therefrom; though one or two offices are understood to be ready so to consider the injuries.

We have hinted that there is some reason to apprehend that the embankments of the Thames, which are said to make in the total 300 miles, are not as they should be for security. The danger averted,—and, as we have shown, through a most fortuitous conjuncture of circumstances,—will, we hope, incite the several Commissioners, and the Conservators of the Thames, to consider whether better survey, and more frequent reparation of the embankments and sluices are desirable; and whether it would not be well to prevent such risk as is proved to be incurred by the storage of gunpowder in a position where explosion may cause a breach.

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For the Journal of the Franklin Institute.

*General Problem of Trussed Girders.* By DE VOLSON WOOD, Prof. of C. E., University of Michigan.

(Continued from page 236.)

If the diagonal bars act as ties, and *b c*, Fig. 5 is subjected to a strain, we will make a section just at the left of *b*. We might use the former section and take the origin at *c*; or we might let the origin remain at *a*, but in the latter case we would have two moments, one the moment of *b c*, the other of *c d*. The first being the simplest, we will take the origin at *b*.

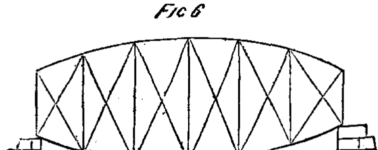
Considering as before the forces at the right of the section, and we have to consider the strains on the infinitely short parts of the bars *a b*, *b c*, and *c d*. To produce tension on this part of *b c* the force acts from *b* towards *c*. Therefore we have $\alpha_2 = 90° + \theta$ and the other angles as before. These in (10) give

$$\left.\begin{array}{l} F\cos i - F_1\cos i_1 - F_2\sin\theta = 0 \\ F\sin i - F_1\sin i_1 + F_2\cos\theta = V - \Sigma_0^{x_1} P \\ F_1 h\cos i_1 = Vx_1 - \Sigma_0^{x} Px \end{array}\right\} \quad . \quad . \quad (12)$$

Equations (12) may be deduced from (10) by substituting minus $\theta$ for $\theta$. Equations (10) or (12) are applicable to Fig. 4 by observing the signs of the angles. It must be observed that at the left of the middle the signs of $i$ and $i_1$ change and become negative.

9°. *Let the lower chord be convex downwards*, as in Fig. 6. Equations (10) are immediately applicable by observing that at the right of the middle, $i$ is positive, $i_1$ negative and $\theta$ positive or negative according to the inclination of the bar; while at the left of the middle, $i$ is negative and $i_1$ positive.

FIG 6



Example. Let the quantities be as in the preceding example, only observe that the inclination of the lower chord is—20°, and we find from (10) that for a load over the whole length;

$$F = 218, \; F_1 = 143, \text{ and } F_2 = -107.$$

The negative value of $F_2$ indicates that it is a tie.

10°. The same equations apply to the case where the upper chord is convex downwards and the lower convex upward as in Fig. 7, by observing the signs of the angles.

Fig 7

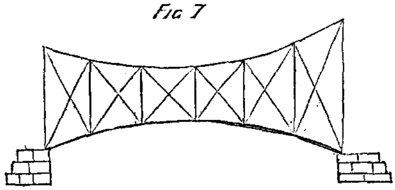
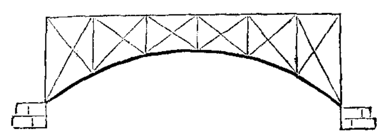


Fig 8

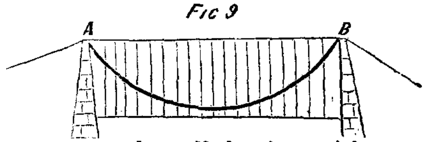


11°. Let the upper chord be horizontal and the lower one convex upwards as in Fig. 8.

