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Equilibrium Values in a Competitive Power Exchange Market

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Abstract. We consider an open electricity market with demand uncertainty. In this market, the generators each decide on a bidding price to maximize profit. Units are dispatched in order of the bid from lowest to highest until demand is satisfied. The market clearing price is the highest bid among the dispatched units. All dispatched units are then sold at this market clearing price. Under a market stability assumption, we derive Nash equilibrium solutions, i.e., bidders' optimal bidding strategies and the resulting market clearing price.

Key words: bidding, Nash equilibrium, electric power

1. Introduction

In a deregulated electricity market, generators sell most of their electric power into the wholesale market, where utilities that distribute to consumers purchase the power. Deregulation is expected to eliminate the monopoly system and encourage competition among generators. A desired result is that generators operate their plants in the most efficient way to sell power at the cheapest rate to consumers. In this paper, we will show conditions for equilibrium prices and demonstrate that efficient generation plans do not always result from deregulated markets.

Electricity industries around the world are going through deregulation. The United Kingdom began to deregulate its electricity industry in 1990, followed by Australia in 1994, Norway and Sweden in 1996, and New Zealand in 1996. Many states in the U.S. are currently restructuring their electricity industries. In March 1998, when the California Power Exchange (PX) began operating, it became the largest deregulated retail electricity market in the world (CAPX-FAQ, 1998). In the California Power Exchange, generators and buyers submit their bids to construct supply and demand curves. The supply curve provides the amount of power available as a function of the selling price. The demand curve reflects the electric demand as a function of the buying price. The point where these two curves intersect determines the spot price – the trading price of electric power for all participants in the market.

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In California, there are 2 types of electricity markets: the Power Exchange (PX) and real-time Balancing Market. In a day-ahead Power Exchange, generators and buyers submit their bids 24 hours in advance. The market is cleared on the day before the actual dispatch. Since the PX is cleared before the actual dispatch, the actual consumption of electricity may not be the same as the amount traded in the PX market. Therefore, suppliers also bid in a real time balancing market before the actual demand is revealed so that trade adjustment can be made in real time to satisfy the demand. More details about the California markets can be found in CAPX-FAQ (1998), Jacobs and Singh (1997), Moore and Anderson (1997), and Wilson (1997).

In such markets, it is expected that generators and buyers will play games, i.e., bid strategically to take advantage of market conditions. Even though stranded cost recovery legislation (Nix, 1999) may make generators unlikely to play games during the four-year initial period (Borenstein and Bushnell, 1997), but, in Year 2002, they are expected to do so. An interesting question is then to find the market spot price. The painful experience in the Midwestern market on 28 June 1998, when the spot price normally at $30 per Megawatt-hour surged to $7,000 (Kranhold and Emshwiller, 1998), is one of the reasons why the answer to this question is critical. Futures are also now available in most electricity markets so that a wide range of investors require pricing information. To find the market spot price, we need to understand the bidding behavior of the market participants.

Most of the work related to open electricity markets has focused on the structure of efficient markets and the characteristics of market participants in the UK market. Several economists model the UK power market as an oligopoly or duopoly and use the supply function equilibrium approach (see Klemperer and Meyer, 1989) to prove the existence of equilibrium points (for examples, see Bolle (1992) and Green and Newbery (1992)). To do so, the bids of one generator, called the supply function, are assumed to be continuously differentiable. Von der Fehr and Harbord (1993), on the other hand, assume a step supply function. They model the UK market as a sealed-bid multiple-unit auction and find that pure strategy equilibria do not always exist in their model. Von der Fehr and Harbord (1993) also show that the particular types of equilibria under the assumption of continuously differentiable supply functions cannot be generalized to a model with a step supply function assumption, which is more realistic. In another study, Anderson and Philpott (1998) study a duopoly model with a continuously differentiable supply function and derive sufficient conditions for the existence of symmetric supply function equilibria. They also derive bounds on the loss of revenue when actual supply functions are step functions.

Instead of assuming the UK market to be an oligopoly or duopoly, Gross and Finlay (1996) assume perfect competition, i.e., the bid of any bidder has a negligible effect on the spot price. They formulate the UK market as a nonlinear programming problem and show that generators will optimally bid at costs.

Simulation approaches have also been applied in several studies of open electricity markets. Richter and Sheblé (1997), for example, use genetic algorithms to evolve the bidding strategies of participants in a double auction market. MacGill and Kaye (1998) propose a decentralized coordination framework and use their evolutionary programming approach to simulate the Australian market. By using simulation, however, equilibrium strategies of participants may not always be derived.

In most non-sealed bid markets, one of the activity rules indicates that a new bid price submitted into the market must be at least the previous winning bid minus a small margin to avoid infinite adjustments of the bid prices. This rule motivates us to assume the finite and discrete set of possible bid prices, which represents a departure from previous work that assumes all possible bid prices are in a continuous range. This assumption allows bidders to undercut each others' bids and, therefore, destroys most potential Nash equilibria that can be obtained from a discrete set assumption.

The goal of this paper is to find a Nash equilibrium value of the market spot price, under the assumption that all participants behave optimally. By 'optimal', we mean that the expected profit of each participant is maximized. We propose both deterministic and stochastic demand models and then present the market stability condition in Section 2. The characteristics of pure strategy Nash equilibrium points in both models are derived in Section 3. Algorithms for finding Nash equilibrium points are developed in Section 4. Numerical examples are presented in Section 5. For simplicity, in the rest of the paper, the word 'equilibrium' always refers to Nash equilibrium.

2. Model Description

To model open electricity markets, we first consider a simplified structure that captures key properties. The insights drawn from these results can help us understand the real system. We model the Power Exchange and real time balancing markets as multi-round non-sealed bid auctions. In the PX market, the total demand d is known to all bidders and can be appropriately modeled as deterministic; however, in the Balancing Market, where generators submit their bids before demand is realized, demand is assumed to be stochastic. We denote the balancing stochastic demand by a random variable D. We also assume that all bidders have the same belief on the demand distribution, which we denote by F. To be consistent with the case of deterministic demand, the realization of the demand in the stochastic case is also denoted by d.

In our models, a bidder submits a quantity of power that he/she is willing to generate and an acceptable price for a unit of this power, called the *bid price*. Bidders who offer lower prices are dispatched before the ones with higher prices. We assume that demand must be satisfied. The *market spot price* or *market clearing* *price (MCP)* is the highest bid price among the dispatched units. All dispatched units are sold at the same price, i.e., the spot price.

We consider a market with N bidders. We assume that Bidder i incurs a generation cost of c_i for each unit of electricity and bids for a fixed quantity x_i at a price p_i. We denote a vector of bids $[(x_1, p_1), \ldots, (x_N, p_N)]$ by $\mathbf{b}$, called a *bid vector*. The goal of Bidder i is to choose x_i and p_i so that his/her expected profit is maximized.

When demand D is revealed, the market clearing price π, as a function of the realization d and the bid vector $\mathbf{b} = [(x_1, p_1), \ldots, (x_N, p_N)]$, can be computed mathematically as:

$$\pi(\mathbf{b}, d) = \left\{ \min_j p_j : \sum_{i \in I(j)} x_i \geq d \right\}, \tag{1}$$

where $I(j) = \{i : p_i \leq p_j\}$.

A bidder whose bid price is equal to the market clearing price is called a *marginal bidder*. A bidder whose bid price is less than the market clearing price is called an *under-bidder*. An under-bidder i is fully dispatched; i.e., the market consumes x_i. A bidder i whose bid price is higher than the spot price π is not dispatched. Marginal bidders are dispatched in the order of the submission time of their bids. That is, earlier bidders are dispatched before later bidders. Let $\mathcal{S}$ be the set of marginal bidders $\{i | p_i = \pi\}$. The notation $|\mathcal{S}|$ denotes the number of bidders in $\mathcal{S}$ and $x(\mathcal{S})$ denotes the set of x_i submitted by marginal bidders; i.e., $x(\mathcal{S}) = \{x_i | i \in \mathcal{S}\}$. We assume that all bidders in $\mathcal{S}$ are equally likely to bid first, second, third, and so on.

In order to compute the expected dispatch of Bidder i, we first define the recursive function:

$$\overline{q}_i(d', x(\mathcal{S}), n) = \begin{cases} 0 & \text{if } d' \leq 0, \\ \frac{1}{n}\min(x_i, d') \\ \quad + \frac{1}{n}\sum_{j=1, j\neq i}^{n} \overline{q}_i(d' - x_j, x(\mathcal{S} - \{j\}), n-1) & \text{otherwise} . \end{cases}$$

Then, the expected dispatch quantity of Bidder i for given $\mathbf{b}$ and d is:

$$q_i(\mathbf{b}, d) = \begin{cases} 0 & \text{if } p_i > \pi(\mathbf{b}, d), \\ x_i & \text{if } p_i < \pi(\mathbf{b}, d), \\ \overline{q}_i(d - \sum_{j \in \{i | p_i < \pi(\mathbf{b}, d)\}} x_j, x(\mathcal{S}), |\mathcal{S}|) & \text{if } p_i = \pi(\mathbf{b}, d). \end{cases} \tag{2}$$

We assume that each bidder knows perfectly the other bidders' costs and bid quantities and that all bidders behave optimally in order to maximize their expected profits. Our goal is to find equilibrium prices and bid quantities for all bidders or agents, $i = 1, \ldots, N$. Let

$$f_i(\mathbf{b}) = E_D[\ (\pi(\mathbf{b}, D) - c_i)\ q_i(\mathbf{b}, D)\ ] \tag{3}$$

be the expected payoff to Agent i given a bid vector $\mathbf{b}$, a possibly random demand D, a marginal price π, that depends on the bids and demand, and a quantity q_i that represents Agent i's production or *dispatch quantity* that also depends on all bids and the demand.

