PSYCHOMETRIKA—VOL. 13, NO. 2
JUNE, 1948

A RATIONALE FOR THE MEASUREMENT OF TRAITS IN INDIVIDUALS*

CLYDE H. COOMBS
UNIVERSITY OF MICHIGAN

Hypotheses presented in a previous paper conceive of an individual's position on an attitude scale as represented by a mean position (status score) and a sigma (dispersion score). The testing of these hypotheses requires a determination of these two scores for each individual. A rationale is presented here for the determination of these two scores for an individual from his forced choice responses to pairs of items of nearly equal scale value.

Certain hypotheses concerning the measurement of psychological traits have been presented in a previous paper.† These hypotheses were based on the two fundamental parameters of an individual obtainable from his performance on a mental test, attitude scale, neurotic inventory, or other suitably designed instruments. The two parameters were called the individual's status score and dispersion score and were each given particular psychological interpretation. Further hypotheses were developed based on the distribution of each of these parameters obtained from a test, scale, or inventory administered to a group of individuals.

The basic problem is the determination of the individual's two parameters in a reliable manner. This is particularly difficult for the dispersion score. The individual's status score is a result of the individual's response to a relatively large number of items, whereas the dispersion score may be reflected in only a small fraction of all the items. The purpose of this paper is to develop a rationale such that the opinions of an attitude scale or the items of a neurotic inventory may be presented to each subject in the form of paired comparisons and both a status and dispersion score may be approximated. Suggestions are also made for a type of mental ability test which would also permit a more reliable determination of dispersion scores.

Another advantage to such a technique lies in the degree to which it removes voluntary control of the individual's score. It is

* This study was made possible by the Bureau of Psychological Services, Institute for Human Adjustment, Horace H. Rackham School of Graduate Studies, University of Michigan.

† Coombs, C. H. Some hypotheses for the analysis of qualitative variables. *Psychol. Rev.*, (in press).

well recognized that an intelligent individual can appear "neurotic" or "stable" as he wills on most existing inventories. Similarly, an individual responding to a set of opinions on an attitude scale can voluntarily choose to have a "pro" or "con" attitude independently of his "true" attitude. However, by presenting an individual with pairs of opinions or statements of the proper distance apart on the scale and forcing a choice between them, he finds himself in a position in which he is less able to guess the "right" answer for any ulterior purpose.

The point from which we begin is the completed attitude scale or inventory with the items' scale positions determined by one of the usual scaling methods. It will be assumed that each item has an exact and known scale position and that its dispersion about that scale position for any cause peculiar to itself (such as ambiguity, etc.) is zero.

Let us consider that the items run the gamut from "pro" to "con." It is postulated further that an individual also has a scale position (S_i) on this same continuum. The individual's scale position, however, is not necessarily a stable one but under different stimulus situations fluctuates about his S_i. We shall assume that the frequency distribution of scale positions taken by an individual follows the normal law (cf. Fig. 1). The mean of this distribution we shall designate as his status score (S_i) and the standard deviation of this distribution as his dispersion score (D_i).

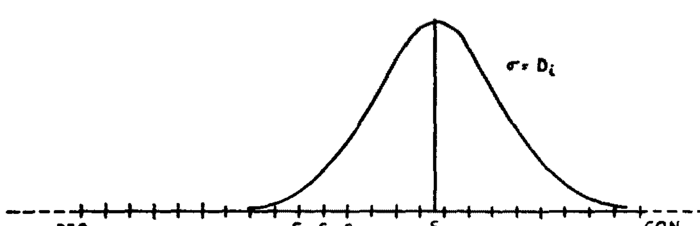


FIGURE 1

Let us designate stimuli with successive scale positions as α, β, γ, δ, $\cdots$, and their respective scale values as S_α, S_β, S_γ, S_δ, $\cdots$. Let us combine these items in pairs such that the members of a pair are not obviously discriminable and administer the inventory with instructions for the individual to select that member of each pair most nearly expressing his own attitude.

