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ON THE APPEARANCE OF MACH BANDS IN GRADIENTS OF VARYING COLOR

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INTRODUCTION

THE DARK and light lines called Mach bands which appear in an illuminated field wherever the second derivative of luminance with respect to position is perceptibly large have been the subject of much investigation (see RATLIFF, 1965). One aspect of Mach bands which has been somewhat controversial is the effect of color. While there seems to be general agreement that the appearance of the bands is independent of the color of the inducing field, there have been various conflicting reports about whether analogous bands are produced by a gradient in color rather than in luminance (FRY, 1948; ERCOLES-GUZZONI and FIORENTINI, 1958; DAW, 1964; VAN DER HORST and BOUMAN, 1967; JACOBSON and MACKINNON, 1969).

The results of experiments on chromatic Mach bands are of particular importance because reported failure to find such effects has been interpreted as indicating that lateral inhibitory effects do not occur in the color mediating channels (VAN DER HORST and BOUMAN, 1967; VAN DER HORST, DEWEERT and BOUMAN, 1967). This conclusion directly contradicts current explanations for simultaneous color contrast phenomena (ALPERN, 1964; JAMESON and HURVICH, 1964; CORNSWEET, 1970). In addition, failure to find chromatic Mach bands is surprising in view of the findings from experiments designed to explore the spatial interactions occurring within the isolated color receptive systems of the eye by determining thresholds for patterns of one color against a bright background of another color. In these experiments the color of the background field is selected to adapt two of the color mechanisms and the dominant wavelength of the test pattern is selected to stimulate preferentially the other remaining color mechanism so that only one color receptive system can detect the stimulus. Several different experimental designs utilizing chromatic adaptation indicate that lateral interactions are based in part on selective changes that occur within each of the three cone mechanisms (ALPERN and RUSHTON, 1965; MATTHEWS, 1967; GREEN, 1968; MCKEE and WESTHEIMER, 1970).

It thus seemed to us that further investigation into Mach bands in color was required. To do this we have interlaced a red with a green triangular-wave grating distribution of luminance. With each of these patterns alone light and dark Mach bands appear at the peaks and troughs of the intensity distribution. A color gradient with no luminance gradient was created by arranging the positions of the two equiluminous Mach band inducing patterns so each bright colored band was overlaid with a dark band from the other colored field. It was our initial intent simply to adjust the two fields until they were equally bright and then see if

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Mach bands were produced. It quickly became apparent that no matter how we adjusted the intensities of the two fields the composite always appeared to contain prominent red and/or green bands. The problem was, however, to be sure that these were not due to some residual brightness unbalance. Therefore, the approach which follows has been used. The luminances of the two fields were equated using heterochromatic brightness matching. Rather than simply having observers look at the color gradient and describe what they saw we have attempted to have them quantify their sensations using a modification of the technique used by LOWRY and DE PALMA (1961). This was done by placing a narrow slit of light just below the interlaced colored patterns. The subject adjusted the brightness and color of the slit until it matched that of the gradient immediately above. The slit was then moved and a new reading was taken on a different part of the pattern. By having the observer report the appearance of the pattern by adjusting the color and brightness of a comparison slit we were able to obtain quantitative data suitable for comparing the differences between the appearance of gradients of luminance and gradients of color.

METHODS

Figure 1 shows a diagram of the apparatus used. Vertical triangular-wave or square-wave grating targets were generated on each of two cathode-ray oscilloscope tubes (CRT_1 and CRT_2) using the methods described by CAMPBELL and GREEN (1965). That is, the vertical sweep was derived from a high frequency oscillator with a triangular output. The horizontal sweep was driven from an oscilloscope time base at a frequency of 50 c/s or higher. The grating patterns were formed by applying voltages to the brightness control grids of the cathode-ray tubes. The response of the *CRT*'s is nearly linear. Deviations from linearity are of the order of only a few per cent for 60 per cent modulated patterns used in these experiments.

