the same as given above.

Then for a uniform load in which the weight of the bridge is considered, we have

$$\begin{aligned} (N-1)(p+w_1) &= \text{the total load,} \\ v &= \frac{1}{2}(N-1)(p+w_1), \\ \Sigma_o^x \theta P &= (n-1)(p+w_1), \\ x_1 &= n l \text{ for a section just at the right of } g, \\ \Sigma_o^x P x &= \frac{1}{2} n l (n-1)(p+w_1). \end{aligned}$$

These in (40), give

$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= \frac{1}{2}(N-2n+1)(p+w_1) \\ H_1 D &= \frac{1}{2}(N-n)nl(p+w_1) \end{aligned} \right\} \quad (45).$$

The third of these equations shows that the strain on the lower chord varies as the product of the segment into which the span is divided by the vertical bar which passes through the upper end of the brace (or tie) which belongs to the panel considered.

For maximum shearing there will be $(n-1)$ unloaded joints, and hence $(N-n)$ loaded joints, but still the weight of the frame must remain as an uniform load throughout. For this case we readily find, in the same way that we found equations (44) that,

$$\left. \begin{aligned} H + F_2 \sin \theta &= H_1 \\ F_2 \cos \theta &= \frac{1}{2}(N-2n+1)w_1 + \frac{(N-n)(N-n+1)}{2N}p \\ H_1 D &= \frac{1}{2N} \left[(N-n)(Nw_1 + (N-n+1)p) \right] nl \end{aligned} \right\} \quad (46)$$

For n less than $\frac{1}{2}N$, the second of (45) is less than the second of (46); hence, we use the latter to compute the strains on the braces. For the strain on the chords use (45).

There is a point at which the shearing stress is zero; and it may be found by placing the second of (46) equal zero and solving for n . The general expression thus found is long and inconvenient, but it may be easily shown that there is one value between $\frac{1}{2}N$ and N —this we will call n_o . The other value exceeds N , and hence is inadmissible.

For example, if $p = w_1$, and $N = 12$ we find $n_o = 7.9$
 $p = 2w_1$, and $N = 10$ we find $n_o = 6.8$

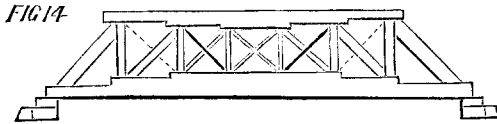
In this way a table might be formed with the arguments p , w_1 and N .

The second of (46) is positive for all values if n between 0 and n_o ; and negative for all values between n_o and N .

To show more clearly the force of this equation, let us suppose that the frame is loaded uniformly throughout, and that it moves off in the direction from A towards B . The strain on Af will be greatest when

all the load is on and is found by making $n=1$ in the second of (46); the strain on ag is greatest when the rear end of the surcharge has reached a , and is found by making $n=2$ in the same equation: the strain on bi is greatest when the rear end has reached b , and is found by making $n=3$; and so on to $n=n_0$, beyond which the expression becomes negative which shows that ties must be used instead of braces; or what is equivalent, the brace must be inclined the other way, like di , for instance. The same phenomenon is observed if the load moves from B towards A. We observe that, counting from either end, the braces beyond n_0 incline in an opposite direction to those within that value. From 0 to $\frac{1}{2}N$, or $\frac{1}{2}(N+1)$, the bars are usually called *main braces* or *main ties*; from $\frac{1}{2}N$ to n_0 , *counter braces*, because they incline in an opposite way from main braces.

Fig. 14 is a graphical representation of equations (46) and the principles which have here been developed.



We observe that the first term of the second member, of the second equation of (46) is positive, for n less than $\frac{1}{2}(N+1)$ and negative for n greater than that value, therefore, when the surcharge extends over half or more than half the bridge it conspires with the weight of the frame to produce strains upon the main braces; but they act against each other to produce the strains on the counter-braces. When there is a strain on a counter-brace, there is none on the main brace in the same panel. Between n_0 and N , the effect of the surcharge is merely to relieve the main braces of a portion of the strain which is produced by the weight of the frame alone.

For a queen post truss we have $N=3$, in (46);

$$\therefore F_2 \cos \theta = (2 - n) w_1 + \frac{1}{3} (3 - n) (4 - n) p,$$

and for the end brace or rafter make $n=1$,

$$\therefore F_2 \cos \theta = w_1 + p.$$

For the braces in the middle panel, make $n=2$,

$$\therefore F_2 \cos \theta = \frac{1}{3} p.$$

Braces are rarely put in the middle panel in the queen post truss, for the stiffness of the frame is relied upon to resist the concentrated load.

