Comment on "Fracture resistance of paper"

The paper written by Drs Seth and Page [1] is indeed interesting. It is delightful to see that the quasi-static crack propagation concept originally developed by Professor Charles Gurney has been confidently extended to describe the fracture resistances of various papers. In particular, these authors have shown that the specific work of fracture* (R) obtained from the quasi-static method is experimentally equivalent to the critical strain energy release rate (G_c) measured from linear elastic fracture mechanics (see Fig. 7 in [1]). Because of the simplicity with which R can be measured, Seth and Page imply that this should provide a basis for a standard test method for paper samples.

While working in the fracture group of Professor Charles Gurney in Hong Kong several years ago, I gained some experience in using the quasi-static method to describe fracture resistances of various materials including paper. We found that, in general, the Gurney-approach could not be applied to all kinds of papers, of which tracing paper was one. As pointed out in the various publications by Gurney and co-authors [2-5], the determination of R requires that cracking be stable and controlled and that at failure irreversible deformations should be contiguous with the crack front surfaces. It was obvious that using the specimen geometry of these authors and under the application of a uniform tensile stress field, cracking was stable unless $\mathrm{d}R/\mathrm{d}a$ was enormously negative. Note that $\mathrm{d}a$ is the incremental increase in crack length. However, it was unfortunate that the authors just assumed that the d_{crit} requirement mentioned in Gurney and Hunt [2] was satisfied so that cracking always took place without premature yielding in parts remote from the crack surfaces. Whether such an assumption is realistic or not can be experimentally testified by unloading to check the reversibility of deformation other than at the crack front*. If zero load and displacement coincide, the d_{crit} condition is satisfied. Because the specimens were pulled to complete failure and no unloading after cracking could be done, it was doubtful whether premature yielding had occurred prior to cracking in the experiments of Seth and Page. Any gross yielding occurring during quasi-static cracking would apparently increase the fracture resistance of the paper. Perhaps this explains why the datum point for tracing paper falls off the correlation line shown in their Fig. 7.

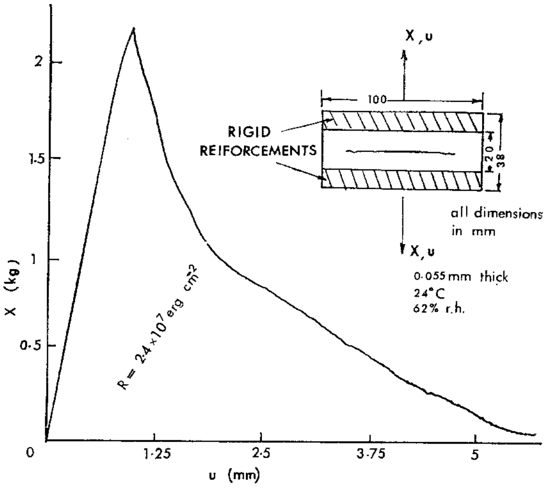

Figure 1 Load-deflection curve for quasi-static cracking in tracing paper.

We have performed some similar cracking experiments on a commercial tracing paper (0.055 mm thick) using a beam-like test geometry with a central crack of length 25 mm. The overall specimen dimensions are shown in the inset of Fig. 1. Rigid flange-reinforcements were mounted on the specimen and a point load applied as shown. Fig. 1 records the results of such an experiment. The specific work of fracture (R) was obtained by dividing the area under the load–deflection curve with the product of the crack length increment and the nominal thickness of the paper. R is found to be 2.4×10^7 erg cm^{-2} and is in excellent agreement with the reported value of 2.57×10^7 erg cm^{-2} by Seth and Page. However, no unloading was allowed to check the reversibility of deformation other than at the crack tips. Fig. 2 shows the results of another cracking experiment performed on a similar testpiece. Both loading and unloading were allowed during the fracture process. Two interesting phenomena were instantly observed: (1) upon unloading at zero load, the displacement was not reversible; (2) large hysteresis loops were formed and bounded by the loading and unloading curves at constant crack lengths.

*In the nomenclature of the authors' original paper, R is incorrectly defined as the work of fracture. In fact, in Gurney's definition, R refers to work per unit area of crack spreading.

*For materials with large irreversible deformations at the crack tips only, a saw-cutting technique is introduced in [5] which checks the deformation reversibility after cracking.

