CLIMATIC OSCILLATIONS DURING THE PRECAMBRIAN ERA

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Abstract. A remarkably regular cyclicity with a fundamental period of ~11–12 cycles is preserved in the 680 million year old Elatina formation of South Australia. All but one of the many periods present can be interpreted as resulting from the combined influences of the sunspot cycle and the lunar nodal tide – in particular, beating between these two cycles gives rise to a long period phase alternation. Available paleontological evidence is used to constrain the lunar distance 680 Ma ago, thereby constraining the length of the lunar nodal tide. We then infer from the beat period that the sunspot cycle was 10.8 ± 0.2 years, which is in agreement with independent astronomical evidence suggesting that the sunspot cycle would then have been some 3–10% shorter than it is at present. Although this interpretation is not consistent with the 12.0 year sunspot cycle counted by Williams and Sonett (1985), we demonstrate that unavoidable random errors made in discriminating unusually indistinct varves gives rise to a systematic overcount of varves of this magnitude. The clarity of this evidence for solar and lunar signals in the climate 680 Ma ago lends support to reports of their importance today.

1. Introduction

Deep sea sediments have preserved an excellent record of climate change during the Pleistocene Epoch. Indeed, this record has provided a secure observational foundation for the Milankovitch Theory of climatic oscillations on a time scale of tens of thousands of years as a consequence of fluctuations in Earth's orbit (e.g., Hays *et al.*, 1976). It is strange that there is very little convincing evidence of Milankovich periodicities in the rock record prior to the last few million years, although speculative identifications have been presented by Anderson (1982), Heckel (1986), and Walker and Zahnle (1986). On the other hand, a remarkable occurrence of regular fluctuations with a much shorter period, decades rather than tens of thousands of years, has been described by Williams (1981; 1983; Williams and Sonett, 1985) from the 680 million year old Elatina formation of South Australia, apparently the product of deposition in a glacial lake. The periodicity in this formation is in the thickness of layers in a clastic

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sedimentary rock, so the connection to climate is not as direct as in the case of the micropaleontological and isotopic record from Pleistocene deep sea cores, but the remarkable regularity of the oscillations in the layering of the Elatina formation argues very strongly for an astronomical influence, presumably mediated by climate. The data presented by Williams and Sonett (1985) are based on measurements of the thickness of some thousands of consecutive layers. The dominant periodicities are very obvious in the record.

A rather comparable record of regular oscillations with a period of tens of years has been reported by Trendall (1973, 1983) for the Weeli-Wolli formation of the Hamersley group in Western Australia. The age of this formation is approximately 2450 million years. This formation is not a clastic deposit like the Elatine formation, but is instead a banded iron-formation, a chemical sediment produced by the precipitation of dissolved constituents from water in response to fluctuating chemical and physical conditions. Periodic fluctuations occur in the thickness of individual layers in this sediment, which are interpreted to be annual deposits, demarcated by seasonal change. The thickness of these annual deposits varies with a period of 23.3 ± 0.3 years as measured by Trendall over some 500 consecutive layers.

No comparable occurrences of regular oscillations of the properties of sedimentary rocks with periods of tens of years have yet been reported. Varved sediments are reasonably common in both chemical and clastic deposits. A number of these deposits of various ages have been statistically examined by Anderson and Koopmans (1982) without revealing any periodic fluctuations in the thickness of the individual layers. One of us (JCGW) has searched for the oscillations reported by Trendall in cores from the Kuruman iron-formation of South Africa, a formation that is similar in many respects, including age, to the Weeli-Wolli formation studied by Trendall. These cores reveal layering of the sediments on a number of different scales, but no regular variation in the thickness of the individual layers.

It appears that quite exceptional conditions are needed to yield a sedimentary deposit that can reveal what are presumably quite small perturbations in the climate system. These conditions probably have more to do with the paleogeography of the depositional basin than any particular stage in the history or evolution of the atmosphere or sun. Of course, even when the sedimentary deposits reflect a very strong periodicity at a period of several years it should not be assumed that the climate in the environment of deposition was equally strongly periodic. The sediments record the response of an integrated climatic and depositional system. The amplification of regular astronomical forcing of the system may have occurred more strongly in the depositional elements of the system than in the climatic elements.

In spite of these qualifications, it does appear that we have been fortunate enough to find two deposits that have preserved remarkably clear records of astronomical factors influencing climate in the remote geological past. Even though the reasons for the unusual sensitivity of these deposits to astronomical influence are not understood, we may still learn about the evolution of the solar system by interpreting the periods recorded in these deposits. This is the goal of this paper.

There are two obvious sources of astronomical influence on the climate with periods in the right range, and both have been invoked in connection with these data. Williams (1981, 1983) and Williams and Sonett (1985) attributed the periods in the Elatina formation to solar activity changes, principally the sunspot cycle and the Hale (magnetic) cycle. Walker and Zahnle (1986) have suggested that the 23.3 year period recorded by Trendall from the Weeli-Wolli formation reflects the period of the lunar nodal tide. We propose to draw on both of these suggestions to reconcile some apparent inconsistencies in the record and to arrive at an improved estimate for the sunspot period at the time of the Elatina formation 680 Ma ago.