Make $i = 0$ in (10) and call the horizontal force in the upper chord H and (10) becomes

$$\left.\begin{array}{l} F_1 \cos i_1 + F_2 \sin\theta = H \\ F_1 \sin i_1 + F_2 \cos\theta = V - \Sigma_0^{x_1} P \\ F_1 h \cos i_1 \quad V x_1 - \Sigma_0^{x} P x \end{array}\right\} \quad . \quad . \quad (13)$$

Fig 9




At the left of the middle, $i_1$ is negative.

12°. Let the lower chord be convex downwards.

In this case we have only to change the sign of $i_1$ in (13). If we suppose that all the ties and braces are omitted—or what is equivalent, suppose $\theta = 0$—we shall have the analytical condition of the suspension bridge. Making $i_1$ negative and $\theta = 0$ in (13) and we have

$$\left.\begin{array}{l} H = F_1 \cos i_1 = 0 \\ \quad F_1 \sin i_1 + F_2 = V - \Sigma_0^{x_1} P. \\ \quad F_1 h \cos i_1 \quad = V x_1 - \Sigma_0^{x} P x \end{array}\right\} \quad . \quad . \quad (14)$$

In the suspension bridge the land cables resist the strain which is now thrown upon the chord AB, Fig. 9. From the first of these we observe that for $i_1 = 0$; $H = F_1$ *i.e.* at the lowest point the tension of the cable equals the compression of the upper chord. Also from the first $F_1 \cos i_1 = H$, that is, the horizontal component of the tension at any point equals the tension at the lowest point, and this is true whether the curve be parabolic, catenarian, or any other form.

From the second of (14) we have

$$F_1 \sin i_1 = V - \Sigma_0^{x_1} P - F_2$$

in which $F_2$ may be included in $\Sigma_0^{x_1} P$, and thus not appear as a separate term. Hence we have

$$F_1 \sin i_1 = V - \Sigma_0^{x_1} P \quad . \quad . \quad (15)$$

which shows that the vertical force resolved by the arch (or cable) at any point equals the portion of the load which is supported by the pier generally diminished by all the load between it and the same pier; or equal the load between the lowest point of the curve and the point considered.

If the load and ties be considered continuous, and the loading follow any algebraic law, the cable will be a continuous curve, and the preceding equations may be more conveniently expressed in the following way:

Let $f\ (x\ y)$ be the equation of the curve,

$s$ any portion of the arc,

$T = F_1 =$ tension at any point

then $\cos i_1 = \frac{dx}{ds}$ $\sin i_1 = \frac{dy}{ds}$

Hence, (14) and (15) become

$$\left.\begin{aligned} T\frac{dx}{ds} &= H \\ T\frac{dy}{ds} &= V - \Sigma_0^{x_1} P \\ Th\frac{dx}{ds} &= V^{x_1} - \Sigma_0^{x} Px \end{aligned}\right\} \quad . \quad . \quad (16)$$

From (16) we may determine the mechanical conditions of the cable such as tension and deflection.

For instance suppose the load is uniformly distributed over the span, and

Let $L =$ the length of span,

$w =$ the load on a unit of length, and take the origin at the lowest point.

Then $V = \frac{1}{2} w L$; $\Sigma_0^{x^1} P = w(\frac{1}{2} L - x)$. These substituted in the second of (16) and then differentiated gives

$$dT\frac{dy}{ds} = wdx \quad . \quad . \quad . \quad . \quad (17)$$

The value of T from the first of (16) substituted in (17) gives

$$dH\frac{dy}{dx} = wdx \text{ the integral of which}$$

is $$x^2 = \frac{2H}{w}y \quad . \quad . \quad . \quad . \quad (18)$$

which is the equation of a parabola.

Equation (18) aids us in using the third of equation (16), but the latter is rarely necessary, when the equation of the curve is known.