For single rafters make $N=2$, and $n=1$, in the second of (46);

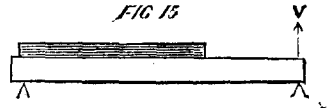
$$\therefore F_2 \cos \theta = \frac{1}{2} (w + p).$$

16°. The principles just stated are more forcibly illustrated by supposing that the ties are so near each other as to be considered continuous, and that the load is continuous and uniform.

For this, let x = the length of surcharge which is continuous from one end, see Fig. 15,
 r = weight of surcharge on a unit of length,
 L = length of frame,
 w = weight of a unit of length of the frame.

Then
$$V = \frac{1}{2}wL + \frac{rx^2}{2L}$$

$$\Sigma_0^x P = w(L-x),$$



and the second of (40) becomes

$$F_2 \cos \theta = \frac{1}{2}w(2x-L) + \frac{rx^2}{2L} = y \text{ (say)} \quad (47)$$

This may be considered as the equation of a curve.

For $x=0$; $y = -\frac{1}{2}wL$

$$y=0$$
; $x = \left(-\frac{rw}{r} \pm \sqrt{\frac{w^2}{r^2} + \frac{w}{r}} \right) L \quad (48)$

$$\frac{dy}{dx} = 0$$
; $x = -\frac{w}{r}$.

Fig. 16 is a geometrical illustration of equation (47).

From (48) we see that the curve crosses at D, a point between A and the middle of the span. For instance,

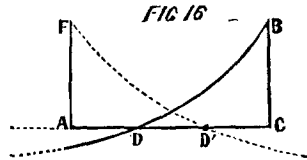
if $r=10w$; $x=0.231L$

$r=5w$; $x=0.299L$

$r=2w$; $x=0.366L$

$r=w$; $x=0.414L$

$r=0$; $x = \frac{1}{2}L$. (See equation (47).



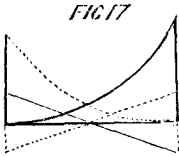
From c to D the braces incline from the support c, and from A to D, they incline from A. If the load moved in the opposite direction, the point of no shearing would be at D', and between D and D' braces must incline both ways to resist the strains arising from loads moving both ways.

Equation (47) may be separated into two parts; one being the strains due to the permanent load; the other, to the surcharge. Call the former y_1 ; the latter y_2 ; and we have

$$y_1 = w(x - \frac{1}{2}L) \quad (49)$$

$$y_2 = \frac{rx^2}{2L} \quad (50)$$

Equation (49) shows, that for an uniform load extending over the whole span, the strains increase uniformly from the middle towards the ends.



Equation (50) shows that they increase as the ordinates of a common parabola, counting from one end. The axis of the parabola is perpendicular to the span. Fig. 17, represents equations (49) and (50).

17°. It is desirable to have some easy mode of determining whether a bar is subjected to a thrust or pull. It is evident that the bars of the upper chord are all subjected to a thrust, while those in the lower chord are subjected to a pull or tension. But the inclined bars may be subjected to either, and in varying circumstances to both kinds of strain. To determine to which strain any bar is subjected under any given condition; we observe, first, that the shearing stress is

{ positive or upwards }
 { negative or downwards } when the second members of (40), is

{ positive }
 { negative } . If we conceive the load to be divided by a vertical

plane so that the part between A and the plane shall equal v, see fig. 12 or 13, then will the shearing stress between v and the plane, be positive; and beyond the plane it will be negative. Or, this principle may be

stated thus: the shearing force will be { positive }
 { negative } according as the

plane section is { nearer }
 { more remote } than a plane which divides the load

into two parts respectively equal to the re-action of the supports.

We next observe that all the bars which are similarly situated on either side of the dividing plan will be subjected to like strains.

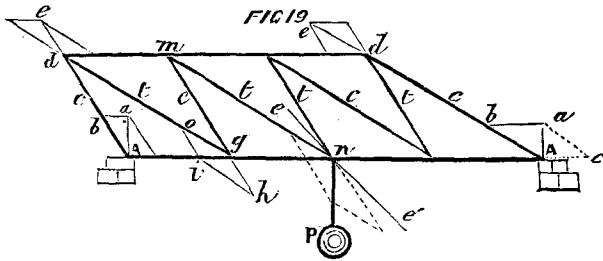
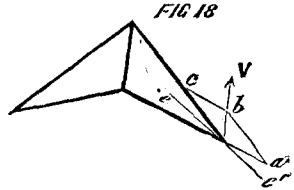
Finally, the brace Ae (fig. 12, or Af fig. 13) is subjected to compression, and its angle of inclination and the shearing stress are both positive; hence we have this general principle; for ordinary trussed bridges:

A bar is subjected to a { thrust }
 { pull } when the sign of its angle of inclination is { the same as }
 { contrary to } the sign of the shearing stress on a plane section which passes through the bar.

