CRACK TIP EFFECTIVE STRAIN RATES IN RATE SENSITIVE MATERIALS

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Simultaneous pairs of values of K (stress intensity factor) and R (fracture toughness, or equivalently strain energy release rate G) for quasi-statically running cracks in compact tension testpieces of polymethylmethacrylate have been determined at various crack velocities ($\dot{a}$) and temperatures (T) [1,2]. K were arrived at from the Gross-Srawley expression (see, for example [3]) using current crack lengths and loads, and R was measured by Gurney's segmental area methods [4].

Presuming that they are related by $K^2 = ER$ (forgetting the Poisson's ratio term), where E (Young's modulus) and R are both rate and temperature sensitive, it follows that E ($\dot{a}$,T) can be determined from K^2/R and references to E found in independent simple tensile tests carried out at various T and $\dot{\varepsilon}$ (tensile strain rates). In this way, experimental effective $\dot{\varepsilon}(\dot{a},T)$ at the crack tip can be established. Crossplotting $(E = K^2/R)_{T,\dot{a}}$ and $(E_{\text{tensile test}})_{\dot{\varepsilon},T}$ in the range $10^{-4} < \dot{a} < 10^{-2}$m/s, gave

$$\dot{\varepsilon} \approx 0.13\dot{a} \qquad (1)$$

where the relationship was independent of temperature within our accuracy. For $\dot{a} > 10^{-2}$m/s, $\dot{\varepsilon}$ rose less steeply, again independent of T and flattened off at about $\dot{\varepsilon} = 10^{-2}$ as seen in Figure 1.

Williams [5] has given for the strain rate at a moving crack tip,

$$\dot{\varepsilon} \approx \pi\varepsilon_y{}^3(E/K)^2\dot{a} \qquad (2)$$

where ε_y is the yield strain, and where E and K are rate dependent. A similar expression may be arrived at from Irwin's crack tip stress rate equation [6]. In [1,2] independent relationships were derived for R($\dot{a}$,T) and the tangent modulus E($\dot{\varepsilon}$,T), which are a toughness-biased Ree-Eyring expression

$$\dot{a} = A_1 \exp[-(U - \lambda R)/kT] \qquad (3)$$

and

$$E = 12.12\dot{\varepsilon}^{0.0087} - 0.0268T \qquad (\text{GN/m}^2) \qquad (4)$$

where A_1 is a constant, k is Boltzmann's constant, λ is the activation area, and U is the activation energy. Amending (2) to

$$\dot{\varepsilon} \approx \pi\varepsilon_y^{\,3}(E/R)\dot{a} \tag{5}$$

and substituting for E and R, we obtain a relation between $\dot{\varepsilon}$ and $\dot{a}$, viz:

$$\dot{a} \approx (U/\lambda)\dot{\varepsilon}/\{\pi\varepsilon^3 10^9 (12.12\dot{\varepsilon}^{0.0087} - 0.0268T) \tag{6}$$

where $T(k/\lambda)\ \ell n\ \dot{a}/A_1$ is omitted since it is small in comparison with U/λ, which is some 1.62 kJ/m^2 [2]. Notice that this $\dot{\varepsilon}$ *vs* $\dot{a}$ relation is dependent upon temperature, whereas our experimental relation is too coarse to pick that up. On Fig. 1 are superimposed the predictions of (6) using $\varepsilon_y \approx 0.003$, the value of which was found by trial and error to bracket the experimental results. The trends are acceptable, but two comments must be made.

The value of ε_y seems very low in comparison with the critical strain level for room temperature craze initiation of 0.013 quoted by Kambour [7], or typical PMMA yield strains of $\varepsilon_y = 0.02$. In fact $\varepsilon_y = 0.003$ corresponds with the *offset* which produces $\varepsilon_y = 0.02$ in PMMA. Again, (6) could be made to agree with the experimental $\dot{\varepsilon}$ *vs* $\dot{a}$ relation, independent of temperature, by using different ε_y at every temperature. For example, at $\dot{\varepsilon} = 10^{-5}s^{-1}$, $\varepsilon_y \to 0.0027$ for T = 283 deg K, but $\varepsilon_y \to 0.0035$ for T = 353 deg K. Such changes are very small, and demonstrate the sensitivity of (2) to cubing ε_y. A discussion of the definition of yield stress and strain in polymers, and how secant moduli (used by Williams [5]) may be affected, is presented in [8].

Secondly, (6) predicts a continuously increasing $\dot{\varepsilon}$ *vs* $\dot{a}$ relation whereas in fact there are limiting velocities for each temperature beyond which crack tip adiabatic heating produces instabilities (see e.g., [2,9]. Thus, there are cut-off points in the $R(\dot{a})$ relation (3); for example, at T = 283 deg K, $\dot{a} \not> 3\times10^{-2}m/s$; at T = 353 deg K, $\dot{a} \not> 5$ m/s. It is interesting that these limiting velocities coincide with the region of the experimental $\dot{\varepsilon}$ *vs* $\dot{a}$ data where the $\dot{\varepsilon}$ level off, to some $\dot{\varepsilon} = 10^{-2}s^{-1}$. The fact that such lower $\dot{\varepsilon}$ occur than are predicted by (6) fits in with the occurrence of adiabatic heating, because in general terms the equation predicts that lower $\dot{\varepsilon}$ are produced at higher T.

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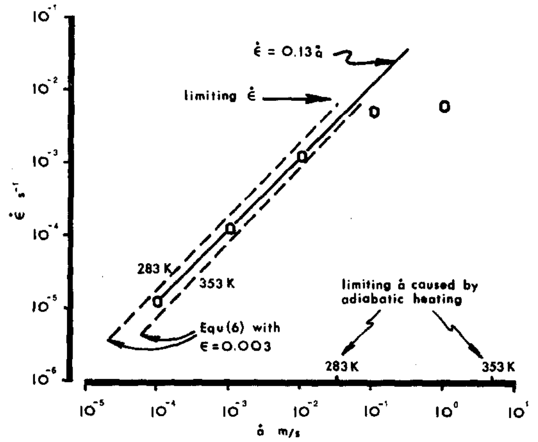

Figure 1. Experimental and Theoretical Relations Between $\dot{\varepsilon}$ and $\dot{a}$. 0-experimental Points