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Extended Producer Responsibility (EPR) and Remanufacturable Product Design

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1 Introduction and Literature Review

Environmental policy-making has recently followed trends of emphasizing preventive measures over end-of-pipe approaches, focusing on environmental performance of products throughout their life-cycles, and favoring goal-oriented and market-based approaches over traditional command-and-control approaches. An excellent example of such policy-making is Extended Producer Responsibility (EPR), which is a goal-oriented approach to improve total life-cycle environmental performance of products (Tojo 2004). The Organization for Economic Cooperation and Development (OECD) defines EPR as an environmental policy approach in which a producer's responsibility for its product is extended to the post-consumer stage of the product's life-cycle. There are two related features of any EPR policy - the shifting of responsibility (physically and/or economically, fully or partially) *upstream* toward the producer and away from municipalities, and, providing incentives to producers to incorporate environmental considerations in the design of their products. A broader definition of EPR according to Lindhqvist (1992) is as follows: *Extended Producer Responsibility is an environmental protection strategy to reach an environmental objective of a decreased total environmental impact from a product, by making the manufacturer of the product responsible for the entire life-cycle of the product and especially for the take-back, recycling and final disposal of the product. Extended Producer Responsibility is implemented through administrative, economic and informative instruments. The composition of these instruments determines the precise form of Extended Producer Responsibility.*

Under conventional waste management practices, local or municipal governments are responsible for the collection and disposal of end-of-life products, and the costs of collection and disposal are generally financed through taxes. EPR shifts financial responsibility for the costs of waste management upstream

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to the beneficiaries of the product, and away from the municipality and the taxpayer. EPR is designed to confront the producer with the costs of product disposal, and to, hence, provide incentives for the producer to take these costs into account in product design and marketing. Under EPR, firms have an incentive to design products that result in lower waste disposal costs or that facilitate material and/or energy recovery. The principle of EPR has been reflected in many waste management policy initiatives, starting with the German *Duales System Deutschland* (DSD), launched by the packaging industry in response to the German Packaging Ordinance of 1991. Since then, EPR policies have been implemented and applied to municipal solid wastes, hazardous wastes, and special wastes from both the residential as well as the commercial sectors.[1] EPR helps realize objectives of sustainable development by reducing waste, reducing the release of potentially toxic chemicals into the environment, reducing the use of virgin material inputs, and reducing energy consumption (OECD 2005).

EPR instruments can be categorized into three broad types - *take-back requirements*, *economic instruments*, and *performance standards*. Take-back requirements assign responsibilities to the beneficiaries of products for end-of-life product management. Economic instruments such as deposit/refund systems, advance disposal fees, and material taxes are incentive-based and provide flexibility in establishing the means to accomplish the EPR target. Performance standards, such as minimum recycled content, can be set to specify a particular percentage of materials to be recovered and reused. To date, OECD member governments have used EPR to stimulate changes in three key priority areas - *resource efficiency*, *cleaner products*, and *waste management*. Examples of EPR instruments from practice include:[2]

i. *Product Take-Back* and/or *Recovery Targets*: Specified Home Appliance Recycling (SHAR) Law in Japan mandating recovery/reuse targets in terms of product weight; Ordinance on Producer Responsibility for Cars in Sweden making manufacturers and importers of cars in Sweden responsible for accepting end-of-life vehicles; German Packaging Ordinance mandating recycling/reuse targets for product packaging; Used Oil/Containers/Filters Industry Management Program of Western Canada for the collection and processing of used oil, oil containers and oil filters.

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[1] EPR programs have thus far been applied to packaging, paint, batteries, electronics, cell phones, tires, used engine oil, appliances, and vehicles.

[2] Sources: US Environmental Protection Agency (*http://www.epa.gov/otaq/mpg.htm*), UK Department for Environment, Food and Rural Affairs (*http://www.defra.gov.uk/environment/climatechange/trading/eu/*), New Zealand Business Council for Sustainable Development (*http://www.nzbcsd.org.nz/story.asp?id=13*), Tojo (2004), OECD (2001), *http://www.publications.parliament.uk/pa/cm199900/cmbills/043/2000043.htm*, *http://www.environment-agency.gov.uk/business/444304/444641/595811/136872/?lang=_e*, *http://www.colby.edu/personal/t/thtieten/Dep.htm*, *http://www.ilsr.org/recycling/epr/tools.html*, *http://www.green-alliance.org.uk*.

ii. *Deposit/Refund Systems:* Deposit/Refund Systems for beverage containers, batteries and tires in certain states in the US; Deposit/Refund Systems for food containers, tires, batteries, lubricants, pesticide containers, and plastics in South Korea.

iii. *Advance Disposal Fees:* Advance disposal fees charged by manufacturers facing the SHAR law to consumers of white goods in Japan; Paint Stewardship Program in British Columbia where "eco-fees" are charged to customers at the point of sale.

iv. *Product Design Standards:* Recycled Content of Newsprint Bill in the UK specifying minimum recycled content for newsprint; European Directive on Packaging and Packaging Waste which limits concentration levels of lead, cadmium, mercury and hexavalent chromium in packaging or packaging components[3]; Corporate Average Fuel Economy (CAFE) standards in the US requiring vehicle manufacturers to comply with fuel economy standards set by the Department of Transportation.

v. *Costs/Charges for Environmental Impact during Product Use*: Costs for Carbon Dioxide emissions incurred by firms in the European Union subject to the EU Greenhouse Gas Emissions Trading Program; Gas Guzzler Tax in the US, imposed on manufacturers on the sale of cars failing to meet fuel economy standards.

vi. *Material Taxes*: "Eco-tax" on PVC in Belgium and Denmark aimed at shifting consumption away from PVC.

vii. *Other Measures*: In addition to the above instruments, a variety of measures have been implemented in practice to complement and support the goals of EPR policies and programs. Examples include eco-labelling, green procurement, and product stewardship.

Thus, it is evident that EPR has been implemented through a variety of instruments across countries. The OECD, in its Guidance Manual for Governments (OECD 2001), describes anticipated actions by producers in direct response to EPR instruments. The principal rationale behind EPR is that manufacturers have the capacity to effect changes at the product and process design stage in order to reduce the environmental impacts of products. However, very little research has been conducted to

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[3] Over thirty-five states in the US have enacted some type of product or packaging restrictions. For example, Rhode Island prohibits non-biodegradable plastic carrier rings, packaging containing potentially toxic heavy metals, metal beverage containers with detachable flip tops, plastic food or beverage containers composed of more than one resin, degradable plastic containers which interfere with recycling, and telephone directory binders which interfere with recycling. See *http://www.ilsr.org/recycling/epr/tools.html*.

evaluate the influence EPR programs have on the design of products and product systems, i.e., upstream changes (Tojo 2004). In a survey-based empirical study covering twenty one manufacturers of Electrical and Electronic Equipment (EEE) and cars in Japan and Sweden subject to EPR legislation, Tojo (2004) concludes that upstream measures, in terms of reduction of hazardous substances and source reduction of material consumption through reuse and recycling, have been undertaken in the two industry sectors in both countries. We paraphrase the survey[4] by Tojo (2004) in Appendix A, to elucidate the nature of EPR legislation in Japan (for EEE) and in Sweden (for automobiles), and the measures taken by manufacturers in response to EPR legislation.

According to the OECD Guidance Manual for Governments (OECD 2001), the application of an EPR instrument to a particular product, product group, or waste stream should take into account the feasibility of steering producer and consumer behavior in a particular direction; the allocation of physical and financial responsibility will affect the applicability of the EPR instrument. It is clear that manufacturers have responded to EPR instruments by changing product designs in order to minimize environmental impacts of products both during use (e.g., through improved energy efficiency) as well as after use (e.g., through design for disassembly and reuse). In other words, manufacturers have responded by taking proactive and preventive upstream measures rather than reactive and topical ones such as treatment of waste. Having empirically described the nature of upstream responses by manufacturers to EPR instruments, we proceed to analytically establish how various implementations of EPR influence upstream actions by a manufacturer producing and selling a remanufacturable product. Specifically, we model two environmental design attributes of a durable, remanufacturable product - a one-dimensional "more is better" measure of environmental performance (such as energy efficiency) q, and a measure of remanufacturability θ (modeled as the fraction of the product that can be recovered after use). q influences the environmental performance of the product during use by the customer while θ influences the end-of-life environmental performance of the product. Thus, q and θ together constitute measures of environmental performance of the product - both during, and at the end of the product's economic life.

Prior research on waste policy instruments and environmentally favorable product design examines economic and social efficiencies of various policy instruments such as taxes, subsidies, standards, combined taxes/subsidies, and take-back. For example, see Calcott & Walls 2002, Eichner & Pethig 2001, Calcott & Walls 2000, Fullerton & Wu 1998, Palmer & Walls 1997, Dinan 1993. The typical objective in this stream of research is for the social planner to maximize net social surplus subject to resource

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[4] Due to the confidential nature of the information obtained and as per requests by interviewees, references to individual companies are not made in the report by Tojo (2004).

constraints, material balance constraints, and production functions. Material input consumption is treated as a surrogate for product design; an environmentally favorable design implies lower material consumption. Customer demand for the manufacturer's product is a function of the quantity produced, product price, income, and quantity of waste disposed. A consistent finding is that a combined tax/subsidy, where there is a tax on the consumption good supply and a subsidy on the demand for material input, can yield the socially optimal product design and quantity of waste. However, Calcott & Walls (2000) argue that the infeasibility of paying households for recycling, and taxing products according to their recyclability implies that the first-best outcome is no longer attainable from a practical standpoint.[5] They set up a constrained second-best optimum and solve for policy instruments that achieve this outcome and find that a disposal fee combined with upstream instruments such as recycled content standards and subsidies can achieve the second-best optimum. We, however, do not engage in such debate, nor do we restrict our analysis to a specific policy instrument. We characterize optimal upstream design and price choices by firms in response to any given EPR legislation.

We model a manufacturer supplying a remanufacturable, durable good to a knowledgeable customer.[6] The customer has a continuing need for the services of the product and optimizes between the costs of replacement and the costs of equipment operation. We assume that product and process technologies are fairly stable and product life-cycles are reasonably long so that remanufacturing is viable to both the manufacturer as well as the customer. An example is a diesel truck engine manufactured by Cummins or Detroit Diesel sold to truck fleets such as UPS and FedEx. Product deterioration implies that operating costs increase with time. We assume that the price and profit advantage afforded by remanufacturing is a valid inducement for the knowledgeable customer to demand a remanufactured product and for the manufacturer to offer a remanufactured product. In contrast to the aforementioned literature in which customer demand or utility depends upon the output produced and the amount of material consumed or disposed, our model explicitly incorporates the manufacturer's design choices of performance and remanufacturability into an individual customer's product replacement decision, given that these design choices directly influence costs to the customer. Demand for the manufacturer's

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[5] Nor do we observe combined taxes/subsidies being implemented in practice.

[6] The reader is directed to Lund (2003) for a comprehensive account of about 275 firms in the remanufacturing industry. The report covers aspects such as product and process design, sales and marketing, workforce, capital investment, costs, and strategic barriers and opportunities in the industry. According to the report, remanufactured products are tested to original performance specifications, they often carry warranties comparable to that of new products, and they are sold in the market at about 45% to 65% of equivalent new product prices. Frequent and expert buyers who possess substantial ongoing experience in purchasing remanufactured products and in evaluating a remanufactured product's performance objectively are most often found in commercial and industrial markets. These are the markets in which remanufactured products are most common.

product via customer replacements is thus connected with design choices through the costs faced by the customer. We explore how various attributes and implementations of EPR, in terms of the magnitudes of environmental costs during product use and waste disposal costs post customer-use, the distribution of waste disposal costs between the manufacturer and the customer, design standards, and recovery/reuse requirements, influence upstream choices of q, θ, and product price by the manufacturer, given that the customer makes equipment replacement decisions optimally. Thus, our model is flexible enough to treat EPR instruments such as product take-back, recovery targets, deposit/refund systems, advance disposal fees, design standards, charges for environmental impact during product use, and any combination of these.[7] Since product take-back is an essential feature of most EPR programs, we assume that the manufacturer is legally responsible for the physical take-back of the product post customer-use. We view profit-maximization as the objective and consider two distinct supply-chain structures - the *uncoordinated* case, in which the manufacturer and customer share common knowledge about q and θ but optimize their profits separately, and the *coordinated case*, in which the manufacturer and customer are one integrated firm which optimizes the sum total of the profits of the customer and the manufacturer.

From a firm's perspective, we present a methodology to incorporate environmental regulation and customer replacement behavior into strategic product design decisions. From a regulatory standpoint, the research effort addresses the impacts of environmental policy parameters on upstream environmental design choices by the manufacturer. In both the uncoordinated as well as the coordinated cases we find that the optimal level of remanufacturability increases in the cost of waste disposal as well as in the environmental costs during product use. The optimal level of performance increases in response to increasing waste disposal costs. When the cost of waste disposal is relatively large, it is profitable for the firm to provide better performance in response to an increase in environmental costs during product use. When the cost of waste disposal is relatively small, it is profitable for the firm to sacrifice performance and save on design costs in response to an increase in environmental costs during product use. However, a numerical exercise demonstrates that, from an environmental standpoint, coordination in the supply chain is beneficial. Design choices in the coordinated case are environmentally superior to those in the uncoordinated case. From the viewpoint of firm profitability as well, we find that the integrated firm always secures a profit larger than the corresponding supply chain profit in the uncoordinated case. Therefore, we suggest contracts that can help achieve coordination in the

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[7] Although not explicitly modeled, a material tax has a net effect similar to that of a unit cost for waste disposal. The instruments accommodated in our model are the predominant ones used in EPR programs across countries. An example from practice of firms being subject to a combination of EPR instruments include firms in the EU subject to product take-back legislation as well as costs for emissions through the EU greenhouse gas emissions trading program.

supply chain because coordination results in both higher supply chain profit as well as environmentally superior product design choices.

