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HOW SIMILAR ARE THE DIFFERENT RESULTS?

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Introduction

An increasing number of studies in the social sciences utilize Smallest-Space Analysis-I (SSA-I) (e.g., Laumann and Guttman, 1966; Schlesinger and Guttman, 1969; Levy and Guttman, 1975a, b; Elizur and Guttman, 1976; Levy, 1976; Guttman and Guttman, 1976; Ben-Sira, 1977; Maimon, 1978). This paper deals with some aspects of comparisons between SSA-I results, and analyzes existing methods for comparing two or more SSA-I solutions obtained from different populations or from different samples of the same population. Alternative measures of the degree of similarity between the various solutions are also discussed.

We begin by very briefly discussing the SSA-I technique. Then, we move to the questions involved in comparing SSA-I solutions. A particular technique for comparison, PINDIS, is then explained in some detail. Finally, the results of an empirical study utilizing four SSA-I solutions are analyzed by PINDIS and by other approaches.

Smallest-Space Analysis-I (SSA-I)

The SSA-I technique (or algorithm) is a member of a family of SSA techniques developed by Guttman (1968) and Lingoes (1973), which represents a symmetric matrix of the coefficients of similarity or dissimilarity among variables by a configuration of points in Euclidean space, where each variable corresponds to a point in this space. The algorithm places each variable in the space according to the rule: if $r_{ij} > r_{kl}$, then $d_{ij} < d_{kl}$, where r is the coefficient of similarity and d is the Euclidean distance between the two variables. The technique is designed to arrange the points in the least number of dimensions. The goodness of fit, for each number of dimensions, is given by a coefficient of alienation which varies from 0 to 1, lower values indicating better fits.

SSA is mainly aimed at testing hypotheses about the structure of interrelationships and is not intended as a cluster analysis. Such hypotheses are tested in terms of the partitioning of the space into separate regions having specific geometric shapes. The analysis is greatly facilitated if the definitional system for the variables is presented by a Mapping Sentence (Guttman, 1970; Levy, 1976; Borg, 1977a) and the hypotheses relate the definitional system to the empirical structure of the results (see for example, Levy and Guttman, 1975a; Levy, 1976).

The comparison between any two SSA-I solutions has often been performed by identifying a certain pattern in the structure of the interrelationships, such as a RADEX (explained below; see also Guttman, 1954), and comparing visually the degree to which the geometric representations of the two solutions are similar (see for example, Levy, 1976). Such a method has the advantage of relating the two solutions to a theoretical framework and a set of hypotheses, since the hypothesis of similarity of the two solutions is expressed in terms of the similar geometric representation. While this is the approach advocated by the users of SSA-I, it may be problematic, since the number of dimensions in each solution may differ. A common practice of SSA-I users has sometimes been to use only a two-dimensional projection from a three-dimensional solution, or a three-dimensional representation of a four-dimensional solution (e.g., Levy and Guttman, 1975a). A four-dimensional solution, however, provides the analyst with six two-dimensional projections, and there is no widely agreed criterion for choosing the appropriate planar projection from the six. One may, therefore, be tempted to choose from the six projections (in the four-dimensional solution) that particular projection which is most similar to the projection from the two-dimensional solution. And while it may be easily seen from the geometric representation that the two two-dimensional projections are, indeed, similar, one may still feel somewhat uncomfortable about disregarding the remaining five projections. An additional problem posed by approaches such as the above is the subjectivity implicit in any judgments involving similarity among configurations.

Another method of comparing two or more SSA-I solutions could be to compute the degree to which the rank order of the cells, in each one of the matrices to be analyzed by SSA-I, is the same; that is, to compute some weak monotonicity measure of association (such as Goodman and Kruskal's Gamma, or Guttman's correlation coefficient of weak monotonicity) between any two matrices having the same number of elements. We note, however, that if such a measure of weak monotonicity association is unity, the SSA-I solution must be similar; the reverse, however, is not true. The SSA-I solutions can still be similar geometrically when the two matrices are only weakly related to each other (see Lingoes, 1977, for uses of CS-I and the limitations of such global measures of similarity).

