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Reply to 'Comments on "Determining the toughness of plastically deforming joints"'

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A paper by Thouless *et al.* [1] reported an analysis and experimental results for the wedge-peel test. In this test, two bonded metal strips are fractured by means of a wedge inserted along the interface. The strips are sufficiently thin to ensure that plastic bending of the metal accompanies fracture. By measuring the radius of curvature of this plastic deformation, the critical bending moment for fracture can be deduced. A steady-state energy-balance that relates the toughness of the joint to the radius of curvature was reported [1]. This analysis permits an estimate of the joint toughness to be calculated from a measurement of the radii. Kinloch and Williams argued that a correction should be made to this approach [2]. Unfortunately, the proposed correction is incorrect and contains inherent logical flaws.

At the heart of the proposal by Kinloch and Williams [2] is the concept of what is sometimes known as "root rotation". This concept is demonstrated in Fig. 1a which shows a beam of modulus E bonded to a substrate and loaded by a force P applied to one end. The length of the crack is a. When calculating the energy-release rate for this geometry, it is convenient to assume that the adherend acts like a beam with a built-in end at the crack-tip. This gives a result for the energy-release rate of $\mathcal{G} = 6P^2a^2/Eh^3$, which is an excellent approximation when the crack is very long. However, in practice, deformation occurs in a "process zone" immediately ahead of the crack. This deformation may result from the adhesive being stretched before failure, or it may be associated with the effect of the bonding stresses acting on the substrate or adherend. The simplifying assumption that the arm is "built-in" at the crack tip over-constrains the system, and under-estimates the energy-release rate. This effect can be corrected for by incorporating an elastic-foundation model proposed by Kanninen [3] into the analysis [4, 5]. The arm is modeled as a beam supported on an elastic foundation, as shown in Fig. 1b. The displacement of the beam ahead of the crack-tip can be derived from simple-beam theory, and the energy-release rate can be shown to be [5]

$$\mathcal{G} = \frac{6P^2a^2}{Eh^3}\left[1 + \frac{\Delta}{a}\right]^2 \qquad (1)$$

Δ can be taken as an estimate of the length of the process zone, and is approximately equal to $2h/3$ for a homogeneous system. In other words, the effective crack length can be considered to be increased by the presence of a process zone.

It is very important to appreciate that the presence of a process zone causes complications only when the crack length needs to be measured to determine the energy-release rate. There are a whole class of important steady-state problems in which the process zone translates in a self-similar fashion with the crack tip, and the energy-release rate is independent of crack length. For these geometries, the energy-release rate can be calculated by an energy-balance approach, and the result is not affected by the existence of a process zone. An illustration of the notion that root-rotation only influences solutions to non-steady state problems is provided by a numerical analysis of the elastic 90°-peel test. This analysis is performed using an embedded-process-zone (EPZ) model [6–8]. The normalized peel forces, P/Γ_0, for two simulations corresponding to root-rotation angles of 56° and 23° are plotted in Fig. 2 as a function of the normalized displacement along the peel direction (normalized by the adherend thickness). It is evident that the steady-state value of the peel force is independent of the root-rotation angle, and is given by $P/\Gamma_0 = 1.0$, as predicted by an energy-balance analysis [9]. However, the peel force is influenced by root

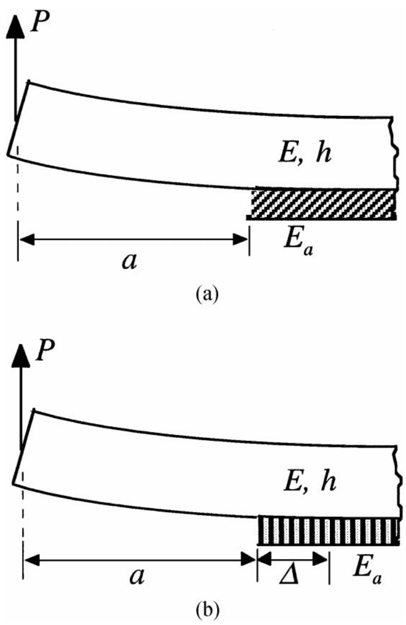

Figure 1 Elastic-foundation model for an adhesively-bonded cantilever beam.

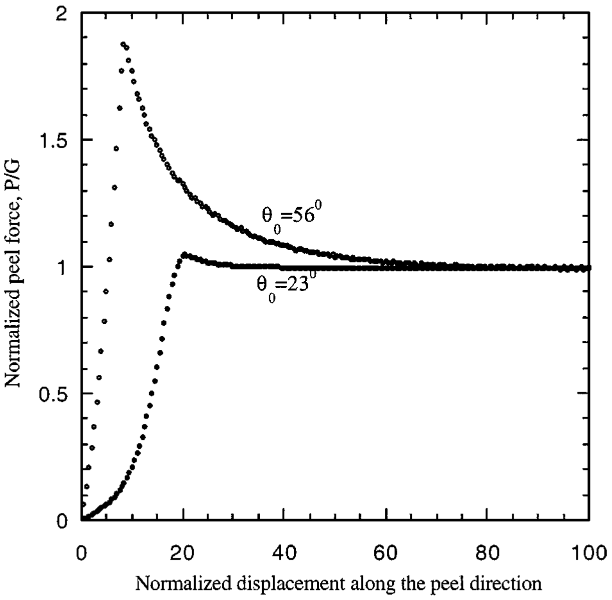

Figure 2 Numerical results of the normalized peel force plotted as a function of the normalized displacement in the peel direction. The material and interface properties were chosen so that the root rotations are 56° and 23° in the two simulations.