The paper is organized as follows. Section 2 presents the model. Section 3 presents the analysis for the uncoordinated case where the manufacturer and customer optimize their profits separately. Section 4 presents the analysis for the coordinated case where the manufacturer and the customer are one integrated firm that maximizes the sum total of profits. Section 5 provides comparisons between the results in the uncoordinated and coordinated cases through a numerical example. Section 6 discusses coordinating contracts between the manufacturer and the customer. Section 7 concludes with insights for firms as well as regulators and provides directions for future research.

2 The Model

A single manufacturer supplies a remanufacturable, durable good to a single customer. The customer has a continuing need for the services of the product. The manufacturer faces the decision of choosing optimal levels of two attributes of the product - one which determines the environmental performance of the product during product use, and the other which determines the product's environmental impact post customer-use. We model a one-dimensional "more is better" measure of environmental performance during product use q, analogous to the modeling of product performance or quality in Chen (2001), Kornish (2001), Kim & Chhajed (2000), and Moorthy & Png (1992). The product's remanufacturability is determined by the second attribute θ. Debo et al. (2003) and Fleischmann et al. (2001) model the choice of remanufacturability by the manufacturer as the fraction of products that can be remanufactured after use. In an analogous manner, and similar to Fullerton & Wu (1998), we model $\theta \in [0, 1]$ as the fraction of the product (say, by weight) that can be re-used, recovered, or remanufactured after use. As in Debo et al. (2003) and Calcott & Walls (2002), θ determines the cost of production; the cost of production is bounded above by the cost of manufacturing a new product, and decreases in θ. To provide a connection between the manufacturer's optimal choices of performance and remanufacturability, and the customer's replacement decision, we utilize an *economic-life* model to characterize the customer's replacement decision. The economic-life model involves determining the optimum point in time to replace equipment (Dean 1961). We use the replacement model suggested by Clapham (1957), which determines the economic life of equipment by minimizing the average sum of capital and (increasing) operating costs per period. Let τ denote the optimal time interval derived using the Clapham model, between successive replacements by the customer.

A higher level of remanufacturability lowers the unit cost c of production. c is assumed to decrease linearly in θ. Additionally, the unit cost to the manufacturer of providing performance is assumed to be convex increasing in q. Hence, the unit cost is given by $c(q,\theta)$, with $c_q > 0$, $c_{qq} > 0$, $c_\theta < 0$, $c_{\theta\theta} = 0$, $c_{q\theta} < 0$.[8] The initial design cost to the manufacturer of providing q and θ is $k(q,\theta)$ with $k_q > 0$, $k_{qq} > 0$, $k_\theta > 0$, $k_{\theta\theta} > 0$, and $k_{q\theta} = 0$. These assumptions are consistent with those in Debo et al. (2003), Kim & Chhajed (2002), Kouvelis & Mukhopadhyay (1995), and Moorthy & Png (1992). Also, we assume that a higher performance level translates into lower product operating costs. In other words, $u(q_1) < u(q_2)$ for $q_1 > q_2$. We denote the cost gradient[9] of the product's environmental impact during use (e.g., monetary charges for emissions) as $e(q)$, and the cost for waste disposal (e.g., landfilling costs) at the end of the product's economic life as $w(\theta)$. We assume that $e_q < 0$, $e_{qq} > 0$. In other words, we assume that performance (such as energy efficiency) decreases the cost of environmental impact during product use in a convex manner. Landfilling costs are typically linear in the amount of waste to be disposed of (for example, see EPA 1998).[10] Hence, we assume that $w(\theta)$ decreases linearly in θ, or $w_\theta < 0$, $w_{\theta\theta} = 0$. Denote α and $\beta = 1 - \alpha$ as the fractions of the cost of waste disposal borne by the customer and manufacturer respectively. EPR instruments differ in the magnitudes of e, w and α, and possibly in the specification of lower bounds on q and θ. We explore incentives for upstream design choices by the manufacturer for different implementations of EPR under two scenarios - when the customer and manufacturer are separate entities and optimize their profits separately (uncoordinated case), and when the customer and manufacturer are one integrated entity that optimizes total profit (coordinated case).

The manufacturer's design and price decisions impact the customer's optimal replacement policy. Performance and remanufacturability choices by the manufacturer, in turn, depend on the trade-offs in terms of the relative frequencies and magnitudes of revenue or profit earning instances corresponding to product replacements by the customer.[11] The sequence of decisions is as follows. In a given EPR scenario, the manufacturer first chooses q and θ jointly. He then chooses the price $r > c$ to be charged to the customer for product replacements. The customer buys the product at price r if she makes her reservation profit from employing an optimal replacement policy, given q, θ, and r. We proceed by backward induction according to the sequence of decisions in order to arrive at the optimal values of

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[8] Incorporating variable costs independent of θ and q does not change the analysis or affect the nature of results. An implicit assumption is that replacements are readily available to the customer. Since we model a single product being remanufactured and sold in each replacement instance, incorporating fixed costs into the model will not change the analysis or the nature of results.

[9] We model a linear degradation of the product's environmental performance with respect to time (and, hence, a linear increase in environmental costs during product use with respect to time), implicitly assuming a uniform usage pattern.

[10] Information on landfilling in the US is available at *http://www.epa.gov/epaoswer/non-hw/muncpl/disposal.htm*.

[11] An implicit assumption in the model is that the customer and the manufacturer have common knowledge about q and θ. This is very reasonable, since we assume that the customer is a frequent or expert buyer.

the decision variables.

3 Uncoordinated Case

In this section, we explore optimal design and price choices by the manufacturer when the manufacturer and customer share common knowledge about q and θ but optimize their profits separately. Using backward induction, we first solve for the customer's optimal replacement policy as a function of the manufacturer's design and price choices. The optimal replacement policy is then fed into the manufacturer's optimal price decision. Finally, we deduce the manufacturer's optimal design choices of performance and remanufacturability, given the optimal price to be charged to the customer and the customer's optimal replacement policy.

3.1 Customer's Problem

We use Clapham's model to represent the customer's replacement problem. Clapham's model determines the economic life of equipment by minimizing the average sum of capital and (increasing) operating costs per period. In order to demonstrate that customer replacement behavior is not qualitatively specific to our choice of the replacement model, we show in Appendix B that the properties of the optimal replacement interval in Clapham's model are similar to those in the richer model by Bellman (1955) which includes discounting but lacks tractability. The customer's profit rate is given by

$$\begin{aligned} \Pi_C &= \phi - \frac{r}{t} - \frac{1}{t}\int_0^t (u+e)x\,dx - \frac{\alpha w}{t} && (1) \\ &= \phi - \frac{(r+\alpha w)}{t} - \frac{(u+e)t}{2} && (2) \end{aligned}$$

where ϕ is the revenue earned by the customer per period by employing the product, t is the replacement interval, and u is the inferiority gradient or the linear rate (with respect to time) of cost increase due to operating inferiority. The second term on the right hand side of (1) represents the capital cost per period for the customer. Notice that environmental cost during product use is incurred in each period whereas the cost for waste disposal is incurred when the product is replaced. The optimal time interval between replacements is the age which maximizes Π_C, and is given by

$$\tau = \sqrt{\frac{2(r+\alpha w)}{u+e}} \qquad (3)$$

The above expression is analogous to the familiar EOQ formula for choosing the optimal order quantity, given the fixed cost of ordering, the unit cost of the product being ordered, and the inventory holding cost rate. The sum of the price of the product and the cost of waste disposal in our model is analogous to the fixed ordering cost in the EOQ model. The sum of the operating costs and the costs of environmental impact during product use per period is analogous to the holding cost rate in the EOQ model.[12] Proposition 1 provides properties of the customer's optimal replacement interval with respect to the replacement cost r, the operating cost gradient u, the cost of waste disposal w, and the environmental cost gradient e.

Proposition 1

i. $\frac{\partial \tau}{\partial r} > 0$; $\frac{\partial^2 \tau}{\partial r^2} < 0$.

ii. $\frac{\partial \tau}{\partial u} < 0$; $\frac{\partial^2 \tau}{\partial u^2} > 0$.

iii. $\frac{\partial \tau}{\partial w} > 0$; $\frac{\partial^2 \tau}{\partial w^2} < 0$.

iv. $\frac{\partial \tau}{\partial \alpha} > 0$; $\frac{\partial^2 \tau}{\partial \alpha^2} < 0$.

v. $\frac{\partial \tau}{\partial e} < 0$; $\frac{\partial^2 \tau}{\partial e^2} > 0$.

Proof:

i. $\frac{\partial \tau}{\partial r} = \frac{1}{\sqrt{2(u+e)(r+\alpha w)}} > 0$; $\frac{\partial^2 \tau}{\partial r^2} = -\frac{1}{2}\frac{1}{\sqrt{2(u+e)(r+\alpha w)^3}} < 0.$ $\blacksquare$

ii. $\frac{\partial \tau}{\partial u} = -\frac{1}{2}\sqrt{\frac{2(r+\alpha w)}{(u+e)^3}} < 0$; $\frac{\partial^2 \tau}{\partial u^2} = \frac{3}{4}\sqrt{\frac{2(r+\alpha w)}{(u+e)^5}} > 0.$ $\blacksquare$

iii. $\frac{\partial \tau}{\partial w} = \frac{\alpha}{\sqrt{2(u+e)(r+\alpha w)}} > 0$; $\frac{\partial^2 \tau}{\partial w^2} = -\frac{\alpha^2}{2\sqrt{2(u+e)(r+\alpha w)^3}} < 0.$ $\blacksquare$

iv. $\frac{\partial \tau}{\partial \alpha} = \frac{w}{\sqrt{2(u+e)(r+\alpha w)}} > 0$; $\frac{\partial^2 \tau}{\partial \alpha^2} = -\frac{w^2}{2\sqrt{2(u+e)(r+\alpha w)^3}} < 0.$ $\blacksquare$

v. $\frac{\partial \tau}{\partial e} = -\frac{1}{2}\sqrt{\frac{2(r+\alpha w)}{(u+e)^3}} < 0$; $\frac{\partial^2 \tau}{\partial e^2} = \frac{3}{4}\sqrt{\frac{2(r+\alpha w)}{(u+e)^5}} > 0.$ $\blacksquare$

Proposition 1.i implies that the optimal time interval τ between replacements increases in a concave manner with the customer's replacement cost r. A higher replacement cost translates into less frequent replacements, and the frequency of replacements can be expected to be decreasingly influenced by increasing replacement costs. Proposition 1.ii implies that τ is convex decreasing in the operating cost gradient. Larger operating cost gradients with respect to time result in more frequent replacements and the frequency of replacements can be expected to be decreasingly influenced by increasing operating

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[12] The EOQ formula for the optimal order quantity is given by $Q^* = \sqrt{\frac{2DS}{H}}$ where D is the demand rate, S is the fixed ordering cost, and H is the holding cost rate per unit (Chase et al. 2001). The optimal order interval is, therefore, $\frac{Q^*}{D} = \sqrt{\frac{2S}{DH}}$. This is very similar to the expression for the optimal replacement interval in (3). In our model, one unit of the product is ordered in each replacement instance.

cost gradients. Propositions 1.iii and 1.iv imply that the replacement interval increases concavely in the cost of waste disposal and in the customer's share of the cost of waste disposal. This is intuitive since a larger cost of waste disposal can be expected to delay replacement and this effect can be expected to diminish as the cost of waste disposal increases. Proposition 1.v implies that τ is convex decreasing in the environmental cost gradient. This again is intuitive because larger environmental cost increases over time prompt more frequent replacements. Also, the frequency of replacements can be expected to be decreasingly influenced by increasing environmental costs. Substituting τ from (3) for t in (2) we have

$$\Pi_C = \phi - \sqrt{2(u+e)(r+\alpha w)} \tag{4}$$

Without loss of generality, the participation constraint for the customer is $\Pi_C \geq 0$, or

$$\phi - \sqrt{2(u+e)(r+\alpha w)} \geq 0 \tag{5}$$

Having analyzed the customer's optimal replacement policy as a function of the manufacturer's decisions and the parameters of the model, we proceed to solve the manufacturer's problem.

3.2 Manufacturer's Problem

For consistency with the customer's objective and for analytical convenience, we assume that the manufacturer's objective is to maximize average profit per period.[13] Let T denote the planning horizon. For instance, the planning horizon could be the duration of time for which a particular product version or model is offered. The optimization problem for the manufacturer subject to the customer's participation constraint in (5) is

$$\max_{q,\theta,r} \Pi_M = \frac{(r-c)}{\tau} - \frac{\beta w}{\tau} - \frac{k}{T} \tag{6}$$

Manufacturer's Price Decision

We work by backward induction and evaluate the manufacturer's optimal price decision given design choices of q and θ. The customer's optimal replacement interval can be plugged into (6). Proposition 2 provides a useful result.

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[13] The problem becomes analytically intractable when discounting is considered. However, the qualitative nature of results remains unchanged.

Proposition 2 *Given design choices of q and θ, the manufacturer's profit increases concavely in the price $r > c$ charged to the customer.*

Proof:

$$\begin{aligned}
\Pi_M &= \frac{r-c-\beta w}{\sqrt{\frac{2(r+\alpha w)}{u+e}}} - \frac{k}{T} \\
\frac{\partial \Pi_M}{\partial r} &= \frac{(r+c+2\alpha w+\beta w)\sqrt{u+e}}{2\sqrt{2}(r+\alpha w)^{3/2}} > 0 \\
\frac{\partial^2 \Pi_M}{\partial r^2} &= -\frac{(r+3c+4\alpha w+3\beta w)\sqrt{u+e}}{4\sqrt{2}(r+\alpha w)^{5/2}} < 0. \quad \blacksquare
\end{aligned}$$

The above result, though straightforward, is not obvious. This is because it is possible for the manufacturer to trade away the magnitude of revenue in each replacement instance for a higher frequency of customer replacements. Corollary 1 follows directly from Proposition 2.