2. Evolution of the Sunspot Period

Williams (1981, 1983) reported a basic ~11 varve periodicity in the thickness of laminae preserved in the 680 million year old Elatina Formation of South Australia. Longer periodicities of ~2, ~13, and ~26 basic cycles were also evident. In addition to these amplitude cycles there is also a long period phase alternation with a period of ~29 basic cycles (Williams and Sonett, 1985). Williams identifies the laminae with glacial varves; i.e., annual deposits in a 'periglacial' lake. Accordingly, Williams interpreted the ~11 and ~22 varve periodicities as the ~11 year sunspot and the ~22 year Hale (magnetic) cycles. The much longer cycles, called the 'Elatina cycles', were interpreted as hitherto unrecognized solar periods.

More recent developments may have put this intriguing interpretation in doubt. Noyes *et al.* (1984) present empirical evidence that, other things being equal, the length of starspot cycles for sun-like stars is proportional to the star's rotation period. This is in essential agreement with predictions of simple $\alpha\omega$-dynamos (cf., Stix, 1982; Robinson and Durney, 1982). As rotation among sun-like stars slows with age, it follows that the sunspot cycle would have been shorter 680 Ma ago than it is today. In the specific case of the Elatina varves, Noyes *et al.* have estimated that the solar cycle would then have been some 3% to 11% shorter than today, corresponding to ~10.8 and ~10.0 years, respectively; the former estimate assumes the modern rate of angular momentum loss via the solar wind, the latter the empirical $t^{-1/2}$ law for stellar rotation (Skumanich, 1972).

Although these estimates could probably have been reconciled with the 11.2 year periodicity originally reported, given the inevitable uncertainties in the geological record, Williams and Sonett (1985) now report that the basic cyclicity is 12.0 years. The resulting 1.2–2.0 year discrepancy between the astronomical and geological sunspot cycles is troubling enough; what is worse is that the astronomical estimate is shorter than the modern sunspot cycle, while the Elatina period is longer.

2.1. *Starspot Cycles*

Starspot cycles appear to be a universal attribute of older, slowly rotating ($P_{rot} > 20$ days), sun-like stars (Wilson, 1978; Vaughan *et al.*, 1981). The evidence for stellar cycles has been obtained directly by monitoring stars in the emission cores of the Ca II H and K lines. These emission cores are a measure of chromospheric activity, the variation of which is one conspicuous aspect of the solar cycle. On a shorter timescale, these emissions are correlated with active regions. As active regions are not symmetrically distributed in longitude and can last for several rotation periods, it is common for chromospheric emissions to be rotationally modulated, and thus accurate stellar rotation periods can be derived as well (Stimets and Giles, 1980; Vaughan *et al.*, Hallam and Wolff, 1981).

Although there is as yet no definitive theory of the solar cycle, the standard $\alpha\omega$-dynamos have enjoyed some successes (e.g., Yoshimura, 1983). These predict that both the α (helicity from rotation acting on convection) and ω (differential rotation) effects should be proportional to the rotation rate (among other things), and hence that the period of a cyclical $\alpha\omega$-dynamo should be linearly proportional to the rotation period (cf., Stix, 1982; Robinson and Durney, 1982). This basic prediction has now received some observational support from a sample of 13 stars. The empirical relation between stellar rotation and starspot periods may be expressed as (Noyes *et al.*, 1984)

$$P_{cyc} \alpha (P_{rot}/\tau_c)^{1.25 \pm 0.5}, \tag{1}$$

where P_{cyc} and P_{rot} are the starspot and rotation periods, respectively, and where τ_c is "the convective overturn time near the base of the stellar convection zone". This latter parameter varies as a function of spectral type. Following Noyes *et al.*, we will use Equation (1) to estimate the period of the solar cycle 680 Ma ago, at which time τ_c was only about 0.5% greater than today.

2.2. *Stellar Rotation*

The sun is spinning down through the loss of angular momentum to the solar wind. Its rotation period 680 Ma ago can be estimated by using either the increasingly well-determined empirical age-rotation relation for sun-like stars, or by extrapolating from the modern rate of solar angular momentum loss. The age-rotation relation has been expressed in the following forms, according to two recent surveys of stellar rotation among sun-like stars (Rengarajan, 1984; and Soderblom, 1985; respectively):

$$\Omega/\Omega_{\odot} = (t_{\odot}/t)^{0.46 \pm 0.10} \tag{2}$$

and

$$\Omega/\Omega_{\odot} = \frac{1.48}{\log_{10}(t_{\odot}/t) + 1.48}, \tag{3}$$

where $\Omega_{\odot}$ represents the modern solar rotational angular velocity, and $t_{\odot} \approx 4.6$ billion years is the present age of the sun. Although the two expressions fit the data equally well, they give somewhat discordant estimates of solar rotation 680 Ma ago. According to Equation (2), the sun was rotating $\sim 7.7 \pm 1.7\%$ faster, while according to Equation (3), the sun rotated $\sim 5.0\%$ faster. Taking the modern sunspot cycle to be 11.1 years, when used in Equation (1), these predict solar cycles of $\sim 10.1 \pm 0.5$ and $\sim 10.5 \pm 0.3$ years, respectively. The quoted error largely reflects the uncertainty in Equation (1).