We search for a Nash equilibrium, $\{p_i^*, i = 1, \ldots, N\}$, such that

$$\begin{aligned} & f_i([(x_1, p_1^*), \ldots, (x_i, p_i), \ldots, (x_N, p_N^*)]) \leq \\ & \quad \leq f_i([(x_1, p_1^*), \ldots, (x_i, p_i^*), \ldots, (x_N, p_N^*)]) \end{aligned} \tag{4}$$

for all feasible bid prices, p_i, for Agent i, and all $i = 1, \ldots, N$. Under a differentiability assumption on f, the condition in (4) is equivalent to finding a solution to the variational inequality,

$$(\mathbf{b} - \mathbf{b}^*)^T V(\mathbf{b}^*) \leq 0, \tag{5}$$

for all $\mathbf{b} = [b_i = (x_i, p_i), i = 1, \ldots, N]$ feasible, where $\mathbf{b}^* = [(x_i^*, p_i^*), i = 1, \ldots, N]$ and

$$V = \begin{bmatrix} \nabla_{b_1} f_1 & 0 & 0 & 0 \\ 0 & \nabla_{b_2} f_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \nabla_{b_N} f_N \end{bmatrix}.$$

In most non-sealed bid markets, the activity rules indicate that a new bid price submitted into the market must be at least the previous winning bid plus a small margin. Therefore, we assume that the bid price must be in the discrete set $\{l\epsilon \mid l = 0, 1, \ldots, O\}$, where O is a large integer and ϵ is a small number. This set is called the set of *possible bid prices*. The discrete bid price set assumption is a change from previous analyses, which assume a continuous set of possible bid prices. We assume also that ϵ is so small that $|c_i - c_j| > 2\epsilon$ for all $i, j \in \{1, \ldots, N\}$ and $i \neq j$. Furthermore, O is so large that $O\epsilon > c_i$ for all $i \in \{1, \ldots, N\}$.

Since $\mathbf{b}$ is practically in discrete space and is not continuous, instead of solving the variational inequality in (5), we solve (4) directly. We give cases where such solutions exist and characterize the structure of the solutions. In future work, we may consider the continuous version as given in (5) to solve the variational inequality directly. The discrete version, however, gives us insight into the structure of the equilibria and reflects reality.

We assume that the exchange market is *stable*, which is defined as the condition when the market clearing price is strictly less than the highest possible bid price, $O\epsilon$. The following proposition gives the necessary condition for market stability.

PROPOSITION 1. *If a market is stable, then the following condition*

$$\overline{D} \leq \sum_{\forall i \neq j} x_i \text{ , for } j = 1, \ldots, N, \tag{6}$$

must be met, where $\overline{D}$ *is the maximum possible realization of demand* D.

Proof. Suppose that $\overline{D} > \sum_{\forall i \neq j} x_i$ for some j, then Bidder j can bid at the maximum price, $O\epsilon$. In this case, there is a positive probability that the spot price π will be equal to $O\epsilon$ causing the market to become unstable. □

The market stability condition (6) can be enforced by adding new bidders into the market and excluding bidders with large capacity. The assumption of market stability is realistic. For example, the US has more than 30% excess capacity of power generation.

In a real system, the electricity spot price also depends on the transmission capacity and transmission cost of the network. Even though the market stability condition is satisfied, outage of a power line can cause the electric spot price to surge; however, for simplicity of our model, we assume that there is no loss of the power line and that the transmission capacity is enough to transmit the necessary amount of power to satisfy the demand. Moreover, the transmission cost is assumed to be negligible.

If $E_D[\ q_i(\mathbf{b}, D)\ ] = x_i$, Bidder i is *completely dispatched*; otherwise, when $E_D[\ q_i(\mathbf{b}, D)\ ] < x_i$, Bidder i is *incompletely dispatched.*

LEMMA 1. *If more than one bidder bids at the same price and one of them is incompletely dispatched, then all marginal bidders bidding at that price are incompletely dispatched.*

Proof. If one of the marginal bidders is incompletely dispatched, then $\sum_{\{j|p_j=\pi(\mathbf{b},d)\}} x_j > d - \sum_{\{i|p_i<\pi(\mathbf{b},d)\}} x_i$. Thus, the last bidder who bids at the spot price cannot be fully dispatched. Since each bidder can be the last one to be dispatched with positive probability, the expected dispatch of each bidder is strictly less than the bid quantity. Hence, all bidders at the spot market price are incompletely dispatched. □

Lemma 1 implies the following corollary.

COROLLARY 1. *If there exists a completely dispatched marginal bidder, then all marginal bidders bidding at the same price are completely dispatched.*

3. Equilibrium Points

We begin by giving conditions on the equilibrium points for the case where every bidder knows the others' bids. At each iteration, bidders can adjust their bid prices from the previous iteration, but they cannot adjust their bid quantities. We assume that they can adjust their bid prices infinitely often. An equilibrium point is reached when no bidder is willing to adjust his/her bid price.

If everyone behaves optimally, the market will be closed at a Nash equilibrium point as in (4). In other words, bidders will adjust their bids until a Nash equilibrium is reached. Since our model is a non-sealed-bid auction market, we restrict ourselves to the class of equilibrium points where everyone knows the others' bid prices.

The analysis for known demand environment and stochastic demand environment are given in Subsections 3.1 and 3.2, respectively. The detailed analysis of the market stability condition is given in Subsection 3.3. The proofs may be skipped for readability without losing the flow of the paper.

3.1. EQUILIBRIUM POINTS WITH KNOWN DEMAND

In this subsection, we give the characteristic of bidders in the market with known demand. We show that a marginal bidder must bid just below some other bidder's cost to achieve the highest spot price Nash equilibrium point. We show that if there are multiple marginal bidders in an equilibrium point, they must all be completely dispatched. Another fact that we mention is that a bidder gets dispatched if and only if his/her cost is less than or equal to the market clearing price.

We focus on one bidder and label him/her as Bidder 0. We now have $N + 1$ bidders. Without loss of generality, we label the remaining bidders in ascending order of their bid prices, that is, $p_1 \leq p_2 \leq \cdots \leq p_N$.

Given a number z, the value $\gamma(z)$ denotes the largest integer such that $p_{\gamma(z)} \leq z$, i.e., $\gamma(z) = \max\{j : p_j \leq z,\ j = 1, \ldots, N\}$. In other words, $\gamma(z)$ is the highest indexed bidder who bids less than or equal to z. Let $y(\mathbf{b}, d, p)$ be the unfulfilled demand left after being supplied by bidders who bid at a price less than or equal to p, excluding Bidder 0, i.e.,

$$y(\mathbf{b}, d, p) = \max\left\{0,\ d - \sum_{j=1}^{\gamma(p)} x_j\right\}.$$

Furthermore, given demand d and a bid price p_0, the random variable $Y(\mathbf{b}, d, p_0)$ denotes the unfulfilled demand left to Bidder 0 if Bidder 0 bids at p_0. If no other bidder bids at p_0, the random variable $Y(\mathbf{b}, d, p_0)$ equals $y(\mathbf{b}, d, p_0)$ with probability 1.

We define $L(d)$ to be the smallest integer such that $\sum_{j=1}^{L(d)} x_j + x_0 \geq d$. Furthermore, we define $U(d)$ to be the smallest integer such that $\sum_{j=1}^{U(d)} x_j \geq d$. Note that $L(d)$ can equal $U(d)$.

LEMMA 2. *A bidder, labeled as Bidder 0, is a marginal bidder if and only if* $p_{L(d)} \leq p_0 \leq p_{U(d)}$.

Proof. To make sure that bidders who bid at p_0 or less have enough capacity to satisfy the demand, we must have $p_0 \geq p_{L(d)}$ to ensure $y(\mathbf{b}, d, p_0) \leq x_0$. Bidder 0 can have positive expected dispatch quantity if and only if $p_0 \leq p_{U(d)}$. When Bidder 0 has positive expected dispatch quantity and the bidders who bid higher than p_0 cannot be dispatched, Bidder 0 is the marginal bidder. □

Lemma 2 implies that Bidder 0 is not dispatched if $p_0 > p_{U(d)}$ and Bidder 0 is an under bidder if $p_0 < p_{L(d)}$. From (4), we have the following.

LEMMA 3. *At a Nash equilibrium point, a bidder is(not) dispatched if his/her unit cost is lower(higher) than the equilibrium market clearing price.*

Proof. If there is a bidder j whose cost is less than the MCP and is not dispatched, Bidder j must decrease p_j to the MCP, violating (4). If there is a bidder j who has cost higher than the MCP and is dispatched, Bidder j must increase p_j to $O\epsilon$, violating (4). □

Lemma 3 implies that the market clearing price can be viewed as a separation between the bidders with higher costs than the MCP and the bidders with lower costs than the MCP. It also implies that a marginal bidder in a known demand environment must bid at least his/her cost. This is not true, however, in a stochastic demand environment. In the following, we use the notation, given a real number x, $\lfloor x \rfloor$ ($\lceil x \rceil$), to denote the maximum (minimum) possible bid price that is less (greater) than or equal to x.

LEMMA 4. *At a Nash equilibrium point in a market with known demand, all optimal bid prices of a marginal bidder must be in the set* $\{p_j - \epsilon \mid j = \gamma(p_{L(d)}) + 1, \dots, U(d)\} \cup \{p_j \mid j = L(d), \dots, U(d)\}$. *Optimal bid prices of an under-bidder can be anything (strictly) less than* $p_{L(d)}$. *Optimal bid prices of an non-dispatched bidder can be anything (strictly) greater than* $p_{U(d)}$. *If Bidder 0 is not dispatched, then an optimal bid price for him/her is* $p_0^* = \lfloor c_0 \rfloor + \epsilon$.