Another problem of comparing SSA-I solutions has to do with the considerable latitude that may possibly exist in drawing the regional boundaries if the number of variables in the analysis is small. The smaller the number of variables, other things being equal, the higher the chances of success in drawing similar geometric shapes from the configuration of points in the space. In such a case a visual comparison may be insufficient to convince the reader that the two configurations are, indeed, similar.

We shall now propose one possible solution to the above difficulties by adapting an individual differences model to the assessment of configurational similarity.

The Procrustean Individual Differences Scaling (PINDIS) Technique

The questions raised above led us to believe that the methods described for comparing SSA-I results should be supplemented by a mathematical comparison of the properties of the matrices analyzed and produced by SSA-I (see also Lingoes, 1977, p. 677). The PINDIS procedure is very much suited to such a task. The Lingoes–Borg model hierarchy (Borg, 1977b; Lingoes and Borg, 1978) is an algorithm intended to compare structures of interrelationships among variables for individuals or groups of individuals (see the Appendix at the end of this paper for an algebraic explanation of this procedure; further details are given in Lingoes and Borg, 1978).

The procedure starts from a set of individual configurations, the X_i's given by a prior analysis such as SSA-I. There is a separate configuration for each member of the set of N individuals. Each configuration consists of n points in m dimensions [1]. As in SSA-I, only the comparative distances between the points in the configuration are meaningful. Thus, any such configuration may be re-scaled or transformed (e.g., centrally dilated or shrunk) as long as such a manipulation does not change the rank order of distances among points. In those analyses where only *relative* distances matter (e.g., SSA-I), such a transformation will not change the meaning of the configuration.

After transforming all the configurations X_i such that their Euclidean norm is unity and their column sums are zero, an average configuration is computed. This average configuration is labelled the *centroid configuration*, or Z. Of course, Z has the property that the sum of the squared distances between its points and the corresponding points over all the X_i's is minimal.

The question now arises as to how well this average configuration represents each and all of the X_i's. The PINDIS algorithm uses the following two measures of similarity between Z and X_i. The first measure is inversely related to the squared Euclidean distance between Z and X_i, $r^2(Z, X_i)$. Since all matrices have a norm of unity and zero column sums, this measure is equivalent to the square of Pearson's product-moment correlation between the coordinate values of Z and X_i. In the language of regression analysis, $r^2(Z, X_i)$ is the proportion of variance shared by these two configurations. An overall measure of fit, or similarity, is the second measure generated by PINDIS, which is the average of the squared correlations between Z and all the X_i's, namely $\bar{r}^2(Z, X.)$.

One possible way of stressing the degree of configurational similarity is by dimensional weighting. Recall that each configuration is given in an m-dimensional space. It may well be that an inherent similarity could show up more clearly if the dimensions of Z were weighted differently. Note again that such a weighting procedure may enable us to identify or stress aspects of similarity that might otherwise be more difficult to notice, such as differential saliences or importances of the dimensions among configurations. The dimensional weighting may be carried out, for example, by dilating or shrinking differentially the various axes of Z. Here again, we have two measures of similarity plus the corresponding overall measures. The first denotes the proportion of shared variance between Z and the X_i's, where Z is dimensionally weighted and the X_i's are rotated optimally to Z, namely, $r^2(ZW_i, X_i)$, where W_i denotes the dimensional weights. The second is the following: since dimensional weighting does not change the meanings of the configurations, one may wish to weight Z differently for each X_i. Such a procedure may yield a further similarity between Z and the X_i's.

PINDIS, therefore, provides us with another measure, $r^2(Z_i^r W_i^r, X_i)$, where, for each i, Z is in an optimal orientation towards X_i, and $W_i^r \neq W_i$ in general.