By contrast, the comparable exercise using the modern rate of solar angular momentum loss is subject to huge uncertainties. According to Pizzo *et al.* (1983), the solar wind in the plane of the ecliptic is presently removing about $2 - 3 \times 10^{29}$ dynes-cm sr^{-1}. As the observations are necessarily confined to the ecliptic, there is a large uncertainty associated with the integration over solid angle. The magnitude of the relevant solar angular momentum reservoir seems to be at least as uncertain, since the question involves both the possibility that the core rotates much more rapidly than does the outer convective envelope (cf., Gough, 1982), and the degree of coupling between the core and the envelope, since the latter contributes only $\sim 1\%$ to the total moment of inertia (Newkirk, 1980). By making the simplest assumptions – that the angular momentum loss is spherically symmetric and that the sun rotates as a rigid body with moment of inertia 0.066 $M_{\odot} R_{\odot}^2$ – we get an estimate that the Elatina sun rotated $\sim$3–4% faster than today, corresponding to a $\sim$10.6 year solar cycle.

There is, therefore, a conflict between the astronomical evidence suggesting that the sunspot period was shorter in the distant past than it is today, and the evidence reported by Williams and Sonett suggesting that the period was longer. In the next section we suggest that the estimate of Williams and Sonett may be too large as a result of a systematic error that appears to be unavoidable in the counting of layers in an imperfect sedimentary record.

3. Counting of Varves

Counting errors arise because individual varves cannot always be unambiguously identified. There are irregular structures in the sediment as well as the regular annual variations. Temporary variations in precipitation conditions can eliminate the marker that divides the deposits of one year from those of the next, causing the accumulation of two years to be counted as one. Alternatively, a temporary fluctuation may be misinterpreted as the marker that divides one year from another, causing the deposits of a single year to be counted as two. Williams and Sonett estimate that this kind of uncertainty may introduce an error of 5% in the varve count. This estimate is apparently subjective.

Uncertainties introduced by irregularities in the sediment would undoubtedly be much larger were it not that the choice of what constitutes a varve can be guided by the thickness. A varve should be about as thick as its neighbours. The counter can ignore variations in the sediment that would suggest unusually thick or thin layers.

However, an insidious and asymmetrical error arises when the demarcation of the layers is ambiguous and there are natural, climatological fluctuations in the thickness of the varves. Substantial variations from year to year in varve thickness can be anticipated by analogy with such climatological parameters as precipitation, runoff, and ice melt. The true distribution of thicknesses of the Elatina varves is not known and is perhaps not knowable in view of the difficulties in identifying varves of unusually great or small thickness, but Williams and Sonett report that the varves at the minimum of the sunspot cycle have an average thickness of 0.19 mm with a standard deviation of 0.07 mm, and the varves at the maxima have an average thickness of 1.20 mm with a standard deviation of 0.47 mm. These standard deviations may be too small because of a tendency to overlook the values far removed from the mean ($\overline{\mu}$), splitting unusually thick layers and lumping unusually thin layers. Nevertheless, let us assume an average ratio of standard deviation to mean of 0.38 and assume that the layer thicknesses are log-normally distributed. Then, fully 7% of the layers have thicknesses greater than $2\overline{\mu}$ and another 7% have thicknesses less than $1/2\overline{\mu}$. When the demarcation of the layers is unclear, there must be a strong tendency to count the unusually thick layers as two varves and to combine the unusually thin layers into one varve.

Just how frequently an observer will split thick layers or lump thin layers is not known. The frequency of these errors must depend on the quality of the record. One reasonably objective criterion is to make the choice that yields the lowest standard deviation for the distribution of layer thicknesses. For a large number of layers in the distribution this criterion results in the splitting of layers with thickness in excess of $\sqrt{2}\overline{\mu}$ and the lumping of adjacent layers with thicknesses less than $\overline{\mu}/\sqrt{2}$. For purposes of illustration, let us use this objective criterion.

Let the probability of one unusually thick layer or one unusually thin layer be P. Then the probability of splitting – of the single layer being counted as two layers – is P. But the probability of lumping is P^2, because two layers can be counted as one layer only if both layers are unusually thin. Splitting and lumping therefore results in a systematic error in the count, with splitting inherently more probable than lumping. This systematic error is a relative overcount of $P - P^2$. Table I presents illustrative values of the error for a range of assump-

TABLE I. Counting error

Split/lump criterion (thickness relative to mean)	Probability P	Relative overcount $P-P^2$
1.4	0.15	0.13
1.6	0.13	0.11
1.8	0.10	0.09
2.0	0.07	0.07
Log-normal distribution.	$\frac{\text{Standard deviation}}{\text{mean}} = 0.38$	

tions concerning the criteria for splitting and lumping, assuming a log-normal distribution of varve thickness with the relative standard deviation quoted by Williams and Sonett. It appears likely that an overcount by 10% can quite easily occur unless the individual layers are very clearly demarcated. It is unlikely that the demarcation is invariably clear in a clastic sediment with varves as thin as 0.2 mm. Our suggestion of the possible importance of counting errors is supported by the observation by Williams and Sonett that solar cycles of thicker, more easily counted varves tend to have shorter periods while thinner, less easily discriminated varves yield periods apparently longer. Overcounting should introduce the same relative error in all directly measured periods in the time series, but will not affect values of the ratios of periods such as we use below.