Corollary 1 *It is optimal for the manufacturer to price the product at $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$.*

Proof: From Proposition 2, we have that Π_M is increasing in r. Hence it is optimal for the manufacturer to charge as high a price as possible, subject to the participation constraint of the customer. The result follows from setting (5) as an equality. $\blacksquare$

Thus, we have solved for the manufacturer's optimal price and the customer's optimal replacement policy as a function of q, θ, and the parameters of the model. We now proceed to the first stage in the sequence of decisions where the manufacturer chooses optimal values of performance and remanufacturability.

Manufacturer's Design Decision

We have $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ and $\tau = \frac{\phi}{u+e}$. The behavior of the manufacturer's profit function with respect to its design choices of q and θ can be dissected with the help of the following observations.

$$\frac{\partial \Pi_M}{\partial q} = \frac{1}{\tau}\frac{\partial r^*}{\partial q} + \frac{-1}{\tau}\frac{\partial c}{\partial q} + \frac{-(r^*-c-\beta w)}{\tau^2}\frac{\partial \tau}{\partial q} + \frac{-1}{T}\frac{\partial k}{\partial q} \tag{7}$$

$$\frac{\partial \Pi_M}{\partial \theta} = \frac{1}{\tau}\frac{\partial r^*}{\partial \theta} + \frac{-1}{\tau}\frac{\partial c}{\partial \theta} + \frac{-\beta}{\tau}\frac{\partial w}{\partial \theta} + \frac{-(r^*-c-\beta w)}{\tau^2}\frac{\partial \tau}{\partial \theta} + \frac{-1}{T}\frac{\partial k}{\partial \theta} \tag{8}$$

We note the following:

i. $\frac{\partial c}{\partial q} > 0$; $\frac{\partial k}{\partial q} > 0$.

ii. $\frac{\partial c}{\partial \theta} < 0$; $\frac{\partial k}{\partial \theta} > 0$; $\frac{\partial w}{\partial \theta} < 0$.

iii. $\frac{\partial r^*}{\partial q} = -\frac{\phi^2}{2\sqrt{(u+e)}}\left[\frac{\partial u}{\partial q} + \frac{\partial e}{\partial q}\right] > 0$, since $\frac{\partial u}{\partial q} < 0$ and $\frac{\partial e}{\partial q} < 0$.

iv. $\frac{\partial \tau}{\partial q} = -\frac{\phi}{\sqrt{(u+e)}}\left[\frac{\partial u}{\partial q} + \frac{\partial e}{\partial q}\right] > 0$, since $\frac{\partial u}{\partial q} < 0$ and $\frac{\partial e}{\partial q} < 0$.

v. $\frac{\partial r^*}{\partial \theta} = -\alpha\frac{\partial w}{\partial \theta} \geq 0$ for $\alpha \geq 0$, since $\frac{\partial w}{\partial \theta} < 0$.

vi. $\frac{\partial \tau}{\partial \theta} = 0$.

The first term on the right hand side of (7) is > 0 since $\frac{\partial r^*}{\partial q} > 0$. This term represents the increase in profit due to a higher price which the manufacturer can charge for a superior performance product. The second term on the right hand side of (7) is < 0 since $\frac{\partial c}{\partial q} > 0$. This term represents the decrease in profit due to a higher unit cost for a superior performance product. The third term on the right hand side of (7) is < 0 since $\frac{\partial \tau}{\partial q} > 0$. This term represents the decrease in profit due to less frequent replacements by the customer if the product has superior performance and, consequently, lower operating and environmental costs. The fourth term on the right hand side of (7) is < 0 since $\frac{\partial k}{\partial q} > 0$. This term represents the decrease in profit due to higher design costs for a superior performance product. The first term on the right hand side of (8) is ≥ 0 since $\frac{\partial r^*}{\partial \theta} > 0$ if $\alpha > 0$, and $\frac{\partial r^*}{\partial \theta} = 0$ if $\alpha = 0$. When the customer incurs a non-zero fraction of the cost of waste disposal, a greater level of remanufacturability reduces this cost and enables the customer to afford a higher product price, thereby contributing positively to manufacturer profit. The second term on the right hand side of (8) is > 0 since $\frac{\partial c}{\partial \theta} < 0$. This term represents the increase in profit due to lower production cost for a product with greater remanufacturability. The third term on the right hand side of (8) is > 0 since $\frac{\partial w}{\partial \theta} < 0$. This term represents the increase in profit due to a decrease in waste disposal costs from a greater level of remanufacturability. The fourth term on the right hand side of (8) is $= 0$ since $\frac{\partial \tau}{\partial \theta} = 0$. The optimal replacement interval is unaffected by the remanufacturability level. This seems surprising because the expression for the optimal replacement interval in (3) includes a term in w which depends on θ. However, this term vanishes when the optimal price r^* is substituted for r in (3). Note that $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$. Therefore, $r^* + \alpha w$ is independent of θ. In other words, any benefit of lower waste disposal costs[14] to the customer from greater product remanufacturability is extracted back by the manufacturer via a higher product price with the net result being that the optimal replacement interval is independent of product remanufacturability. The fifth term on the right hand side of (8) is < 0 since $\frac{\partial k}{\partial \theta} > 0$. This term represents the decrease in profit due to higher design costs for a product with greater remanufacturability.

While we are able to individually describe the factors influencing manufacturer profit, we now

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[14] Recall that the customer's share of waste disposal costs is αw.

require functional forms for k, c, u, e, and w in order to facilitate exposition and provide collective insights into the problem. We assume that the fixed cost of design is separable in q and θ and is of the form $k := k_1q^2 + k_2\theta^2$. We assume that the cost of production is $c := c_0q^2(1-\theta)$. Thus, the cost of production decreases linearly in the remanufacturability of the product and is bounded above by the cost c_0q^2 of manufacturing a new product. We assume $u := \frac{u_0}{q}$. Thus, u decreases convexly in q but does not depend on θ. Other possibilities exist for the customer's operating cost gradient u and the customer's revenue ϕ. u could possibly depend on θ if there is a perceived increase in operating inferiority from using a remanufactured product which feeds into the customer's optimal replacement decision. Also, we assume that ϕ is constant in each period, independent of q. The case when the customer's revenue is independent of the performance level q is apt in situations where q impacts operating costs but not revenue. An example is when q represents energy efficiency. Examples where ϕ would increase in q include situations where q impacts equipment uptime or a feature like torque. A higher value of q would then imply that the customer can earn larger revenues for the same duration of product usage. Combinations of different functional assumptions yield various scenarios. However, we focus on the assumed functional forms. We assume w to be of the form $w_0(1-\theta)$, implying that the cost of waste disposal decreases linearly in the amount of waste disposed. Finally, we assume $e := \frac{e_0}{q}$. Thus, the cost of environmental impact during product use decreases convexly in performance. With these functional forms, we now look at optimal design choices by the manufacturer. In the first stage of the sequence of decisions, the manufacturer optimizes his profit with respect to q and θ, given the optimal price to be charged for the product and the customer's optimal replacement policy. Substituting $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ (from Corollary 1) and $\tau = \frac{\phi}{u+e}$ into the manufacturer's profit function in (6), we have

$$\Pi_M = \frac{\phi^2 - 2(c+w)(u+e)}{2\phi} - \frac{k}{T} \tag{9}$$

Proposition 3 provides properties of the manufacturer's profit function with respect to q, θ, and the parameters of the model.

Proposition 3

i. Assume $k_2 > \max\left\{\frac{(u_0+e_0)(c_0q^2-w_0)^2T}{4w_0\phi q(1-\theta)}, \frac{(u_0+e_0)(c_0q^2+w_0)T}{2\phi q}\right\}$, and denote $q^* = q : \frac{\partial \Pi_M}{\partial q} = 0$, $\theta^* = \theta : \frac{\partial \Pi_M}{\partial \theta} = 0$. The manufacturer's profit function is jointly concave in q and θ, and, hence, (q^*, θ^*) uniquely maximizes Π_M.[15]

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[15] The reader should bear in mind that in the presence of design standards, q or/and θ would be bounded below by the standard(s). For expositional convenience, we do not incorporate possible design standards into the expressions for q and θ. We also ignore the rather obvious effects of design standards in subsequent analysis.

ii. $\frac{\partial \Pi_M}{\partial w_0} < 0$; $\frac{\partial^2 \Pi_M}{\partial w_0^2} = 0$.

iii. $\frac{\partial \Pi_M}{\partial e_0} < 0$; $\frac{\partial^2 \Pi_M}{\partial e_0^2} = 0$.

iv. $\frac{\partial \Pi_M}{\partial \phi} > 0$; $\frac{\partial^2 \Pi_M}{\partial \phi^2} < 0$.

Proof:

i. $\frac{\partial \Pi_M}{\partial q} = -\frac{c_0(u_0+e_0)q^2(1-\theta)T-(u_0+e_0)w_0(1-\theta)T+2k_1\phi q^3}{\phi q^2 T}$; $\frac{\partial^2 \Pi_M}{\partial q^2} = -\frac{2[(u_0+e_0)w_0(1-\theta)T+k_1\phi q^3]}{\phi q^3 T} < 0$.
$\frac{\partial \Pi_M}{\partial \theta} = \frac{c_0(u_0+e_0)q^2T+(u_0+e_0)w_0T-2k_2\phi q\theta}{\phi q T}$; $\frac{\partial^2 \Pi_M}{\partial \theta^2} = -\frac{2k_2}{T} < 0$.
The determinant of the Hessian matrix of $\Pi_M(q,\theta) = \frac{-(u_0+e_0)^2(c_0q^2-w_0)^2T^2+4k_2\phi(u_0+e_0)w_0q(1-\theta)T+4k_1k_2\phi^2q^4}{\phi^2q^4T^2}$ > 0 if $k_2 > \frac{(u_0+e_0)(c_0q^2-w_0)^2T}{4w_0\phi q(1-\theta)}$. Thus, with the assumption on k_2, the Hessian matrix of Π_M with respect to q and θ is negative definite, yielding the result. ∎

ii. $\frac{\partial \Pi_M}{\partial w_0} = -\frac{(u_0+e_0)(1-\theta)}{\phi q} < 0$; $\frac{\partial^2 \Pi_M}{\partial w_0^2} = 0$. ∎

iii. $\frac{\partial \Pi_M}{\partial e_0} = -\frac{(c_0q^2+w_0)(1-\theta)}{\phi q} < 0$; $\frac{\partial^2 \Pi_M}{\partial e_0^2} = 0$. ∎

iv. $\frac{\partial \Pi_M}{\partial \phi} = \frac{\phi^2+2(u+e)(c+w)}{2\phi^2} > 0$; $\frac{\partial^2 \Pi_M}{\partial \phi^2} = -\frac{2(u+e)(c+w)}{\phi^3} < 0$. ∎

The optimal values of q and θ can be derived from the first order conditions $\frac{\partial \Pi_M}{\partial q} = 0$ and $\frac{\partial \Pi_M}{\partial \theta} = 0$. Thus, we have

$$q^* = \frac{\sigma^2 - c_0\sigma(u_0+e_0)(1-\theta)T + c_0^2(u_0+e_0)^2(1-\theta)^2T^2}{6k_1\phi\sigma} \tag{10}$$

Where $\sigma = \Big[(u_0+e_0)(1-\theta)T[54k_1^2w_0\phi^2 - c_0^3(u_0+e_0)^2(1-\theta)^2T^2 - 6k_1\phi(81k_1^2w_0^2\phi^2 - c_0^3w_0(u_0+e_0)^2(1-\theta)^2T^2)^{1/2}]\Big]^{1/3}$.

$$\theta^* = \frac{(u_0+e_0)(c_0q^2+w_0)T}{2k_2\phi q} \tag{11}$$

Note that $\sigma > 0$ and $q^* > 0$. Also, $0 < \theta^* < 1$ since $k_2 > \frac{(u_0+e_0)(c_0q^2+w_0)T}{2\phi q}$ by assumption. Thus, the only assumption needed for the manufacturer's profit to be jointly concave in q and θ is that the coefficient of the design cost of remanufacturability be sufficiently large - which is a reasonable assumption. An interesting observation is that the optimal values of q and θ do not depend upon the distribution of the waste disposal cost between the manufacturer and the customer. This has to do with optimal product price $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ charged by the manufacturer. The manufacturer extracts any decrease in the customer's share of the waste disposal cost by charging a higher product price. Proposition 3.ii states that the manufacturer's profit decreases in the cost of waste disposal. This is because of two effects. A larger cost of waste disposal decreases the price which the manufacturer can charge to the customer and also increases the manufacturer's share of the waste disposal cost. Proposition 3.iii states that the manufacturer's profit decreases in the environmental costs to the customer during product use. This, again, is because a larger environmental cost depresses the price which the manufacturer can charge to the customer. From Proposition 3.iv we have that the manufacturer's profit increases in the customer's revenue per period from the use of the product. A larger customer revenue from product use enables the manufacturer to charge a higher product price. This alludes to a possible benefit to the manufacturer in sharing effort to help increase customer revenue. Proposition 4 describes the behaviors of the optimal performance and remanufacturability levels for the uncoordinated case, with respect to the parameters of the model.