Proof. Given the bid vector $\mathbf{b} = [(x_i, p_i), i = 1, \dots, N]$ and the demand d, the profit of Bidder 0 as a function of p_0 is denoted by $f_0(p_0 \mid x_0, \mathbf{b}, d)$ and can be written as:

$$f_0(p_0 \mid x_0, \mathbf{b}, d) = \begin{cases} (\pi(\mathbf{b}, d - x_0) - c_0)x_0 & \text{if } p_0 < p_{L(d)}, \\ (p_0 - c_0)E[\min(x_0, Y(\mathbf{b}, d, p_j))] & \text{if } p_{L(d)} \leq p_j = p_0 \leq p_{U(d)}, \\ & \quad j = L(d), \dots, U(d), \\ (p_0 - c_0)\min(x_0, y(\mathbf{b}, d, p_j)) & \text{if } p_{L(d)} \leq p_j < p_0 < p_{j+1} \leq p_{U(d)}, \\ & \quad j = L(d), \dots, U(d) - 1, \\ 0 & \text{if } p_{U(d)} < p_0. \end{cases} \tag{7}$$

From Lemma 2, if Bidder 0 is a marginal bidder, then $p_{L(d)} \leq p_0 \leq p_{U(d)}$. Moreover, since Bidder 0 is a marginal bidder, Bidder 0's profit must be positive, otherwise Bidder 0 must bid above $p_{U(d)}$ and receive zero profit. Thus, $p_0 \geq c_0$. Since the term $(p_0 - c_0)\min(x_0, y(\mathbf{b}, d, p_j))$ is increasing in p_0, the optimal bid price p_0^* that gives a maximum value of the function $f_0(p_0 \mid x_0, \mathbf{b}, d)$ must be in the set $\{p_j - \epsilon \mid j = \gamma(p_{L(d)}) + 1, \dots, U(d)\} \cup \{p_j \mid j = L(d), \dots, U(d)\}$.

If Bidder 0 is an under-bidder, then $p_0 < p_{L(d)}$. Bidder 0's profit is $(\pi(\mathbf{b}, d - x_0) - c_0)x_0$, where $\pi(\mathbf{b}, d - x_0)$ is the market clearing price when demand is $d - x_0$. This market clearing price can be computed from (1). Bidder 0's profit $(\pi(\mathbf{b}, d - x_0) - c_0)x_0$ is independent of p_0 as long as $p_0 < p_{L(d)}$.

If it is optimal for Bidder 0 not to dispatch, then $p_0 > p_{U(d)}$ by Lemma 2. Bidder 0's profit remains 0 as long as $p_0 > p_{U(d)}$. Moreover, $c_0 > p_{U(d)}$ by Lemma 2 and Lemma 3. Thus, $\lfloor c_0 \rfloor + \epsilon$ is one of Bidder 0's optimal bid prices. □

We denote the set of all bidders bidding at p by $\mathcal{A}(p)$, i.e., $\mathcal{A}(p) = \{i | p_i = p, i = 1, \ldots, N\}$. The following propositions explain the characteristics of the market clearing price at an equilibrium point.

PROPOSITION 2. *At a Nash equilibrium point in a market with known demand, if a marginal bidder, labeled as Bidder 0 where* $p_0 = \pi(\mathbf{b}, d)$, *is completely dispatched, then* p_0 *must be in the set* $\{p_j - \epsilon | p_j - \epsilon \leq c_j, j = \gamma(p_{L(d)}) + 1, \ldots, U(d)\}$.

Proof. Suppose Bidder 0 bids at p_0 and is completely dispatched. By increasing p_0 to $p'_0 = p_0 + \epsilon$, Bidder 0 is still completely dispatched if no one else bids at p'_0. Thus, the marginal Bidder 0 can gain more profit by increasing p_0 to p'_0. At an equilibrium point, p_0 must be in the set $\{p_i - \epsilon, i = 1, \ldots, N\}$. From Lemma 2, however, Bidder 0 can be a marginal bidder only if $p_{L(d)} \leq p_0 \leq p_{U(d)}$. Therefore, p_0 must be in the set $\{p_i - \epsilon, i = \gamma(p_{L(d)}) + 1, \ldots, U(d)\}$.

Now, suppose that $p_0 = p_k - \epsilon$ for some k, then bidders who bid at p_k are not dispatched. Therefore, bidders bidding at p_k can reduce their bid price and get dispatched if their costs are less than $p_k - \epsilon$. Thus, this is not an equilibrium point. Hence, the equilibrium point is the point where $c_i \geq p_k - \epsilon, \forall i \in \mathcal{A}(p_k)$. □

PROPOSITION 3. *At a Nash equilibrium point in a market with known demand, if a marginal bidder, labeled as Bidder 0, is incompletely dispatched, then* p_0 *must be in the set* $\{p_j - \epsilon | p_j - \epsilon \leq c_j, j = \gamma(p_{L(d)}) + 1, \ldots, U(d)\}$ *and no one else bids at this price.*

Proof. If $c_0 \geq p_{U(d)}$, then Bidder 0 incurs a financial loss by bidding below $p_{U(d)}$. Therefore, Bidder 0 is not a marginal bidder, resulting in a contradiction.

We consider the case of $c_0 < p_{U(d)}$. Lemma 4 indicates that the marginal price must be in the set $\{p_j - \epsilon | j = \gamma(p_{L(d)}) + 1, \ldots, U(d)\} \cup \{p_j | j = L(d), \ldots, U(d)\}$. We need to show further that, at any equilibrium point, if Bidder 0 is marginal and $q_0(\mathbf{b}, d) < x_0$, then p_0 is not in the set $\{p_j | j = L(d), \ldots, U(d)\}$. We prove that if p_0 is in the set $\{p_j | j = L(d), \ldots, U(d)\}$, then at least one of the bidders $j \in \{L(d), \ldots, U(d)\}$ can adjust p_j to increase $f_j(\mathbf{b}, d)$, therefore, voiding the equilibrium point.

When Bidder 0 bids at p_j and is incompletely dispatched, then, from Lemma 1, each of the bidders bidding at p_j is incompletely dispatched.

If $p_0 = p_j$ where $p_{L(d)} \leq p_j < p_{U(d)}$, all bidders at p_j decrease their prices in order to be dispatched for all of their bidding quantities at the same selling price. Consequently, only Bidder 0 bids at p_j. Therefore, this point is not an equilibrium.

For the case where $p_0 = p_{U(d)}$, if Bidder j with $p_j = p_{U(d)}$ lowers p_j to p'_j so that $q_j(\mathbf{b}', d) = x_j$ and $\pi(\mathbf{b}', d) = \pi(\mathbf{b}, d)$, where $\mathbf{b}'$ is the vector $\mathbf{b}$ with p'_j

replacing p_j, Bidder j earns a higher profit. Therefore, this is not an equilibrium point. If no bidder at $p_{U(d)}$ can lower his/her bid price without changing the market clearing price, then each bidder at $p_{U(d)}$ must have a bid quantity more than the unfulfilled demand $y(\mathbf{b}, d, p_{U(d)} - \epsilon)$, i.e., $x_j \geq y(\mathbf{b}, d, p_{U(d)} - \epsilon)$, $\forall j \in \mathcal{A}(\mathbf{b}, p_{U(d)})$. If $c_j > p_{U(d)}$ for some $j \in \mathcal{A}(p_{U(d)})$, then $f_j(\mathbf{b}, d) < 0$. Hence, Bidder j will increase p_j, again negating the equilibrium condition.

We now consider the remaining case, where $c_j \leq p_{U(d)}$ and $x_j \geq y(\mathbf{b}, d, p_{U(d)} - \epsilon)$ for all $j \in \mathcal{A}(p_{U(d)})$. Suppose there are $I \geq 2$ bidders bidding at $p_{U(d)}$, i.e., Bidder $U(d) - i$, $i = 0, \ldots, I - 2$ and Bidder 0 bid at $p_{U(d)}$. Since $x_j \geq y(\mathbf{b}, d, p_{U(d)} - \epsilon)$ for each $j \in \mathcal{A}(p_{U(d)})$, each bidder's expected dispatch is $y(\mathbf{b}, d, p_{U(d)} - \epsilon)/I$. If a bidder $j \in \mathcal{A}(p_{U(d)})$ lowers p_j to $p'_j = p_{U(d)} - \epsilon$, then $q_j(\mathbf{b}', d) \geq y(\mathbf{b}, d, p_{U(d)} - \epsilon)$, where $\mathbf{b}'$ is the vector $\mathbf{b}$ with p'_j replacing p_j.

Since Bidder 0's optimal bid price is $p_{U(d)}$, we have that $f_0(p_{U(d)}|x_0, \mathbf{b}, d) = y(\mathbf{b}, d, p_{U(d)} - \epsilon)(p_{U(d)} - c_0)/I$ must be greater than or equal to the lower bound of $f_0(p_{U(d)} - \epsilon|x_0, \mathbf{b}, d)$, which is equal to $(p_{U(d)} - \epsilon - c_0)y(\mathbf{b}, d, p_{U(d)} - \epsilon)$. Simplifying the inequality, we have that $c_0 \geq p_{U(d)} - \epsilon I/(I - 1)$, but $I \geq 2$ gives $I/(I - 1) \leq 2$, therefore $c_0 \geq p_{U(d)} - 2\epsilon$. Since $c_0 < p_{U(d)}$, we have that $p_{U(d)} > c_0 \geq p_{U(d)} - 2\epsilon$, but ϵ is small enough so that $|c_i - c_j| > 2\epsilon$ for all i, j. Hence, $c_{U(d)-i} < p_{U(d)} - 2\epsilon, i = 0, \ldots, I - 2$, which is the same as $(p_{U(d)} - \epsilon - c_{U(d)-i})y(\mathbf{b}, d, p_{U(d)} - \epsilon) > y(\mathbf{b}, d, p_{U(d)} - \epsilon)(p_{U(d)} - c_{U(d)-i})/2$, $i = 0, \ldots, I - 2$. Since $I \geq 2$, we have

$$(p_{U(d)} - \epsilon - c_{U(d)-i})y(\mathbf{b}, d, p_{U(d)} - \epsilon) > \\ > y(\mathbf{b}, d, p_{U(d)} - \epsilon)(p_{U(d)} - c_{U(d)-i})/I, \quad i = 0, \ldots, I - 2\,.$$

That is, for each $i = 0, \ldots, I - 2$, the lower bound of Bidder $U(d) - i$'s expected profit when Bidder $U(d) - i$ bids at $p_{U(d)} - \epsilon$ is greater than Bidder $U(d) - i$'s expected profit at $p_{U(d)}$. Therefore, Bidders $U(d) - i$, $i = 0, \ldots, I - 2$, want to lower their bid prices. As a result, the original bid cannot be an equilibrium point.

We have shown that if $p_0 \in \{p_j | j = L(d), \ldots, U(d)\}$, then at least one of the bidders wants to adjust his/her bid and, therefore, $p_0 \notin \{p_j | j = L(d), \ldots, U(d)\}$ at an equilibrium point. As a result, at an equilibrium point, if it exists, $p_0 \in \{p_j - \epsilon | j = \gamma(p_{L(d)}) + 1, \ldots, U(d)\}$. This also implies that the marginal bidder is the only one bidding at this price.