It seems likely, therefore, that the estimate of the sunspot of the sunspot period by Williams and Sonett (1985) is susceptible to errors that are too large to allow a firm confirmation or contradiction of the astronomical estimate. In the face of the inherent uncertainty concerning systematic errors in the estimation of periods, is it possible to deduce a more accurate value for the sunspot cycle 680 Ma ago from the excellent Elatina record? In the sections that follow we show that it is possible to arrive at an improved estimate by using the longer period fluctuations in the record.

4. Interpretation of Periodicity in the Elatina Formation

The finest bands in the Elatina are identified with the seasonal cycle. These vary in thickness with a regular period of ~11–12 varves. This period has been identified with the sunspot cycle (Williams, 1981). The sunspot cycles themselves show a strong tendency to be alternately thicker and thinner, suggesting the presence of an additional period of approximately twice the sunspot cycle. Indeed, Williams (1981, 1983) and Williams and Sonett (1985) interpret this as the Hale cycle. However, the magnitude of the alternation varies with a period of about 14 solar cycles, and the phase of the alternation reverses at the same interval. This Williams and Sonett themselves point out, calling it the 'sawtooth pattern', for which they find a period of 29.2 sunspot cycles. The phenomenon clearly represents a beat pattern produced by the sunspot cycle and a period nearly but not precisely twice as long; a point that we will illustrate below. We believe that this longer period does not reflect the solar magnetic cycle. The Hale and sunspot cycles are presumed to be commensurate while the relevant periods in the Elatina record are plainly not commensurate. Instead their interaction yields the beat phenomenon analyzed in the following paragraphs. We identify the longer oscillation with the lunar nodal tide, an identification that we shall defend after presenting our analysis of the beat period in the Elatina record.

For a heuristic illustration of the beat phenomenon we write the impact of the two periods – one the solar cycle and the other differing from twice the solar cycle by a small quantity – as the sum of a squared sine wave representing the sunspot cycle and a smaller amplitude ($B < A$) sine wave representing the nodal tide:

$$\text{Forcing} = F(t) = A \sin^2(2\pi\nu_1 t) + B \sin(2\pi\nu_2 t). \tag{4}$$

The period $P_{\text{cyc}} \equiv (2\nu_1)^{-1}$ represents the period of the sunspot cycle; $P_{\text{nod}} \equiv 1/\nu_2$; and as will be shown the beat period P_{beat} is $1/\nu_3$, where $\nu_3 = \nu_2 - \nu_1$. Williams (1983) measures the thickness of solar cycles between successive minima. These occur near $2\pi\nu_1 t = n\pi$, where n is an integer. Writing $t_1 \equiv n/2\nu_1$, $t_2 \equiv (n+1)/2\nu_1$, and $\bar{t} \equiv (t_1 + t_2)/2$, the thickness of the n^{th} solar cycle may be written as

$$S_n = \int_{t_1}^{t_2} F(t)\,\mathrm{d}t = A\,\frac{t}{2} - A\,\frac{\sin(4\pi\nu_1 t)}{8\pi\nu_1} - B\,\frac{\cos(2\pi\nu_2 t)}{2\pi\nu_2}\Bigg]_{t_2}^{t_2} \tag{5}$$

$$= \frac{A}{4\nu_1} + \frac{B}{2\pi\nu_2}\,[\cos(n\pi(1+\nu_3/\nu_1)) - \cos((n+1)\pi(1+\nu_3/\nu_1))],$$

which reduces to

$$S_n = \frac{A}{4\nu_1} + (-1)^n\,\frac{B}{\pi\nu_1}\,\cos\left(2\pi\nu_3\,\frac{t_1+t_2}{2}\right)\cos\left(\frac{\pi}{2}\,\frac{\nu_3}{\nu_1}\right)$$

$$= \frac{A}{4\nu_1} + (-1)^n\,\frac{B}{\pi\nu_1}\,\cos(2\pi\nu_3\bar{t})\cos\left(\frac{\pi}{2}\,\frac{\nu_3}{\nu_1}\right). \tag{6}$$

Since $\nu_3 << \nu_1, \nu_2$, the second cosine factor is a multiplicative factor approximately equal to one. The envelope of the sawtooth pattern is seen to be a cosine wave with period $P_{\text{beat}} = 1/\nu_3$. The relation between the three periods may be written

$$P_{\text{cyc}} = \frac{1}{2}\,P_{\text{nod}}\left(1 \pm 2\,\frac{P_{\text{cyc}}}{P_{\text{beat}}}\right), \tag{7}$$

reflecting the fact that ν_2 may be either greater than or less than ν_1. The ratio P_{beat}/P_{cyc} is the sawtooth pattern of 29.2 solar cycles identified by Williams and Sonett.