Equation (18) and the third of (16) give the same value for H at the lowest point. For at the lowest point $x = \frac{1}{2} L = x_1$, $y = h = D$

$$\frac{dx}{ds} = 1 \qquad \Sigma_0^{x} Px = \tfrac{1}{2}wL \times \tfrac{1}{4}L = \tfrac{1}{8}wL^2$$

$$V = \tfrac{1}{2} wL$$

which in the third of (16) gives

$$T = H = \frac{\frac{1}{4} w L^2 - y w L^2}{D} = \frac{w L^2}{8D} \quad . \quad . \quad (19)$$

which is the same that we obtain directly from (18). The same result may be obtained geometrically.

We would proceed in a similar way if the loading follows any other law. If the loading be the cable itself—forming a catenary—we have $V - \Sigma_0^{x_1} P = \int \delta K ds$, when $\delta$ is the weight of a unit of volume and K the section of the cable.

If the load increases uniformly from the middle each way, then $V_0^{x_1} \Sigma - P = \frac{1}{2} w x^2$

13°. If the lower chord be horizontal and the upper one convex upwards, we have $i_1 = 0, F_1 = H_1$ in (10), which will give

$$\left.\begin{array}{l} F \cos i + F_2 \sin \theta = H_1 \\ F \sin i + F_2 \cos \theta = V - \Sigma_0^{x_1} P \\ H_1 h \quad = V x_1 - \Sigma_0^{x_1} P x \end{array}\right\} \quad . \quad . \quad . \quad (20)$$

The equations thus far deduced are for the most general distribution of the load. Let us now suppose that the load is uniformly distributed over a part or the whole of the span, being continuous from one end to the point considered,—or from end to end,—as the case may be. If the load extends from end to end we readily have

$$\left.\begin{array}{ll} V = \frac{1}{2} w L; & \Sigma_0^{x_1} P = w x_1 \\ V x_1 = \frac{1}{2} w L x_1 & \Sigma_0^{x_1} P x = \frac{1}{2} w x_1^2 \end{array}\right\} \quad . \quad . \quad . \quad (21)$$

If it extends from the remote end to the point considered—thus giving the maximum shearing,—we have

$$\left.\begin{array}{ll} V = \frac{w(L - x_1)^2}{2L} & \Sigma_0^{x_1} P = 0 \\ V x_1 = \frac{w(L - x_1) x_1^2}{2L} & \Sigma_0^{x_1} P x = 0 \end{array}\right\} \quad . \quad . \quad . \quad (22)$$

It is impossible to strictly realize the latter condition, for the weight of the frame always forms an appreciable part of the load, and may be considered a permanent load, while the surcharge may be called *moveable* or *transient.*

These may be considered separately or together. If separately, we shall have for the case of maximum shearing arising from the surcharge, from (20)

$$\left.\begin{array}{l} F \cos i + F_2 \sin \theta = H_1 \\ F \sin i + F_2 \cos \theta = V \\ H h \quad = V x_1 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (23)$$

The 1st and 2d of (23) are the same as (3) and (2) page 224 of the April number of the *Journal* for this year.

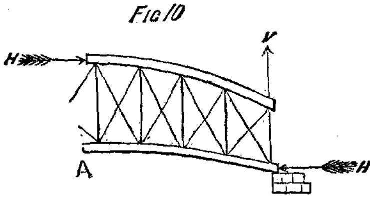
The weight of the frame cannot be definitely known until the strains upon the several parts are known, but these cannot be known until the load is known; hence the strains due to the weight of the frame remain an implicit function of the weight. We may, however, approximate to the weight by assuming that it is an uniform load, and then finding the sections of the principal parts at the middle of the frame. This assumption will not be far from the truth, and will generally be on the side of safety. I give the following as one method of finding the approxmate weight.

Let K = the section of the upper chord at the middle which also equals the section of the lower one,
L = the total length of span,
D = the depth of frame at the centre.
$\delta$ = the weight of a unit of volume of the frame,
$n$ = the ratio between the weight of a chord and the whole weight of the frame.

Then the weight of the frame equals $n\,\delta\,\text{KL}$

and $\text{V} = \frac{1}{2} n\,\delta\,\text{KL}$

Fig 10




Let a section be made at the middle of the frame, and taking A as the origin of moments and we have
$\frac{1}{2}\, n\delta\text{KL} \times \frac{1}{4}\text{L}$ = the moment of the frame,
$\frac{1}{8} w\text{L}^2$ = the moment of the surcharge.

