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LETTERS TO THE EDITOR

DETERMINATION OF MIXING RATIOS FROM THE GAMMA-GAMMA DIRECTIONAL CORRELATIONS IN A TRIPLE CASCADE*

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Formulas for the determination of the three mixing ratios in a triple cascade have been derived and expressed in terms of the directional correlation coefficients of γ_1-γ_2, γ_2-γ_3 and γ_1-γ_3 cascades. The proposed treatment does not employ the usual assumption of neglecting the small mixing in a particular gamma-ray transition.

In the last three decades, gamma–gamma directional correlations have proven to be a powerful tool in determining the spins and parities of nuclear energy levels as well as the multipolarities and mixing ratios of gamma-ray transitions. In the analysis of the correlation data, one usually begins with a double cascade of which one transition is almost pure, e.g. a quadrupole transition. By assuming one transition to be pure, the mixing ratio δ of the other can be determined. However, such an assumption is sometimes unnecessary.

Furthermore, the directional correlation coefficients $A_{kk}(\gamma_i\gamma_j)$ of the cascade chosen according to the above criterion may not be the one that has been measured most accurately. Starting with such a cascade, we will propagate the large error to the mixing ratios in the subsequent cascades. For some nuclei, a triple cascade as shown in fig. 1 exists and the γ_1–γ_2, γ_2–γ_3, and γ_1–γ_3 (intermediate γ-ray unobserved) directional correlations can all be measured with accuracy. In such a case, the mixing ratios of the three transitions can be calculated by using the directional correlation coefficients simultaneously. The assumption of any transition being pure is therefore eliminated.

We outline here the formulas used for the determination of the three mixing ratios δ_1, δ_2 and δ_3. The theory of γ-γ directional correlation has been presented in ref. 1 along with extensive references. There has been some confusion concerning the sign of δ. Dzhelepov et al.[2]) suggested the adoption of a convention in which the sign of δ given by the formulas of Biedenharn and Rose[3]) is reversed if the transition is the second in the cascade and otherwise left unchanged. This is the convention adopted here. The signs of all the mixing ratios in this article have been adjusted so that they can be compared readily with values obtained by other authors. We will also employ the notation used in refs. 1 and 4.

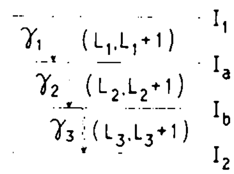

Fig. 1. γ-ray transitions in a triple cascade.

We define Q_ν as the fraction of the $L_\nu+1$ multipole in the νth (mixed) transition,

$$Q_\nu = \frac{\delta_\nu^2}{1+\delta_\nu^2}; \quad 0 \leq Q_\nu \leq 1. \tag{1}$$

Or, one obtains

$$\delta_\nu = \pm\left(\frac{Q_\nu}{1-Q_\nu}\right)^{\frac{1}{2}} + \tag{2}$$

Our procedure begins with a trial value of Q_2 which results in two roots in δ_2. For γ_1–γ_2 cascade, one has

$$A_k(\gamma_2) = (1+\delta_2^2)^{-1}\,[F_k(L_2, L_2, I_b, I_a) - \\ - 2\delta_2 F_k(L_2, L_2+1, I_b, I_a) + \\ + \delta_2^2 F_k(L_2+1, L_2+1, I_b, I_a)], \tag{3}$$

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where $F_k(L_i, L_i', I_j, I_k)$ are the F-coefficients as defined and tabulated by Ferentz and Rosenzweig[5]). Furthermore, one has

$$A_k(\gamma_1) = A_{kk}(\gamma_1\gamma_2)/A_k(\gamma_2), \tag{4}$$

where the normalized coefficients $A_{kk}(\gamma_1\gamma_2)$ can be determined from the least squares fit to the experimental data in the unperturbed angular correlation function $W(\gamma_1\gamma_2;\theta)$ of the form

$$W(\gamma_1\gamma_2;\theta) = 1 + A_{22}(\gamma_1\gamma_2)\,P_2(\cos\theta) + A_{44}(\gamma_1\gamma_2)\,P_4(\cos\theta). \tag{5}$$

For γ_2–γ_3 cascade, one uses

$$A_k'(\gamma_2) = (1+\delta_2^2)^{-1}\,[F_k(L_2, L_2, I_b, I_a) + 2\delta_2 F_k(L_2, L_2+1, I_b, I_a) + \delta_2^2 F_k(L_2+1, L_2+1, I_b, I_a)]. \tag{6}$$

Furthermore, one has

$$A_k(\gamma_3) = A_{kk}(\gamma_2\gamma_3)/A_k'(\gamma_2). \tag{7}$$

The mixing ratio δ_3 is

$$\delta_3 = [A_k(\gamma_3) - F_k(L_3+1, L_3+1, I_2, I_b)]^{-1} \times \{-F_k(L_3, L_3+1, I_2, I_b) \mp [F_k^2(L_3, L_3+1, I_2, I_b) - (A_k(\gamma_3) - F_k(L_3+1, L_3+1, I_2, I_b)) \times (A_k(\gamma_3) - F_k(L_3, L_3, I_2, I_b))]^{\frac{1}{2}}\}. \tag{8}$$

