PSYCHOMETRIKA—VOL. 22, NO. 1
MARCH, 1957

THE DETAILED METHOD OF OPTIMAL REGIONS*

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The detailed method of optimal regions is an extended form of the method of optimal regions which has been found effective in solving the personnel classification problem when the number of job categories is small. The automatic determination of the successive values of the v_j, made possible by the more exact techniques of the detailed method, provide easier solutions for the more complex problems and provide solutions, which, for the most part, can be mechanized. In a sense the detailed method of optimal regions is more than a detailed form of the method of optimal regions. It is essentially a method of transformations by which the original matrix is reduced to a matrix from which the solution is easily obtained.

1. *Introduction*

The personnel classification problem [1] deals with the assignment of individuals to jobs, where the contribution to the common effort of each individual i if he is placed in position j is the known quantity, c_{ij}. Two recommended methods of solution are the simplex method [3] and the method of optimal regions [2]. The reader is referred to these references for the statement of the problem, the derivation of important properties, and descriptions of methods of solution.

The method of optimal regions is especially effective when, as is common, the number of different positions, k, is small. The method is based on the determination of a constant, v_j, for each position. In the detailed method of optimal regions, more specific rules are given for determining the v_j. Since these rules demand the calculation of auxiliary matrices, the detailed method is especially effective with machines, but it is also recommended when non-trivial problems are to be worked by hand.

Let the number of individuals to be assigned to the k positions be N, and let c_{ij} be entries in a matrix with N rows and k columns. The quota, q_j, the number of men to be assigned to position j, is exhibited in a row at the top of the matrix. This matrix is illustrated in Table 1, where ten men are to be assigned to four positions with quotas 4, 1, 4, 1, respectively. The problem is to make the assignment so that the sum of the corresponding c_{ij} values is as large as possible.

*Much of the basic research covered in this paper was carried out while the author was working on the problem of personnel classification in his capacity as Consultant, Personnel Research Branch, Department of the Army. The author wishes to express his appreciation to the Department of the Army for permission to use these materials in this paper. The opinions expressed are those of the author and are not to be construed as official or as those of the Department of the Army.

2. *The Conditions of Solution*

The basic conditions of solution, fundamental to the simplex method and other methods as well as to the method of optimal regions, imply the existence of u_i and v_j [2, p. 20] such that

$$(2.1) \qquad \begin{aligned} c_{ij} &= u_i + v_j \text{ for assigned values,} \\ c_{ij} &\leq u_i + v_j \text{ for unassigned values.} \end{aligned}$$

If J_i denotes the position to which individual i is assigned, the first expression of (2.1) is

$$(2.2) \qquad c_{iJ_i} = u_i + v_{J_i} .$$

Subtracting the second expression of (2.1) from (2.2) gives a (necessary) condition for solution:

$$(2.3) \qquad c_{iJ_i} - v_{J_i} \geq c_{ij} - v_j .$$

(2.3) may be called the generalized Brogden condition [2, pp. 20-21]. The method of optimal regions is based on the v_j of (2.3). The detailed method of optimal regions also uses the u_i of (2.2).

3. *The Determination of the Initial v_j*

Given the values c_{ij} and the quotas q_j , the first step of the detailed method of optimal regions (and of the method of optimal regions) is the determination of $v_j^{(0)}$, the initial values of v_j . Count out the q_j largest values in each column j and take the smallest of them. In Table 1, this process leads to the values, $v_1^{(0)} = 29$, $v_2^{(0)} = 49$, $v_3^{(0)} = 27$, $v_4^{(0)} = 41$. Then $c_{ij} - v_j^{(0)} \geq 0$ for at least q_j elements in column j.

In problems worked by hand, it is commonly useful to indicate those values which are equal to or greater than the $v_j^{(0)}$. Asterisks have been used to indicate those values.

The $v_j^{(0)}$ may be determined with the use of punched cards. One card is punched for each individual, indicating the c_{ij} values for all positions. The cards are then sorted for each position and the $v_j^{(0)}$ determined from the sorter card count or from a tabulator run using cumulated frequencies.

4. *The Determination of the Initial Assignment and the $u_i^{(0)}$ Values*

The initial assignment, $J_i^{(0)}$, is then made with the use of (2.3). Thus, in Table 1, compare $c_{ij} - v_j^{(0)}$ for successive values of j for each i and make the initial assignment $J_i^{(0)}$ to that column for which $c_{ij} - v_j^{(0)}$ is largest. Individual 1 is initially assigned to job category 1 since -6 is greater than -36, -11, or -27. In case of a tie for the largest value of $c_{ij} - v_j^{(0)}$, both values of j are recorded in the column for $J_i^{(0)}$.

With hand methods, many of the assignments can be made with a simpler rule. If there is a single asterisk in a given row, the assignment is made to the column in which the asterisk appears. If there are two or more asterisks in a given row, only those columns with asterisks need be considered in applying the criterion.

The number of initial assignments to each position is then determined. This number is indicated by $q_j^{(0)}$ and is placed, for comparison, above the q_j values. The first number indicates the number of definite initial assignments and the second the number of ties. Then form $q_j^{(0)} - q_j$, which indicates an excess of assignments if positive and a deficiency if negative. In Table 1 there is an excess of two assignments in column 1 and deficiencies of single assignments in columns 2 and 4.