Nevertheless, the similarity may still be somewhat blurred because all the variables in the matrix X_i are weighted identically for a given dimension. If the variables could be weighted differently, we might be able to identify certain regularities of structure that would otherwise be more difficult to see. PINDIS generates here a measure of vectorially weighted similarity (the perspective model), $r^2(V_i Z, X_i)$, which is the proportion of shared variance between Z and the X_i's after weighting the different variables in Z and rotating the X_i's optimally with respect to Z. Corresponding to the idiosyncratic rotation of Z, i.e. Z_i^r above, we can have idiosyncratic origins for Z, denoted by Z_i^t, which gives rise to a further measure of fit, $r^2(V_i^t Z_i^t, X_i)$, where $V_i^t \neq V_i$.

Business Studies and the Development of Managerial Skills – A Comparison of Four SSA-I Solutions

The problem of evaluating the quality of training given to students of business administration has drawn some attention in recent years (Livingstone, 1971; Harrell, 1972; Weinstein and Srinivasan, 1974; Leavitt, 1975). However, it is difficult to find reports on empirical investigations concerning the degree to which academic study of Business Administration develops in the students the necessary skills and qualifications required for the successful performance of managerial roles; or, put somewhat differently, the degree to which Business Studies do, indeed, assist the graduates in doing their jobs. Here, we re-analyze the results of a previous study conducted in 1976 among graduates of a business school in Israel (Maimon and Guttman, 1976; Maimon, 1977). We use this study to demonstrate the analysis of data either using common methods or with the aid of the PINDIS procedure.

The population studied consists of four groups: 200 graduates of an M.B.A. (Master of Business Administration) program; 47 superiors at the work-places of those graduates (practically all M.B.A. students in Israel are fully employed during their studies), who had known the graduates at least during and after the studies; 100 middle-level managers who, for a year, had participated in a management development program in a business school; and 20 superiors at the work-places of those managers, who had known the managers before and after the one-year training program (for full details of the study, see Maimon and Guttman, 1976; Maimon, 1977).

A major question in that study dealt with the degree to which the studied population felt that participation in Business Studies programs helped the graduates to develop skills that are required for the performance of managerial tasks. The conceptual framework was conveniently summarized in the following mapping sentence (for a recent discussion of the concept of mapping sentence and facet analysis, see Borg, 1977a).

A mapping sentence for the evaluation of the degree to which business administration studies have contributed in the development of the skills required by managerial jobs

Respondent (X) evaluates the degree to which business administration studies helped the graduate develop A. *Type of Skill*
a_1 instrumental
a_2 cognitive
skills in the area of B. *Area of Skills*
b_1 communication
b_2 leadership
b_3 analysis
b_4 professional ability

The evaluation is $\left\{\begin{array}{c}\text{very favorable}\\ \text{to}\\ \text{very unfavorable}\end{array}\right\}$ in terms of the contribution of the

particular skill to the performance of managerial jobs.

After four extensive pre-tests among the four populations, nine indicators were operationalized according to the mapping sentence, as follows:

$a_1 b_1^*$ the ability to establish contact with people and the ability to negotiate with people
$a_1 b_1^*$ the ability to express oneself
$a_1 b_2^{**}$ the ability to take initiative
$a_1 b_2^{**}$ leadership
$a_1 b_2^{**}$ the ability to control and organize people
$a_1 b_4$ the ability to solve professional problems
$a_2 b_2$ the ability make decisions

a_2b_3 the ability to judge correctly
a_2b_4 the ability to diagnose existing problems and to identify potential ones

Items marked * and ** have the same facet structure and, thus, could be considered replicates. In another design they would probably be differentiated on other facets; here they are not. One would expect from facet theory, however, that replicates would be more highly correlated among themselves than they would be with non-replicates (see Borg, 1977a).

Data were gathered by a structured questionnaire for each of the four populations. The questions concerning skill development were phrased in the following way:

"In a previous study of the Israel Institute of Applied Social Research about the management climate in Israel, the following qualifications, which are necessary for a manager, were mentioned among several others. In your opinion, to what degree did your studies in the M.B.A./Diploma program help you develop these qualifications?"

The same questions were presented to the superiors at work with a small modification:

"... to what degree, based on your experience with graduates of the M.B.A./Diploma program, did the studies help develop these qualifications?"