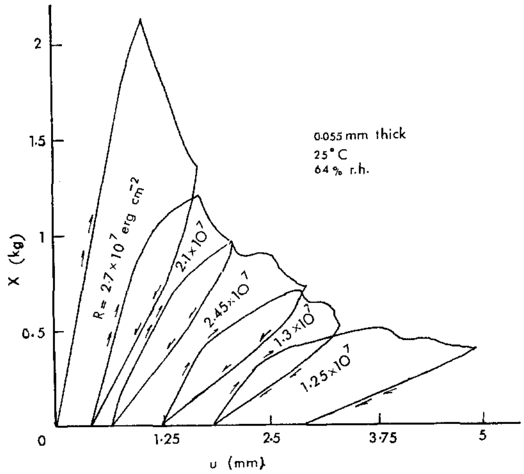

Figure 2 Load-deflection curve for quasi-static cracking in tracing paper showing deformation irreversibility and hysteresis loss.

TABLE I Values of ER/σ_Y^2 for paper and some typical materials

Bond paper 1	35.0 (mm)
Bond paper 2	74.0
Tracing paper	87.8
Writing paper	77.5
Newsprint 1	36.9
Newsprint 2	28.0
Semi-bleached kraft paper	37.2
Unbleached kraft paper	83.2
Mineral glasses	2.50×10^{-5}
Timber	0.4
Brittle polymer	5.3
High tensile steel	4.2
Al 7075	175.0
Low carbon steel	500.0

These observations showed that the necessary condition for the quasi-static method of specific work of fracture determination was not satisfied, so that inelastic deformation or premature yielding had occurred before cracking in parts remote from the crack tips. Only invalid R values as shown in the figure could be obtained from these results. From these arguments, we believe that the R value shown in Fig. 1 is possibly invalid. It overestimates the true specific work of fracture.

It is also very likely that Seth and Page have overestimated the R value for tracing paper. In a recent paper, Gurney *et al.* [5] have described a parameter (ER/σ_Y^2) to characterize the transition between cracking and general yielding failure of structures. Large ER/σ_Y^2 materials are tough and ductile while low ER/σ_Y^2 materials are weak and brittle. Since d_{crit} increases with ER/σ_Y^2 [2, 5], it follows that for tracing paper with ER/σ_Y^2 = 87.8 mm, d_{crit} will be prohibitively large and becomes impracticable in laboratory tests. The dimension of d used in our test and that of Seth and Page, is about 20 mm only and obviously does not satisfy this size requirement. It is, therefore, not surprising that general yielding may have preceded cracking in tracing paper.

Table I gives the ER/σ_Y^2 values for the eight papers used in [1]. Values for some other materials are also included. It may be seen that, as claimed by Seth and Page, papers do possess great resistances to cracking. It must also be recognized that the three papers (Bond paper 2, tracing paper and unbleached kraft paper) that do not give good correlation in the results of Seth and Page are those with the largest ER/σ_Y^2 values. Perhaps these results could be rechecked to see if reversibility of deformation was observed and hysteresis loss was negligible.

In the K_c measurements, the authors have apparently taken into account effects of the plastic zone size (r_Y). However, it appears that r_Y for those papers with large ER/σ_Y^2 values is not at all small compared with the crack length and therefore, it is not clear how linear elastic fracture mechanics can be used to evaluate K_c in such cases.

In so far as R and G_c are independent of specimen geometry and crack velocity, good correlation should exist between these two quantities. But are R values for the eight papers considered really crack velocity independent? If not, R and G_c must be compared on a common crack velocity basis.

Finally, as a suggestion, since the ER/σ_Y^2 values for paper samples are often greater than 20 mm, the beam-like testpiece geometry used by Seth and Page would not be desirable [5]. Instead, the test rig described in [5] would provide an alternative testing method for such materials. The reinforcements provided to the paper testpiece will eliminate all possible yielding at regions remote from the crack tip. Valid R values for all these papers with high ER/σ_Y^2 ratios would then be easily determined using the quasi-static method. Because of the relatively low Young's modulus of paper, the test rig can be made of Perspex or polycarbonate. A similar test apparatus described by Weitzmann and Finnie [6] may also be worth considering.

We very much hope that these are some points Drs Seth and Page could consider before setting a standard testing method for paper samples based on the quasi-static method of Gurney and Hunt [2].