One *CRT* had a green phosphor (P31) which was used with a No. 3 Kodak Wratten filter to produce an output with dominant wavelength of about 540 nm. The other *CRT* had a red phosphor (P22R) which when used in conjunction with a No. 32 Kodak Wratten filter produced a monochromatic output at a wavelength of 619 nm. The patterns produced on the two oscilloscopes were optically combined by means of a beam splitting cube (BC_1). Two colored lights which were metermeric to the *CRT* outputs were obtained by filtering two tungsten light sources with a No. 26 Kodak Wratten filter (red) and a No. 57A Kodak Wratten filter (green). The outputs from the two tungsten sources (*S*) were imaged onto a diffusing screen behind a narrow

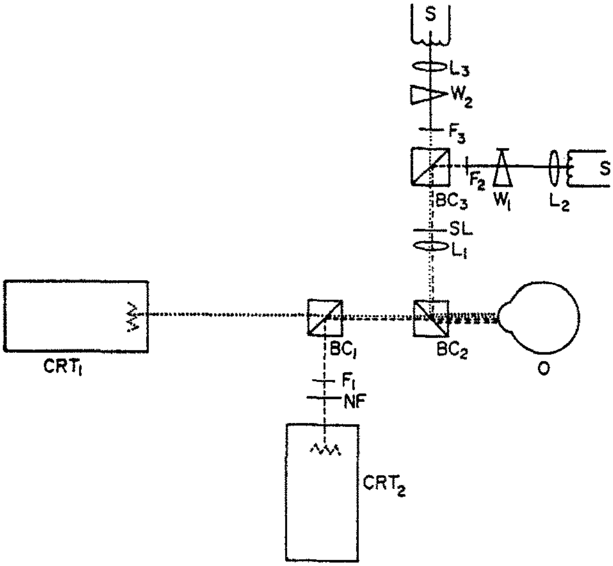

FIG. 1. Diagram of apparatus.

slit (*SL*) by means of lenses (L_2,L_3) and a beam splitting cube (BC_3). Two neutral density wedges (W_1, W_2) allowed the brightness of each component in the mixture to be varied independently. The illuminated slit was viewed through a quality photographic objective (L_1). The slit and *CRT*'s were viewed simultaneously through a beamsplitter cube (BC_2). The position of the slit behind L_1 and the distance of the *CRT*'s from the observer were adjusted so that they all appeared at the same optical distance (114 cm). The slit subtended a visual angle of 4 min of arc and one cycle of the grating patterns occupied 2° of visual angle. A system of mechanical screws controlling the horizontal position of the slit allowed its position to be varied with respect to the patterns above it.

The subject's head was firmly held by a bite bar. His pupil was dilated by instilling a drop or two of 0·5% Mydriacyl (tropicamide) into the right eye. The targets were viewed through a 4 mm artificial pupil and spectacle lens correction. Both of the subjects whose results are reported here had normal color vision.

RESULTS

In the first set of experiments alternating dark and light stripes of equal angular subtense (1°) were produced on each *CRT*. The *CRT*'s were positioned so that the square wave grating patterns were 180° out-of-phase. That is, a dark stripe in one field fell on a bright stripe in the other field. The intensity of the green field was adjusted by the observer to produce an equiluminous pattern of alternating red and green stripes. The luminance of the field was measured by brightness matching to the white field of an SEI photometer to be 6 cd/m². By presenting each of the colored fields separately and positioning a slit of the same color just below selected portions of the square-wave grating the luminances of both the dark and bright portions of each pattern were determined by having the subject adjust the brightness of the slit to match that of the middle of the bar above it. By this procedure the contrast, *C*, defined in the usual manner using the relationship that

$$C = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

was determined to be about 0·6.

The square-wave grating was then replaced with a triangular grating of equal amplitude and the same mean luminance. That is, each uniformly illuminated stripe was replaced by a uniformly increasing gradient and each dark stripe by a uniformly decreasing gradient (see Fig. 2). As before a slit of light of the same color as the pattern was positioned below the grating. The upper edge of the illuminated slit just touched the lower edge of the triangular grating. In early experiments using a slit of detectable width the subjects were distracted by the fact that in attempting to make matches they could use one of two criteria. Either they could attempt to make the boundary between the slit and the pattern disappear or they could make brightness matches. This difficulty was overcome by decreasing the width of the measuring slit. Then, in general, when a match was achieved there was no apparent discontinuity and the slit appeared to be an extension of the pattern. In all measurements, other than those used to equate the contrast of the two displays, the narrower slit was used with both the triangular and square-wave gratings.