We continue with

$$A_k'(\gamma_3) = (1+\delta_3^2)^{-1}\,[F_k(L_3, L_3, I_2, I_b) + 2\delta_3 F_k(L_3, L_3+1, I_2, I_b) + \delta_3^2 F_k(L_3+1, L_3+1, I_2, I_b)]. \tag{9}$$

Then, we can calculate $U_{kk}(\gamma_2)$ in the unperturbed 1–3 directional correlation function,

$$U_{kk}(\gamma_2) = \frac{A_{kk}(\gamma_1\gamma_3)}{A_k(\gamma_1)\,A_k'(\gamma_3)}. \tag{10}$$

On the other hand, the mixing ratio δ_2 is related to $U_{kk}(\gamma_2)$ and the 6-j symbols by the following expression,

$$\bar{\delta}_2 = \pm\left[(-1)^{I_a+I_b}\,(2I_a+1)^{\frac{1}{2}}\,(2I_b+1)^{\frac{1}{2}}\,(-1)^{L_2} \times \begin{Bmatrix} I_a & I_a & k \\ I_b & I_b & L_2 \end{Bmatrix} - U_{kk}(\gamma_2)\right]^{\frac{1}{2}} \left[U_{kk}(\gamma_2) - (-1)^{I_a+I_b} \times (2I_a+1)^{\frac{1}{2}}\,(2I_b+1)^{\frac{1}{2}}\,(-1)^{L_2+1} \begin{Bmatrix} I_a & I_a & k \\ I_b & I_b & L_2+1 \end{Bmatrix}\right]^{-\frac{1}{2}}, \tag{11}$$

where the symbol $\bar{\delta}_2$ is used for δ_2. Thus, our procedure begins with a trial value for δ_2 and terminates when the value of $\bar{\delta}_2$ calculated via eqs. (3) through (11) is equal to the input value of δ_2. Finally, one obtains

$$\bar{Q}_2 = \frac{\bar{\delta}_2^2}{1+\bar{\delta}_2^2}. \tag{12}$$

Our procedure in determining Q_2 can be represented schematically by fig. 2. The superscript in $\delta_2^{(+)}$ signifies that the positive root of δ_2 in eq. (2) is used as δ_2 in eq. (3), while the superscripts in $\delta_3^{(+-)}$ denote that the value of δ_3 resulted from $\delta_2^{(+)}$ and the choice of the negative sign in eq. (8). The symbol $\bar{Q}_2^{(+-)}$ stands for the value of $\bar{Q}_2$ obtained from $\delta_2^{(+)}$ and $\delta_3^{(+-)}$. For each of the four branches, one can plot the input value Q_2 against the output value $\bar{Q}_2^{(ij)}$ where $i, j = \pm$. An intercept of the plot and the straight line $Q_2 = \bar{Q}_2^{(ij)}$ gives a solution for $\bar{Q}_2^{(ij)}$ which in turn determines a set of solutions for $\delta_2^{(i)}$ and $\delta_3^{(ij)}$. One possible situation which occurs in the triple cascade of ^{175}Lu is shown in fig. 3. Once δ_2 and δ_3 are determined, δ_1 can be

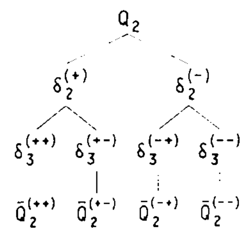

Fig. 2. Schematic diagram showing the procedure in determining Q_2.

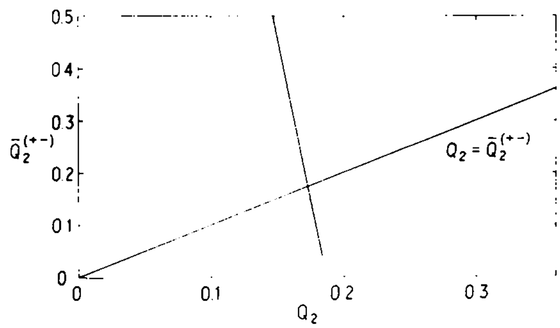

Fig. 3. An intercept in the Q_2-$\bar{Q}_2^{(+-)}$ diagram locates a solution for $\bar{Q}_2^{(+-)}$.

obtained by

$$\delta_1 = [A_k(\gamma_1) - F_k(L_1+1, L_1+1, I_a, I_1)]^{-1} \times$$
$$\times (F_k(L_1, L_1+1, I_a, I_1) \pm \{F_k(L_1, L_1+1, I_a, I_1) -$$
$$-[A_k(\gamma_1) - F_k(L_1+1, L_1+1, I_a, I_1)] [A_k(\gamma_1) -$$
$$-F_k(L_1, L_1, I_a, I_1)]\}^{\frac{1}{2}}). \quad (13)$$

Since in directional correlation experiments, $A_{22}(\gamma_i\gamma_j)$ are usually more accurately determined than $A_{44}(\gamma_i\gamma_j)$, one should start with the former. The latter can be used to rule out some sets of false solutions.

Formulae for the evaluation of uncertainties associated with the mixing ratios δ_1, δ_2 and δ_3, originating from the experimental errors in the directional correlation coefficients will be included in the report to the National Science Foundation.

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