level and E_c is the bottom of the conduction band.

If the conduction band in CdO has basically the usual parabolic shape modified by the addition of β levels per oxygen vacancy at energy E_c, then η can be calculated from:

$$n = 2\left[\frac{2\pi\,(m^*/m_e)kT}{h^2}\right]^{\frac{3}{2}} F_{\frac{1}{2}}(\eta) + \frac{\beta[V_{ox}^{++}]}{1+\exp(-\eta)}\,, \qquad (3)$$

where $F_{\frac{1}{2}}(\eta)$ is the Fermi-Dirac integral of order $\frac{1}{2}$. A graph of $A \equiv \{\ln[V_{ox}^{++}] + 2\eta\}$ versus $-\ln P$ (calculated from our experimental data at five different annealing temperatures and using $\beta = 2$ and $m^*/m_e = 0.14$) which should have a slope of $\frac{1}{2}$, is shown in fig. 1. Calculations using different values of β and m^* do not remove the discrepancy, apparent in fig. 1, between theory and experimental data; at low carrier concentration the density of states around E_c is larger than indicated by eq. (3), whereas for large n the theory overestimates the number of states actually present.

Since diffusion experiments [3] support the validity of the basic defect reaction (1), we conclude that the conduction band in oxygen-deficient CdO does not have a simple parabolic shape.

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UNIQUE SET OF CRITICAL EXPONENTS AND ORDER-PARAMETER THEORIES FOR 1, 2, 3-DIMENSIONAL SUPERFLUIDS*

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A unique set of critical exponents [$\alpha = \alpha' = 0$; $\beta = \frac{1}{2}$, $\gamma = \gamma' = 1$; $\delta = 3$; $\nu = \nu' = 2/d$; $\eta = \frac{1}{2}(4-d)$] is motivated theoretically and used to determine order-parameter theories for d = 1, 2, 3-dimensional Bose and Fermi superfluids.

In their strongest form, the Widom-Kadanoff [1] scaling laws give seven relations among nine critical exponents. Recently, several additional relations have been proposed [2-4] for the critical exponents of Bose (Fermi) superfluids in the bulk. These additional relations are based on an expansion of the free energy functional in terms of the complex order parameter ψ and its gradient $\nabla\psi$, i.e., $f = f_0 + f\{\psi\} + f\{\nabla\psi\}$. Here $|\psi|^2$ is proportional to the particle (pair) *condensate* density [3], not the superfluid density. For convenience, define a $\delta = 2n-1$ order-parameter theory as one based on a free energy $f\{\psi\}$ expanded to $|\psi|^{2n}$ (n is taken as an integer), and define all critical exponents as in ref. 1.

The purpose of this note is to provide, within this order-parameter framework, a theory for d-dimensional superfluids near the transition, which agrees with the critical exponents observed experimentally and/or predicted microscopically, and is not overrrun by large fluctuations in the *amplitude* and *phase* of the order parameter. In the process, we extend the additional relations [2,3] for the bulk superfluid to smaller dimensional systems. The collection of *all* the relations

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for critical exponents infers, without any input of experimental exponents, a *unique* set of exponents and a *unique* form of order-parameter theories for d-dimensional superconductors and superfluid helium.

Based on the success of the classical Ginzburg-Landau theory, we consider a $\delta = 3$ order-parameter theory. The conditions [2] of non-divergent amplitude fluctuations and a finite specific-heat jump at the transition give $\zeta \geqslant \frac{2}{3}$, $d = 3$; $\zeta = 0$, $d = 2$, $\zeta \leqslant -2$, $d = 1$. The thermodynamic restriction that fluctuations must increase the entropy yields $\zeta \leqslant \frac{2}{3}$, $d = 3$; $\alpha' = 0$, $d = 2$; $\zeta \geqslant -2$, $d = 1$. Combining these results with the relation [1,3], $\zeta = (2-\alpha')(d-2)/d$, which is derived solely from considerations on $f\{\nabla\psi\}$, we find:

$$\alpha' = 0; \quad \delta = 3; \quad \zeta = 2(d-2)/d; \quad d = 1, 2, 3. \tag{1}$$