In Figure 1 we have plotted the difference in width between adjacent solar cycles in the more precisely measured data presented by Williams (1983). We have alternated the sign between successive cycles to isolate the cosine wave. It is immediately apparent that the data would indeed approximate a sine wave but for an abrupt phase discontinuity at $n = 24$. This discontinuity is almost certainly due simply to a miscount; the cycle concerned also happens to be the widest in the published record. Furthermore, in their more recent paper Williams and Sonett implicitly state that there are in fact no such abrupt phase reversals when they write: "As any miscount would break the observed pattern of phase reversals, the persistence of the pattern confirms the accuracy of the long sequence". The result of replacing the thick cycle with two thinner ones is shown in Figure 2. The period of the matching cosine wave shown in Figure 2 is 28 solar cycles; the best fit was 28.4 P_{cyc}.

This period can also be identified in the published Fourier transform (Figure

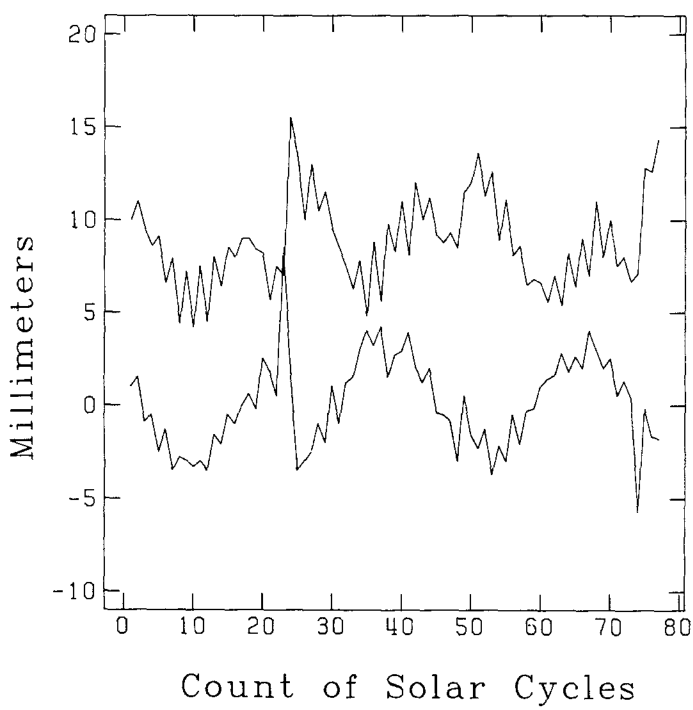

Fig. 1. The top curve is the thickness in millimeters of successive varve cycles (which are interpreted as sunspot cycles) as preserved in the Elatina formation of South Australia. The data have been adapted from the more accurately measured subset presented by Williams (Figure 4, 1983). The lower curve is the difference between successive sunspot cycles while alternating the sign; i.e., $\Delta \text{cycle}_n = (-1)^n (\text{cycle}_n - \text{cycle}_{n-1})$. We interpret the peak at $n = 24$ in the upper curve and the corresponding discontinuity in the lower curve as a missed (uncounted) cycle.

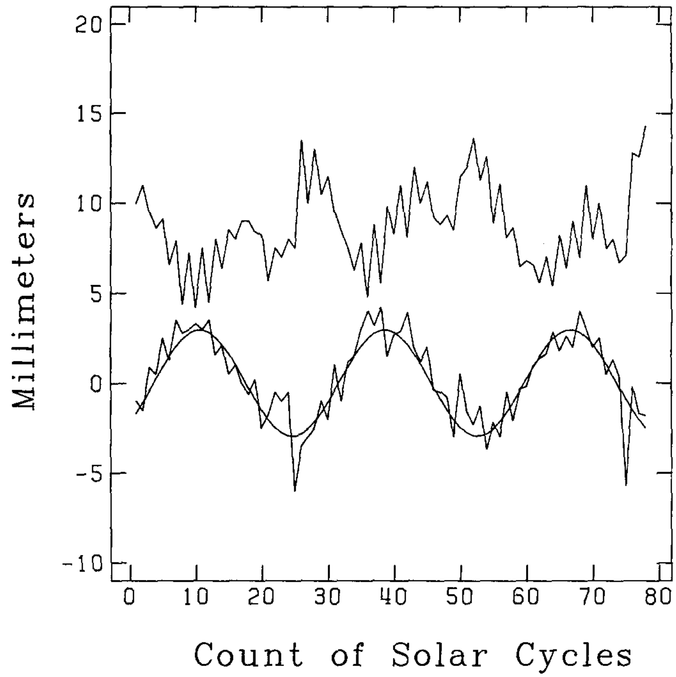

Fig. 2. Figure 2 is the same as Figure 1, save that it has been corrected for the presumed missed count by replacing the peak at $n=24$ with two cycles adding to the same total. The lower curve is then obtained in the same way as the lower curve in Figure 1. A sine wave of period 28.4 sunspot cycles provides a good fit. It represents the beating between the sunspot cycle and a cycle nearly but not exactly twice as long. We identify the longer period with the lunar nodal tide.