Red and green triangular and square-wave gratings were separately presented and the subject adjusted a red or green measuring line until its brightness matched the brightness of the portion of the pattern directly above the slit. Subjects had little difficulty in making these matches except when the slit was positioned immediately below the dark Mach band. Here, the Mach band above appeared dark, but not black. Yet the luminous slit below never had quite the same quality as the dark Mach band no matter how the intensity of the slit was varied. If the subject was instructed to carry-on and to ignore the differences he was able to then make repeatable settings. This "match" setting was determined not so much by making

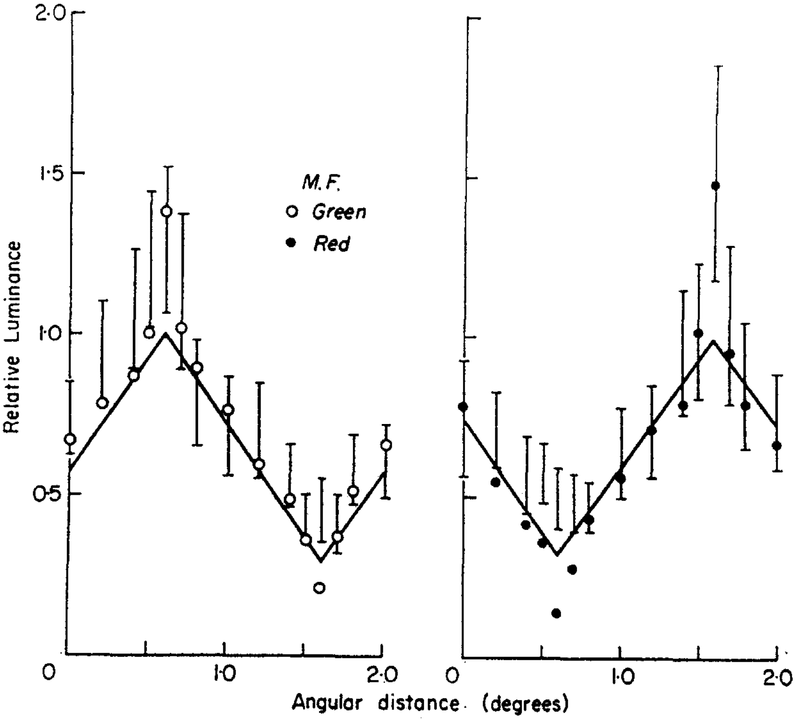

Fig. 2. Relative luminance of comparison lights that match triangular gratings. The ordinate shows the intensity of the matching field relative to the intensity that matched the bright portion of a square wave grating of the same mean luminance and contrast. The open circles to the left are for the green triangular grating presented alone. The closed circles to the right are for the red triangular grating. The vertical lines to the left and right are for interlaced green and red triangular gratings. These lines give the relative luminance of the green and red matching lights which in combination matched the chromatic gradient. The length of the lines defines the limits for an acceptable match. For further details see text.

things appear identical but more by the process of noting when the slit was clearly too bright or clearly too dim and then setting the intensity in the slit between these two extremes. The use of this kind of psychophysics and only two subjects means that the exact values of the settings probably should not be given much weight. The settings do prove that a dark band exists, even though its magnitude is not clear.

The results of the complete matching procedure determined at several points along each triangular-wave grating are shown for one subject in Fig. 2. The solid lines in Fig. 2 plot the physical intensity distribution inferred from matches made to the square-wave gratings of the same contrast. These lines connect the experimentally determined matches of the comparison slit to the bright and the dark bar of the square-wave grating. In addition, all the settings have been normalized so that the match between the slit and the bright bar of the grating has value one. The matches determined with the green triangular grating are shown as open circles and those for the red triangular grating as closed circles. The matches to both triangular patterns were brighter at the peak than the bright bar of the square-wave and dimmer at the trough than the dark bar of the square-wave. These data are in good agreement with subjects' verbal reports of the appearance of triangular gratings. All observers reported seeing sharply delineated dark and light lines in both colored fields. There was some disagreement among observers about whether the bands in both fields were equally prominent. Interestingly, subject M.F.'s matching confirmed his subjective impression that the red Mach bands were slightly more apparent than the green.

Next, the equiluminous red and green triangular gratings were presented simultaneously. The gratings were optically combined in antiphase so as to create a chromatic gradient. Proceeding as before, the subject saw the interlaced colored patterns with a slit below which was illuminated by variable amounts of the red and green lights used to match the patterns presented separately. We had hoped that a slit illuminated with the combination of lights that matched the separate gratings would also match the combined gratings. This would have provided an elegant demonstration that Mach bands were produced in gradients of color. Unfortunately, it quickly became apparent that matters were not this simple and that combining the gratings and matching lights in this way did not always produce a satisfactory match. We therefore have made detailed measurements directed at quantifying the appearance of the chromatic gradient.