Note that relation (1) also gives rise to non-divergent phase fluctuations, in the sense that the factor due to phase fluctuations [5] in the order-parameter correlation function is temperature independent when evaluated at the amplitude coherence length. Hence a local order parameter can be defined right up to the transition. The collection of relation (1) and the scaling laws determines uniquely the following set of critical exponents:

$$\begin{gathered}\alpha = \alpha' = 0; \; \beta = \tfrac{1}{2}; \; \gamma = \gamma' = 1; \; \delta = 3; \\ \nu = \nu' = 2/d; \; \eta = \tfrac{1}{2}(4-d); \; \zeta = 2(d-2)/d;\end{gathered} \tag{2}$$

or $x = 3d/4$, $y = d/2$, where x and y are defined in ref. 1.

For *bulk* superfluid helium, the set (2) appears to be valid as $\alpha = \alpha' = 0$, $\zeta = \frac{2}{3}$ agree with experiments [1] and $\beta = \frac{1}{2}$, $\eta = \frac{1}{2}$ with microscopic theories [6]. We now compare (2) with the recent order-parameter theories [7-9]. A $\delta = 5$ theory [8] cannot be excluded on the basis of (2) since $\delta = 3$ was assumed. However, the combination of the relation [4], $(1+\gamma)(2-\eta) = 3\gamma$, and the scaling laws gives the unique set for $\delta = 5$: $\alpha = \alpha' = -1$; $\beta = \frac{1}{2}$; $\gamma = \gamma' = 2$; $\delta = 5$; $\nu = \nu' = 1$; $\eta = 0$; $\zeta = 1$; which is contrary to experiments. Furthermore, the fact that $\beta = \frac{1}{2}$ in both the $\delta = 3$ and $\delta = 5$ sets rules out the superfluid density as the order-parameter [7,8].

The order-parameter theory of ref. 9, which satisfies (2) for $d = 3$, can now be extended to smaller dimensions:

$$-A_d \frac{\hbar^2}{2m}\left|\frac{\tau}{\tau_{cd}}\right|^a \nabla^2\psi + B_d\tau\psi + C_d|\psi|^2\psi = 0\,, \tag{3}$$

where $a = -(4-d)/d$; A_d, B_d, C_d are phenomenological constants; $\tau = T/T_c - 1$; T_c is the transition temperature; and τ_{cd} is the Ginzburg critical range for d-dimensional superfluids. As an application of eq. (3), we calculate the critical superfluid velocity v_{sc} due to thermally activated fluctuations and find $v_{sc} \sim |\tau|^{\frac{2}{3}(4-d)}$ of $2(4-d)/3$. Hence at the transition of a thin helium film, we expect ρ_s to be a constant [10,11] and $v_{sc} \sim |\tau|^{1/3}$ to vanish [10]. Furthermore, we estimate the depletion of $\rho_s(d{=}2)$ due to phase fluctuations and obtain $\rho_s(d{=}2) \approx 0.4\, \rho_s(d{=}3)$, which agrees with experiments [11].

For the superconducting transition, the classical Ginzburg-Landau theory does not satisfy (2) and must be modified in the critical range $(\tau < \tau_{cd})$ to give eq. (3). For bulk superconductors, we ascertain that the penetration length diverges, $\lambda \sim |\tau|^{-1/3}$, the Ginzburg-Landau parameters vanish, $\kappa_1 \sim \kappa_2 \sim |\tau|^{1/3}$, and $H_{c3}/H_{c2} \sim |\tau|^{1/3}$. Hence, sheath and type II superconductivity both vanish in the critical range because of large fluctuations. Extending the theory to dynamic properties, we find that the conductivity diverges in the critical range above T_c as τ^a, $a = -(4-d)/d$, which was previously obtained [12] assuming $\alpha = 0$. Details will be presented elsewhere.

Finally, we emphasize that *exact* long-range order does not exist in one and two dimensions [5] and that application of eq. (3) to $d = 1, 2$ superfluids should be made with care.

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