The nine skills were worded in the same way as the nine profiles mentioned above. The responses were coded on a five-point scale from "very much" to "very little".

One of the hypotheses that we had planned to test in the original study was that the structure of interrelationships among the nine skills, in all four populations, was that of a RADEX (Guttman, 1954; Schlesinger and Guttman, 1969; Levy, 1976; Lingoes and Borg, 1977), i.e., that it is a combination of a Simplex and a Circumplex [2]. In facet-analytic terms, it means the combination of at least two facets, one operating as a modulating facet, while the other one is a polarizing facet (see below).

In this study, we hypothesized that facet A should turn out to be a modulating facet. A modulator's elements correspond to the relative distance from the origin [3]. The hypothesis was that all the skills with the element a_2, that is, cognitive skills, would form, in SSA-I terms, a circular region with a smaller diameter than the circular region of the skills having the element a_1. A smaller circle, in SSA-I terms, means that the intercorrelations among the variables in that circle are, generally, larger than the intercorrelations among the variables in a circle with a

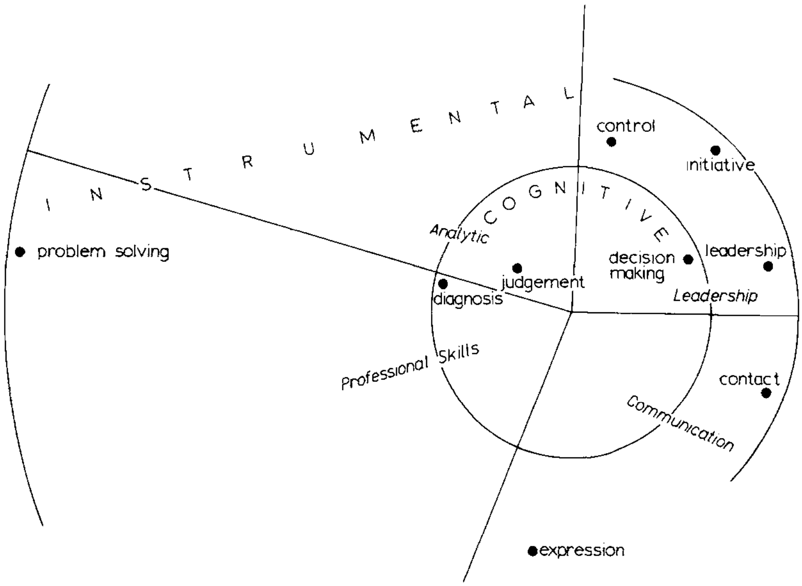

Fig. 1. A two-dimensional SSA-I solution for the M.B.A. (coefficient of alienation = 0.10).

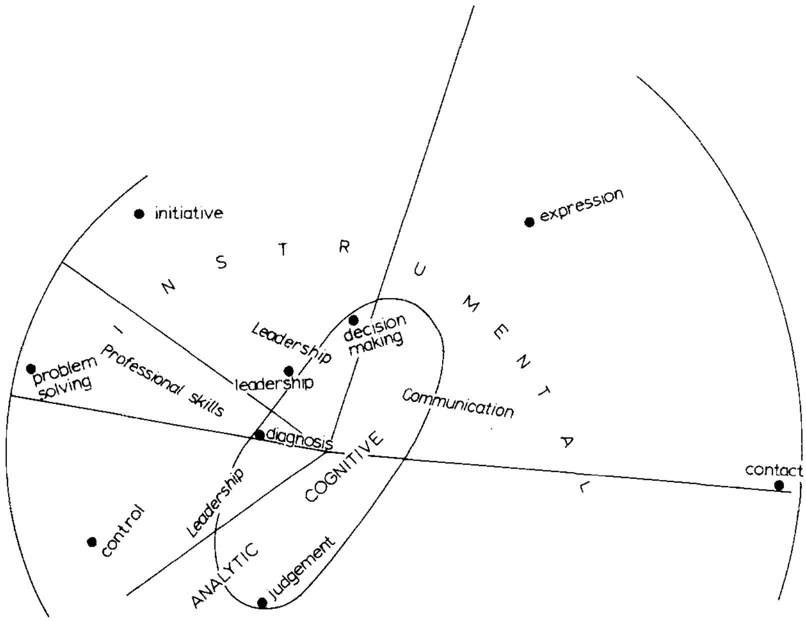

Fig. 2. A two-dimensional SSA-I solution for the M.B.A. superiors (coefficient of alienation = 0.12).

