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The Schur multiplier of McLaughlin's simple group

By

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Let G be the simple group of McLaughlin, $|G| = 2^7 3^6 5^3 7 \cdot 11$. We show that A, the Schur multiplier, has order 3. Since the Lyons group contains a 3-fold covering of G, [3], it suffices to bound $|A|$ by 3.

For $p \geqq 7$, the Sylow p-group of G is cyclic, whence $O_p(A) = 1$. The subgroups $U_3(5)$ and $2.A_8$ (perfect) have indices relatively prime to 5 and 2, respectively, whence $O_5(A) = O_2(A) = 1$ by the well-known result of Gaschütz [2] and the facts that $U_3(5)$ has multiplier of order 3 (hence relatively prime to 5) and that $2.A_8$ has trivial Schur multiplier; see [1] and [4].

The remaining prime is 3. Let H be the normalizer in G of a Sylow 3-center; thus $H \cong 3^{1+4}\,SL(2, 5)\,2$, H is 3-constrained and $L = H/O_3(H)$ operates absolutely irreducibly on $O_3(H)/O_3(H)'$. Let $1 \to A \to \hat{H} \to H \to 1$ be the extension of H induced by the covering sequence of G. Set $P = O_3(\hat{H})$. Since L has trivial Schur multiplier and since L contain an involution which inverts $O_3(H)/O_3(H)'$, the extension $1 \to A \to P \to O_3(\hat{H}) \to 1$ is nonsplit, $A \leqq P'$ and $[P, P, P] = 1$. It follows that A is elementary abelian, is an L-invariant direct factor of P', and corresponds to alternating $\mathbb{F}_3$-valued bilinear maps on $O_3(H)/O_3(H)'$ which are L-invariant. Absolute irreducibility then implies that $|A| \leqq 3$. This completes the argument.

The result of this paper should be included in [1].

References

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