The observer's task was to adjust the contributions from the two colored sources until both the intensity and color of the slit matched that of the pattern immediately above. When a satisfactory match had been attained, the contribution of one of the colored lights was increased and then decreased until the match was no longer satisfactory in either brightness and/or color. The neutral density wedge controlling the light was set at the midpoint between

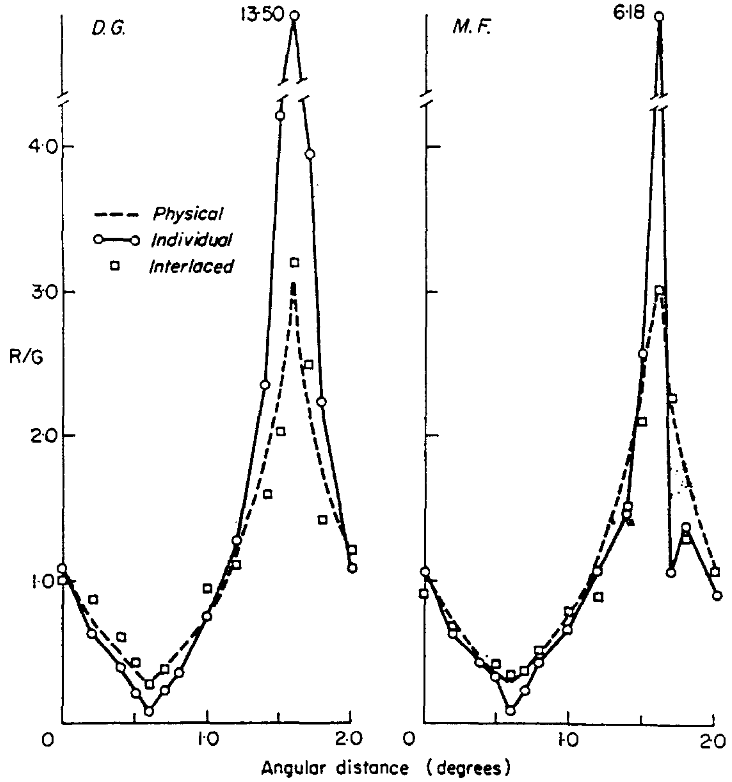

Fig. 3. Relative chromaticity of matches to chromatic gradient. The symbols show for two subjects the ratio of the red to the green components in the matching slit. The solid curves show the *R/G* ratio to be expected from complete independence and the dashed lines show the chromaticity expected from the physical amounts of red and green in the pattern.

these two extremes and the limits of the acceptable match were determined by varying the other light. The results of this procedure are indicated in Fig. 2 where the bars plot the range of acceptable matches. As before, the settings are plotted on a scale which has been normalized by the matches made to the interlaced square-wave gratings. Since it made no difference with the square-waves whether the matches were determined with interlaced or separate patterns the same normalizing factor has been used in both parts of the experiment. The limits were determined by averaging four individual settings determined at two separate experimental sessions. The bars on the left in the figure indicate for several positions along the grating the range between too much green and too little green and the bars to the right indicate the range between too much red and too little red. These matches show that the combined pattern contains slightly less prominent but still readily evident red and green stripes at the locations of the Mach bands in the individual patterns.

By analogy with the apparent dark and light bands produced by an inflection in luminance gradient one expects to see bands of accentuated redness or greenness in the abruptly changing gradient of color. To characterize the chromaticity of the perceived pattern the ratio of the red to green components in the matching slit has been plotted in Fig. 3 for two subjects. For comparison two theoretical curves are shown. The solid curves show the red/green ratio to be expected from complete independence; that is, if the matches for the individual patterns were, in combination, matches to the superimposed patterns (Grassmann's law of additivity for cross-context matches). The other curves (dashed lines) are the matches to be expected from the physical amounts of red and green at each position. The squares in Fig. 3 are the measured red/green variations calculated from data such as that of Fig. 2 by averaging the two limits for an acceptable match. There appears to be no significant difference between the measured red/green ratios and those to be expected from the physical amounts of red and green light in the chromatic gradient.