Proposition 4

i. $\frac{\partial \theta^*}{\partial w_0} > 0$; $\frac{\partial \theta^*}{\partial e_0} > 0$; $\frac{\partial \theta^*}{\partial c_0} > 0$; $\frac{\partial \theta^*}{\partial u_0} > 0$; $\frac{\partial \theta^*}{\partial \phi} < 0$; $\frac{\partial \theta^*}{\partial k_2} < 0$.

ii. $\frac{\partial q^*}{\partial w_0} > 0$; $\frac{\partial q^*}{\partial e_0} < 0$ if $w_0 < c_0 q^{*2}$, and > 0 if $w_0 > c_0 q^{*2}$; $\frac{\partial q^*}{\partial c_0} < 0$; $\frac{\partial q^*}{\partial u_0} < 0$ if $w_0 < c_0 q^{*2}$, and > 0 if $w_0 > c_0 q^{*2}$; $\frac{\partial q^*}{\partial \phi} > 0$ if $w_0 < c_0 q^{*2}$, and < 0 if $w_0 > c_0 q^{*2}$; $\frac{\partial q^*}{\partial k_1} < 0$.

Proof:

i. $\frac{\partial \theta^*}{\partial w_0} = \frac{(u_0+e_0)T}{2k_2 \phi q} > 0. \quad \blacksquare$

$\frac{\partial \theta^*}{\partial e_0} = \frac{(c_0 q^2+w_0)T}{2k_2 \phi q} > 0. \quad \blacksquare$

$\frac{\partial \theta^*}{\partial c_0} = \frac{(u_0+e_0)qT}{2k_2 \phi} > 0. \quad \blacksquare$

$\frac{\partial \theta^*}{\partial u_0} = \frac{(c_0 q^2+w_0)T}{2k_2 \phi q} > 0. \quad \blacksquare$

$\frac{\partial \theta^*}{\partial \phi} = -\frac{(u_0+e_0)(c_0 q^2+w_0)T}{2k_2 \phi^2 q} < 0. \quad \blacksquare$

$\frac{\partial \theta^*}{\partial k_2} = -\frac{(u_0+e_0)(c_0 q^2+w_0)T}{2k_2^2 \phi q} < 0. \quad \blacksquare$

ii. Denote $f_1 := \frac{\partial \Pi_M}{\partial q}$. q^* satisfies $\frac{\partial \Pi_M}{\partial q} = 0$; i.e., $f_1(q^*) = 0$. Using the *Implicit Function Theorem*, we have $\frac{\partial q^*}{\partial x} = -\frac{(\partial f_1/\partial x)}{(\partial f_1/\partial q^*)}$. Since Π_M is concave in q, $(\partial f_1/\partial q^*) < 0$. Hence, $\frac{\partial q^*}{\partial x} = \frac{(\partial f_1/\partial x)}{|\partial f_1/\partial q^*|}$.

$\frac{\partial q^*}{\partial w_0} = \frac{(u_0+e_0)(1-\theta)q^*T}{2[(u_0+e_0)(1-\theta)w_0 T + k_1 \phi q^{*3}]} > 0. \quad \blacksquare$

$\frac{\partial q^*}{\partial e_0} = \frac{(w_0 - c_0 q^{*2})(1-\theta)q^*T}{2[(u_0+e_0)(1-\theta)w_0 T + k_1 \phi q^{*3}]} < 0$ if $w_0 < c_0 q^{*2}$, and > 0 if $w_0 > c_0 q^{*2}$. $\blacksquare$

$\frac{\partial q^*}{\partial c_0} = -\frac{(u_0+e_0)(1-\theta)q^{*3}T}{2[(u_0+e_0)(1-\theta)w_0 T + k_1 \phi q^{*3}]} < 0. \quad \blacksquare$

$\frac{\partial q^*}{\partial u_0} = \frac{(w_0 - c_0 q^{*2})(1-\theta)q^*T}{2[(u_0+e_0)(1-\theta)w_0 T + k_1 \phi q^{*3}]} < 0$ if $w_0 < c_0 q^{*2}$, and > 0 if $w_0 > c_0 q^{*2}$. $\blacksquare$

$\frac{\partial q^*}{\partial \phi} = \frac{(u_0+e_0)(c_0 q^{*2} - w_0)(1-\theta)q^*T}{2\phi[(u_0+e_0)(1-\theta)w_0 T + k_1 \phi q^{*3}]} > 0$ if $w_0 < c_0 q^{*2}$, and < 0 if $w_0 > c_0 q^{*2}$. $\blacksquare$

$\frac{\partial q^*}{\partial k_1} = -\frac{\phi q^{*4}}{(u_0+e_0)(1-\theta)w_0 T + k_1 \phi q^{*3}} < 0. \quad \blacksquare$

Note that $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$. θ^* increases in w_0 because of two effects. A larger waste disposal cost reduces the price which the manufacturer can charge to the customer for the product and also increases the waste disposal cost incurred by the manufacturer. A larger remanufacturability level reverses these effects. As the coefficient c_0 increases, θ^* increases in order to lower the cost of production. Similarly, q^* decreases as c_0 increases, in order to keep the cost of production low. Both θ^* and q^* decrease in the cost of design. This is intuitive since larger design costs should result in lower optimal choices of θ^* and q^*. θ^* increases in e_0 and u_0 in order to decrease waste disposal costs and, hence, increase the price that can be charged for the product. θ^* decreases in ϕ because a larger customer revenue per period increases the price that can be charged by the manufacturer and diminishes the incentive for the manufacturer to reduce waste disposal costs by providing greater remanufacturability. As w_0 increases, q^* increases in order to decrease u and e and, hence, increase manufacturer revenue from the sale of the product. Note that the customer's optimal replacement interval is $\tau = \frac{\phi}{(u+e)}$. There is an important difference in the manner in which q and θ impact the manufacturer's total revenue over the planning horizon. While r^* increases in both q as well as θ, τ increases in q but is unaffected by θ. Thus, an increase in θ always increases the manufacturer's total revenue over the planning horizon whereas the impact of a change in q on the manufacturer's total revenue over the planning horizon is more intricate. When the cost of waste disposal is relatively high, r^* is relatively low and it is profitable for the manufacturer to increase q and, hence, increase r^*. Thus, q^* increases in e_0 and u_0, and decreases in ϕ when w_0 is relatively large. These incentives are reversed when w_0 is relatively small because when w_0 is small, an increase in q increases the replacement interval τ but does not significantly increase r^*.

We now proceed to analyze the coordinated case. Sections 3 and 4 are supplemented by a numerical example in Section 5, in which comparisons between the outcomes in the two cases are made and insights for firms as well as regulators are provided.

4 Coordinated Case

In this section, we determine the optimal replacement policy and design choices for the integrated firm which optimizes the sum total of the profits of the customer and the manufacturer. Using backward induction, we first solve for the integrated firm's optimal replacement policy as a function of its design choices. Note that since the firm is integrated, the price decision is absent. We then deduce the firm's optimal design choices of performance and remanufacturability, given the optimal replacement policy.

4.1 Replacement Decision

Since the manufacturer and customer are one integrated entity, the replacement decision is to choose an optimal replacement time interval that maximizes the sum total of the customer's and manufacturer's profits, given design choices of q and θ. Thus, the integrated firm chooses t which maximizes Π, where Π is given by

$$\Pi := \Pi_C + \Pi_M = \left[\phi - \frac{r+\alpha w}{t} - \frac{(u+e)t}{2}\right] + \left[\frac{r-c}{t} - \frac{k}{T} - \frac{\beta}{w}t\right] = \phi - \frac{(c+w)}{t} - \frac{(u+e)t}{2} - \frac{k}{T} \quad (12)$$

The replacement interval which maximizes Π is, therefore,

$$\tau_i = \sqrt{\frac{2(c+w)}{u+e}} \quad (13)$$

A comparison of (13) and (3) shows that the integrated firm's effective unit cost, i.e., the sum of production cost and the cost of waste disposal affects the replacement interval in the coordinated case, while in the uncoordinated case, the replacement interval depends upon the customer's effective unit cost, i.e., the sum of the price charged by manufacturer and the customer's share of the waste disposal cost. Proposition 5 provides properties of the optimal replacement interval with respect to the production cost c, the operating cost gradient u, the cost of waste disposal w, and the environmental cost gradient e.

Proposition 5

i. $\frac{\partial \tau_i}{\partial c} > 0$; $\frac{\partial^2 \tau_i}{\partial c^2} < 0$.

ii. $\frac{\partial \tau_i}{\partial u} < 0$; $\frac{\partial^2 \tau_i}{\partial u^2} > 0$.

iii. $\frac{\partial \tau_i}{\partial w} > 0$; $\frac{\partial^2 \tau_i}{\partial w^2} < 0$.

iv. $\frac{\partial \tau_i}{\partial e} < 0$; $\frac{\partial^2 \tau_i}{\partial e^2} > 0$.

Proof:

i. $\frac{\partial \tau_i}{\partial c} = \frac{1}{\sqrt{2(u+e)(c+w)}} > 0$; $\frac{\partial^2 \tau_i}{\partial c^2} = -\frac{1}{2}\frac{1}{\sqrt{2(u+e)(c+w)^3}} < 0$. ∎

ii. $\frac{\partial \tau_i}{\partial u} = -\frac{1}{2}\sqrt{\frac{2(c+w)}{(u+e)^3}} < 0$; $\frac{\partial^2 \tau_i}{\partial u^2} = \frac{3}{4}\sqrt{\frac{2(c+w)}{(u+e)^5}} > 0$. ∎

iii. $\frac{\partial \tau_i}{\partial w} = \frac{1}{\sqrt{2(u+e)(c+w)}} > 0$; $\frac{\partial^2 \tau_i}{\partial w^2} = -\frac{1}{2\sqrt{2(u+e)(c+w)^3}} < 0$. ∎

iv. $\frac{\partial \tau_i}{\partial e} = -\frac{1}{2}\sqrt{\frac{2(c+w)}{(u+e)^3}} < 0$; $\frac{\partial^2 \tau_i}{\partial e^2} = \frac{3}{4}\sqrt{\frac{2(c+w)}{(u+e)^5}} > 0$. ∎

Explanations for the above behaviors are similar to those provided for Proposition 1 *mutatis mutandis*. Having solved for the integrated firm's optimal replacement policy, we proceed to the first stage in the sequence of decisions and deduce the firm's optimal design choices of performance and remanufacturability.

4.2 Design Decision

The behavior of the integrated firm's profit function with respect to its design choices of q and θ can be described with the help of the following observations.

$$\frac{\partial \Pi}{\partial q} = \frac{-1}{\tau_i}\frac{\partial c}{\partial q} + \left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right]\frac{\partial \tau_i}{\partial q} + \frac{-\tau_i}{2}\frac{\partial u}{\partial q} + \frac{-\tau_i}{2}\frac{\partial e}{\partial q} + \frac{-1}{T}\frac{\partial k}{\partial q} \tag{14}$$

$$\frac{\partial \Pi}{\partial \theta} = \frac{-1}{\tau_i}\frac{\partial w}{\partial \theta} + \frac{-1}{\tau_i}\frac{\partial c}{\partial \theta} + \left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right]\frac{\partial \tau_i}{\partial \theta} + \frac{-1}{T}\frac{\partial k}{\partial \theta} \tag{15}$$

We note the following:

i. $\frac{\partial c}{\partial q} > 0; \frac{\partial k}{\partial q} > 0; \frac{\partial u}{\partial q} < 0; \frac{\partial e}{\partial q} < 0.$

ii. $\frac{\partial c}{\partial \theta} < 0; \frac{\partial k}{\partial \theta} > 0; \frac{\partial w}{\partial \theta} < 0.$

iii. $\frac{\partial \tau_i}{\partial q} = \frac{1}{\sqrt{2(u+e)(c+w)}}\frac{\partial c}{\partial q} - \sqrt{\frac{(c+w)}{2(u+e)^3}}\left[\frac{\partial u}{\partial q} + \frac{\partial e}{\partial q}\right] > 0.$

iv. $\frac{\partial \tau_i}{\partial \theta} = \frac{1}{\sqrt{2(u+e)(c+w)}}\left[\frac{\partial w}{\partial \theta} + \frac{\partial c}{\partial \theta}\right] < 0.$

v. $\left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right] = 0$, since $\tau_i = \sqrt{\frac{2(c+w)}{u+e}}$.

The first term on the right hand side of (14) is < 0 since $\frac{\partial c}{\partial q} > 0$. This term represents the decrease in profit due to a higher unit cost for a superior performance product. The second term on the right hand side of (14) is $= 0$ since $\left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right] = 0$, although $\frac{\partial \tau_i}{\partial q} > 0$. The replacement interval increases in performance because of higher production cost, lower operating costs, and lower environmental costs during product use. However, this effect vanishes because the replacement interval is optimally chosen such that the marginal benefit of decreased operating, environmental, and waste disposal costs from better performance and remanufacturability equates the marginal cost of production. The third term on the right hand side of (14) is > 0 since $\frac{\partial u}{\partial q} < 0$. This term represents the increase in profit due to lower operating costs for a superior performance product. The fourth term on the right hand side of (14) is > 0 since $\frac{\partial e}{\partial q} < 0$. This term represents the increase in profit due to lower environmental costs for a superior performance product. The fifth term on the right hand side of (14) is < 0 since $\frac{\partial k}{\partial q} > 0$. This term represents the decrease in profit due to larger design costs for a superior performance product. The first term on the right hand side of (15) is > 0 since $\frac{\partial w}{\partial \theta} < 0$. This term represents the increase in profit due to a decrease in waste disposal cost with greater remanufacturability. The second term on the right hand side of (15) is > 0 since $\frac{\partial c}{\partial \theta} < 0$. This term represents the increase in profit due to lower production cost for a product with greater remanufacturability. The third term on the right hand side of (15) is $= 0$ since $\left[\frac{c+w}{\tau_i^2} - \frac{u+e}{2}\right] = 0$, although $\frac{\partial \tau_i}{\partial \theta} < 0$. The replacement interval decreases in remanufacturability because of lower production and waste disposal costs. However, this effect vanishes again because the replacement interval is optimally chosen such that the marginal benefit of decreased operating, environmental, and waste disposal costs from better performance and remanufacturability equates the marginal cost of production. The fourth term on the right hand side of (15) is < 0 since $\frac{\partial k}{\partial \theta} > 0$. This term represents the decrease in profit due to larger design costs for a product with greater remanufacturability. Substituting the optimal replacement interval from (13) for t in the integrated firm's profit function in (12), we have

$$\Pi = \phi - \sqrt{2(c+w)(u+e)} - \frac{k}{T} \tag{16}$$

Proposition 6 provides properties of the integrated firm's profit function with respect to q, θ, and the parameters of the model. We use the same functional forms as those assumed in Section 3.