4, Williams and Sonett, 1985). In terms of the notation used there, we would identify the 22.4 vt ('varve time') cycle with the nodal tide. The solar cycle itself spans the range from 9.7 vt to 14.4 vt, centered at 12.0 vt. The two remaining short periods of 25.8 vt and 8.0 vt might then be explained as the sum and difference of the two fundamental frequencies: i.e., letting $P_{\text{cyc}} = 1/2\nu_1 = 12$ vt and $P_{\text{nod}} = 1/\nu_2 = 22.4$ vt, these periods are $P_{\text{diff}} \equiv 1/(2\nu_1 - \nu_2) = 25.8$ vt and $P_{\text{sum}} \equiv 1/(2\nu_1 + \nu_2) = 7.8$ vt. These periods would naturally arise if the climatic effects of the two fundamental periods were multiplicative, rather than simply additive. As an illustration, we might extend the reasoning behind Equation (4) to include these terms:

$$\text{Forcing} = F(t) = (1 + A\sin^2(2\pi\nu_1 t))\,(1 + B\sin(2\pi\nu_2 t))$$

$$F(t) = \left(1 + \frac{A}{2}\right) - \frac{A}{2}\cos(4\pi\nu_1 t) + \left(1 + \frac{A}{2}\right) B\sin(2\pi\nu_2 t)$$

$$- \frac{AB}{2}\sin(2\pi(2\nu_1 - \nu_2)t) - \frac{AB}{2}\sin(2\pi(2\nu_1 + \nu_2)t). \quad (8)$$

An advantage of an analysis concentrating on the beat phenomenon is that the ratio P_{beat}/P_{cyc} is not subject to the systematic error described above. Precise estimates of this ratio are possible. Our analysis of the published data of Williams (1983) yielded a value of 28.4 for this ratio. In what follows we will use the value of 29.2 deduced by Williams and Sonett (1985) from the entire database. If precise knowledge of this ratio can be combined with independent information on the longer period P_{nod}, then Equation (7) can be used to derive an estimate of the sunspot period P_{cyc} that is free of counting errors. We shall now present our reasons for identifying the longer period with the lunar nodal tide, and an estimate of the period of this tide at the time of deposition of the Elatina formation.

5. Lunar Nodal Tide

The period of the lunar nodal tide, presently 18.6 years, has been reported in several modern climate records. These include temperature and rainfall records from western North America (Currie, 1981, 1984b; Vines, 1982) and eastern Australia (Vines, 1982); tree ring data from both western North America (Stockton and Meko, 1983; Currie, 1984a, 1984b) and Patagonia (Currie, 1983); and perhaps the Indian monsoons (Campbell *et al.*, 1983), River Nile flooding (Hameed, 1984), and Beijing's rainfall (Hameed *et al.*, 1983; a claim disputed by Clegg and Wigley, 1984).

The lunar nodal tide arises from the precession of the moon's orbital plane about the ecliptic. As the moon's orbital plane is inclined by ~5 deg to the ecliptic, and the ecliptic inclined ~23.5 deg to the equator, the inclination of the lunar orbit to the equator varies between ~18.5 deg and ~28.5 deg. The precession period depends on the Earth-Moon distance. It can be represented by the expression (cf., Kaula, 1969)

$$P_{nod} = 18.6\left(\frac{\cos(I_0)}{\cos(I)}\right)\left(\frac{a_0}{a}\right)^{3/2} \text{ years,} \quad (9)$$

where the modern values of the lunar distance, $a_0 = 3.844 \times 10^{10}$ cm $\approx 60.27\ R_\oplus$, and the inclination of the lunar orbit to the ecliptic I are denoted by the subscript 'o'. As a practical matter tidal evolutionary changes in I are negligible compared to changes in a for $a > 40\ R_\oplus$ (cf., Goldreich, 1966), which encompasses virtually all of Earth's history.

5.1. *Evolution of Lunar Distance*

Evidence for the rate of lunar recession is available on several different timescales. As these matters are a major topic of three recent books (Lambeck, 1980; Brosche and Sündermann, 1978, 1983), we will not dwell on them here. As determined by various direct, indirect, and historical means, the moon is currently receding with a velocity of $\sim 1.1 - 1.3 \times 10^{-7}$ cm sec^{-1}. This is helpful, but the only important source of long term information is paleontological (cf. Scrutton, 1978, and Lambeck, 1980, for thorough reviews). In brief, fine banding found in certain fossil corals and molluscs is interpreted as daily growth increments or as the record of the semidiurnal or diurnal tides, in accordance with the growth habits of their modern descendants. Modulation of the fine banding by the fortnightly or monthly tidal cycles and/or by the yearly seasonal cycle is also evident. From these data Lambeck (1980) has deduced an average Phanerozoic lunar recession velocity of $\langle \dot{a}_0 \rangle \approx \sim 1.0 \times 10^{-7}$ cm sec^{-1}.