We now make the further assumption that an individual chooses that member of a pair of statements which is nearest his own momentary position on the attitude scale.

Theoretically, one could present any pair of stimuli, γ and δ, a large number of times, and from the proportion of time ($P_{\gamma p \delta}$), S_γ was chosen in preference to S_δ, the x/σ value ($Z_{\gamma\delta}$) corresponding to this proportion may be readily obtained from normal probability tables. The following equation may then be written:

$$Z_{\gamma\delta} = \left(\frac{S_\gamma + S_\delta}{2} - S_i \right) \frac{1}{D_i}. \tag{1}$$

Where S_γ and S_δ have been previously determined by scaling, $Z_{\gamma\delta}$ is known, and S_i and D_i are the two unknowns.

Obviously, with a similar equation from another pair of stimuli, λ and μ,

$$Z_{\lambda\mu} = \left(\frac{S_\lambda + S_\mu}{2} - S_i \right) \frac{1}{D_i}, \tag{2}$$

it would be easy to solve for S_i and D_i. By subtracting equation (2) from equation (1) and solving for D_i:

$$D_i = \frac{S_\gamma + S_\delta - (S_\lambda + S_\mu)}{2(Z_{\gamma\delta} - Z_{\lambda\mu})}, \tag{3}$$

and by substituting its value from equation (3) for D_i in equation (1) or (2), S_i would be immediately given.

Application of the above formulas, however, would require that successive judgments on the same pair of stimuli be independent of each other. In the situation in which stimuli are statements of opinion, this condition would not hold. Consequently, some other device must be developed to determine $P_{\gamma p \delta}$.

Let us consider the pair of stimuli γ and δ. It is desired to determine $P_{\gamma p \delta}$ in spite of the fact that γ and δ can be presented to an individual only once. Let us designate all other pairs of stimuli λ and μ such that λ is the member of the pair that has the lower scale value. For purposes of simplicity, we shall regard the origin as being on the left end of the scale and the scale values of the stimuli as increasing to the right.

On the basis of the assumptions made so far, the response of an individual to stimuli λ and μ may be reinterpreted and adjusted in a quantitative manner so that it may be regarded as a response to another pair of stimuli, γ and δ. Suppose, for example, that λ and μ are below γ and δ in scale value and that an individual states that he prefers λ to μ. The scale position of such an individual at that moment is below the abscissa $\dfrac{S_\lambda + S_\mu}{2}$. In order for an individual to make the judgment γ preferred to μ, $(\gamma p \mu)$, he must be below the point $\frac{S_\gamma + S_\delta}{2}$ (cf. Fig. 2). In this instance, then, that $\frac{S_\lambda + S_\mu}{2} < \frac{S_\gamma + S_\delta}{2}$ and $\lambda\ p\ \mu$, this judgment on this pair of stimuli may be regarded as a judgment $\gamma\ p\ \delta$.

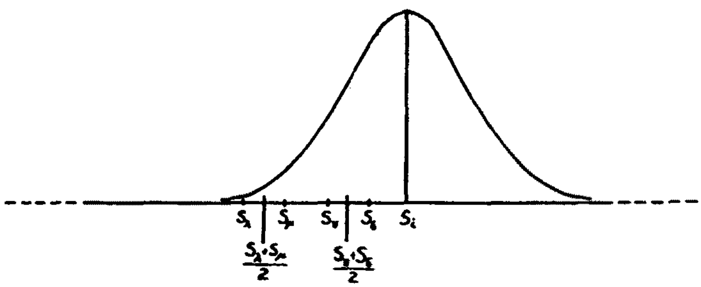


FIGURE 2

In a similar manner, it may be shown that if $\frac{S_\lambda + S_\mu}{2} > \frac{S_\gamma + S_\delta}{2}$ and $\mu\ p\ \lambda$, then this judgment may be regarded as a judgment $\delta\ p\ \gamma$.