Suppose the marginal bidder bids at $p_k - \epsilon$ for some $k \in \{L(d), \ldots, U(d)\}$; all bidders who bid at p_k are not dispatched. Therefore, they can reduce their bid prices if their costs are less than $p_k - \epsilon$. Thus, this cannot be an equilibrium point. Hence, an equilibrium point is a point where $c_j \geq p_k - \epsilon$ for all $j \in \mathcal{A}(p_k)$. □

Proposition 2, Proposition 3, and Lemma 1 imply the following corollary.

COROLLARY 2. *At a Nash equilibrium point in a market with known demand, the market clearing price must be in the set* $\{p_j - \epsilon | p_j - \epsilon \leq c_j, j = 1, \ldots, N\}$. *Moreover, if there is more than one marginal bidder, then all of these marginal bidders are completely dispatched.*

Table I. Data for EXAMPLE 1.

i	c_i	x_i
1	1.0	5
2	6.0	5
3	7.0	1

PROPOSITION 4. *At a Nash equilibrium point in a market with known demand, the highest possible equilibrium spot price must be in the set* $\{\lfloor c_i \rfloor, i = 1, \ldots, N\}$.

Proof. Since $\lfloor c_k \rfloor = \max\{p_k - \epsilon | p_k - \epsilon \leq c_k,\ p_k \in$ *the set of possible bid prices*$\}$, by Corollary 2, the set $\{\lfloor c_i \rfloor, i = 1, \ldots, N\}$ contains the highest possible equilibrium spot price. □

It is not necessary, however, that all equilibrium spot-market prices are in the set $\{\lfloor c_i \rfloor, i = 1, \ldots, N\}$. (See the following example.)

EXAMPLE 1. *Consider the costs and bidding quantities in Table I, where* $\epsilon = 0.01$ *and demand* $d = 5$.

The point $p_1 = 5$, $p_2 = 5.01$ and $MCP = 5$ is an equilibrium point.

Proposition 4 and Lemma 3 imply the following.

COROLLARY 3. *Suppose bidders bid at their costs and Bidder k is the lowest cost bidder who is not dispatched. Then,* $\lceil c_{k-1} \rceil$ *is a lower bound on the equilibrium market clearing price and* $\lfloor c_k \rfloor$ *is a lower bound on the highest possible equilibrium market clearing price.*

Proof. Since $\sum_{i=1}^{k-1} x_i > d$, if the equilibrium MCP is less than or equal to $\lceil c_{k-1} \rceil - \epsilon$, then there must be at least one dispatched bidder whose cost is higher than the MCP, which violates Lemma 3. From Proposition 4, we have that $\lfloor c_k \rfloor$ is a lower bound on the highest possible equilibrium MCP. □

In summary, for a market with known demand, a marginal bidder must bid just below some other bidder's cost to achieve the highest spot price Nash equilibrium point. Thus, the spot market price is bounded above by the highest bidder's cost. All bidders whose costs are higher than this market clearing price are not dispatched. All bidders whose costs are less than the market clearing price bid at or below this price. Furthermore, if this marginal bidder is incompletely dispatched, then there is only one marginal bidder.

We discuss the characteristics of bidders in a market with stochastic demand in the following subsection.

3.2. EQUILIBRIUM POINTS WITH STOCHASTIC DEMAND

In the stochastic demand environment, a *marginal bidder* is someone who bids at the market clearing price in one or more realizations of demand.

We focus on one bidder and label him/her again as Bidder 0. Without loss of generality, we label the remaining bidders in ascending order of their bid prices, $p_1 \leq p_2 \leq \cdots \leq p_N$. Using similar notation to the deterministic case, at any given demand realization d, the smallest integer $L(d)$ such that $\sum_{j=1}^{L(d)} x_j + x_0 \geq d$ is denoted by $L(d)$ and the smallest integer $U(d)$ such that $\sum_{j=1}^{U(d)} x_j \geq d$ is denoted by $U(d)$. We denote the set of the realizations $\{d | p_{L(d)} \leq p_0 \leq p_{U(d)}\}$ by $\mathcal{D}$. When $D = d \in \mathcal{D}$, Bidder 0 bids at the market clearing price.

When demand is stochastic, bidders cannot forecast the spot-market price exactly. This uncertainty causes some changes in optimal bidding strategies. In this subsection, we explore the characteristic of bidders at a Nash equilibrium point. We show that, if the marginal bidder is always completely dispatched, then there must be a bidder bidding just above the marginal bidder. Moreover, we show that if there is no bidder bidding just above the marginal bidder, then there must be two marginal bidders. In fact, among these two marginal bidders, the one with lower bid quantity must be bidding at most 2ϵ above his/her cost. Furthermore, we find the upper bound of the market clearing price.

The following lemma explains the marginal bidder's characteristics at an equilibrium point.

LEMMA 5. *At a Nash equilibrium point in a market with stochastic demand, if there are I, $I > 1$, marginal bidders who bid at p_0 and there exists a demand realization that makes one marginal bidder in $\mathcal{A}(p_0)$ incompletely dispatched, then $I - 1$ marginal bidders have costs at least $p_0 - 2\epsilon$ and the other bidder has the highest bid quantity among the I bidders in $\mathcal{A}(p_0)$.*

Proof. Suppose I bidders bid at p_0. We label the one with the highest bid quantity as Bidder 0, i.e., $x_0 = \max\{x_i \mid i \in \mathcal{A}(p_0)\}$. Since there are marginal bidders bidding at p_0, $\mathcal{D} \neq \emptyset$.

Since there exists a demand realization $d' \in \mathcal{D}$ such that one marginal bidder in $\mathcal{A}(p_0)$ is incompletely dispatched, from Lemma 1, all marginal bidders in $\mathcal{A}(p_0)$ are incompletely dispatched, i.e., $q_i(\mathbf{b}, d') < x_i \forall i \in \mathcal{A}(p_0)$.

We prove this lemma by contradiction. Suppose there is a bidder $k \in \mathcal{A}(p_0)$ such that $c_k < p_0 - 2\epsilon$ and $k \neq 0$. We show that Bidder k earns more expected profit by reducing p_k to $p'_k = p_0 - \epsilon$, which violates the equilibrium condition. We will show this by first conditioning on the demand and then showing that, for at least one realization of demand, Bidder k earns higher profit by reducing p_k and gains no less profit in all other realizations.

When $D = d \in \mathcal{D}$, the demand realization d is such that $p_{L(d)} \leq p_k < p_{U(d)}$ or $p_k = p_{U(d)}$. In the case where $p_{L(d)} \leq p_k < p_{U(d)}$, if $q_k(\mathbf{b}, d) < x_k$, Bidder k can get a higher dispatch at the same selling price by reducing p_k. If $q_k(\mathbf{b}, d) = x_k$,

Table II. Payoff table of Bidder k.

Case		$f_k(p_k \mid x_k, \mathbf{b}, d)$	$f_k(p'_k \mid x_k, \mathbf{b}, d)$
(1)	$p_{L(d)} > p_k$	$(\pi - c_k)x_k$	$(\pi - c_k)x_k$
(2)	$p_{L(d)} \leq p_k < p_{U(d)}$ and $q_k(\mathbf{b}, d) < x_k$	$(p_0 - c_k)q_k(\mathbf{b}, d)$	$(p_0 - c_k)x_k$
(3)	$p_{L(d)} \leq p_k < p_{U(d)}$ and $q_k(\mathbf{b}, d) = x_k$	$(p_0 - c_k)x_k$	$(p_0 - c_k)x_k$
(4)	$p_k = p_{U(d)}$ and $y(\mathbf{b}, d, p_0 - \epsilon) > x_k$	$(p_0 - c_k)q_k(\mathbf{b}, d)$	$(p_0 - c_k)x_k$
(5)	$p_k = p_{U(d)}$ and $y(\mathbf{b}, d, p_0 - \epsilon) \leq x_k$	$\leq (p_0 - c_k)$ $y(\mathbf{b}, d, p_0 - \epsilon)/2$	$\geq (p_0 - \epsilon - c_k)$ $y(\mathbf{b}, d, p_0 - \epsilon)$
(6)	$p_k > p_{U(d)}$	0	≥ 0

however, Bidder k gets the same dispatch at the same selling price by reducing p_k. Consider the case where $p_k = p_{U(d)} = p_0$. If $x_k < y(\mathbf{b}, d, p_0 - \epsilon)$, then Bidder k is willing to lower p_k to $p'_k = p_0 - \epsilon$ to get a higher dispatch at the same selling price. We now consider the case where $x_k \geq y(\mathbf{b}, d, p_0 - \epsilon)$. Because $x_0 \geq x_k \geq y(\mathbf{b}, d, p_0 - \epsilon)$, we have that $y(\mathbf{b}, d, p_0 - \epsilon)/2 \geq q_k(\mathbf{b}, d)$. The inequality $c_k < p_0 - 2\epsilon$ is the same as the inequality $(p_0 - \epsilon - c_k)y(\mathbf{b}, d, p_0 - \epsilon) > (p_0 - c_k)y(\mathbf{b}, d, p_0 - \epsilon)/2$, that is, the lower bound of $f_k(p'_k \mid x_k, \mathbf{b}, d)$ is greater than the upper bound of $f_k(p_k \mid x_k, \mathbf{b}, d)$. Therefore, Bidder k wants to lower p_k. Bidder k's payoff is shown in Table II.