Tidal energy dissipation occurs today primarily in the oceans (Zschau, 1978; Lambeck, 1980; Platzmann, 1985). If it is assumed that tidal energy dissipation is directly proportional to the tidal energy (i.e., a constant Q factor), and if solar tides and the inclination and eccentricity of the lunar orbit are neglected, the recession of the moon due to the semidiurnal tide may be approximated by

$$\dot{a} = \langle \dot{a}_0 \rangle \, (a/a_0)^{-11/2}. \tag{10}$$

The lunar distance is therefore

$$a(t) = a_0 \left(1 - \frac{13}{2} \frac{\langle \dot{a}_0 \rangle}{a_0} \, t \right)^{2/13}. \tag{11}$$

Equation (11) has the disadvantage of predicting an Earth-Moon collision 1.9 billion years B.P. Since there is no evidence in the geological record of such a cataclysm, it is safe to conclude that tidal friction was smaller during the Precambrian than during the Phanerozoic. Interpreted as the lunar nodal period, Trendall's 23.3 ± 0.3 year periodicity in the Weeli-Wolli banded iron-formation places the moon at $a/a_0 = 0.861 \pm 0.008$ at 2.5 Ga (Walker and Zahnle, 1986). The result of combining a single average rate of Precambrian tidal friction consistent with this datum and a single average Phanerozoic rate consistent with the paleontological data can be expressed

$$\frac{a(t)}{a_0} = \frac{a_1}{a_0} \left(1 - \frac{13}{2} \, (t - t_1) \, \frac{\langle \dot{a}_1 \rangle}{a_1} \right)^{2/13}, \tag{12}$$

where $a_1 = a(t_1 = 550 \text{ Ma}) = 0.948 \, a_0$, and where

$$\langle \dot{a}_1 \rangle = 0.32 \langle \dot{a}_0 \rangle \, (a_1/a_0)^{-11/2} = 0.43 \, \langle \dot{a}_0 \rangle.$$

This particular tidal history implies that the Earth-Moon collision took place at 4.7 Ga, which is in accord with the stability of the lunar orbit over the lifetime of the solar system. According to Equation (12), the lunar distance 680 Ma ago was $\sim 0.943 \, a_0$, so that at that time $P_{nod} = 20.3$ years.

6. Length of the Sunspot Cycle 680 Million Years Ago

In the preceding sections, we have summarized astronomical evidence that predicts the sunspot cycle to have been around 10.3 years 680 Ma ago, and the (largely) paleontological evidence constraining the lunar distance at this time. We have presented compelling evidence that the shorter periods observed in the Elatina Formation arise from two fundamental periodicities, one slightly less than twice as long as the other. If we identify the longer of these