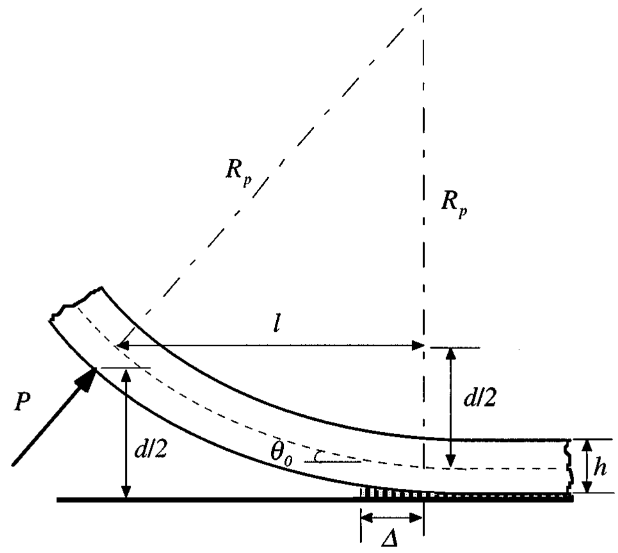

Figure 3 Illustration of the steady-state wedge-peel configuration. d is the diameter of the wedge tip, R_p is the radius of curvature after deformation of the arms, Δ is the process zone length, and l is crack length calculated by Kinloch and Williams [2] to be given by $l^2 = R_\mathrm{p}d$.

rotation under non-steady-state conditions where the geometry is evolving. An energy-balance argument for elasto-plastic peel tests can be used to a similar effect [8]. Details of the geometry near the crack-tip can affect neither the work done by the steady-state peel force nor the energy dissipated by bending within the adherend.*

The analysis of the wedge-peel test by Thouless *et al.* [1] was based on the assumption that the wedge imposes a bending moment on the sample, and that failure occurs in response to this bending moment. Since the analysis was based on a steady-state energy balance, no assumptions were made about the location at which the force applied by the wedge acts. As discussed above, this means that root-rotation effects are of no concern. In their correction, Kinloch and Williams [2] used the measured radii to back out the critical moment, as was done in the original paper [1]. They then used the geometry of the problem to deduce l, the distance from the point-of-action of the wedge to the point-of-adhesion. They calculated this as $l = \sqrt{R_\mathrm{p}d}$, where R_p is the radius of curvature and d is the diameter of the wedge-tip (Fig. 3). It can be seen from Fig. 3 that this calculation of l assumes that the point-of-adhesion is the point-of-zero-rotation of the adherends. Therefore, l already includes the length of the process zone. However, in Kinloch and Williams [2], the length of the process zone (assumed to be equal to $2h/3$) was added again to l, and a new effective crack length was obtained. This new effective crack length was used to adjust the value of the critical bending moment that had been calculated from the experimental measurement of R_p. An increased interface toughness was then calculated from this new moment. In summary, the proposed correction to the toughness is associated with two physical inconsistencies. In the first place, the equation used to calculate l implicitly assumes that l includes any possible process zone. It is incorrect to add the length of the process zone a second time. In the second place, the "corrected" moment is inconsistent with the measured radii of curvature from which the original calculation of the moment had been deduced in the first place.

The results of some recent embedded-process-zone model calculations for the wedge-peel test shed some additional interesting light on this problem [7]. These numerical analyses show that the energy-balance calculation presented in Ref. [1] are correct provided that fracture is dominated by bending and that the peak stress supported by the adhesive is negligible. Under these conditions, the toughness is given by

$$\frac{\Gamma_0}{Ah} = \frac{2n}{(n+2)(n+1)}\left(\frac{h}{\sqrt{3}R_\mathrm{p}}\right)^{n+1} \tag{2}$$

where R_p is the measured radius of curvature after deformation, and h is the thickness of the adherends which are assumed to deform according to a power-law relationship of the form $\sigma = A\varepsilon^n$, A and n are material constants, and σ and ε are stress and strain.† As described in Ref. [7], there are two competing effects that affect the use of this equation. If the adhesive exerts a substantial stress on the adherends, the resulting hydrostatic constraint hinders yielding of the metal. Equation 2 then gives an over-estimate of the toughness. On the other hand, if the wedge is relatively thin, then the assumption that bending dominates the fracture process becomes invalid. Equation 2 then gives an under-estimate of the toughness.

* The analysis of the peel test by Kinloch *et al.* [10] implicitly assumes that only the energy associated with bending beyond the crack tip is dissipated. Any energy that may be dissipated by bending of the peel arm within the process zone is incorporated into the quantity denoted as the adhesive fracture energy. This approach of distinguishing between bending in the two regions naturally leads to a root-rotation effect.

† This equation has been corrected to account for plane-strain deformation which is appropriate when the arms of the wedge-peel test are wide.

The results of Ref. [7] suggest that the wedge-peel test is perhaps best performed using a sequence of increasing wedge sizes until a constant radius of curvature is obtained. Equation 2 can then be used to obtain an upper-bound for the toughness. (The numerical calculations also suggest that this upper-bound estimate becomes closer to the toughness as the adherend thickness decreases. In practice, this may mean that the experiment should be set up to obtain the smallest possible radius of curvature.) Re-analyzing the data of Ref. [1] using the embedded-process-zone technique has shown that the values of toughness originally quoted in the paper were approximately correct for the aluminum joints. Apparently, the geometry and deformation were such that the two competing effects described in the previous paragraph effectively canceled each other out. However, it has also been shown that the assumption of bending-dominated fracture was not met for the results pertaining to steel joints. A numerical analysis of the experimental results has shown that the toughness of the aluminum and steel joints (which failed cohesively) were comparable [11]. Finally, it should be noted that the values of toughness calculated using the proposed correction of Ref. [2] are not consistent with the values calculated by the numerical analysis.

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