References

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Reply to 'Comment on "Fracture resistance of paper"'

We would like to thank Dr Mai for his helpful comments on our paper. It is gratifying to see such interest from one who has worked extensively on the quasi-static crack propagation technique. We were, of course, aware of the papers quoted by him, and of the very problem that he has outlined. The apparatus and shape of test specimens required for measuring fracture resistance with precision is unfortunately different for every material. We are proposing an apparatus and specimen dimension that, we hope, is suitable for most papers, and particularly for those for which the property of fracture resistance seems important. We did a considerable amount of preliminary work to ensure that as far as possible the sample dimension satisfied the condition that the net load at failure was less than the yield load. We determined the effect of sample width on the work of fracture and found it to be almost unaffected over the range 8 to 18 cm as shown for three papers in Fig. 1. We concluded, therefore, that our specimens were of adequate dimension for fracture resistance measurements.

In his Figs. 1 and 2, Mai shows that for his sample of tracing paper our procedure would not give valid results. We would like to make two points about this.

(a) Tracing paper is anisotropic, being much more ductile in the cross direction than the machine direction as shown by the load-elongation curves of Fig. 2. From Mai's Fig. 1, it is apparent that he has chosen the cross direction, for the displacement, u, at failure is 25% of the initial span length. Our work was done using, as stated, the machine direction when the

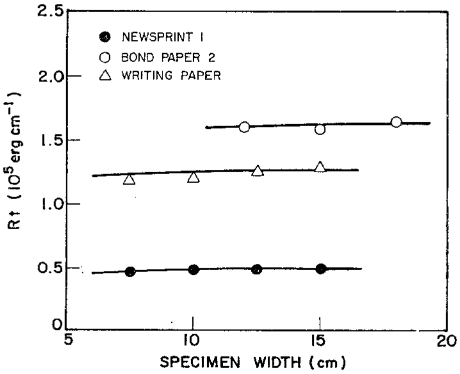

Figure 1 Variation of work of fracture (Rt) with specimen width.

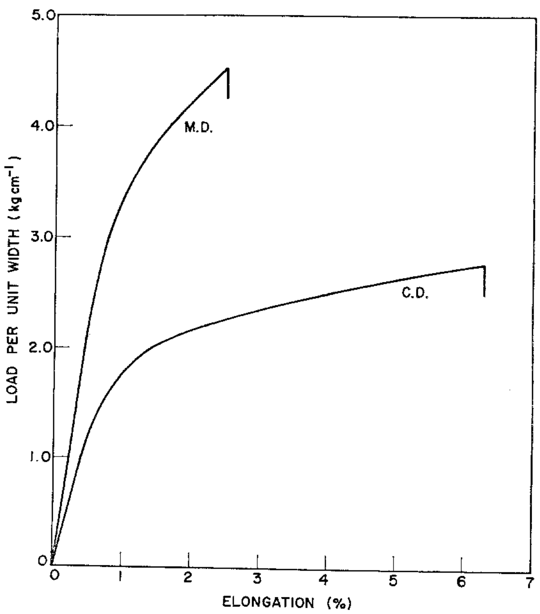

Figure 2 Load-elongation curve for tracing paper in the machine and cross directions.

displacement at failure was only 2.5%. For all our specimens the recycled load–displacement curves were not as shown in Mai's Fig. 2. In our case, reversibility of the curves to the origin was approximately achieved.

(b) Paper is hygroscopic and its properties strongly depend on relative humidity. All papers are more ductile at the 64% R.H. used by Mai, rather than at the North American standard test humidity of 50% that we used.

We think, therefore, that Mai has chosen an extreme example, for which a very large specimen width would be necessary.

It is important to note the original purpose of our work. In practice, when paper is drawn through a printing press from a reel, it occasionally breaks under the drawing stress. The breaks are almost always associated with flaws at the edge of the sheet. We hoped that a fracture mechanics approach might lead to a test that would measure the ability of paper to carry these flaws without failure. We are primarily interested, therefore, in the fracture resistance of paper strained in the machine direction. We are also primarily interested in printing grade papers, such as newsprint, which are among the most brittle of papers.

Regrettably, in practice, compromises must be made, and for industrial acceptance of a new test, speed and simplicity are important. Although, therefore, we recognize the points made by Mai, it may be prudent to accept a test which, while not satisfying exactly the necessary conditions, is a good approximation to them. Before proceeding we will certainly consider alternative test rigs of the type suggested by him.

The question raised by Mai on the dependence of R on crack velocity is a good one. We have not measured crack velocity, but we have found that R is unaffected if the rate of strain is changed by one order of magnitude, and this indicates that for paper, the effect of crack velocity might be small.

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