Proposition 6

i. Assume $k_1 > \max\left\{\sqrt{\frac{(u_0+e_0)(c_0q^2-w_0)^4(1-\theta)T^2}{8q^5(c_0q^2+w_0)^3}}, \frac{T}{2}\left[\frac{\sqrt{2(u_0+e_0)(1-\theta)}(c_0q^2-w_0)(5c_0q^4+3w_0q^2)}{4[q^3(c_0q^2+w_0)]^{3/2}} - \frac{c_0\sqrt{2(u_0+e_0)(1-\theta)}}{\sqrt{q(c_0q^2+w_0)}}\right]\right\}$ and $k_2 > \max\left\{\sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)T^2}{8q(1-\theta)^3}}, \sqrt{\frac{27u_0T^2(c_0q^2+w_0)}{32q}}\right\}$, and denote $q_i^* = q : \frac{\partial \Pi}{\partial q} = 0$, $\theta_i^* = \max\left\{\theta : \frac{\partial \Pi}{\partial \theta} = 0, 0\right\}$. The integrated firm's profit function is jointly concave in q and θ, and, hence, (q_i^*, θ_i^*) uniquely maximizes Π.

ii. $\frac{\partial \Pi}{\partial w_0} < 0$; $\frac{\partial^2 \Pi}{\partial w_0^2} > 0$.

iii. $\frac{\partial \Pi}{\partial e_0} < 0$; $\frac{\partial^2 \Pi}{\partial e_0^2} > 0$.

iv. $\frac{\partial \Pi}{\partial \phi} > 0$; $\frac{\partial^2 \Pi}{\partial \phi^2} = 0$.

Proof:

i. $\frac{\partial \Pi}{\partial q} = \frac{\sqrt{(u_0+e_0)(1-\theta)}(w_0-c_0q^2)}{\sqrt{2q^3(c_0q^2+w_0)}} - \frac{2k_1q}{T}$; $\frac{\partial^2 \Pi}{\partial q^2} = -\frac{c_0\sqrt{2(u_0+e_0)(1-\theta)}}{\sqrt{q(c_0q^2+w_0)}} + \frac{\sqrt{2(u_0+e_0)(1-\theta)}(c_0q^2-w_0)(5c_0q^4+3w_0q^2)}{4[q^3(c_0q^2+w_0)]^{3/2}} - \frac{2k_1}{T} < 0$ if $k_1 > \frac{T}{2}\left[\frac{\sqrt{2(u_0+e_0)(1-\theta)}(c_0q^2-w_0)(5c_0q^4+3w_0q^2)}{4[q^3(c_0q^2+w_0)]^{3/2}} - \frac{c_0\sqrt{2(u_0+e_0)(1-\theta)}}{\sqrt{q(c_0q^2+w_0)}}\right]$.
$\frac{\partial \Pi}{\partial \theta} = \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)}{2q(1-\theta)}} - \frac{2k_2\theta}{T}$; $\frac{\partial^2 \Pi}{\partial \theta^2} = \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)}{8q(1-\theta)^3}} - \frac{2k_2}{T} < 0$ if $k_2 > \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)T^2}{32q(1-\theta)^3}}$.
The determinant of the Hessian matrix of $\Pi(q, \theta)$
$= \frac{-(u_0+e_0)^2(1-\theta)(c_0q^2+w_0)^2w_0T^2+4\sqrt{2q}k_2\left[(u_0+e_0)(1-\theta)(c_0q^2+w_0)\right]^{3/2}(1-\theta)w_0T+8k_1k_2(u_0+e_0)(c_0q^2+w0)^2(1-\theta)^2q^3}{2(u_0+e_0)(c_0q^2+w0)^2(1-\theta)^2q^3T^2}$
$-\frac{\sqrt{2(u_0+e_0)^3(c_0q^2+w_0)(1-\theta)q}\left[k_1q^2(c_0q^2+w_0)^2+k_2(1-\theta)^2(c_0q^2-w0)^2\right]T}{2(u_0+e_0)(c_0q^2+w0)^2(1-\theta)^2q^3T^2} > 0$ if $k_1 > \sqrt{\frac{(u_0+e_0)(c_0q^2-w_0)^4(1-\theta)T^2}{8q^5(c_0q^2+w_0)^3}}$ and $k_2 > \sqrt{\frac{(u_0+e_0)(c_0q^2+w_0)T^2}{8q(1-\theta)^3}}$. Note that the lower bound on θ is 0. With the assumptions on k_1 and k_2, the Hessian matrix of Π with respect to q and θ is negative definite, yielding the result.

∎

ii. $\frac{\partial \Pi}{\partial w_0} = -\sqrt{\frac{(u_0+e_0)(1-\theta)}{2q(c_0q^2+w_0)}} < 0$; $\frac{\partial^2 \Pi}{\partial w_0^2} = \sqrt{\frac{(u_0+e_0)(1-\theta)}{8q(c_0q^2+w_0)^3}} > 0$. ∎

iii. $\frac{\partial \Pi}{\partial e_0} = -\sqrt{\frac{(c_0q^2+w_0)(1-\theta)}{2q(u_0+e_0)}} < 0$; $\frac{\partial^2 \Pi}{\partial e_0^2} = \sqrt{\frac{(c_0q^2+w_0)(1-\theta)}{8q(u_0+e_0)^3}} > 0$. ∎

iv. $\frac{\partial \Pi}{\partial \phi} = 1$; $\frac{\partial^2 \Pi}{\partial \phi^2} = 0$. ∎

The optimal values of q_i and θ_i can be derived from the first order conditions $\frac{\partial \Pi}{\partial q} = 0$ and $\frac{\partial \Pi}{\partial \theta} = 0$. Thus, we have that q_i^* satisfies

$$8k_1^2q^5(c_0q^2 + w_0) - u_0T^2(c_0q^2 - w_0)^2(1 - \theta) = 0 \tag{17}$$

And θ_i^* (if > 0) satisfies

$$\theta^3 - \theta^2 + \frac{u_0T^2(c_0q^2 + w_0)}{8k_2^2q} = 0 \tag{18}$$

Note that $q_i^* > 0$. The left hand side of equation (18) is a polynomial (cubic) function of odd degree with real coefficients. Hence, it has at least one real root. Using *Descartes' Rule of Signs* we have that the above cubic function has one negative root and either two positive real roots or two complex conjugate roots. We require θ_i^* to be a non-negative fraction less than or equal to one. In order that the cubic equation have one negative and two positive roots, we require that the discriminant of the cubic equation be < 0; i.e., we require that $27\xi^2 - 4\xi < 0$, where $\xi = \frac{u_0T^2(c_0q^2+w_0)}{8k_2^2q}$. The assumption $k_2 > \sqrt{\frac{27u_0T^2(c_0q^2+w_0)}{32q}}$ ensures that $27\xi^2 - 4\xi < 0$ and, hence, that the cubic equation has one negative and two positive roots. $\theta \geq 1$ cannot satisfy $\theta^3 - \theta^2 + \xi = 0$. Hence $\theta_i^* > 0$ implies that $\theta_i^* < 1$. In the coordinated case, the assumptions needed for the integrated firm's profit to be jointly concave in q and θ are that the coefficients of the design costs of performance and remanufacturability be sufficiently large. These are reasonable assumptions. Propositions 6.ii and 6.iii show that the integrated firm's profit decreases convexly in w_0 and e_0. In contrast to the linear decrease of supply chain profit with respect to w_0 and e_0 in the uncoordinated case, the integrated firm's profit in the coordinated case is decreasingly affected by increasing costs of waste disposal and environmental costs during product use. Thus, the effects of increasing environmental costs on firm profits are felt more strongly in the uncoordinated case than in the coordinated case. In addition, Proposition 6.iv shows that the integrated firm's profit increases linearly with per-period revenue ϕ, in contrast to the concave increase in supply chain profit with respect to ϕ in the uncoordinated case. Proposition 7 describes the behaviors of the optimal performance and remanufacturability levels for the coordinated case, with respect to the parameters of the model.

Proposition 7

i. $\frac{\partial \theta_i^*}{\partial w_0} > 0$; $\frac{\partial \theta_i^*}{\partial e_0} > 0$; $\frac{\partial \theta_i^*}{\partial c_0} > 0$; $\frac{\partial \theta_i^*}{\partial u_0} > 0$; $\frac{\partial \theta_i^*}{\partial \phi} = 0$; $\frac{\partial \theta_i^*}{\partial k_2} < 0$.

ii. $\frac{\partial q_i^*}{\partial w_0} > 0$; $\frac{\partial q_i^*}{\partial e_0} < 0$ if $w_0 < c_0 q_i^{*2}$, and > 0 if $w_0 > c_0 q_i^{*2}$; $\frac{\partial q_i^*}{\partial c_0} < 0$; $\frac{\partial q_i^*}{\partial u_0} < 0$ if $w_0 < c_0 q_i^{*2}$, and > 0 if $w_0 > c_0 q_i^{*2}$; $\frac{\partial q_i^*}{\partial \phi} = 0$; $\frac{\partial q_i^*}{\partial k_1} < 0$.

Proof:

i. Denote $f_2 := \frac{\partial \Pi}{\partial \theta}$. θ_i^* satisfies $\frac{\partial \Pi}{\partial \theta} = 0$; i.e., $f_2(\theta_i^*) = 0$. Using the Implicit Function Theorem, we have $\frac{\partial \theta_i^*}{\partial x} = -\frac{(\partial f_2/\partial x)}{(\partial f_2/\partial \theta_i^*)}$. Since Π is concave in θ, $(\partial f_2/\partial \theta_i^*) < 0$. Hence, $\frac{\partial \theta_i^*}{\partial x} = \frac{(\partial f_2/\partial x)}{|\partial f_2/\partial \theta_i^*|}$.

$$\frac{\partial f_2}{\partial w_0} = \sqrt{\frac{(u_0+e_0)}{8q(w_0+c_0q^2)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial w_0} > 0. \quad \blacksquare$$
$$\frac{\partial f_2}{\partial e_0} = \sqrt{\frac{(w_0+c_0q^2)}{8q(u_0+e_0)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial e_0} > 0. \quad \blacksquare$$
$$\frac{\partial f_2}{\partial c_0} = \sqrt{\frac{(u_0+e_0)q^3}{8(w_0+c_0q^2)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial c_0} > 0. \quad \blacksquare$$
$$\frac{\partial f_2}{\partial u_0} = \sqrt{\frac{(w_0+c_0q^2)}{8q(u_0+e_0)(1-\theta_i^*)}} > 0 \Rightarrow \frac{\partial \theta_i^*}{\partial u_0} > 0. \quad \blacksquare$$
$$\frac{\partial f_2}{\partial \phi} = 0 \Rightarrow \frac{\partial \theta_i^*}{\partial \phi} = 0. \quad \blacksquare$$
$$\frac{\partial f_2}{\partial k_2} = -\frac{2\theta_i^*}{T} < 0 \Rightarrow \frac{\partial \theta_i^*}{\partial k_2} < 0. \quad \blacksquare$$

ii. Denote $f_3 := \frac{\partial \Pi}{\partial q}$. q_i^* satisfies $\frac{\partial \Pi}{\partial q} = 0$; i.e., $f_3(q_i^*) = 0$. Using the Implicit Function Theorem, we have $\frac{\partial q_i^*}{\partial x} = -\frac{(\partial f_3/\partial x)}{(\partial f_3/\partial q_i^*)}$. Since Π is concave in q, $(\partial f_3/\partial q_i^*) < 0$. Hence, $\frac{\partial q_i^*}{\partial x} = \frac{(\partial f_3/\partial x)}{|\partial f_3/\partial q_i^*|}$.

$$\frac{\partial f_3}{\partial w_0} = \frac{(3c_0 q_i^{*2}+w_0)\sqrt{(u_0+e_0)(1-\theta)}}{\sqrt{8q_i^{*3}(w_0+c_0q_i^{*2})^3}} > 0 \Rightarrow \frac{\partial q_i^*}{\partial w_0} > 0. \quad \blacksquare$$
$$\frac{\partial f_3}{\partial e_0} = \frac{(w_0-c_0q_i^{*2})\sqrt{1-\theta}}{\sqrt{8q_i^{*3}(u_0+e_0)(w_0+c_0q_i^{*2})}} \Rightarrow \frac{\partial q_i^*}{\partial e_0} < 0 \text{ if } w_0 < c_0q_i^{*2}, \text{ and } > 0 \text{ if } w_0 > c_0q_i^{*2}. \quad \blacksquare$$
$$\frac{\partial f_3}{\partial c_0} = -\frac{(c_0q_i^{*2}+3w_0)\sqrt{q_i^*(u_0+e_0)(1-\theta)}}{\sqrt{8(w_0+c_0q_i^{*2})^3}} < 0 \Rightarrow \frac{\partial q_i^*}{\partial c_0} < 0. \quad \blacksquare$$
$$\frac{\partial f_3}{\partial u_0} = \frac{(w_0-c_0q_i^{*2})\sqrt{1-\theta}}{\sqrt{8q_i^{*3}(u_0+e_0)(w_0+c_0q_i^{*2})}} \Rightarrow \frac{\partial q_i^*}{\partial u_0} < 0 \text{ if } w_0 < c_0q_i^{*2}, \text{ and } > 0 \text{ if } w_0 > c_0q_i^{*2}. \quad \blacksquare$$
$$\frac{\partial f_3}{\partial \phi} = 0 \Rightarrow \frac{\partial q_i^*}{\partial \phi} = 0. \quad \blacksquare$$
$$\frac{\partial f_3}{\partial \phi} = -\frac{2q_i^*}{T} < 0 \Rightarrow \frac{\partial q_i^*}{\partial k_1} < 0. \quad \blacksquare$$

The behavior of the optimal remanufacturability level θ_i^* and the optimal performance level q_i^* with respect to c_0, k_1 and k_2 is intuitive and the explanation for the behavior is similar to that provided for Proposition 4 in Section 3. θ_i^* increases in w_0 in order to lower the cost of waste disposal. From our earlier discussion, we have that $\frac{\partial \tau_i}{\partial \theta} < 0$, and $\frac{\partial \tau_i}{\partial q} > 0$. Any increase in θ increases the frequency of replacements by the firm. Thus, the response to either increasing operating costs (i.e., u_0) or increasing environmental costs during product use (i.e., e_0) is an increase in the frequency of replacements through greater remanufacturability. The response to increasing waste disposal costs (i.e., w_0) is a decrease in the frequency of replacements through better performance. When w_0 is relatively large, it is profitable for the firm to provide better performance and decrease the frequency of replacements in response to an increase in u_0 or e_0. However, when w_0 is relatively small, the replacement frequency is correspondingly large, and it is beneficial to sacrifice performance and save on design costs. Hence, when w_0 is relatively small, q_i^* decreases in u_0 and e_0. The integrated firm's revenues from product use are independent of design choices, and these revenues contribute entirely to the integrated firm's profit. This is in contrast to the uncoordinated case where the customer's revenue from product use, together with q, θ, and r, governs the revenue earned by the manufacturer through the sale of the product to the customer. We provide additional insights into the results in the uncoordinated and coordinated cases with the help of a numerical example in Section 5.

