INTERFEROMETRIC RECONSTRUCTION OF PHASE OBJECTS USING DIFFUSE 'CODING' AND TWO HOLOGRAMS

G. W. STROKE and A. E. LABEYRIE
The University of Michigan, Ann Arbor, Michigan

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In contrast with previously described multiply-exposed *single* holograms, used for interferometry according to a principle first described by Gabor, Stroke, Restrick, Funkhouser and Brumm, interferometric image 'coding' and 'decoding' may be accomplished in diffuse light with *two* separately recorded holograms.

The possibility of adding the complex amplitudes of the electric field vectors in two wavefronts by successively double-exposing a *single* hologram was first suggested by Gabor, Stroke, Restrick, Funkhouser and Brumm [1], and demonstrated by them for an 'image-synthesis' application. Several interferometric applications (e.g. of the two-beam type) suggested by that work have also been demonstrated in a number of cases [2-4]. Another early example of a multiple-exposed hologram was described by Stroke and Falconer [5] in connection with 'multi-directional illumination and moving scatterers'. In these and other comparable experiments, *single* holograms were used for interferometric and other purposes, with or without 'double' or multiple exposure.

It is of a particular interest to show that complex amplitudes in two wavefronts may be interferometrically added by means of *two separate* holograms, recorded independently in two separate exposures. The interferometric addition may be performed, for example, by placing the two holograms next to each other, with the emulsions in close 'contact' and suitably aligned during the reconstruction process. It is also of interest to demonstrate that an interferometrically 'decoded' image may be obtained, with the aid of a suitable 'decoding' hologram H_2 in a case where the first hologram H_1 of a phase object, of interest, has been recorded by illuminating the object with a randomly diffused wavefront, or indeed by letting the hologram 'look' at the phase object *through* a diffusing glass (or other diffusor) placed next to it.

The mathematical theory of the image-reconstruction obtained with the two 'adjacent' holograms (in physical or equivalent optical contact) is comparable to the theory given in our ref. 1, for the general case, and in our ref. 2 for a special case, for *single* multiply-exposed holograms, provided that it is understood that the intensities in the two 'partial holograms' added in the *single* holograms of our ref. 1 and 2 are

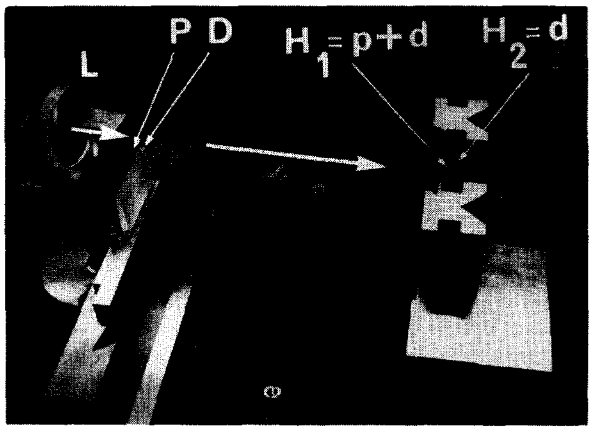

Fig. 1. Holographic interferometry arrangement used both in the recording and in the reconstruction. P = phase object (see reconstructed image in fig. 2), D = diffusor, L = collimating lens, H_1, H_2 = holograms. The recording of H_1 is obtained by interference of the field scattered by P+D with the coherent reference background produced by L. The recording of H_2 is obtained by interference of the reference background with the field scattered by D alone (with P removed): a clear glass plate of thickness comparable to H_1 was placed in front of H_2 during the recording, for compensation (H_1 was recorded through its glass support, a Kodak 649 F microflat plate). The white arrows show the light path used to illuminate H_1+H_2 in the reconstruction (P and D are *not* used in the reconstruction; p+d and d, in H_1 and H_2 are symbolical). The 75 mm length of D gives the scale of the arrangement.

now simply added by means of the *two* separately recorded holograms, placed in close contact with each other. Let a_R be the complex amplitude distribution produced on the hologram H_1 by a reference source R (situated in the focal plane of the lens L in fig. 1), and let a_{p+d} be the complex amplitude distribution produced on H_1 by the arrangement object P + diffusor D, where the field scattered by the phase object P may actually be recorded *through* the diffusor, as in fig. 1. (For the theory of holography with diffusely illuminated phase objects, see e.g. refs. 5 and 6). Let a_d be the complex amplitude produced in the plane of the holograms by the diffusor D alone (with the object P removed). The intensity recorded in the first hologram H_1 is $I_1 = a_R^2 + a_{p+d}^2 + a_R a_{p+d}^* + a_R^* a_{p+d}$, and the intensity recorded in the second hologram H_2 is $I_2 = a_R^2 + a_d^2 + a_R a_d^* + a_R^* a_d$. In the reconstruction process, the two holograms are placed carefully aligned, with the emulsions (i.e. I_1 and I_2) in contact. The complex amplitude of the wave transmitted through H_1 and H_2 in the two separated image-forming side-band terms [6] is $(H_{1+2})_+ = a_R(a_{p+d}^* + a_d^*)$ and $(H_{1+2})_- = a_R^*(a_{p+d} + a_d)$, which will give by holographic (Fourier or Fresnel [6]) transformation complex-amplitude distributions in the side-band images proportional to $(A_{p+d}^* + A_d^*)$ and $(A_{p+d} + A_d)$, where $A_{p+d} = A_d A_p$, and A_p respectively A_d are the complex amplitude distributions in the waves transmitted through the phase object P, respectively the diffusor D, when they are illuminated by a plane spatially coherent monochromatic wave (here 6328 Å laser light). It immediately follows that the complex amplitudes in the reconstructed images will be proportional to $(A_p^* + 1)$, respectively $(A_p + 1)$, showing that an interferometrically decoded image has indeed been reconstructed. Fig. 2 shows an interferometrically decoded image of a phase object letter "φ" and word "phase") obtained according to this principle, in the arrangement of fig. 1.

Physically, the complex addition obtained with two holograms may be understood, also, by noting that the complex addition results from a superposition of the image-forming wavefront, reconstructed by the first hologram H_1, on the image-forming wavefront reconstructed by the second hologram H_2. In a sense, our two-hologram holography arrangement may be considered as an interferometric 'image decoding' arrangement, in which a 'coded' hologram H_1 of a phase object P may be made to produce a 'decoded' image with the aid of a separately recorded 'decoding' hologram. Clearly, with a phase object having no steep phase gradients (compared to the gradients in the diffusor), the coded hologram H_1 will not be capable of revealing its 'key' (produced by the diffusor) by any holographic or interferometric process. The second hologram H_2, with the recording of the 'key' is required for the reconstruction of the image of the phase object.

Fig. 2. Interferometrically decoded reconstructed image obtained with the two "coded" (H_1) and "decoded" (H_2) holograms in fig. 1. A similar interferogram can be obtained by double-exposure of a *single* hologram (with or without the use of the coding diffusor), when no decoding by a second hologram is of interest.

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