DISCUSSION

There are several reasons for believing lateral interactions which probably cause Mach bands can occur within the three color systems of trichromatic vision. These reasons range from the simple observation that Mach bands can be produced by colored intensity gradients to elaborate extensions of the Stiles two-color threshold technique. MATTHEWS (1967), for example, has shown that Mach bands occur within each of the color mechanisms by determining increment thresholds for a red, green, or blue Mach band inducing field superimposed on a background of a complementary color. In two further applications of the two-color approach GREEN (1968) has shown that the sine contrast sensitivity for the separate color mechanisms shows the usual low frequency decrease in sensitivity, and WESTHEIMER (1970) has shown that the inhibitory effects of varying the size of an adapting spot occur within the individual color systems.

If one accepts the usual explanation that Mach bands arise from the fact that the receptive fields of cells within the human visual system have excitatory areas surrounded by zones which are inhibitory, then it may be meaningful to examine the kinds of spatial organization required of color sensitive cells to produce the reported results. The question of whether chromatic Mach bands exist depends on whether lateral interactions occur before or after color information is coded in the nervous system. For example, suppose that at some place in the visual system one finds two kinds of channels, one of which relays brightness signals and the other, color signals. The brightness signals might be expected to arise from some additive combination of signals from the various receptors. Color signals might be formed by encoding the relative photon catch in the three kinds of cones. In the above system, color Mach bands would not be produced in chromatic gradients if the lateral interactions occurred only in brightness channels. This kind of simple model is not consistent with our finding that luminous bands are produced by the chromatic gradient. In addition, these notions leave simultaneous color contrast phenomena completely unexplained. Moreover, there is clear evidence from single cell recordings showing lateral interactions between signals from different types of cones (WIESEL and HUBEL, 1966: DAW, 1967; GOURAS, 1968).

If the only lateral inhibitory interactions were between signals arising from like kinds of cones, then one would expect that in our experiments a region having a negative spatial second derivative in color, but not luminance, would appear more intensely colored than a corresponding uniform field. In our experiments the shifts should have been in the direction of making the red areas appear redder and the green areas greener. This was not found. On the other hand our data clearly show a Mach band type effect in a light distribution of constant luminance and varying chromaticity. It is worth noting that these bands cannot be due to a failure to equate the red and green triangular grating in luminance. Errors in equating the brightness of the two colored field might be expected to produce a bright band at the positions corresponding to the peak of the more luminant grating but, by the same arguments, a dark band should appear at the troughs in this grating, i.e. the peaks of the other field. Since bright bands occur at both positions the above cannot be the explanation. One might also wonder whether these bands might not be caused by small deviations from linearity in the intensity gradients generated on the oscilloscope. This is unlikely to be the cause for several reasons. First, the deviations from linearity are in the wrong direction to produce Mach bands. The response of the oscilloscope to a triangular modulating voltage tends, if anything, to fall below the input at the peaks of the waveform. Second, the luminances of the triangular waveforms were determined from matches made to squarewaves of the same peak luminance and, therefore, distortions in the relationship between oscilloscope modulation voltage and light ouput should not produce bands more luminant than the bright bars of the square wave grating.

In trying to understand the above results we have thought of two relatively simple schemes that seem potentially capable of fitting our findings. First, at the earliest stages of processing the lateral interactions could be cone specific, in the sense outlined above. In addition, at some later stage there might be cross-coupling so that an increase in the signal coding redness causes a concomitant increase in the signal for green. A second alternative is that in addition to selective interactions there might be a spatial color opponent system. With such a system a negative flection in color could locally cause an apparent color shift in opposition to those produced by selective interactions. To see this, consider a hypothetical cell, having an excitatory center connected to red cones and an opponent surround connected to green cones positioned at the red peak in the interlaced red and green triangular waveforms. Since to each side of the peak the relative amount of green in the stimulus is increasing, the cell would be relatively inhibited. If these kinds of cells were signalling color, one might expect the position corresponding to the peak of the red triangular grating and the trough of the green grating to appear relatively greener. It would seem then that the presence in the human visual system of a combination of cells with chromatic spatial antagonisms and cells with selective lateral interactions would provide a possible basis for understanding the present results. The postulated existence of both types of spatial interaction seems to be consistent with the kinds of color-coded cells found in the primate visual system (WIESEL and

Hubel, 1966; Gouras, 1968; Hubel and Wiesel, 1968). The problem, however, for any such theory is to provide a system of interactions which produces simultaneous color contrast, the color shifts found in stepwise changes in chromaticity (Daw, 1964; Jacobson and MacKinnon, 1969), and the failure to find the chromatic analog of Mach bands in continuous gradients of color (Fry, 1948; Ercoles-Guzzoni and Fiorentini, 1958; Van der Horst and Bouman, 1967; and the present study).