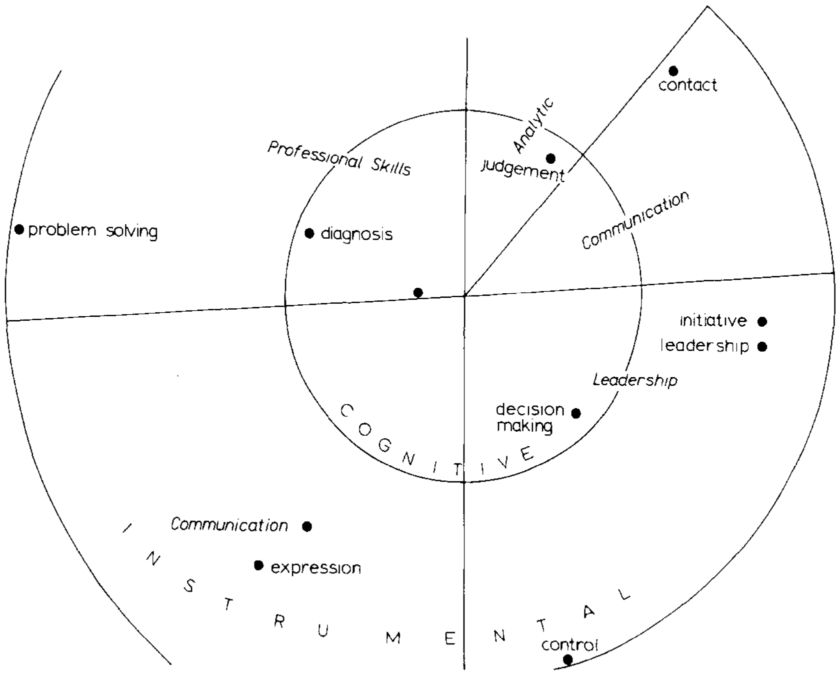

Fig. 3. A two-dimensional SSA-I solution for the Diploma graduates (coefficient of alienation = 0.18).

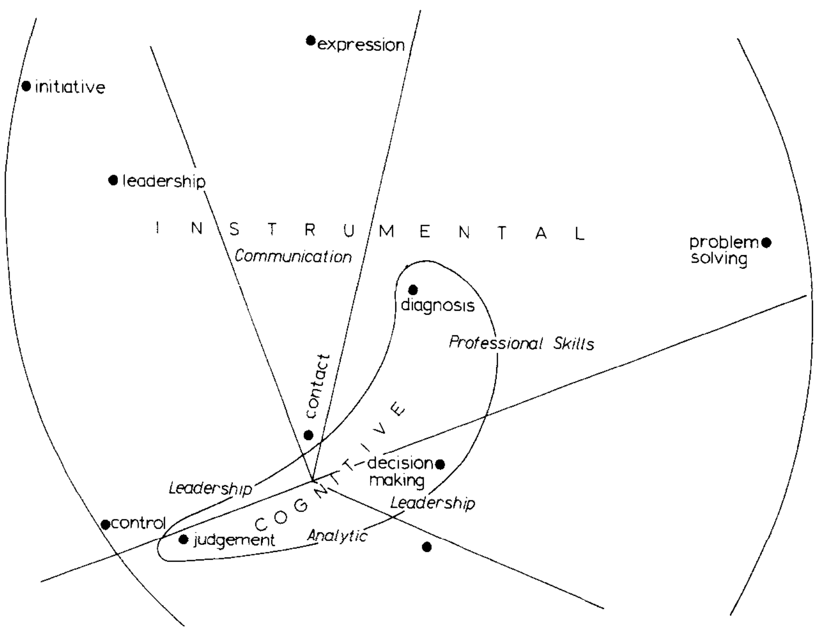

Fig. 4. A two-dimensional SSA-I solution for the superiors of the Diploma graduates (coefficient of alienation = 0.18).