5 Numerical Example

Tables 1 and 2 provide numerical illustrations of the results in the uncoordinated and coordinated cases and facilitate comparisons between the two cases. Note that the values for ϕ, e^*, u^*, $\Pi_M^* + \Pi_C^*$, and Π^* are per-period amounts whereas the values for w^* and r^* are amounts per replacement instance. As seen in Tables 1 and 2, the optimal reaction to increasing w_0 is, for the manufacturer in the uncoordinated case and the integrated firm in the coordinated case, to provide greater performance as well as remanufacturability. The numerical illustrations for the two cases are set up with w_0 being relatively large (see Propositions 4 and 7). Hence, in both cases we have that the optimal response to increasing e_0 is, for the manufacturer in the uncoordinated case and the integrated firm in the coordinated case, to provide greater performance and remanufacturability. From Table 1, we see that with $w_0 = 400$ and $e_0 = 20$, when the customer's revenue per period increases from 1500 to 2500, the manufacturer can charge a price which is 160.51% higher. However, the incentives for the manufacturer to provide performance and remanufacturability decrease when the customer's per-period revenue increases. From Table 1, we see that when $w_0 = 400$ and $e_0 = 20$, the drop in performance is 11.22% and the drop in remanufacturability is 36.22% when the customer's per-period revenue increases from 1500 to 2500. In the coordinated case, design choices are unaffected by the integrated firm's revenue. As seen in Table 2, design choices remain exactly the same despite revenue per-period increasing from 1500 to 2500.

Recall that in the uncoordinated case, the customer's profit is set to 0 without loss of generality. Hence, the total supply chain profit is equal to the manufacturer's profit. Notice that supply chain profit in both the uncoordinated and the coordinated cases decreases with increasing waste disposal costs (i.e., increasing w_0) or increasing environmental costs during product use (i.e., increasing e_0). For the uncoordinated case in Table 1, the 1000 unit increase in customer revenue from 1500 to 2500 per period results in a correspondingly smaller increase in supply chain profit in absolute terms; for

Table 1: Numerical Illustration: Uncoordinated Case*

Scenario	w_0	w^*	e_0	e^*	u^*	q^*	θ^*	τ	$\Pi_M^* + \Pi_C^*$	r^*
Increasing w_0	200	186.18	20	152.73	572.74	0.1309	0.0691	2.07	590.86	1457.63
$\phi = 1500$	400	359.37	20	118.04	442.65	0.1694	0.1016	2.68	514.09	1826.78
	600	523.19	20	102.11	382.92	0.1959	0.1280	3.09	452.81	2057.87
Increasing w_0	200	191.28	20	171.15	641.81	0.1169	0.0436	3.08	1144.18	3748.32
$\phi = 2500$	400	374.07	20	133.02	498.81	0.1504	0.0648	3.96	1090.63	4758.95
	600	550.69	20	115.26	432.21	0.1735	0.0822	4.57	1047.22	5432.75
Increasing e_0	400	363.21	10	60.49	453.70	0.1653	0.0920	2.92	533.51	2006.27
$\phi = 1500$	400	359.37	20	118.04	442.65	0.1694	0.1016	2.68	514.09	1826.78
	400	355.53	30	173.29	433.24	0.1731	0.1112	2.47	495.07	1677.04
Increasing e_0	400	376.43	10	68.39	512.93	0.1462	0.0589	4.30	1103.53	5187.43
$\phi = 2500$	400	374.07	20	133.02	498.81	0.1504	0.0648	3.96	1090.63	4758.95
	400	371.72	30	194.68	486.70	0.1541	0.0707	3.67	1078.00	4400.44

$^*c_0 = 5000, u_0 = 75, k_1 = 75000, k_2 = 50000, \alpha = \beta = 0.5, T = 50.$

example, when $w_0 = 400$ and $e_0 = 20$, this increase in supply chain profit is 576.54. In contrast, for the coordinated case in Table 2, any increase in revenue from product use results in an identical increase in supply chain profit. In addition, the integrated firm's profit in the coordinated case always exceeds the corresponding supply chain profit in the uncoordinated case. The loss to the supply chain from a lack of coordination can be attributed to a close analog of the classic double marginalization problem in a decentralized bilateral monopoly (Spengler 1950). In the uncoordinated case, the manufacturer prices the product above cost and this price determines the customer's product replacement frequency. The optimal replacement interval in the uncoordinated case is given by $\sqrt{\frac{2(r+\alpha w)}{u+e}}$, whereas that in the coordinated case is given by $\sqrt{\frac{2(c+w)}{u+e}}$, where $r > c$. The net effect of optimal design choices on product replacement frequency is that the customer in the uncoordinated case replaces the product less frequently than the integrated firm in the coordinated case. Also, in the uncoordinated case, $r^* = \frac{\phi^2}{2(u+e)} - \alpha w$ and $\tau = \frac{\phi}{u+e}$. Therefore, the average revenue per period earned by the manufacturer from product replacements by the customer, is $\frac{r^*}{\tau} = \phi - \frac{\alpha w(u+e)}{\phi}$, implying that customer revenue from product use does not entirely translate into net supply chain revenue (i.e., $\frac{r^*}{\tau} < \phi$), and also that any increase in the customer's per-period revenue from product use results in a smaller increase in total supply chain profit.

The monetary impact of the lack of coordination decreases as the cost of waste disposal increases

Table 2: Numerical Illustration: Coordinated Case*

Scenario	w_0	w^*	e_0	e^*	u^*	q_i^*	θ_i^*	τ_i	Π^*
Increasing w_0	200	165.57	20	129.33	484.99	0.1546	0.1722	0.93	864.37
$\phi = 1500$	400	314.66	20	101.53	380.75	0.1970	0.2134	1.39	724.93
	600	453.46	20	89.15	334.30	0.2243	0.2442	1.74	626.54
Increasing w_0	200	165.57	20	129.33	484.99	0.1546	0.1722	0.93	1864.37
$\phi = 2500$	400	314.66	20	101.53	380.75	0.1970	0.2134	1.39	1724.93
	600	453.46	20	89.15	334.30	0.2243	0.2442	1.74	1626.54
Increasing e_0	400	319.79	10	51.29	384.64	0.1950	0.2006	1.47	761.42
$\phi = 1500$	400	314.66	20	101.53	380.75	0.1970	0.2134	1.39	724.93
	400	309.70	30	150.95	377.38	0.1987	0.2258	1.32	690.64
Increasing e_0	400	319.79	10	51.29	384.64	0.1950	0.2006	1.47	1761.42
$\phi = 2500$	400	314.66	20	101.53	380.75	0.1970	0.2134	1.39	1724.93
	400	309.70	30	150.95	377.38	0.1987	0.2258	1.32	1690.64

*$c_0 = 5000, u_0 = 75, k_1 = 75000, k_2 = 50000, \alpha = \beta = 0.5, T = 50.$

(i.e., as w_0 increases). The increase in supply chain profit as a result of coordination is 46.29% when $\phi = 1500$, $w_0 = 200$ and $e_0 = 20$, but is 38.37% when $\phi = 1500$, $w_0 = 600$ and $e_0 = 20$. The corresponding increases are 62.94% and 55.32% when $\phi = 2500$. In addition, in Table 1, when $w_0 = 200$ and $e_0 = 20$, the increase in supply chain profit in absolute terms when customer revenue per period increases from 1500 to 2500, is 553.32 whereas the increase in supply chain profit is 594.41 when $w_0 = 600$ and $e_0 = 20$. The monetary impact of the lack of coordination decreases in the environmental cost during product use (i.e., in e_0) too. The increase in supply chain profit as a result of coordination is 42.72% when $\phi = 1500$, $w_0 = 400$ and $e_0 = 10$, and is 39.50% when $\phi = 1500$, $w_0 = 400$ and $e_0 = 30$. The corresponding increases are 59.62% and 56.83% when $\phi = 2500$. In addition, in Table 1, when $w_0 = 400$ and $e_0 = 10$, the increase in supply chain profit when customer revenue per period increases from 1500 to 2500, is 570.02 whereas the increase in supply chain profit is 582.93 when $w_0 = 400$ and $e_0 = 30$.

An important point to note is that optimal design choices in the coordinated case are always more environmentally favorable than the corresponding optimal design choices in the uncoordinated case. The divergence between the optimal design choices in the two cases decreases as either w_0 or e_0 increases. When $\phi = 1500, w_0 = 200, e_0 = 20$, the optimal performance level in the coordinated case is 18.11% higher than that in the uncoordinated case. The difference is 14.50% when $\phi =$

$1500, w_0 = 600, e_0 = 20$. Corresponding differences are 32.25% and 29.28% when $\phi = 2500$. When $\phi = 1500, w_0 = 400, e_0 = 10$, the optimal performance level in the coordinated case is 17.97% higher than that in the uncoordinated case. The difference is 14.79% when $\phi = 1500, w_0 = 400, e_0 = 30$. Corresponding differences are 33.38% and 28.94% when $\phi = 2500$. Likewise, when $\phi = 1500, w_0 = 200, e_0 = 20$, the optimal remanufacturability level in the coordinated case is 149.20% higher than that in the uncoordinated case. The difference is 90.78% when $\phi = 1500, w_0 = 600, e_0 = 20$. Corresponding differences are 294.95% and 197.08% when $\phi = 2500$. When $\phi = 1500, w_0 = 400, e_0 = 10$, the optimal remanufacturability level in the coordinated case is 118.04% higher than that in the uncoordinated case. The difference is 103.06% when $\phi = 1500, w_0 = 400, e_0 = 30$. Corresponding differences are 240.58% and 219.38% when $\phi = 2500$. The differences in the optimal performance levels and in the optimal remanufacturability levels between the uncoordinated and coordinated cases widen as ϕ increases. This is because optimal design choices are invariant with respect to per-period revenue in the coordinated case, in contrast to the uncoordinated case where design choices become increasingly environmentally unfavorable with increase in per-period customer revenue.

Thus, from an environmental standpoint, coordination in the supply chain is beneficial. Design choices in the coordinated case are environmentally superior to those in the uncoordinated case. From the viewpoint of firm profitability as well, we see that the integrated firm always secures a profit larger than the corresponding supply chain profit in the uncoordinated case. Therefore, in Section 6, we discuss contracts that can help achieve coordination in the supply chain since coordination results in both higher supply chain profit as well as environmentally superior product design choices.

6 A Coordinating Contract

We present a contract between the manufacturer and the customer in the uncoordinated case, which achieves the higher supply chain profit as well as the environmentally superior product design of the coordinated case. In the uncoordinated case in Section 3, the distribution of waste disposal costs between the manufacturer and the customer has no impact on the manufacturer's profitability nor does it affect upstream design choices by the manufacturer. It also turns out that different allocations of responsibilities for operating costs and environmental costs during product use do not have a net impact on the manufacturer's profitability and design choices. However, the assignment of responsibility for the replacement interval decision does materially affect design and profit outcomes. We suggest a coordinating contract wherein either the customer or the manufacturer bears the operating and environmental costs during product use while the manufacturer assumes responsibility for the replacement interval decision.[16] The contract reflects the general concepts of leasing and *installed base management*. Installed base management is an arrangement in which the manufacturer bundles maintenance services along with the sale or lease of its product to the customer. Bhattacharya et al. (2005) provide an excellent discussion of the literature on leasing and installed base management. They compare the policy of selling a product to that of installed base management under different time horizons, under stable and improving technologies for product servicing, under competition, and when remanufacturing is possible across product generations. They find that if the remanufacturing option is considered, installed base management does better than selling. They also provide examples from practice and qualitatively justify the use of leasing and installed base management. However, to the best of our knowledge, the existing literature has not studied the impact or potential influence of such arrangements on product design, nor have such arrangements been evaluated in a regulatory context such as EPR. A major contribution of our paper lies in demonstrating that, under EPR legislation, coordination in the supply chain through such arrangements can lead to higher supply chain profits and environmentally superior product designs. An important point to note is that the analysis in this section applies quite generally, without the need to assume specific functional forms for k, c, u, e, and w.