This rule which is essentially that given by Rankine for a similar case, (see his Applied Mechanics, p. 161) fails in many cases when applied to trusses of different form. For instance, in the double rafter, as shown in Fig. 18, the inclination of both the right hand rafters is positive, and the shearing stress is positive; yet the lower one is subjected to tension.

In Fig. 19, all the bars have a *positive* inclination, while the alternate ones are subjected to tension.

Tredgold in his *Carpentry* (Fourth edition p. 12) gives the following rule to distinguish ties from struts.



“Let a parallelogram be constructed on the direction of the straining force as a diagonal, the sides of the parallelogram being parallel to the sustaining forces ; then, let the other diagonal of the parallelogram be drawn ; and, parallel to it, draw a line through the point where the directions of the forces meet. Consider towards which side of this line the straining force would move if left at liberty ; and all supports on that side will be in a state of compression and all those on the other side will be in a state of tension.”

For instance, in Fig. 18, construct a parallelogram on AV as a diagonal, having its sides parallel to AB and AC . Draw ee' through A parallel to ac . Now if v were free to move it would move vertically upward, and as AB is above ee' it would, according to the rule, be compressed and AC extended. The rule draws a correct conclusion in this case ; still it is not infallible. For if we suppose there is a single weight acting on the frame, as shown in Fig. 19, and we apply the rule to it, we will find that all the bars at the right which are marked c , should be subjected to tension ; whereas, they are really subjected to compression. The rule also decides that the lower bar is compressed, whereas it is extended. In justice to the rule, however, I should say that if we suppose that two inclined bars sustained the weight—neglecting the influence of the horizontal tie—it gives correct results. But with this admission I find that it fails in numerous instances in the arched truss. It can then be *relied upon only* in cases where two bars—and only two—are concerned in resisting the force. Tredgold applied it to such cases.

Robison in his *Mechanical Philosophy*, vol. i. p. 504, gives the following rule: “Take notice of the direction in which the piece acts from which the strain proceeds. Draw a line in that direction *from*

the point on which the strain is exerted; and let its length express the magnitude of this action. From its remote extremity draw lines parallel to the pieces on which the strain is exerted. If one of these lines cut the bar, or the bar prolonged in the direction of its remote end, the bar will be compressed; but if it cut the prolongation in the opposite direction it is a tie." This rule is also given in the *Encyclopædia Britannica* in the article on Carpentry.

This rule is more nearly general than either of the preceding, and yet it must be used with caution and with some limitations. To illustrate it, refer to Fig. 19, and let aa represent the re-action of the supports, which is vertically upwards. Through a draw a line parallel to the lower chord; it cuts the bar Ad ; hence Ad is a strut. Now the force in the strut acts from A towards d ; therefore, draw de in that direction from d , and proceed as before, and we see that the upper chord is compressed and the next bar extended. Proceed in this way until we come to the point where P is applied: or generally; *commence at either end and follow the rule to the point where the vertical shearing stress is zero.* The limitations to which I wished to refer, is—*we must not pass the point where the vertical shearing stress is zero.*

I have said the rule must be used with caution. For instance, in Fig. 19, to find the strains upon the bars which concentrate at g , we have found that the bar dg is a tie, and to produce tension on the part go , it must act from g towards o ; through o draw oi parallel to mg ; it cuts Ag at i ; hence, we would be inclined to infer that Ag is compressed, but we found while considering the forces at A , that it was extended. We must therefore infer that the point i belongs to the prolongation of ng ; and hence ng is extended. If these cautions and limitations be observed I think that the rule will always give correct results.

My rule has been to observe that the inclined bars commencing at either end, are alternately compressed and extended up to the point where the shearing stress is zero; and that all parts of the upper chord are compressed, and of the lower are extended. It is not difficult to determine whether the first one is extended or compressed, and hence, the rule is easily applied.

In the next article I shall consider some cases in which the axis of Y intersects more than three bars.

(To be Continued.)

Mons. Crepin's Experiments on Baltic Timber Creosoted under Bethell's Process.

[Annales des Travaux Publics de Belgique.]

From the Lond. Practical Mechanic's Journal, August, 1864.

The experiments undertaken by me in 1857, at Ostend, to ascertain the relative preservation of timber prepared with sulphate of copper, and timber prepared with creosote oil, when placed in the sea, and the relative resistance of such differently prepared timber to the attacks of the Teredo worm, have been previously given to the scientific world.