TABLE I

Weak Monotonicity Intercorrelation Coefficients for the M.B.A. (above the diagonal) and the Superiors of the M.B.A. (below the diagonal)

	Initiative	Judgement	Contact	Expression	Control	Decision making	Leadership	Problem solving	Problem diagnosis
Initiative		0.75	0.62	0.41	0.64	0.77	0.72	0.23	0.69
Judgement	0.68		0.58	0.60	0.78	0.71	0.68	0.51	0.82
Contact	−0.05	0.33		0.61	0.57	0.79	0.79	0.18	0.56
Expression	0.59	0.22	0.63		0.45	0.56	0.53	0.25	0.58
Control	0.57	0.79	0.21	0.21		0.80	0.75	0.28	0.64
Decision making	0.71	0.59	0.51	0.81	0.63		0.88	0.40	0.56
Leadership	0.82	0.71	0.53	0.70	0.73	0.97		0.15	0.58
Problem solving	0.70	0.52	0.13	0.49	0.74	0.59	0.63		0.51
Problem diagnosis	0.56	0.75	0.32	0.59	0.76	0.93	0.98	0.90	

TABLE II

Weak Monotonicity Intercorrelations for the Diploma Graduates (above the diagonal) and the Superiors of the Diploma Graduates (below the diagonal)

	Initiative	Judgement	Contact	Expression	Control	Decision making	Leadership	Problem solving	Problem diagnosis
Initiative		0.77	0.78	0.59	0.66	0.91	0.93	0.56	0.60
Judgement	0.39		0.83	0.68	0.60	0.74	0.76	0.56	0.70
Contact	0.32	0.91		0.75	0.44	0.65	0.78	0.56	0.62
Expression	0.50	0.26	0.61		0.69	0.65	0.58	0.64	0.64
Control	0.07	0.92	0.89	0.40		0.73	0.74	0.49	0.66
Decision making	0.37	0.62	0.89	0.20	0.64		0.80	0.61	0.80
Leadership	0.94	0.28	0.68	0.92	0.69	0.67		0.53	0.69
Problem solving	−0.28	−0.22	0.57	0.48	0.18	0.44	0.23		0.75
Problem diagnosis	0.53	0.80	0.66	0.61	0.31	0.91	0.66	0.42	

larger diameter. The rationale behind this hypothesis is that instrumental skills are more dependent on personality traits and previous socialization experience, which academic training can influence very little, if at all. Since different instrumental skills may be related to different personality characteristics, there is no reason to assume, a priori, that the program will be successful, to similar degrees, in developing these skills; and different levels of success in skill development imply smaller correlations, on the average, than similar levels of success. Cognitive skills, on the other hand, may be better manipulated by the educational process and one would therefore expect to find that the degree of their development is more similar.

Facet *B* was hypothesized to be a polarizing one [4]. The polarizer's elements correspond to the different directions away from the origin in the plane. Operationally, one would expect, in SSA-I terms, to be able to clearly identify four separate regions in the space according to the elements of facet *B*. The interpretation of such a finding would be that the studied populations do, indeed, differentiate among the four areas. Such a differentiation is relevant to the evaluation of the quality and applicability of business training; the training need not be successful to the same degree in all four areas.

To test these hypotheses, a matrix of Guttman's correlation coefficient of weak monotonicity (Guttman, 1977; Lingoes, 1973) for the nine skills was computed for each one of the four populations and then subjected to SSA-I. These matrices are presented in Tables I and II. Figures 1–4 present the SSA-I solutions. To facilitate visual comparisons we chose the two-dimensional solution, although a three-dimensional solution would have produced a "cleaner" presentation for Figs. 3 and 4.