6.1 Analysis of the Coordinating Contract

The sequence of decisions as per the suggested contract is as follows. The manufacturer first chooses q and θ jointly. He then chooses the replacement interval t for the product followed by the price $r > c$ to be charged to the customer for product replacements. The customer buys replacements at price r if she makes her reservation profit given q, θ, t and r. The customer's profit rate is given by

$$\Pi_C = \phi - \frac{(r+\alpha w)}{t} - \delta\frac{(u+e)t}{2} \tag{19}$$

where t is the replacement interval chosen by the manufacturer. $\delta = 1$ if the operating and environmental costs during product use are borne by the customer, and $\delta = 0$ if the manufacturer bears these costs. Without loss of generality, the participation constraint for the customer is $\Pi_C \geq 0$, or

$$\phi t - r - \alpha w - \delta\frac{(u+e)t^2}{2} \geq 0 \tag{20}$$

The optimization problem for the manufacturer subject to the customer's participation constraint in (20) is

$$\max_{q,\theta,t,r} \Pi_M = \frac{(r-c)}{t} - \frac{\beta w}{t} - (1-\delta)\frac{(u+e)t}{2} - \frac{k}{T} \tag{21}$$

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[16] Note, however, that the contract can coordinate the supply chain for any distribution of operating and/or environmental costs during product use between the manufacturer and the customer.

Given design choices of q and θ, and the replacement interval t, the manufacturer's profit increases in the price $r > c$ which he charges to the customer. Hence, it is optimal for the manufacturer to price the product at $r^* = \phi t - \alpha w - \delta \frac{(u+e)t^2}{2}$. Substituting r^* for r in the manufacturer's profit function in (21), we have

$$\Pi_M = \phi - \frac{(c+w)}{t} - \frac{(u+e)t}{2} - \frac{k}{T} \tag{22}$$

Π_M is concave in t. Hence, $\tau := \sqrt{\frac{2(c+w)}{u+e}} = t : \frac{\partial \Pi_M}{\partial t} = 0$ uniquely maximizes Π_M. Substituting τ for t in (22), we have

$$\Pi_M = \phi - \sqrt{2(c+w)(u+e)} - \frac{k}{T} \tag{23}$$

which is identical to the integrated firm's profit function in (16) that is optimized with respect to q and θ. In the first stage of the sequence of decisions under the contract, the manufacturer optimizes his profit in (23) with respect to q and θ. This implies that the resulting optimal values of q and θ are identical to those in the coordinated case. Note that the customer's profit is set to 0 without loss of generality. Hence, a contract in which either the customer or the manufacturer bears the operating and environmental costs during product use while the manufacturer assumes responsibility for the replacement interval decision, achieves the higher supply chain profit as well as the environmentally superior product design of the coordinated case.

An equivalent contract is one in which the manufacturer, instead of charging r per product replacement, charges a fee of f per period to the customer for the use of the product. This is similar to a typical leasing contract for a durable product which specifies a product usage fee per period as well as the duration of the lease. In our model, we assume uniform product usage by the customer over time. Hence, the time duration of product use corresponds to actual product usage. The participation constraint for the customer is $\phi - f - \frac{\alpha w}{t} - \delta \frac{(u+e)t}{2} \geq 0$. The manufacturer's profit Π_M increases in f and, hence, he chooses $f^* = \phi - \frac{\alpha w}{t} - \delta \frac{(u+e)t}{2}$. Substituting f^* for f into the manufacturer's profit function yields $\tau = \sqrt{\frac{2(c+w)}{u+e}}$ and a supply chain profit identical to the integrated firm's profit in (16).

6.2 Other Contracts

For completeness, we also examine the following contracts:

a. Leasing contract which specifies a menu comprising combinations of product usage fees per period and corresponding replacement intervals.

b. Contract which specifies a menu comprising combinations of product replacement prices and corresponding fees or penalties that depend upon the duration of product usage.

6.2.1 Menu of Per-Period Fees and Replacement Intervals

The contract discussed here is representative of leasing contracts for durable products which specify usage fees per period depending upon the duration of the lease. We investigate optimal design choices and supply chain profit in the contract wherein the customer bears the operating and environmental costs during product use while the manufacturer specifies a menu of combinations of per-period product usage fees and corresponding, required replacement intervals. The sequence of decisions is as follows. The manufacturer first chooses q and θ jointly. He then chooses the menu $\{f, t_f\}$ of per-period product usage fees and corresponding replacement intervals. The customer pays a usage fee from the menu and replaces the product at the corresponding replacement interval if she makes an optimal profit given q, θ, and $\{f, t_f\}$. The customer's profit rate is given by

$$\Pi_C = \phi - f - \frac{\alpha w}{t_f} - \frac{(u+e)t_f}{2} \tag{24}$$

where f is the per-period fee charged by the manufacturer and t_f is the corresponding replacement interval required by the manufacturer. The optimization problem for the manufacturer (in the absence of a customer participation constraint) is

$$\max_{q,\theta,\{f,t_f\}} \Pi_M = f - \frac{c}{t_f} - \frac{\beta w}{t_f} - \frac{k}{T} \tag{25}$$

In order that the resulting design choices of q and θ be identical to those in the coordinated case, we require that the manufacturer's profit function being maximized with respect to q and θ be identical to the integrated firm's profit function in (16). Thus, the design-coordinating combinations of the per-period fee and the replacement interval are

$$\{f, \tau_f\} = \left\{ f, \frac{c + \beta w}{f - \phi + \sqrt{2(c+w)(u+e)}} \right\} \tag{26}$$

We require $f > \phi - \sqrt{2(c+w)(u+e)}$ in order that $\tau_f > 0$. In equating the manufacturer's profit to the integrated firm's profit, the customer ends up with a non-positive profit. The customer makes zero profit and is indifferent between participating in the contract and staying out of it if and only if $f = \phi - \frac{\alpha w}{t_f} - \frac{(u+e)t_f}{2}$, or, in other words, if f and τ_f are identical to f^* and τ in Section 6.1, in which case the contract coordinates design choices *and* supply chain profit.

6.2.2 Menu of Replacement Prices and Usage Penalties

In this contract, the customer bears the operating and environmental costs during product use and is responsible for the replacement decision. The manufacturer charges a price for product replacements as well as a penalty corresponding to the duration of product usage in order to induce a replacement interval identical to that in the integrated case. The sequence of decisions is as follows. The manufacturer first chooses q and θ jointly. He then chooses the menu $\{r, \xi_r\}$ of replacement prices and corresponding usage penalties. The penalty charged to the customer is $\xi_r t^2$, where t is the customer's replacement interval.[17] The customer replaces the product at price r and pays the corresponding usage penalty if she makes an optimal profit from employing a replacement policy given q, θ, and $\{r, \xi_r\}$. The customer's profit rate is given by

$$\Pi_C \;=\; \phi - \frac{(r+\alpha w)}{t} - \xi_r t - \frac{(u+e)t}{2} \tag{27}$$

where r is the replacement price charged by the manufacturer and $\xi_r t^2$ is the corresponding usage penalty. Therefore, the combinations of r and ξ_r which induce the customer to replace the product at intervals identical to τ_i in (13) are given by[18]

$$\{r, \xi_r^*\} = \left\{ r, \frac{(r-c-\beta w)(u+e)}{2(c+w)} \right\} \tag{28}$$

The manufacturer's profit rate is given by

$$\Pi_M = \frac{(r-c)}{\tau_i} + \xi_r^* \tau_i - \frac{\beta w}{\tau_i} - \frac{k}{T} \tag{29}$$

And the total supply chain profit is given by

$$\Pi_M + \Pi_C = \phi - \frac{(c+w)}{\tau_i} - \frac{(u+e)\tau_i}{2} - \frac{k}{T} \tag{30}$$

which is identical to the integrated firm's profit in (16) when $\tau_i = \sqrt{\frac{2(c+w)}{u+e}}$ is substituted in (30). However, the contract does not generally coordinate supply chain profit or design choices because the profit function in (29), which the manufacturer optimizes with respect to q and θ, is different from the profit function in (16) optimized by the integrated firm. In order for the contract to coordinate supply chain profit and design, Π_M in (29) should equal Π in (16). In other words, we require that

$$r + \xi_r^* \tau_i^2 = \phi \tau_i - \alpha w - \frac{(u+e)\tau_i^2}{2} \tag{31}$$

The conditions in (28) and (31) together with the customer's optimal replacement interval $\tau_i = \sqrt{\frac{2(c+w)}{u+e}}$, yield the following combination of r and ξ_r that coordinates both supply chain profit as well as design choices:

$$(\tilde{r}, \tilde{\xi}_r^*) = \left(\frac{\phi \tau_i}{2} - \alpha w \;,\; \frac{\phi}{2\tau_i} - \frac{(u+e)}{2} \right) \tag{32}$$

Table 3 summarizes the contracts examined in Section 6.

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[17] Other functional forms of the penalty can easily be treated.

[18] As a special case, note that $\xi_r = 0$ when $r = c + \beta w$. In other words, when r is set to $c + \beta w$ without a penalty being charged, the customer's optimal replacement interval is identical to the replacement interval τ_i in the coordinated case.

Table 3: Summary of Contracts Examined

Contract type	Replacement decision by	Operating costs borne by	Generally coordinates
Coordinating contract - replacement price/per-period fee	Manufacturer	Either	Profit, Design
Menu $\{f, t_f\}$ of per-period fees and replacement intervals	Customer*	Customer	Design†
Menu $\{r, \xi_r\}$ of replacement prices and usage penalties	Customer	Customer	Neither‡

*The customer chooses a fee and the corresponding replacement interval from the menu; †Coordinates supply chain profit and design for a specific combination of f and t_f; ‡Coordinates both supply chain profit and design for a specific combination of r and ξ_r.

7 Discussion and Future Work

EPR is designed to confront manufacturers with the environmental costs of the products they produce. The principal rationale behind EPR is that manufacturers have the capacity to effect changes at the design stage in order to reduce the life-cycle environmental impacts of products. While it is empirically evident that manufacturers have responded to EPR legislation by taking preventive upstream measures such as environmentally favorable product design (Tojo 2004), to the best of our knowledge, ours is the first effort to analytically establish how various implementations of EPR influence upstream actions by a manufacturer producing and selling a remanufacturable product.

In both the coordinated and the uncoordinated cases we find that the optimal level of remanufacturability increases in the cost of waste disposal as well as in the environmental costs during product use. The optimal level of performance increases in response to increasing waste disposal costs. When the cost of waste disposal is relatively large, it is profitable for the firm to provide better performance in response to an increase in environmental costs during product use. When the cost of waste disposal is relatively small, it is profitable for the firm to sacrifice performance and save on design costs in response to an increase in environmental costs during product use. However, the numerical exercise demonstrates that, from an environmental standpoint, coordination in the supply chain is beneficial. Design choices in the coordinated case are environmentally superior to those in the uncoordinated case. From the viewpoint of firm profitability as well, we find that the integrated firm always secures a profit larger than the corresponding supply chain profit in the uncoordinated case. The contracts presented in Section 6 can help achieve coordination in the supply chain.

Our results provide insights to firms subject to or anticipating EPR legislation, as well as to regulators implementing or contemplating EPR legislation. From a firm's perspective, we present a methodology to incorporate environmental regulation and customer replacement behavior into strategic product design decisions. From a regulatory standpoint, the research effort addresses the impacts of environmental policy parameters on upstream environmental design choices by the manufacturer. Increasing environmental costs do not always induce desirable design outcomes. For example, the response to increasing environmental costs during product use when the cost of waste disposal is relatively small, is for the manufacturer to decrease the performance level in the product. However, remanufacturability is always influenced in an environmentally favorable manner with increasing environmental costs. Any increase in waste disposal costs achieves reduction in waste by inducing a higher level of remanufacturability that increases recovery after product use, as well as a higher level of performance that decreases the frequency of equipment replacement. The distribution of waste disposal costs between the manufacturer and the customer has no net impact on design outcomes. We also find that it could be optimal for the manufacturer to go beyond mandated design standards, if any. In order for design standards to have bite, it is important for the regulator to understand what the firm would optimally do in the absence of design standards. Thus, the impacts that EPR instruments have on design outcomes are intricate and not obvious. Coordination in the supply chain better meets the profitability objective of the firm as well as the environmental design goals of the regulator. In addition, the negative impacts of increasing environmental costs on profitability are felt more strongly in the uncoordinated case than in the coordinated case. The seemingly divergent objectives of profitability and environmental benevolence can be harmonized through the coordinating contracts described in Section 6. An important point to note is that these contracts are very generally applicable, irrespective of the specific functional forms assumed for the purpose of analysis. The suggested contracts reflect the general concepts of leasing and installed base management.

Several extensions to this work merit treatment in future research, though we believe that our basic qualitative insights would largely continue to hold. In assuming that remanufacturing is profitable to the manufacturer and is also preferred by the knowledgable customer, we abstracted from the cannibalization effect that remanufactured products have on the sales of new products. Also, we assumed that a single manufacturer sells to a single customer. The effects of considering multiple customers and heterogeneous preferences with regard to new and remanufactured products are worth evaluating. In addition, competition between manufacturers for customer demand can change upstream outcomes in interesting ways in the presence of customers' switching costs. Information asymmetry between the manufacturer and the customer with regard to the manufacturer's production costs and the customer's operating costs could be studied within the context of our model. Uncertainty could be incorporated into the model in different ways. There could be uncertainties in the success of design efforts, in the possibility that a returned product could be remanufactured, and in environmental policies over time. Uncertainties and information asymmetries will render further complexity to the analysis of coordinating contracts. Finally, additional insights into EPR policy design can be facilitated by endogenizing the policy parameters of the model.