The frequency with which pairs of stimuli λ, μ such that $\frac{S_\lambda + S_\mu}{2} < \frac{S_\gamma + S_\delta}{2}$ and $\lambda\ p\ \mu$ occur will be designated $f_{\lambda\mu\gamma\delta}$. Similarly, the frequency with which the pairs of stimuli λ, μ such that $\frac{S_\lambda + S_\mu}{2} > \frac{S_\gamma + S_\delta}{2}$ and $\mu\ p\ \lambda$ occur will be designated $f_{\delta\gamma\mu\lambda}$.

Let us now consider those pairs of stimuli λ, μ such that $\frac{S_\lambda + S_\mu}{2} < \frac{S_\gamma + S_\delta}{2}$ but $\mu\ p\ \lambda$. In this case, this judgment may be broken up so that part of it is regarded as a judgment $\gamma\ p\ \delta$ and the other part of it as a judgment $\delta\ p\ \gamma$. Consider Fig. 3.

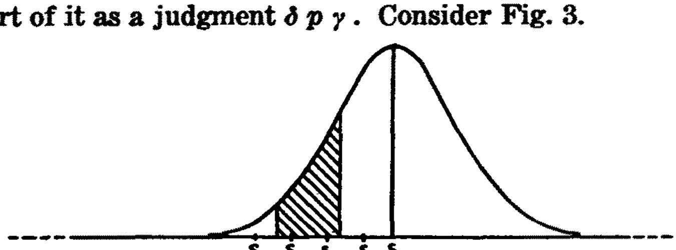


FIGURE 3

An individual who makes the judgment $\mu\ p\ \lambda$ has an attitude at the moment somewhere above $\frac{S_\lambda + S_\mu}{2}$. The probability that the individual would have made the judgment $\gamma\ p\ \delta$ at the time he made the judgment $\mu\ p\ \lambda$ is the cross-hatched area under the curve in Fig. 3 expressed as a fraction of the area under the curve to the right of the ordinate erected at $\frac{S_\lambda + S_\mu}{2}$ and is given by the equation:

$$\frac{\displaystyle\int_{X_3=\left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}}^{X_1=\left(\frac{S_\gamma + S_\delta}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX}{\displaystyle\int_{X_3=\left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}}^{\infty} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX}. \tag{4}$$

The sum of the probabilities given by equation (4) for all pairs of stimuli (λ, μ) such that $\frac{S_\lambda + S_\mu}{2} < \frac{S_\gamma + S_\delta}{2}$ and $\mu\ p\ \lambda$ we shall designate $f_{\mu\lambda\gamma\delta}$.

It is immediately apparent that if the number of pairs of stimuli $\frac{S_\lambda + S_\mu}{2} < \frac{S_\gamma + S_\delta}{2}$ and $\mu\ p\ \lambda$ be designated as K, then $K - f_{\mu\lambda\gamma\delta}$ is the number of times that the judgment $\mu\ p\ \lambda$ may be taken as a judgment $\delta\ p\ \gamma$ and will be designated $f_{\mu\lambda\delta\gamma}$.

Let us now consider those pairs of stimuli $\frac{S_\lambda + S_\mu}{2} > \frac{S_\gamma + S_\delta}{2}$ but $\lambda\ p\ \mu$. In this case also, this judgment may be broken up into two parts such that part of it is regarded as a judgment $\delta\ p\ \gamma$ and the other part of it as a judgment $\gamma\ p\ \delta$.

Consider Fig. 4.