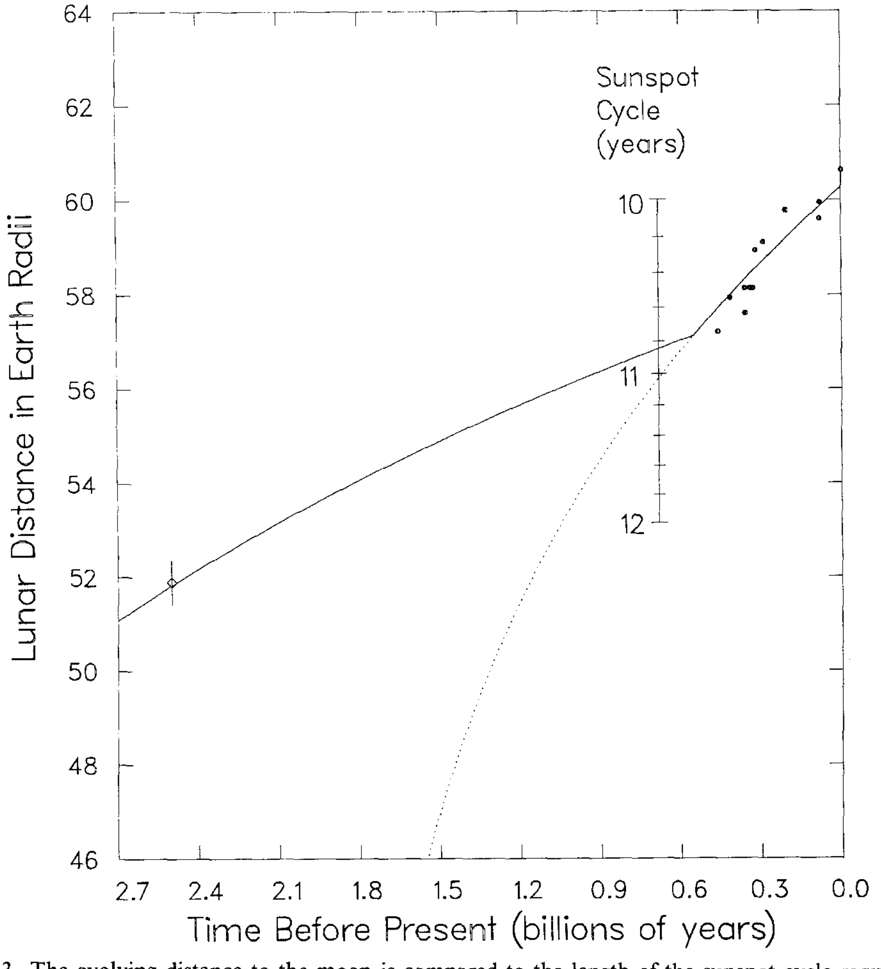

Fig. 3. The evolving distance to the moon is compared to the length of the sunspot cycle required by our interpretation of the periodicities present in the Elatina formation. The individual points represent lunar distances derived from the paleontological data (after Scrutton, 1978); the dotted curve is an extrapolation of the Paleontological data (implying an Earth-moon collision 1.9 Ma ago); and the solid curve, which is consistent with the stability of the lunar orbit over the life of the solar system, we previously obtained by identifying a 23.3 year periodicity in the 2.5 billion year old Weeli-Wolli banded iron-formation with the lunar nodal tide (Walker and Zahnle, 1986). Superimposed is the sunspot period (in years) indicated by equation (7). The beat period seen in Figure 2 relates the length of the sunspot cycle to the period of the lunar nodal tide. The latter is primarily a function of the lunar distance, so that a given lunar distance corresponds to a particular sunspot period. Our estimate of a ~10.8 year sunspot cycle is consistent with independent astronomical estimates that the sunspot cycle would then have been 3–10% shorter 680 Ma ago than it is today (Noyes *et al.*, 1984).

with the lunar nodal tide, we can derive a value for the shorter period, believed to be that of the sunspot cycle. The estimate is illustrated in Figure 3. The two curves representing lunar distance are Equations (11) and (12). Also shown are lunar distances derived from the paleontological data for days per year (after Scrutton, 1978). The length of the sunspot cycle has been recast in terms of lunar distance by means of Equations (7) and (9).

Our best estimate for the length of the sunspot cycle is $\sim 10.8 \pm 0.2$ years, which we obtain by inspection of Figure 3. The small uncertainty in our estimate reflects only its insensitivity to the lunar distance. Although this value is still somewhat longer than the preferred astronomical estimates (viz. 10.1 ± 0.5 and 10.5 ± 0.3 years), there is no longer any conflict. Indeed, it may simply imply that slowly spinning stars lose angular momentum somewhat less rapidly than has heretofore been thought. On the other hand, our estimate, like the astronomical estimate, is considerably shorter than that deduced by Williams and Sonett from direct counting of varves, a discrepancy that we attribute to systematic errors in the counting process.

7. Conclusion

Is the Elatina Formation actually a record of the sunspot cycle 680 million years ago? We do not know. It is puzzling, to say the least, that something as indistinct in the modern climate as the sunspot cycle should apparently dominate a local climate 680 Ma ago, but should have left no other comparable records yet discovered. But no serious alternative to the sunspot explanation has yet been offered.

The apparent preservation of the sunspot cycle and the lunar nodal tide in the Elatina implies enormous amplification of very small changes in climatic forcing. We have elsewhere pointed out (Zahnle and Walker, 1987) that the solar semidiurnal atmospheric tide (these tides are thermally forced by absorption of incoming sunlight by atmospheric water vapor and ozone) was resonant with free oscillations of the atmosphere when the day was ~ 21 hours long. This took place some 600 million years ago. Very large atmospheric tides would have resulted, with associated surface pressure oscillations in excess of 10 mbars. Near resonance the solar gravitational torque on the semidiurnal atmospheric tide was of comparable magnitude and of opposite sign to the lunar torque on the oceanic tides; hence the resonance was very probably in effect 680 million years ago. It is therefore conceivable that a nearly resonant atmospheric tide could be part of the sought-for amplifying mechanism.

Here we have suggested that if the shorter periodicities present in the Elatina are indeed ascribed to modulation of glacial varves by the solar cycle and the lunar nodal tide, then paleontological constraints on the distance to the moon imply that the sunspot cycle 680 Ma ago was 10.8 ± 0.2 years. We point out that systematic errors inherent to counting varves would lead the observer to overestimate the number of varves per cycle, especially for cycles characterized by thinner varves. It is not coincidental that cycles of relatively thick varves average ~11 rather than 12 varves in the analysis of Williams and Sonett. Although our result is still somewhat longer than the astronomically derived estimates of 10.1 ± 0.5 and 10.5 ± 0.3 years, it is not so long as to preclude this explanation of the shorter Elatina periods. On the other hand, the much longer Elatina cycle $P_{\mathrm{Elat}} = 26.2\ P_{\mathrm{cyc}}$ (which is an amplitude cycle to be distinguished from the 29.2 P_{cyc} beat period) is not obviously explained solely in terms of P_{cyc} and P_{nod}. As Williams and Sonett point out, that P_{Elat} is not the same as P_{beat} is shown by secular changes in their relative phase. The regularity of the Elatina cycle is striking, as is its absence from the modern sunspot record.

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