Since there exists $d' \in \mathcal{D}$ such that $q_k(\mathbf{b}, d') < x_k$, at $D = d'$, Bidder k earns higher profit by reducing p_k. To show that Bidder k cannot gain less profit in all other realizations, we consider three types of demand realizations. First, when the bidders at p_0 are marginal bidders (Cases (2)–(5) in Table II), as previously shown, Bidder k's profit is not less by reducing p_k to $p'_k = p_0 - \epsilon$. Second, when the bidders at p_0 are under-bidders (Case (1) in Table II), Bidder k's profit is the same when reducing p_k to $p'_k = p_0 - \epsilon$. Lastly, when the bidders at p_0 is not dispatched (Case (6) in Table II), Bidder k's profit will be less by reducing p_k to $p'_k = p_0 - \epsilon$ only if $c_k > p_0 - \epsilon$ and the current market clearing price is $p_0 - \epsilon$; however, $c_k < p_0 - 2\epsilon$ from the assumption. Thus, Bidder k's profit is at least the same if he/she reduces p_k to $p'_k = p_0 - \epsilon$. □

LEMMA 6. *At a Nash equilibrium point in a market with stochastic demand, if a marginal bidder bids at p_0 and is completely dispatched at all demand realizations that make him/her a marginal bidder, then there exists a bidder k with $p_k = p_0 + \epsilon$.*

Proof. We label the marginal bidder at p_0 as Bidder 0. Since $q_0(\mathbf{b}, d) = x_0, \forall d \in \mathcal{D}$, if no bidder bids at $p_0 + \epsilon$, then Bidder 0 can earn more profit by increasing p_0 to $p'_0 = p_0 + \epsilon$, violating the equilibrium condition. □

Lemma 6 implies that, at a Nash equilibrium point where a marginal bidder bids at p_0, if no bidder bids at $p_0 + \epsilon$, then there exists a realization $d' \in \mathcal{D}$ such that Bidder 0 is incompletely dispatched, i.e., $q_0(\mathbf{b}, d') < x_0$.

LEMMA 7. *At a Nash equilibrium point in a market with stochastic demand, a marginal bidder, labeled as Bidder 0, must: (1) bid higher than or equal to c_0 or (2) bid at $\lfloor c_0 \rfloor < c_0$ and there exists a bidder who bids at $\lfloor c_0 \rfloor + \epsilon$.*

Proof. We label the marginal bidder as Bidder 0. We consider two cases. The first is when Bidder 0 does not incur a financial loss on one of the demand realizations that makes Bidder 0 a marginal bidder. In this case, we have that $p_0 \geq c_0$.

The second case is when Bidder 0 incurs a financial loss on all demand realizations that make him/her a marginal bidder. That is, $p_0 < c_0$. Bidder 0 optimally bids at this price only if there is a demand realization that makes Bidder 0 an under bidder with positive profit. Moreover, the market clearing prices π of this demand realization must be higher than c_0.

In this case, if no bidder bids at $\lfloor c_0 \rfloor + \epsilon$, Bidder 0 can gain higher expected profit by increasing p_0 to $p'_0 = \lfloor c_0 \rfloor + \epsilon$. But if there is a bidder bidding at $\lfloor c_0 \rfloor + \epsilon$, then it is possible that Bidder 0's profit is maximized at $p^*_0 = \lfloor c_0 \rfloor < c_0$. This is the case when Bidder 0 gains high profit from being an under-bidder while losing slightly from being a marginal bidder. □

We know from Lemma 3 that a marginal bidder in a known demand environment must bid at least his/her cost; however, Lemma 7 explains that a marginal bidder in stochastic demand environment can bid just below his/her cost, if there is a bidder bidding above him/her.

Lemma 5 explains that $I - 1$ incompletely dispatched marginal bidders at p_0 must have costs at least $p_0 - 2\epsilon$. The following lemma tightens this result by showing that, if no bidder bids just above p_0, then the $I - 1$ marginal bidders must in fact have costs higher than $p_0 - 2\epsilon$. That is, none of them has a cost of $p_0 - 2\epsilon$.

LEMMA 8. *At a Nash equilibrium point in a market with stochastic demand, if there are I, $I > 1$, marginal bidders who bid at p_0 and there is no bidder bidding at $p_0 + \epsilon$, then $I - 1$ marginal bidders have costs strictly higher than $p_0 - 2\epsilon$ and the other bidder has the highest bid quantity among the I bidders in $\mathcal{A}(p_0)$.*

Proof. We label the bidder with the highest bid quantity as Bidder 0, i.e., $x_0 = \max\{x_i \mid i \in \mathcal{A}(p_0)\}$. By Lemma 6, there exists a demand realization d' that makes one marginal bidder in $\mathcal{A}(p_0)$ incompletely dispatched. By Lemma 5, we only need to show that the case $c_k = p_0 - 2\epsilon$, $k \neq 0$ cannot occur.

Suppose that there exists $k \neq 0$ such that $c_k = p_0 - 2\epsilon$, then, following the same arguments as in the proof of Lemma 5, we have that Bidder k will always gain higher profit by reducing p_k to $p_0 - \epsilon$, except only in the case where $p_k = p_{U(d')}$ and $x_k \geq y(\mathbf{b}, d', p_0 - \epsilon)$.

In this case, the lower bound of $f_k(p'_k|x_k, \mathbf{b}, d')$ is $(p_0 - \epsilon - c_k)y(\mathbf{b}, d', p_0 - \epsilon)$, where $p'_k = p_0 - \epsilon$, while the upper bound of $f_k(p_k|x_k, \mathbf{b}, d')$ is $(p_0 - c_k)$ $y(\mathbf{b}, d', p_0 - \epsilon)/2$. Since $c_k = p_0 - 2\epsilon$, we have that $f_k(p'_k|x_k, \mathbf{b}, d') \geq f_k(p_k|x_k, \mathbf{b}, d')$. If there is a demand realization d_1 such that $x_k < y(\mathbf{b}, d_1, p_0 - \epsilon)$ and $x_k > q_k(\mathbf{b}, d_1)$, then $f_k(p'_k|x_k, \mathbf{b}, d_1) > f_k(p_k|x_k, \mathbf{b}, d_1)$, since Bidder k can obtain a higher dispatch at the same selling price. Thus, Bidder k can gain higher profit by reducing p_k to p'_k.

As a result, the only case that we need to consider is when $x_k \geq y(\mathbf{b}, d, p_0 - \epsilon)$ for all d that makes Bidder k an incompletely dispatched marginal bidder. By Lemma 1, we have that at all d' that makes Bidder 0 an incompletely dispatched marginal bidder, $x_0 \geq x_k \geq y(\mathbf{b}, d', p_0 - \epsilon)$. In this case, the lower bond of $f_0(p'_0|x_0, \mathbf{b}, d')$ is $(p_0 - \epsilon - c_0)y(\mathbf{b}, d', p_0 - \epsilon)$ and the upper bound of $f_0(p_0|x_0, \mathbf{b}, d')$ is $(p_0 - c_0)y(\mathbf{b}, d', p_0 - \epsilon)/2$.

If $c_0 < p_0 - 2\epsilon$, then Bidder 0 can gain higher payoff by reducing his/her bid price to $p'_0 = p_0 - \epsilon$ for all demand realizations that make Bidder 0 an incompletely dispatched marginal bidder. For other realizations, Bidder 0 cannot receive lower profit by reducing p_0 to p'_0. Hence, Bidder 0 has better expected profit by changing p_0, yielding a contradiction.

Now, suppose $c_0 \geq p_0 - 2\epsilon$, since $c_k = p_0 - 2\epsilon$ and $|c_0 - c_k| > 2\epsilon$ by the assumptions, we have that $c_0 > p_0$; however, from Lemma 7, we have that $p_0 = \lfloor c_0 \rfloor$ and there exists a bidder bidding at $p_0 + \epsilon$, yielding a contradiction. □

The following lemma gives a characteristic of the best response function of each bidder. This lemma is the stochastic version of Lemma 4.

LEMMA 9. *At a Nash equilibrium point in a market with stochastic demand, all optimal bid prices of a marginal bidder, labeled as Bidder 0, must be in the set* $\{p_j - \epsilon | j = 1, \ldots, N\} \cup \{p_j | j = 1, \ldots, N\}$.

Proof. If there is no other bidder j who bids $p_j = p_0$ or $p_j = p_0 + \epsilon$, then Bidder 0 can increase p_0 by ϵ without reducing his/her dispatch quantity under any demand. If Bidder 0 is a marginal bidder, then the return increases; otherwise, Bidder 0 earns the same. Thus, at an equilibrium point, we must have $p_0 = p_j$ or $p_0 = p_j - \epsilon$ for some j. □

LEMMA 10. *At a Nash equilibrium point in a market with stochastic demand, if there is only one marginal bidder bidding at* p_0*, then there exists a bidder k bidding at* $p_k = p_0 + \epsilon$.

Proof. We label the marginal bidder at p_0 as Bidder 0. Since there is only one marginal bidder bidding at p_0, then $p_0 \in \{p_j - \epsilon | j = 1, \ldots, N\}$ by Lemma 9. Hence, there exists a bidder k bidding at $p_k = p_0 + \epsilon$. □

COROLLARY 4. *At a Nash equilibrium point in a market with stochastic demand, if a marginal bidder bids at p_0 and no bidder bids at $p_0 + \epsilon$, then there are exactly two marginal bidders bidding at p_0. Label the bidder with lower bid quantity as Bidder k, then $p_0 = \lceil c_k \rceil$ or $p_0 = \lceil c_k \rceil + \epsilon$.*

Proof. Since no bidder bids at $p_0 + \epsilon$, from Lemma 10, there is more than one bidder bidding at p_0. Suppose there are I bidders at p_0. From Lemma 8, $I - 1$ lowest bid quantity bidders at p_0 must have costs higher than $p_0 - 2\epsilon$. Also, their cost is at most p_0 by Lemma 7. From the assumption that $|c_i - c_j| > 2\epsilon,\ \forall i \neq j$, we have that $I = 2$. Since $p_0 - 2\epsilon < c_k \leq p_k = p_0$, we have that $p_0 = p_k \in \{\lceil c_k \rceil, \lceil c_k \rceil + \epsilon\}$. □

The following lemma is a stochastic demand version of Proposition 4.