Note that in all four figures facet *A* is a modulating one while *B* is a

TABLE III

Weak Monotonicity Correlation Coefficients Between the Four Matrices

	M.B.A. graduates	Superiors of M.B.A.	Diploma graduates	Superiors of the Diploma graduates
M.B.A. graduates	–	0.23	0.81	0.53
Superiors of M.B.A. graduates	0.23	–	0.53	0.05
Diploma graduates	0.81	0.53	–	0.43
Superiors of the Diploma graduates	0.53	0.05	0.43	–

TABLE IV

PINDIS Analysis of the Matrices in Figures 1 and 2

Population	$r^2(X,Z_i)$	$r^2(ZW_i,X_i)$	$r^2(Z_i^r W_i^r,X_i)$	$r^2(V_i Z,X_i)$	$r^2(V_i^t Z_i^t,X_i)$
I. M.B.A.	0.82	0.84	0.84	0.93	0.97
II. Superiors of M.B.A.	0.44	0.50	0.56	0.90	0.93
III. Diploma graduates	0.54	0.57	0.58	0.90	0.98
IV. Superiors of the Diploma graduates	0.59	0.60	0.64	0.85	0.99
Mean r^2	0.60	0.63	0.66	0.90	0.97

polarizing facet. However, although the geometric shapes are quite similar, there exist some dissimilarities: in Fig. 2, the variable "control" is located in a different region from the other leadership variables; in Fig. 3 the communication variables are located in two different regions; and in Fig. 4 "decision making" is separated from the other decision-making variables.

Now, there is an additional point to be considered when trying to assess the degree of similarity between the four solutions. The number of variables, nine, is small and allows the analyst considerable freedom in drawing the boundary lines between the different regions. Furthermore, the origin in each figure *is not central* to the empirical distribution of the points in the map. Rather, it is determined by the partitionings related to the two facets, which means that there is a lot of freedom in its determination.

Given the geometrical similarity on the one hand, and on the other keeping in mind both the dissimilarities and the great latitude in shaping the geometrical similarity, the question we pose is: to what extent are the solutions similar?

A simple test was mentioned earlier: compute some weak monotonicity measure of association among the four matrices and check the degree to which the figures approach unity. Table III presents Guttman's weak monotonicity correlation coefficients among the four matrices.

It is easily seen from Table III that whereas there is a high degree of similarity in the structures of the interrelationships of the nine skills between the M.B.A. and the Diploma graduates, the magnitudes of the other coefficients are such that we still cannot assess the degree of similarity.

At this stage, we subjected the four matrices to PINDIS. Table IV summarizes the results of this analysis.

Discussion and Conclusion

Our hypothesis, that the structures of the interrelationships among the nine skills are similar for the four groups, is further supported by PINDIS. This support comes from the column of $r^2(V_iZ, X_i)$ and $r^2(V_i^tZ_i^t, X_i)$'s, which have high values. The first three columns in Table IV corroborate our findings from Table III, namely, that the four matrices have different degrees of similarity to the centroid. This is inferred from the high variability of the r^2's in each column of Table IV. Column 4, that of vector weighting (fixed origin), indicates, on the other hand, that the four configurations have, on the average, 90% of their variance in common with the centroid. Furthermore, the difference between the highest figure in that column and the smallest is 0.08, compared to 0.38 in the first column, 0.34 in the second, and 0.28 in the third. In the fifth column, that of vector weighting (idiosyncratic origins), the differences between the configurations disappear almost completely.

Figures 1–4 indicate a certain regularity in the data, one that can best be expressed in terms of a RADEX. A visual inspection of the figures indicates support both for the hypothesis of similarity of structure, as well as for the proposition that the structure is that of a RADEX. On the other hand, however, the SSA-I solutions require different numbers of dimensions, the number of variables in the analysis is small (which gives the analyst much freedom in drawing the demarcation lines between the various regions in the space), and the rank order of interpoint distances is not very similar in the four matrices (cf. Table III).

Nevertheless, vector weighting of these matrices by the PINDIS procedure results in a very strong pattern of similarity, indicating strong support for the hypothesis in addition to that provided by visual comparison of Figs. 1–4.