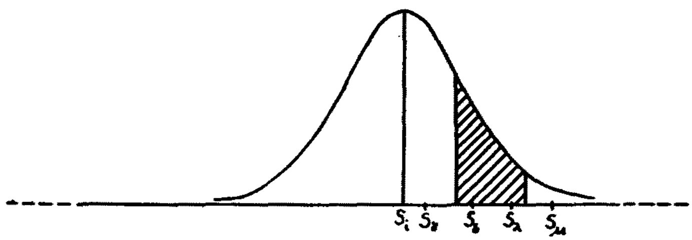


FIGURE 4

An individual who makes the judgment $\lambda\ p\ \mu$ has an attitude at the moment somewhere on the continuum below $\dfrac{S_\lambda + S_\mu}{2}$. The probability that the individual would have made the judgment $\delta\ p\ \gamma$ at the time he made the judgment $\lambda\ p\ \mu$ is the cross-hatched area under the curve in Fig. 4 expressed as a fraction of the area under the curve to the left of the ordinate erected at $\dfrac{S_\lambda + S_\mu}{2}$ and is given by the equation:

$$\frac{\displaystyle\int_{X_1=\left(\frac{S_\gamma + S_\delta}{2} - S_i\right)\frac{1}{D_i}}^{X_2=\left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX}{\displaystyle\int_{-\infty}^{X_2=\left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX} . \tag{5}$$

The sum of the probabilities given by equation (5) for all pairs of stimuli (λ, μ) such that $\dfrac{S_\lambda + S_\mu}{2} > \dfrac{S_\gamma + S_\delta}{2}$ and $\lambda\ p\ \mu$ we shall designate $f_{\delta\gamma\lambda\mu}$.

Again it is immediately apparent that if the number of pairs of stimuli $\dfrac{S_\lambda + S_\mu}{2} > \dfrac{S_\gamma + S_\delta}{2}$ and $\lambda\ p\ \mu$ be designated as L, then $L - f_{\delta\gamma\lambda\mu}$

is the number of times the judgment $\lambda\ p\ \mu$ may be taken as a judgment $\gamma\ p\ \delta$ and will be designated $f_{\gamma\delta\lambda\mu}$.

The proportion of judgments $\gamma\ p\ \delta$ may now be written as a function of all the judgments as follows:

$$P_{\gamma p\delta} = \frac{f_{\gamma\delta} + f_{\lambda\mu\gamma\delta} + f_{\mu\lambda\gamma\delta} + f_{\gamma\delta\lambda\mu}}{f_{\gamma\delta} + f_{\lambda\mu\gamma\delta} + f_{\mu\lambda\gamma\delta} + f_{\gamma\delta\lambda\mu} + f_{\delta\gamma} + f_{\delta\gamma\mu\lambda} + f_{\mu\lambda\delta\gamma} + f_{\delta\gamma\lambda\mu}}, \tag{6}$$

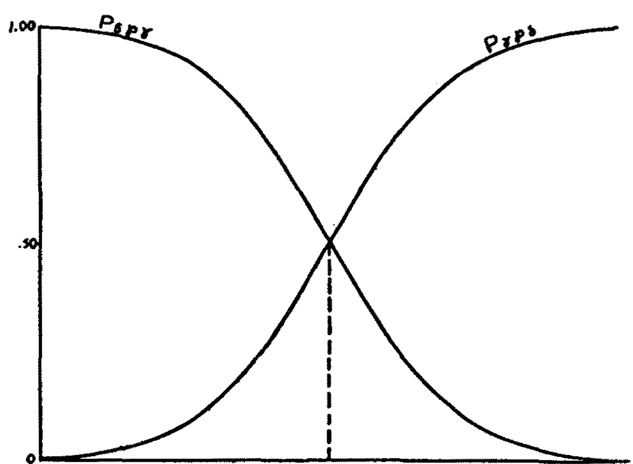
where $f_{\gamma\delta}$ and $f_{\delta\gamma}$ are 0 or 1 and 1 or 0, respectively, depending upon whether the judgment for the pair of stimuli γ and δ was $\delta\ p\ \gamma$ or $\gamma\ p\ \delta$.

The denominator of equation (6) is equal to the number of items. Furthermore, the proportion of judgments $\gamma\ p\ \delta$ is equal to the probability of the individual's S_i being to the left of $\dfrac{S_\gamma + S_\delta}{2}$, which is given by

$$P_{\gamma p\delta} = \int_{-\infty}^{X_1 = \left(\frac{S_\gamma + S_\delta}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX \tag{7}$$

and is illustrated by Fig. 5.