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Appendix A: EPR Legislation and Manufacturer Responses

EPR Legislation for EEE in Japan:

The Specified Home Appliance Recycling (SHAR) Law in Japan, which was enacted in 1998 and which fully came into force in 2001, legally assigns part of the responsibility for the end-of-life management of products to producers. The scarcity of final disposal sites, the increase of EEE in the waste stream and the inadequacy of existing treatment plants for handling EEE were the main driving forces for the enactment of the Law. Under the program, manufacturers as well as importers of four large electrical home appliance categories - televisions, refrigerators, air conditioners, and washing machines, are required to take back their discarded products, dismantle them and recover components and material that can be reused or recycled. The Ministerial Order sets reuse and recycling rate requirements of between 50 to 60% by weight for respective products. This should be fulfilled by product reuse, component reuse, and material recycling with a positive monetary value. Retailers are required by law to take back used products when they sell similar new products, as well as products that they themselves sold. End-users bear the responsibility of covering the costs for the end-of-life management of the products they discard. In addition, the Law for Promotion of Effective Utilization of Resources (in short, the Recycling Promotion Law), enacted in 1991, generally promotes various measures that improve recycling and reuse. Under the law, manufacturers of specific product groups are advised to take various types of measures. For instance, manufacturers of large electrical home appliances that are now governed under the SHAR Law should take measures to facilitate design for ease-of-disassembly and recycling. A revision of the Recycling Promotion Law came into force in April 2001 with its core being the promotion of the 3R (reduce, reuse, recycle) principle. The revision includes the specification of five areas where certain measures should be taken and the type of products/industries that fall under each area. Among these five areas, those that are relevant to EEE are the following: 1) waste reduction through promotion of lesser material use and greater longevity of products, 2) reuse of components and material recycling, and 3) collection and recovery of end-of-life products by industries. Requirements set in the areas under 1 and 2 primarily relate to the design and use of materials in products. Examples of such requirements include design for upgradability and longevity, reduction in the number and quantity of components, use of common parts, use of recycled materials, establishment of repair networks, increase of remanufacturing (use of recovered components), development of plans for use of recovered components, design for ease-of-disassembly and ease-of-replacement, consideration for safety issues, intelligent use of packaging material, and the provision of information. The take-back requirements for computers from private households, with an accompanying advance disposal fee system, started in October 2003.

Responses by Japanese EEE Manufacturers:

The interviewed manufacturers[19] use a combination of different tools, including design guidelines, recyclability assessments, life-cycle assessments (LCA), and checklists, to evaluate the overall environmental performance of products. Three common assessment areas include energy efficiency, reduction of hazardous substances, and resource efficiency and recyclability. In general, product standards take into consideration the content of existing and anticipated legislation, environmental performance of suppliers, and the company's own environmental policy. With regard to energy efficiency, LCA studies conducted with CO_2 emissions are used to assess environmental impacts during different phases of a product's life-cycle. Large home appliances, personal computers, and copying machines are among the products whose CO_2 emissions are highest during the use phase. The manufacturers described the link between improved energy efficiency during the use phase of products and consumers' direct economic interests as a factor that drives them to focus on energy efficiency. Other factors that drive manufacturers to take a conscious look at energy efficiency include the Kyoto Protocol to the UN Framework Convention on Climate Change and revisions of the Japanese legislation on energy efficiency. The enhancement of resource efficiency and recyclability has been addressed through approaches such as reduction of material use, extension of product life, ease of disassembly, reuse and recycling of components and materials, and the use of recycled materials. Modular design, component reuse, design for upgradability, reconditioning of products, and remanufacturing are among the typical measures taken for prolonging a product's economic life. For one manufacturer, 60% by weight of all the materials used in a 1997 model of its copying machine consisted of reused components taken from the 1993 model. A producer of computers also started to collect components from old rental products for use as spare parts. Collection and reuse of toner cartridges has been one of the common initiatives taken by manufacturers of printers.

EPR Legislation for Automobiles in Sweden:

In the year 2000, about 159,000 passenger cars came to the end of their useful lives in Sweden. A deposit-refund type system for cars was introduced in 1975 to deal with the problems of littering and scrappers' improper treatment of materials such as engine fluids. The system had been successful in reducing the problem of littering and it provided scrappers with economic compensation for ensuring environmentally appropriate treatment of scrap. However, the scheme was criticized for not providing incentives to car manufacturers to incorporate considerations for end-of-life management at the design phase. In 1997, an EPR system called the Ordinance on Producer Responsibility for Cars was

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[19] Fujitsu Limited, Hitachi Limited, Matsushita Electric Industrial Corporation Limited, Mitsubishi Electric Corporation, NEC Corporation, Ricoh Company Limited, Sharp Corporation, Sony Corporation, and Toshiba Corporation.

introduced via legislation, replacing the conventional deposit-refund system. The Ordinance makes manufacturers and importers of cars in Sweden responsible for accepting end-of-life vehicles free of charge to the customers. Manufacturers and importers are also responsible for the establishment of a system that takes care of end-of-life vehicles, regardless of their age. A reuse and recycling target of 85% to be achieved by 2002 has been set, which will be extended to 95% beginning 2012.

Responses by Swedish Automobile Manufacturers:

Emissions and fuel consumption during the use phase were identified as important areas which the interviewed manufacturers were working on.[20] Legislation on emissions requirements in various countries, an increasing awareness of climate change, and demand from customers were mentioned as factors explaining the importance of these areas. Materials such as aluminium, thermoset plastics with glass fibers, and thermoplastics which contribute to light-weighting, were used to help improve fuel efficiency. A truck manufacturer mentioned that its customers requested it to increase fuel efficiency and thus help reduce CO_2 emissions. The manufacturers also developed lists of hazardous substances that they wished to phase out. They provided their suppliers with these lists as part of product specifications. With regard to design for disassembly, the decrease of disassembly time was mentioned by manufacturers as an important focus area. Manufacturers have established or are establishing internal workshops where they can learn about the equipment and techniques used in disassembly, and the times and costs involved. This would feed back into the process of design for end-of-life. With regard to reuse of components, one manufacturer reconditioned old components taken out of used cars and sold them as spare parts under its brand name. Another manufacturer employed a large disassembler for the management of reused components, and relied on the disassembler to supply spare parts whenever necessary. The enhancement of recyclability of components was another focus area identified by manufacturers. Apart from metals, which constitute an average of 75% of the weight of a car, the remaining materials from used cars were earlier being shredded and sent to landfills as auto shredder residues. One manufacturer conducted a pilot project with its suppliers where several types of materials were scrapped in batches. These materials were shredded and ground, and were used by some suppliers to make parts such as wheel housings and interiors. Measures such as avoidance and treatment of waste, avoidance of hazardous substances such as lead and organic tin and the use of clean paint shops, were also undertaken to reduce environmental impacts during the production phase.

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[20] The Swedish automobile manufacturers interviewed were Saab Automobile AB, Volvo Car Corporation, and Volvo Truck Corporation.

Appendix B: Bellman Model

To determine whether or not customer replacement behavior is specific to our choice of the replacement model, we examine properties of the customer's optimal replacement interval with respect to the replacement cost r and the operating cost gradient u, in the richer model of Bellman (1955) which considers discounting. For conciseness, we present the analysis for the uncoordinated case in the absence of EPR. Bellman proposes a profit-maximization formulation for the problem of evaluating the optimal equipment replacement interval. Let $f(t)$ denote the overall profit to the customer of operating a product of age t under an optimal replacement policy. Let $u(t)$ denote the cost of operating and maintaining the product during period t and let ϕ denote the revenue earned per period by operating the product. At each epoch t there are two courses of action. The product can be kept (K) for another time period or, a remanufactured product could be purchased (P). In the former case, $f(t)$ satisfies $f_K(t) = \phi - u(t) + \alpha f(t+1)$. In the latter case $f(t)$ satisfies $f_P(t) = \phi - r - u(0) + \alpha f(1)$. Notice that the process renews itself at each replacement instance. The functional equation for $f(t)$ is

$$f(t) = \max\{K : \phi - u(t) + \alpha f(t+1), P : \phi - r - u(0) + \alpha f(1)\}$$

The customer's optimal policy has the form - operate the product for η periods and then purchase a replacement. This yields the following system of equations

$$\begin{aligned} f(0) &= \phi - u(0) + \alpha f(1) \\ f(1) &= \phi - u(1) + \alpha f(2) \\ &\vdots \\ f(\eta - 1) &= \phi - u(\eta - 1) + \alpha f(\eta) \\ f(\eta) &= \phi - r - u(0) + \alpha f(1) \end{aligned}$$

Solving for $f(1)$ recurrently we have,

$$\begin{aligned} f(1) &= [\phi - u(1)] + \alpha[\phi - u(2) + \alpha f(3)] \\ &= [\phi - u(1)] + \alpha[\phi - u(2)] + \alpha^2[\phi - u(3)] + \dots \alpha^{\eta-2}[\phi - u(\eta - 1)] + \alpha^{\eta-1}[\phi - r - u(0) + \alpha f(1)] \\ &= \frac{\phi}{1-\alpha} - \frac{[\sum_{i=1}^{\eta-1} \alpha^{i-1} u(i)] + \alpha^{\eta-1}[r + u(0)]}{1 - \alpha^\eta} \qquad (33) \end{aligned}$$

The optimal replacement interval η^* is the value of η which maximizes $f(0)$ and, hence, $f(1)$. We show similarities in behavior between η^* in the Bellman model and τ in the Clapham model. We assume a linear operating cost gradient u, and that $u(0) = 0$ without loss of generality. After algebraic simplification, we have

$$f(1) = \frac{\phi}{1-\alpha} - \frac{u}{(1-\alpha)^2} + \frac{\eta \alpha^{\eta-1} u}{(1-\alpha)^2(1-\alpha^\eta)} - \frac{\alpha^{\eta-1} r}{1-\alpha^\eta}$$

Therefore,[21]

$$\begin{aligned}
\frac{\partial f(1)}{\partial \eta} &= \frac{(1-\alpha)^2|\ln(\alpha)|r\alpha^{\eta-1} - |\ln(\alpha)|u\alpha^{\eta-1}\eta + u\alpha^{\eta-1}(1-\alpha^{\eta})}{(1-\alpha)^2(1-\alpha^{\eta})^2} \\
\frac{\partial^2 f(1)}{\partial \eta^2} &= -\frac{|\ln(\alpha)|\Big[(1-\alpha)^2|\ln(\alpha)|r\alpha^{\eta-1}(1+\alpha^{\eta}) - |\ln(\alpha)|u\alpha^{\eta-1}(1+\alpha^{\eta})\eta + 2u\alpha^{\eta-1}(1-\alpha^{\eta})\Big]}{(1-\alpha)^2(1-\alpha^{\eta})^3}
\end{aligned}$$

To ensure that there exists a value of η that maximizes $f(1)$, we require that $\frac{\partial^2 f(1)}{\partial \eta^2} < 0$. In other words, we require that

$$u \;<\; \frac{(1-\alpha)^2|\ln(\alpha)|\alpha^{\eta-1}(1+\alpha^{\eta})r}{|\ln(\alpha)|\alpha^{\eta-1}(1+\alpha^{\eta})\eta - 2\alpha^{\eta-1}(1-\alpha^{\eta})} \tag{34}$$

(34) implies that concavity of $f(1)$ can be ensured by assuming u to be sufficiently small. Note that the right hand side of (34) is $> 0 \;\forall\; \eta \geq 1,\; \alpha \in (0,1)$. The optimal value of η obtained by equating $\frac{\partial f(1)}{\partial \eta}$ to 0 is

$$\eta^* = \frac{u[1+\text{LambertW}(-e^{-\xi})] + |\ln(\alpha)|r(1-\alpha)^2}{|\ln(\alpha)|u} \tag{35}$$

where $\xi = \frac{u+|\ln(\alpha)|r(1-\alpha)^2}{u} > 1$. The LambertW function is the inverse of the function $f(W) = We^W$. $W(x)$ is real for $x \geq -1/e$. The LambertW function in (35) returns a real value since $\xi > 1$ and, hence, $-e^{-\xi} > -1/e$. Also note that $-1 < W(x) < 0$ for $x \in (-1/e, 0)$. Thus, the LambertW function in (35) returns a value in the range $(-1, 0)$. Proposition 8 provides properties of the customer's optimal replacement interval with respect to the replacement cost r and the operating cost gradient u.

Proposition 8

i. $\frac{\partial \eta^*}{\partial r} > 0$; $\frac{\partial^2 \eta^*}{\partial r^2} < 0$.

ii. $\frac{\partial \eta^*}{\partial u} < 0$.

Proof:

i. $\frac{\partial \eta^*}{\partial r} = \frac{(1-\alpha)^2}{u[1+\text{LambertW}(-e^{-\xi})]} > 0$; $\frac{\partial^2 \eta^*}{\partial r^2} = \frac{(1-\alpha)^4|\ln(\alpha)|\text{LambertW}(-e^{-\xi})}{u^2[1+\text{LambertW}(-e^{-\xi})]^3} < 0$. $\blacksquare$

ii. $\frac{\partial \eta^*}{\partial u} = -\frac{(1-\alpha)^2 r}{u^2[1+\text{LambertW}(-e^{-\xi})]} < 0$. $\blacksquare$

The above results are similar to those for τ in Proposition 1. The participation constraint for the customer, analogous to that in (5), is $f_{\eta^*}(0) \geq 0$, or

$$\frac{\phi}{1-\alpha} - \frac{\alpha u}{(1-\alpha)^2} + \frac{\eta^* \alpha^{\eta^*} u}{(1-\alpha)^2(1-\alpha^{\eta^*})} - \frac{\alpha^{\eta^*} r}{1-\alpha^{\eta^*}} \geq 0 \tag{36}$$

(36) is, however, analytically intractable to solve for r.

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[21] $f(1)$ is a discrete function of η. However, the duration of a "period" can be reduced to an arbitrarily small length of time; the number of periods in a given time interval would correspondingly increase. In the limit, $f(1)$ would be a continuous function of η. We assume that the technical conditions necessary for interchanging limits and differentiation hold.