LEMMA 11. *At a Nash equilibrium point in a market with stochastic demand, the highest possible equilibrium spot price must be in the set $\{\lfloor c_i \rfloor, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil + \epsilon, i = 1, \ldots, N\}$.*

Proof. Label the marginal bidder in the highest demand realization as Bidder 0. We consider two cases. For Case 1, there is a Bidder k bidding just above Bidder 0. Thus, Bidder k is not dispatched for any demand. If Bidder k bids more than $\lfloor c_k \rfloor + \epsilon$, Bidder k can gain higher profit by reducing p_k; however, if $p_k \leq \lfloor c_k \rfloor + \epsilon$, Bidder k cannot incur a financial loss by increasing p_k to $\lfloor c_k \rfloor + \epsilon$. Since we focus on the highest possible equilibrium spot price, we conclude that Bidder k must bid at $\lfloor c_k \rfloor + \epsilon$. From the hypothesis, Bidder 0 bids at $\lfloor c_k \rfloor$, which is in the set $\{\lfloor c_i \rfloor, i = 1, \ldots, N\}$. Consider Case 2 where no bidder bids above Bidder 0. By Corollary 4, p_0 must be in the set $\{\lceil c_i \rceil, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil + \epsilon, i = 1, \ldots, N\}$. □

It is possible that the equilibrium spot price at the highest demand realization is not in the set $\{\lfloor c_i \rfloor, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil + \epsilon, i = 1, \ldots, N\}$. This is when the undispatched bidder who bids just above the marginal bidder bids less than his/her cost. We give an example of this case in Subsection 5.2.

Note that the sets $\{\lceil c_i \rceil, i = 1, \ldots, N\}$ and $\{\lceil c_i \rceil + \epsilon, i = 1, \ldots, N\}$ appear in the stochastic demand environment but not in the deterministic demand environment. The following example shows that indeed these conditions can occur at an equilibrium point in the stochastic case.

Table III. Data for EXAMPLE 2.

i	c_i	x_i
1	1	40
2	7.9	10
3	13	100

Table IV. Payoff matrix for EXAMPLE 2.

i	p_j	p_i	$f_i(p_i \mid p_j, D = d)$		$E_D[f_i(p_i \mid p_j)]$
			$d = 8$	$d = 40$	
1	$p_2 = 10$	10	36	315	147.6
		8	56	280	145.6
2	$p_1 = 10$	10	8.4	10.5	9.24
		8	0.8	21	8.88

EXAMPLE 2. *Consider a market where demand D is 8 with probability 0.6 and 40 with probability 0.4. Let $\epsilon = 2$. There are 3 bidders with costs and bid quantities shown in Table III.*

Bidder 1's expected profit when bidding at 10 is $0.6(4(10-1))+0.4(35(10-1)) = 147.6$. Bidder 1 earns 145.6 if $p_1 = 8$. Bidder 2 earns 9.24 when $p_2 = 10$ and 8.88 when $p_2 = 8$. Their payoffs are shown in Table IV. Hence, both Bidders 1 and 2 will bid at 10, or $\lceil c_2 \rceil + \epsilon$, satisfying the equilibrium condition. Note that when Bidders 1 and 2 both bid at 12, this is not an equilibrium point because each can gain by lowering his/her bid.

We know from Lemma 3 that the market clearing price in a known demand environment can be viewed as a separation between the bidders with higher costs than the MCP and the bidders with lower costs than the MCP. But in a stochastic demand environment, the MCP for each demand realization may not be a separator, as discussed in Lemma 7; however, the MCP at the highest demand realization does serve as a separator, as shown in the following lemma.

LEMMA 12. *At the highest realization of the demand, bidders who bid below or equal to the marginal price have costs less than or equal to the MCP at this highest demand realization. Moreover, all bidders bidding above the marginal have costs higher than or equal to this MCP.*

Proof. By contradiction, if a bidder with higher cost than the MCP bids less than or equal to the MCP, then he/she can gain more profit by increasing his/her bid price. Likewise, if a bidder with lower cost than MCP bids more than the MCP, he/she is not dispatched for any demand and therefore he/she can gain more profit by reducing his/her bid price. □

The following corollary gives a lower bound on the market clearing price at the highest demand realization.

COROLLARY 5. *Suppose bidders bid at their costs. Let Bidder k be the highest cost bidder who is dispatched at the highest demand realization. Then,* $\lceil c_k \rceil$ *is a lower bound on the market clearing price at the highest demand realization. Moreover, at an equilibrium point, Bidder k is dispatched at this highest demand realization.*

Proof. At the highest demand realization d', if there exists an equilibrium market clearing price less than $\lceil c_k \rceil$, then Bidder k should bid above this price by Lemma 12 and not be dispatched; however, since Bidder k can be dispatched at d' when everyone bids at cost, the total bid quantity of bidders with lower costs than c_k should be less than d'. Hence, there would exist a bidder with higher cost than c_k who gets dispatched at d', violating Lemma 12. □

In summary, for a market with stochastic demand, a marginal bidder must bid at or just below another bidder's bid price at a Nash equilibrium point. Moreover, if the marginal bidder is the only one bidding at this price or if the marginal bidder is always completely dispatched, then there must be a bidder bidding just above the marginal bidder; however, if no one bids just above the marginal bidder, then there is exactly one other marginal bidder bidding at that price. Moreover, among these two marginal bidders, the one with the lower bid quantity must bid less than 2ϵ above his/her cost. Furthermore, we have that the market clearing price is bounded above by the highest bidder's cost plus 2ϵ.

In the following subsection, we introduce a screening process that can be used to determine the bidders that will not be able to dispatch any unit.

3.3. MARKET STABILITY CONDITION AND COMPETITIVE BIDDERS

The market stability condition can help remove some redundant or non-competitive bidders who will not be able to dispatch any unit, thus reducing the size of the problem.

LEMMA 13. *If a set of bidders satisfies the market stability condition (6), then, at an equilibrium point with known demand, at least one bidder is not dispatched. When demand is stochastic, if all bidders are dispatched at least for one demand realization, then the highest equilibrium bid price of the bidders is at most* $\lceil c_k \rceil + \epsilon$*, where k is the highest cost bidder.*

Proof. When demand is known, by Corollary 2, if there is more than one marginal bidder, then all marginal bidders are completely dispatched. Therefore, there must be at least one bidder who is not dispatched; otherwise, the market stability condition is violated. Suppose, when demand is stochastic, all bidders are dispatched at least for one demand realization, then they are dispatched at the highest demand realization as well. Condition (6) ensures that there are multiple marginal bidders at the highest demand realization and no other bidder bids above them. By Corollary 4, the marginal bidders must bid at most $\lceil c_k \rceil + \epsilon$, where k is the highest cost bidder. □

The market stability condition and Lemma 13 help us obtain the following proposition, which gives an upper bound on the equilibrium market clearing price, thus further reducing the size of the problem.

PROPOSITION 5. $\lceil c_k \rceil + \epsilon$ *is an upper bound of the equilibrium market clearing price, where k is the highest cost bidder in a subset of bidders that satisfies the market stability condition.*

Proof. Suppose that, for some demand, the equilibrium market clearing price is higher than $\lceil c_k \rceil + \epsilon$; then every bidder who is not dispatched incurs a financial gain by bidding below or at $\lceil c_k \rceil + \epsilon$. Hence, all bidders can be dispatched. The contradiction follows from Lemma 13, where the highest bid price of the bidders must be at most $\lceil c_k \rceil + \epsilon$. □

From Proposition 5, we can tighten the bound by finding the 'lowest cost' subset of bidders satisfying the market stability condition. This tightening can be done by first setting a subset $\mathcal{C}$ to be an empty set $\emptyset$. Then, while the bidders in Set $\mathcal{C}$ cannot satisfy the market stability condition (6), we can continue adding the lowest cost bidders to Set $\mathcal{C}$. We stop adding bidders when bidders in $\mathcal{C}$ satisfy the market stability condition. Set $\mathcal{C}$ can be called a set of *competitive bidders*. All bidders outside $\mathcal{C}$ cannot be dispatched and can be removed from our consideration. The upper bound on the equilibrium market clearing price is the ceiling of the highest cost bidder in Set $\mathcal{C}$ plus ϵ.

The process of constructing the competitive bidder set is called the *screening* process. Before searching for an equilibrium point, screening should be done to reduce the size of the problem. The formal screening algorithm to construct a set of competitive bidders $\mathcal{C}$ from a set of all bidders $\mathcal{B}$ is described as follows.

ALGORITHM 0.

1. Set $\mathcal{C} \leftarrow \emptyset$.
2. While the bidders in Set $\mathcal{C}$ do not satisfy the market stability condition (6) and $\mathcal{B} \neq \emptyset$, move the lowest cost bidder from Set $\mathcal{B}$ to Set $\mathcal{C}$.

The resulting set $\mathcal{C}$ is the set of competitive bidders. All bidders remaining in Set $\mathcal{B}$ will not be dispatched.

Table V. Data for EXAMPLE 3.

i	c_i	x_i
1	1.0	5
2	6.0	5
3	7.0	1
4	9.0	1
5	10.5	11

In the following section, we describe an algorithm to compute an equilibrium point.

4. Finding Equilibrium Points

The following example shows that a problem can have multiple Nash equilibrium points. Moreover, it is possible that the equilibrium market clearing price is not unique.

EXAMPLE 3. *Consider the costs and bidding quantities in Table V with demand* $d = 10$.

An equilibrium point occurs when Bidder 2 is the marginal bidder bidding at 10.5 and the other bidders bid at their costs. At this point, Bidder 2's profit is 13.5 while Bidder 1's profit is 47.5. Another equilibrium point is when Bidder 1 is the marginal bidder bidding at 9 and the other bidders bid at their costs. At this point, Bidder 1's profit is 32 while Bidder 2's profit is 15.

Both equilibrium points can occur at the end of the market clearing process. Bidder 1 gains higher profit at the first point when Bidder 1 is an under-bidder. Bidder 2 gains higher profit at the second point when Bidder 2 is an under-bidder. The point actually reached depends on the activity rule of the market.

In a market where bidders are allowed to adjust their bids infinitely often without any further restriction on their bid price, the result may become a game of 'chicken'. In this game, bidders hang on to the last moment with bids at cost (or the lowest possible bid price) waiting for someone else to take on the role of marginal bidder.

As in most non-sealed bid markets, the California Power Exchange has activity rules that constrain the adjustment of the bid prices. One of these rules (Wilson, 1997) indicates that a bidder cannot bid more than the MCP in the previous round. Therefore, the first equilibrium point will be reached if Bidder 1 bids before Bidder 2, and vice versa. The first bidder gains the better market position.

Not only cost and capacity determine the market power of a bidder in the competitive power exchange market; this example shows that the bid submission time is also one of the most important factor in determining market power. A bidder with a fast decision making process has an advantage.