Hence, equation (6) may be written out in full as:



$X = \frac{S_\gamma + S_\delta}{2}$

FIGURE 5

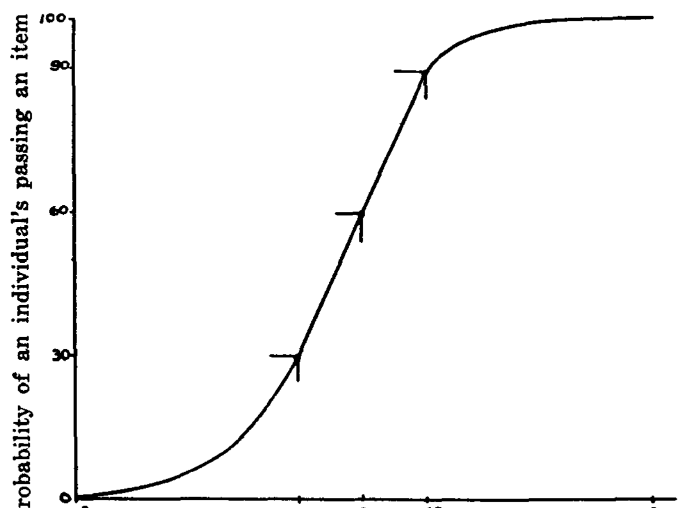
$$N \int_{-\infty}^{X_1 = \left(\frac{S_\gamma + S_\delta}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX =$$

$$f_{\gamma\delta} + f_{\lambda\mu\gamma\delta} + \sum_1^K \frac{\displaystyle\int_{X_3 = \left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}}^{X_1 = \left(\frac{S_\gamma + S_\delta}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX}{\displaystyle\int_{X_3 = \left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}}^{\infty} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX} + \sum_1^L \frac{\displaystyle\int_{-\infty}^{X_1 = \left(\frac{S_\gamma + S_\delta}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX}{\displaystyle\int_{-\infty}^{X_2 = \left(\frac{S_\lambda + S_\mu}{2} - S_i\right)\frac{1}{D_i}} \frac{1}{\sqrt{2\pi}} D_i\, e^{-\frac{(X-S_i)^2}{2D_i^2}}\, dX}, \qquad (8)$$

where N is the number of items.

Equation (8) is an implicit equation in S_i and D_i and insoluble, and only approximate solutions for S_i and D_i can be obtained.

In certain types of situations, equations (1) and (2) can be used directly without the necessity of (6) as an intermediate step. Such situations would include those psychophysical experiments in which it would be possible to present the same pair of stimuli a large number of times without the subject's recognizing the *pair*. Tests of mental ability and achievement also could be constructed which would permit the use of equations (1) and (2). They would, however, have to be especially constructed for the purpose. One could, for example, prepare an arithmetic test with 1/3 of the items at each of three levels of difficulties, or 1/5 of the items at each of five levels of difficulty, etc. With a sufficiently large number of items at each level of difficulty, the proportion of items of that difficulty which can be passed by an individual can be readily computed.



Difficulty of an item = t score equivalent of percentage of a group passing the item

FIGURE 6

This is illustrated by a hypothetical case in Fig. 6. The three points having been obtained experimentally, there are three observation equations and two parameters, the M and σ, or the S_i and D_i, of this individual's curve.

Occasionally the use of equations (1) and (2) without equation (6) might be possible with attitude scales. If the experimenter is fortunate enough to have a lot of items end up at each of several scale values sufficiently close together, then they may be presented in all possible pairs and treated as if the same pair were being retested independently.

One clear and obvious approximation to equation (6) is to neglect the integrals. Then every pair of stimuli can be treated in turn as γ and δ and a $P_{\gamma p \delta}$ obtained for every such pair. This would give a considerable number of equations all in the same two unknowns. A solution by least squares is readily obtained.