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Abstract—Red and green triangular-wave intensity distributions were generated on cathode-ray oscilloscope tubes. When these patterns are viewed separately, light and dark Mach bands appear at the peaks and troughs of the intensity distribution. The perceived brightness distribution was quantified by matching the brightness of the pattern with a narrow slit of light of the same color positioned in varying positions just below the triangular-wave field. The sensations produced by gradients of color rather than luminance have been investigated by interlacing equiluminous red and green triangular gratings 180° out-of-phase (dark bands in one field correspond to bright bands in other field). An illuminated slit just below the chromatic gradients was adjusted in color and brightness to produce satisfactory matches to different parts of the interlaced red and green patterns. Our measurements show that bright red and green bands appear in light distributions of constant luminance and varying chromaticity.

Résumé—On engendre sur des écrans d'oscillographe cathodique des distributions rouge et verte dont l'intensité est une onde triangulaire. Quand ces figures sont vues séparément, des bandes de Mach lumineuses et sombres apparaissent aux pointes et aux creux de la distribution d'intensité. On évalue quantitativement la distribution perçue de luminosité en égalisant la luminosité de la figure avec celle d'une fente étroite de lumière de la même couleur placée dans des positions variées juste au-dessous du champ à onde triangulaire. On étudie les sensations produites par des gradients de couleur plutôt que de luminance en entrelaçant des réseaux triangulaires rouge et vert de même luminosité et en opposition de phase (les bandes obscures d'un champ correspondant aux bandes brillantes de l'autre). On réglait en couleur et luminosité une fente lumineuse juste en-dessous des gradients chromatiques, jusqu'à égalisation satisfaisante avec les diverses parties des distributions entrelacées rouge et verte. On trouve que des bandes brillantes vertes et rouges apparaissent dans les distributions de lumière de luminance constante et chromaticité variable.

Zusammenfassung—Rote und grüne dreieckförmige Intensitätsverteilungen wurden mit Kathodenstrahl-Oszillographenröhren erzeugt. Wenn die Testmuster getrennt gesehen wurden, erschienen helle und dunkle Machbänder im Maximum und Minimum der Intensitätsverteilung. Die wahrgenommene Helligkeitsverteilung wurde durch Angleichen der Helligkeit eines schmalen Lichtspalts der gleichen Farbe in verschiedenen Positionen genau unter dem Dreieckgitter meßbar gemacht. Die Empfindungen, mehr durch Farbgradienten als durch Leuchtdichte hervorgerufen, sind bei gekreuzten roten and grünen Dreieck-Gittern gleicher Leuchtdichte bei um 180° verschobener Phasen untersucht worden (dunkle Bänder in einem Feld entsprechen hellen Bändern im anderen Feld). Ein beleuchteter Spalt genau unter den chromatischen Gradienten wurde in Farbe und Helligkeit angeglichen. Messungen an verschiedenen Stellen der gekreuzten roten und grünen Teste zeigen, daß helle rote und grüne Bänder bezüglich der Lichtverteilung konstant in der Leuchtdichte und in variierender Farbigkeit erscheinen.

Резюме—На экране катодно-лучевого осциллоскопа генерировались изменяющиеся по интенсивности красные и зеленые треугольные волны. Когда эти объекты были видимы по отдельности, светлые и темные полосы Маха наблюдались на пиках и впадинах распределения интенсивности. Воспринимаемое распределение светлот было определено количественно путем сравнения этого объекта с узкой щелью света того же самого цвета, находящейся в разных позициях чуть ниже "треугольно-волнового" поля. Ощущения продуцируемые скорее градиентами цвета, чем яркостью, были исследованы путем размещения равнояркой красной и зеленой треугольных решеток, в противофазе на 180° (темные полосы в одном поле соответствовали ярким полосам в другом поле). Светящаяся щель, расположенная хуть ниже хроматических градиентов, была отюстирована по цвету и яркости так. что позволяла производитъ удовлетворительное сравнение различных участков сменяющих друг друга красного и зеленого объктов.

Наши измерения показывают, что яркие красные и зеленые полосы возникают при постоянной яркости света и переменной хроматичности.