Since a problem can have multiple Nash equilibrium points, we shall focus our interest on the equilibrium point that gives the highest market clearing price π. The following subsections present algorithms to find these Nash equilibrium points of interest.

4.1. FINDING EQUILIBRIUM POINTS WITH DETERMINISTIC DEMAND

When the problem with known demand d has multiple Nash equilibrium points, we are interested in finding the equilibrium point that gives the highest market clearing price π. Since consumers must pay πd for the supply, this point is worst for consumers.

Proposition 4 reduces the decision space of the marginal bidder into N possible market clearing prices. An equilibrium point is derived from finding the marginal bidder and finding this bidder's bid price, i.e., the market clearing price. For any given marginal bidder, an optimal bid price and dispatch can be computed. Therefore, we first find each bidder's optimal bid price and dispatch, given that this bidder is the marginal bidder. Some bidders whose costs are relatively high may not be able to become a marginal bidder. These bidders will have zero dispatch. Hence, we can pick the bidder with the highest optimal bid price to be the marginal bidder. The market clearing price π is the optimal bid price of the marginal bidder. An equilibrium point occurs when the marginal bidder bids at π and the others bid at their costs. The formal algorithm for finding the equilibrium points follows.

ALGORITHM 1.

1. For each $i = 1, \ldots, N$
   (a) Let $\mathbf{K(i)}$ be the set of indices $\{k| \sum_{j=1, j\neq i}^{k} x_j < d\}$.
   (b) Set $g_i(k) = \min\{x_i, d - \sum_{j=1, j\neq i}^{k} x_j\}$ for all $k \in \mathbf{K(i)}$.
   (c) Compute $k^* = \arg\max_{k\in\mathbf{K(i)}}(\lfloor c_{k+1}\rfloor - c_i)g_i(k)$.
   (d) Set $p_i^* = \lfloor c_{k^*+1}\rfloor$ and $q_i = g_i(k^*)$.
2. Let

$$\pi = \max\{p_i^* | q_i > 0, i = 1, \ldots, N\}.$$

3. Set $\mathbf{I}$ to be the set of integers $i \in \{1, \ldots, N\}$ such that $p_i^* = \pi$. This is the set of possible marginal bidders.
4. The equilibrium points are the points where one of the bidders in $\mathbf{I}$ bids at π and the others bid at cost, i.e., bidder j bids at $\lfloor c_j\rfloor + \epsilon$.

The value of $g_i(k)$ can be viewed as the minimum dispatch if i bids at $\lfloor c_{k+1} \rfloor$. The values of p_i^* and q_i computed from Step 1 are the optimal bid price and dispatched quantity of the marginal Bidder i.

It can also be proven that if all bidders have the same bid quantities, i.e., $x_i = x \forall i$, then the optimal strategy of Bidder i is to bid at the next higher bidder's cost, $\lfloor c_{i+1} \rfloor$.

The following proposition confirms that the points computed from Algorithm 1 are the Nash equilibrium points. Moreover, the resulting spot price π is the highest possible equilibrium spot price.

PROPOSITION 6. *The points computed from Algorithm 1 are pure strategy Nash equilibrium points. Moreover, there is no other Nash equilibrium point where a spot price is higher than* π.

Proof. First, we show that the points computed from Algorithm 1 are Nash equilibrium points. To prove this, we need to show that no one can obtain higher expected profit by changing his/her bid price.

Since the bid price of the marginal bidder, π, is the optimal bid price computed from Step 1 in Algorithm 1, the marginal bidder cannot obtain higher expected profit by changing his/her bid price. For any bidder j with $c_j \geq \pi$, he/she is optimally not dispatched.

For any under-bidder j with $c_j < \pi$, Bidder j's payoff will be the same if Bidder j changes p_j to $p'_j < \pi$ and may be even less if $p'_j = \pi$. When $p'_j > \pi$, Bidder j may not be able to be dispatched, hence earning a lower payoff than with a bid of p_j. If Bidder j can still be dispatched, Bidder j will become the marginal bidder with market clearing price π' and dispatch quantity q'_j. Step 1 in Algorithm 1 shows that if j is the marginal bidder, $(\pi' - c_j)q'_j \leq (p_j^* - c_j)q_j$; however, from Step 2 in Algorithm 1, $p_j^* \leq \pi$. Therefore, $(p_j^* - c_j)q_j \leq (\pi - c_j)x_j$. Thus, Bidder j cannot achieve a higher expected profit by changing p_j.

As a result, the points computed from Algorithm 1, satisfying (4), are Nash equilibria. To prove that no other equilibrium point has a spot price higher than π, we proceed by contradiction. Suppose a point with spot price $\pi' > \pi$ is an equilibrium point. Then, if a bidder j whose cost is less than π' bids more than π', Bidder j can earn higher profit by bidding at $p'_j < \pi'$, violating the equilibrium condition. Consider the case where all bidders whose costs are less than π' bid at π' or less. By Step 1 in Algorithm 1, if the marginal bidder k is the only one bidding at π', Bidder k can receive higher profit by reducing p_k to $p_k^* \leq \pi'$ in Step 1d, violating the equilibrium condition. If there is more than one bidder bidding at π', from Corollary 2, these marginal bidders are completely dispatched. Following Step 1 in Algorithm 1, the marginal bidders can secure higher profit by reducing their bid prices to $p^* \leq \pi'$ in Step 1d, violating the equilibrium condition. Note that $p_k^* < \pi'$ since $p_k^* \leq \pi$ from Step 2 of Algorithm 1 and $\pi < \pi'$ from assumption. □

Proposition 6 shows that the equilibrium market clearing price computed from Algorithm 1 is the highest possible equilibrium market clearing price. Since consumers have to pay πd for the supply, the points computed from Algorithm 1 are the worst points for consumers.

PROPOSITION 7. *In a market with known demand, a pure strategy Nash equilibrium point exists.*

Proof. Steps 2 and 3 in Algorithm 1 imply that the algorithm always terminates with at least one equilibrium point. Proposition 6 confirms that this point is a pure strategy Nash equilibrium point. □

4.2. FINDING EQUILIBRIUM POINTS WITH STOCHASTIC DEMAND

As in the known demand case, a market with stochastic demand may also have many equilibria. The bidders' choice of equilibrium point is not clear. In fact, they might choose to play mixed strategies. Although pure strategy equilibrium points can be found by the algorithm described at the end of this subsection, the resulting point is still unclear. For this reason, we do not focus on constructing an efficient algorithm to find a pure strategy equilibrium point with stochastic demand. The point at which the bidders choose to play must first be clarified. We leave that question for future research.

The results from Subsection 3.2 greatly reduce the size of bidders' decision spaces and, therefore, reduce the time to search for equilibrium points. The key arguments are as follows.

From Corollary 4, a marginal bidder k in the stochastic demand environment must either bid just below other bidders' bid prices or bid at $\lceil c_k \rceil$ or $\lceil c_k \rceil + \epsilon$. Suppose the marginal bidder bids just below Bidder j's bid price. If Bidder j is never marginal at any demand, then Bidder j can gain higher profit by reducing p_j if $c_j < p_k$. Hence, $c_j \geq p_k$ and Bidder j can gain no less profit by bidding at $\lceil c_j \rceil$; however, if Bidder j is also a marginal bidder in one of the demand realizations, then either there is a bidder bidding just above Bidder j or $p_j = \lceil c_j \rceil$ or $\lceil c_j \rceil + \epsilon$.

Lemma 11 also tells us that the highest possible equilibrium MCP must be in the set $\{\lfloor c_i \rfloor, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil, i = 1, \ldots, N\} \cup \{\lceil c_i \rceil + \epsilon, i = 1, \ldots, N\}$.

One simple search algorithm to find pure strategy equilibrium points is the following. First, the screening process or Algorithm 0 as described in Subsection 3.3 is implemented to reduce the size of the problem. Second, a bidder, labeled as Bidder 1, is picked from the competitive bidder set and this Bidder 1 is assumed to be a marginal bidder at the highest demand realization. The bid price of Bidder 1 must be either $p_1 = \lceil c_1 \rceil$ or $\lceil c_1 \rceil + \epsilon$. Next, another bidder, labeled as Bidder 2, is picked from the competitive bidder set and this Bidder 2 is assumed to bid at or below Bidder 1. The possible bid prices of Bidder 2 are p_1, $p_1 - \epsilon$, $\lceil c_2 \rceil$, or $\lceil c_2 \rceil + \epsilon$. Next, another bidder, labeled as Bidder 3, is picked from the competitive bidder set and this Bidder 3 is assumed to bid at or below Bidder 2. The possible bid prices of Bidder 3 are p_2, $p_2 - \epsilon$, $\lceil c_3 \rceil$, or $\lceil c_3 \rceil + \epsilon$. Note that $p_3 \neq p_2$ if $p_2 = p_1$. The process is then repeated until the set is empty. We then enumeratively find a Nash equilibrium from their possible bid prices described above. If no point is found, the whole selection process is repeated. In fact, the above algorithm can be substantially improved by using the fact from Lemma 12 that the MCP at the highest demand realization is a separator between the bidders with higher costs than this MCP and the bidders with lower costs than this MCP. That is, Bidder 1 should be selected from the highest cost bidder first. Moreover, since all bidders with costs less than Bidder 1 can be dispatched at the highest demand realization, we can also check if the selected Bidder 1 can supply the demand. If not, the selected Bidder 1 can never be dispatched and can be removed from the competitive bidder set.

5. Numerical Results

Consider the costs and bidding quantities in Table V and $\epsilon = 0.01$. Demand D is stochastic which is uniformly distributed over 7, 9, and 11.

The following subsections show the numerical results when the demand is realized before bidders make their decisions and when the demand is realized after bidders make their decisions.

5.1. DEMAND IS KNOWN BEFORE BIDDERS MAKE THEIR DECISIONS

In this subsection, demand D is known before the bidders make their decisions. Thus, we can model the problem as three deterministic problems with respect to each realization. The results from Algorithm 1, i.e., the equilibrium point with the highest spot price, are compared with the results from two other strategy profiles, which are: (1) bidders bid at the next higher bidder's costs and (2) bidders bid at their costs.