The degree of similarity of structures, in SSA-I terms, should be determined by the extent to which a specific set of hypotheses about the configuration of the points in the space, and the partitioning of the space into distinguishable regions, is repeatedly supported for the different populations. But the use of PINDIS in order to further substantiate the results becomes necessary, the more freedom one has in shaping particular geometric forms from the points in the space.

Acknowledgement

We are grateful to Pergamon Press for permission to reprint Tables 2–5 and Fig. 1 from Maimon, Z. (1980), "Business Studies and the Development of Managerial Skills", *Studies in Educational Evaluation, Vol. 6*, pp. 83–97.

Notes

1 The analysis is also valid if not all the X_i's have the same dimensionality.
2 A Simplex is (in SSA-I terms) a simple ordering of the variables or subsets of the variables, in lines that do not bend back upon themselves. A Circumplex is a circular ordering of the variables in a geometric shape that approximates a circle (see Lingoes and Borg, 1977, for a more rigorous definition of these manifolds).
3 An example of a modulating facet is the case where the variables are represented by two circumplexes: an inner one and an outer one.
4 An example of a polarizing facet is a group of four Simplexes each in a different direction from the origin.

Appendix – An Algebraic Explanation of the PINDIS Procedure

The data which the PINDIS procedure uses is a set of N matrices X_i, where each X_i of dimension $n \times m$ characterizes some individual or a set of individuals.

The first step which PINDIS takes is in centering each matrix X_i, so that its column sums vanish, and normalizing each matrix to 1. Then the program transforms each X_i as follows:

$$\hat{X}_i = X_i R_i + j t_i' \tag{1}$$

where R_i is an $(m \times m)$ matrix, j is an $(n \times 1)$ vector of ones, t_i is an $(m \times 1)$ vector, and the prime on t denotes the transposition operator. The matrices R_i and the vectors t_i are chosen to minimize

$$L = \mathrm{tr}(\hat{X}_i - Z)(\hat{X}_i - Z)' \tag{2}$$
$$= \sum_a \sum_b (\hat{X}_i^{ab} - Z^{ab})$$

where $Z = (1/N)\ \Sigma\ X_i$; tr is the trace operator (which gives the diagonal sum of a squared matrix); and Z^{ab}, X_i^{ab}, are the (a, b) elements of Z and X_i, respectively.

The matrix Z obtained from (1) represents the centroid matrices.

The matrices $\hat{X}_i$ obtained from (1) will replace the X_i's in the subsequent analysis (they will become the "new" X_i's and the "hat" on them will be omitted). Two measures of closeness will be obtained from (1): the proximity of population i to the centroid will be measured by

$$r^2(Z, X_i) = \sum_a \sum_b (Z^{ab} - X_i^{ab})^2$$
$$= \operatorname{tr}(Z - X_i)(Z - X_i)' \; ; \qquad (3)$$

and the overall similarity of the individuals to each other will be measured by

$$r^2(Z, X.) = N^{-1} \sum_{i=1}^{N} r^2(Z, X_i) \qquad (4)$$

The dimensional weights are obtained by the following minimization:

$$\min_{Q_i, W_i} \operatorname{tr}(XW_i - X_iQ_i)(ZW_i - X_iQ_i)' \quad i = 1, ..., N \qquad (5)$$

where W_i and Q_i are $(m \times m)$ diagonal matrices. The result of (5) is denoted by $r^2(ZW_i, X_i)$.

The individually chosen dimensional weights are obtained by the following minimization:

$$\min_{Q_i, W_i, S_i} \operatorname{tr}(ZS_iW_i - X_iQ_i)(ZS_iW_i - X_iQ_i)' \qquad (6)$$

where Q_i, W_i, S_i are $(m \times m)$ diagonal matrices. The result of (6) is denoted by $r^2(Z_iW_i, X_i)$.

The vector weights are obtained by solving

$$\min_{V_i} \operatorname{tr}(V_iZ - X_i)(V_iZ - X_i)' \qquad (7)$$

where the matrices V_i are $(n \times n)$ diagonal matrices. The result of (7) is denoted by $r^2(V_iZ, X_i)$.

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