Hence the equation

$$\text{HD} = \tfrac{1}{8} n\delta\text{KL}^2 + \tfrac{1}{8} w\text{L}^2$$

$$\therefore \text{H} = \frac{n\delta\text{KL}^2 + w\text{L}^2}{8\text{D}} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

But this horizontal force must be resisted by the material. If C be the resistance to crushing of a unit of section, we have

$$\text{CK} = \text{H} = \frac{n\delta\text{KL}^2 + w\text{L}^2}{8\text{D}}$$

$$\therefore \text{K} = \frac{w\text{L}^2}{8\text{CD} - n\delta\text{L}^2} \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

For cast iron $\delta = 0{\cdot}25$ lbs. per cubic inch,
for wrought iron $\delta = 0{\cdot}27$ lbs. per cubic inch,
for wood $\delta = 0{\cdot}03$ lbs. per cubic inch,

and $n$ may be 3, 4, 5, or 6, according to the nature of the structure.

Although the weight of the frame may be considered an uniform load, yet it produces very different strains in different structures. For instance, if the upper chord be a parabola, so much of the chord as is really an uniform load produces no strains upon the ties and braces; while the lower chord and ties strain the ties and braces.

If the chords are parallel they produce the same or *very nearly* the same, strains as an uniform surcharge. It is *safe* to consider the effect in most practical cases the same as for an uniform surcharge, and I shall so consider it hereafter.

Although the load may be continuous, yet it is really supported at the joints where the ties and braces are connected with the chords; and the same effect would be produced upon the trussing if the load were divided into as many equal weights as there are bays, and each weight were supported directly by the ties and braces. This hypothesis will enable us to slightly modify the preceding equations. They may also be modified for many forms of trusses so as to be more convenient for computation. These modifications will form the subject of the next article.

(To be Continued.)

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## MECHANICS, PHYSICS, AND CHEMISTRY.

### *Note on the Variations of Density produced by Heat in Mineral Substances.* By Dr. T. L. Phipson, F.C.S.

From the London Chemical News, No. 235.

That any mineral substance, whether crystallized or not, should diminish in density by the action of heat, might be looked upon as a natural consequence of dilation being produced in every case and becoming permanent. Such diminution of density occurs with idocrase, Labradorite, feldspar, quartz, amphibole, pyroxene, peridote, Samarskite, porcelain, and glass. But Gadolinite, zircons, and yellow obsidians augment in density from the same cause. This again may be explained by assuming that, under the influence of a powerful heat, these substances undergo some permanent molecular change. But in this note I have to show that this molecular change is not permanent, but intermittent, at least as regards the species I have examined, and probably with all the others. Such researches, while tending to elucidate certain points of chemical geology, may likewise add something to our present knowledge of the modes of action of heat. My experiments were undertaken to prove an interesting fact announced formerly by Magnus—namely, that specimens of idocrase after fusion had diminished considerably in density without undergoing any change of composition: before fusion their specific gravity ranged from 3·349 to 3·45, and after fusion only 2·93 to 2·945. Having lately received specimens of this and other minerals brought from Vesuvius in January last by my friend Henry Rutter, Esq., I determined upon repeating this experiment of Magnus. I found, first, that what he stated for idocrase and for a specimen of reddish-brown garnet was also the case with the whole family of garnets as well as for the minerals of the idocrase groupe; secondly, that it is not necessary to melt the minerals: it is sufficient that they should be heated to redness without fusion, in order to occasion this change of density; thirdly, that the diminished density thus produced by the action of a red heat is not a permanent state, but that the specimens, in the course of a month or less, resume their original specific gravities. These curious results were first obtained by me with a species of lime garnet in small yellowish crystals, exceedingly brilliant and resinous, almost granular, fusing with difficulty to black enamel, accompanied with very little