The first part of Table VI shows the optimal bid price and the optimal expected profits computed from Algorithm 1. The last column contains the expected profit for each bidder and the bolded figures represent the market clearing prices. The expected equilibrium market clearing price is 10. The second and third parts of Table VI show the bid prices and the expected profits when the bidders bid at the next higher bidder's costs and when the bidders bid at their costs, respectively. The resulting expected market clearing prices are 7.67 and 6.33, respectively.

It is clear from the example that, when all the bidders behave optimally, they achieve higher expected profits than when they bid at their own costs or at the next higher bidder's costs. It is possible, however, that some bidders can gain higher profit, if other bidders do not behave optimally. An example of this situation is when Bidder 2 bids at 6 and the others bid at 10. In this case, Bidder 2 gets 20, as long as demand is greater than 5.

The dispatches of bidders are *socially optimal* when they minimize total generation cost. Hence, socially optimal dispatches can be obtained when bidders bid at

Table VI. Bid prices and expected profits under three strategy profiles.

i	$d = 7$		$d = 9$		$d = 11$		$E[f_i]$
	p_i	f_i	p_i	f_i	p_i	f_i	
Alg. 1							
Bidder 1	1.01	40	1.01	47.5	1.01	47.5	45
Bidder 2	**9**	3	**10.5**	9	**10.5**	18	10
Bidder 3	7.01	2	7.01	3.5	7.01	3.5	3
Bidder 4	9.01	0	9.01	1.5	9.01	1.5	1
Bidder 5	10.51	0	10.51	0	10.51	0	0
Next higher							
Bidder 1	6	30	6	30	6	40	33.33
Bidder 2	**7**	2	**7**	4	7	15	7
Bidder 3	9	0	9	0	**9**	2	0.67
Bidder 4	10.5	0	10.5	0	10.5	0	0
Bidder 5	$O\epsilon$	0	$O\epsilon$	0	$O\epsilon$	0	0
At cost							
Bidder 1	1	25	1	25	1	30	26.67
Bidder 2	**6**	0	**6**	0	6	5	1.67
Bidder 3	7	0	7	0	**7**	0	0
Bidder 4	9	0	9	0	9	0	0
Bidder 5	10.5	0	10.5	0	10.5	0	0

their costs or at the next higher bidder's costs; however, the dispatches of bidders when they each behave optimally, called *individually optimal*, may not be socially optimal. For example, when demand $d = 9$, Bidder 2, who has capacity of 5 units, prefers to bid at 10.5 and is dispatched for only 2 units. This dispatch has 2 units less than the socially optimal dispatch. Bidders 3 and 4 who are not dispatched in the socially optimal condition get completely dispatched in the individually optimal condition. This situation shows that unregulated markets can often produce costlier generation plans than a regulated monopoly.

5.2. DEMAND IS REALIZED AFTER BIDDERS MAKE THEIR DECISIONS

In this subsection, demand D can be 7, 9, or 11 with equal probability. There are many pure strategy equilibrium points for this problem. Several of them can be found from the algorithm described in Subsection 4.2. For example, the points where $p_5 = 10.5$, $p_4 = 9.01$, $p_3 = 7$, $p_2 = 9$, $p_1 \leq 7$ are pure strategy equilibrium points. Bidders 1, 2, and 3 get payoffs of 40, 9, and 2, respectively. Bidders

Table VII. Costs and bid quantities of bidders.

i	c_i	x_i
1	3	5
2	5	3
3	6	3
4	10	3
5	12	11
6	15	8

4 and 5 get nothing. This point is similar to their optimal bidding decisions when demand is known to be 7.

There are other pure strategy equilibrium points that cannot be found from the algorithm as well. For example, the point where $p_1 = 1$, $p_2 = 7$, $p_3 = 7.01$, $p_4 = p_5 = 7.02$ is an equilibrium. This is the point when Bidders 4 and 5 bid less than their costs. They cannot be affected by doing this because they cannot be dispatched in any event; however, their bid affects other bidders since the other bidders receive lower profits.

From Table VI, when demand is realized before bidders make their decisions, Bidder 2 is always the marginal bidder in all three demand realizations; however, when demand is realized after bidders make their decisions, Bidder 2 might be optimal without being a marginal bidder. For example, when $P(D = 7) = P(D = 9) = 0.25$ and $P(D = 11) = 0.5$, the points where $p_5 = 10.5$, $p_4 = 9.01$, $p_2 = 9$, $p_3 = 7$, $p_1 \leq 7$ are not equilibrium points since Bidder 2 can gain higher profit by reducing p_2. In fact, one equilibrium point is where $p_1 = 9$, $p_2 = 6.01$, $p_3 = 7.01$, $p_4 = 9.01$, $p_5 = 10.52$ and Bidder 1 is a marginal bidder in all realizations.

The following is another example of interest. Consider the costs and bidding quantities in Table VII and $\epsilon = 0.01$.

Demand D is stochastic and can take values 5 with probability 0.25, 10 with probability 0.5, and 15 with probability 0.25.

This example has many pure strategy equilibrium points. Some of them are shown in Table 8.

Points 1–3 cannot be obtained from the algorithm described in Subsection 4.2 because Bidder 4 bids less than c_4. At these points, Bidder 4 cannot be dispatched when $D = 5, 10$ but Bidder 4 is an under-bidder when $D = 15$. Hence, Bidder 4's profit cannot change as long as $6.01 < p_4 < 12$; however, Bidder 4's bid price can greatly affect Bidders 1-3's profits. The same situation holds for Bidder 6 at Point 3.

Point 4 can be obtained from the algorithm described in Subsection 4.2. This is the only equilibrium point where every bidder bids above their costs. We suggest

Table VIII. Some pure strategy equilibrium points.

No.	p_1	f_1	p_2	f_2	p_3	f_3	p_4	f_4	p_5	f_5	p_6	f_6
1	5	25.025	6.01	8.51	5.01	6.765	6.02	3.75	15	0.75	15.01	0
2	6	24.025	3.01	9.765	6.01	6.76	6.02	3.75	15	0.75	15.01	0
3	6	20.2875	3.01	7.5225	6.01	4.5175	6.02	1.5075	12.01	0.0025	12.02	0
4	6	36.25	10	12.5	6.01	12.75	10.01	3.75	15	0.75	15.01	0

that this point is expected to be played if all bidders are both rational and 'generous' to others. Bidder 6 who cannot be dispatched bids at $\lfloor c_6 \rfloor + \epsilon$. Bidder 5 who is the marginal at $D = 15$ bids just below Bidder 6. Bidder 4 who can be dispatched only when $d = 15$ is an under-bidder so $p_4 = \lfloor c_4 \rfloor + \epsilon$. Bidder 3 chooses to play an under-bidder role when $D = 10$ and bids just at $p_3 = \lfloor c_3 \rfloor + \epsilon$. This forces Bidder 2 to take the marginal role and $p_2 = p_4 - \epsilon$.

One interesting issue about Point 4 is, that Bidder 2 who has lower cost than Bidder 3 and the same bidding quantity as Bidder 3 gains less profit than Bidder 3. Suppose Bidder 2 can submit the bid before Bidder 3 and wants to play an under-bidder role when $D = 10$; Bidder 2 might try to bid at $p_2 = \lfloor c_2 \rfloor + \epsilon = 5.01$. This forces Bidder 3 to take a marginal role at $D = 10$ by bidding at $p_3 = 10$p; however, Bidder 1 is not happy with this and can gain higher profit by increasing p_1 to be higher than p_2. So, suppose Bidder 2 instead of bidding at 5.01, moves p_2 up to 6 so that Bidder 1 can bid at 5.99. This does make Bidder 1 happy; however, after seeing $p_1 = 5.99$, Bidder 2, who currently gains a profit of 15, notices that he/she can gain higher profit by bidding below p_1. Thus, it is not an equilibrium point. If Bidder 2 reduces p_2 to be below p_1, Bidder 1 will again lose. In fact, both Bidder 1 and Bidder 2 never settle down at their equilibrium if Bidder 3 bids at 10.

Bidder 2 never wants to be settled at an equilibrium point as in Point 4 by taking the marginal role at $D = 10$. This is because Bidder 2 receives only 12.5 while he/she can get 15 at a non-equilibrium point for him/her. As a result, Bidder 2 should be satisfied with the profit of 15 when he/she bids at 6 and the market closes at a non-equilibrium point.

6. Conclusion

We model a selling-at-spot electricity market as a multi-round non-sealed bid auction. Both stochastic and deterministic versions of the demand for electricity are considered. Assuming that bidders are rational, we provide the characteristics and behavior of the bidders in this market as well as the resulting market clearing price at an equilibrium point. Furthermore, the market stability condition is introduced as a requirement for a market to be operable. This condition can also be used to determine the redundant bidders in the market and, hence, reduce the size of the problem.

Algorithms for finding pure strategy equilibrium points in both deterministic demand and stochastic demand cases are developed. We prove also that a pure strategy equilibrium point will always exist in a known demand environment.

Using these algorithms, we find equilibrium points for some numerical examples and show that the resulting dispatches when each bidder behaves optimally may not be socially optimal. Moreover, we show that unregulated markets can often produce costlier generation plans than result from a regulated monopoly.

The market power of a bidder is the capability of the bidder to control the market or determine the market clearing price. In general, there are multiple equilibrium points that can be reached when the market is cleared. The market power of a bidder also includes the capability of the bidder to make the market clear at the bidder's highest payoff equilibrium point. In general, a bidder with lower generation cost and higher capacity has high market power; however, we found that the bid submission time is an important factor in determining the market power. This is because the bidder who bids earlier has better opportunity to select the equilibrium point that is beneficial to him/her. Furthermore, it is also possible that a market is not closed at a Nash equilibrium point when demand is stochastic. A bidder may choose to play a non-equilibrium point if he/she notices that his/her profit will be less than the result of playing at an equilibrium point.

We would like to apply our models to the California Power market. Our future research will focus on models that: (1) allow bidders to submit more than one bid; (2) include a multi-period problem, where the fixed generation cost will be incorporated; (3) establish necessary and sufficient conditions for the individually optimal dispatch to be equal to the socially optimal dispatch; and (4) determine bidders' market power.

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