Next determine the $u_i^{(0)}$ values. From (2.2)

$$u_i = c_{iJ_i} - v_{J_i}, \tag{4.1}$$

and then $u_i^{(0)} = c_{iJ_i}^{(0)} - v_{J_i}^{(0)}$. The results are placed in the column labelled $u_i^{(0)}$. In practice, it is commonly convenient to determine the values of u_i simultaneously with the values of J_i.

5. *The Determination of the First Transformed Matrix*

The first transformed matrix is computed using the formula

$$c_{ij}^{(1)} = c_{ij} - u_i^{(0)} - v_j^{(0)}. \tag{5.1}$$

Every element is either zero or negative since $u_i^{(0)}$ and $v_j^{(0)}$ are determined so that $c_{ij} \leq u_i^{(0)} + v_j^{(0)}$. The values $c_{ij}^{(1)}$ resulting from the application of (5.1) to the problem of Table 1 are shown in Table 2.

The negative signs in this matrix (and the following ones) can be eliminated by using the alternative transformation

$$t_{ij}^{(1)} = -c_{ij}^{(1)} = u_i^{(0)} + v_j^{(0)} - c_{ij}. \tag{5.2}$$

This is illustrated in Table 3. The value of $v_j^{(1)}$ is then the value of the q_jth *smallest* $t_{ij}^{(1)}$ in column j. The values of $J_i^{(1)}$ are then determined using

$$t_{iJ_i^{(1)}}^{(1)} - v_{J_i^{(1)}}^{(1)} \leq t_{ij}^{(1)} - v_j^{(1)}, \tag{5.3}$$

as illustrated in Table 3. The values of $J_i^{(0)}$ are indicated by the zero values of $t_{ij}^{(1)}$. The summary values $q_j^{(0)}$ and $q_j^{(1)}$ are recorded in the top rows. Examination shows that the transformation process is not yet completed since there is an excess of at least one assignment in position 1. Hence an additional transformation is carried out.

6. *The Determination of Successive Transformations*

Since the values of $v_j^{(1)}$ are available, only the values $u_i^{(1)}$ are needed to complete the transformation. Now

$$(6.1) \qquad u_i^{(1)} = c_{iJ_i^{(1)}}^{(1)} - v_{J_i^{(1)}}^{(1)} ,$$

and the next transformation is given by

$$(6.2) \qquad t_{ij}^{(2)} = t_{ij}^{(1)} - u_i^{(1)} - v_j^{(1)} .$$

The application of this transformation to the matrix of Table 3 leads to the matrix of Table 4. The symbol θ is used for each of the zero terms appearing in the same row. Thus the ties of Table 3 are indicated by the θ's of Table 4.

The values of $q_j^{(1)}$ show an excess of at least 1 in column 1. Hence one of the men tentatively assigned to column 1 must be assigned to one of the other columns. This is accomplished by subtracting from column 1 the smallest non-zero entry in any of the rows corresponding to individuals tentatively assigned to position 1. In Table 4, this value is 2; so $v_1^{(2)} = -2$. The remaining values of $v_j^{(2)}$ are 0, but they need not be recorded since nothing is to be subtracted.

The values of $J_i^{(2)}$ are then determined and the summary $q_j^{(2)}$ values. There are no excesses or deficiencies indicated either in the single columns or in the combinations of columns. The obvious assignment of ties leads to the set of J_i values identifying the solution.

In some cases it is necessary to make transformations on combinations of columns, since the method leads to a solution only when every combination of columns, as well as each column separately, has no deficiency [4, p. 16]. The technique for finding a suitable transformation when there is a deficiency in several columns differs slightly from that described above. In Table 3, note that an excess in column 1 indicates a deficiency in columns 2, 3 and 4. Indeed, a summary of the $J_i^{(1)}$ column shows only five men with 0 in columns 2, 3 or 4. But $q_2 + q_3 + q_4 = 6$. Hence there is a deficiency of 1 in this subset. A common positive amount can be subtracted from each of these columns to introduce an additional term, provided the negative of this amount is subtracted from every row which has at least one zero term in columns 2, 3, 4. In this way the tentative assignments to the columns having a net deficiency is maintained, while adding at least one new assignment to these columns. The amount to subtract from the columns is the smallest (non-zero) number in those columns which is not in a row tentatively assigned to column 2, column 3 or column 4. In this way the transformation leads to a matrix having the desired property that every element is non-negative.

In Table 4, $t_{43}^{(2)} = 2$; so $v_2^{(2)} = v_3^{(2)} = v_4^{(2)} = 2$ with $v_1^{(2)} = 0$. These values of $v_j^{(2)}$ lead to values of $J_i^{(2)}$ which are identical with those of Table 4. The two transformations are essentially equivalent transformations since they lead to the same matrix. This is the $t_{ij}^{(3)}$ matrix of Table 5. Assignments satisfying the quotas can be made to the zero terms of this matrix.

The method is designed, at each step, to decrease the number of deficiencies in some particular column or combination of columns without increasing the number of deficiencies in the remaining columns. The method necessarily converges since the total number of deficiencies is finite and since a sufficient condition for solution is an assignment with no deficiencies in any column or combination of columns [4, p. 16]. The process converges very rapidly in the common case in which the number of job categories is small. Experience has led to the empirical conclusion that, for small values of k, the number of transformations required for solution is approximately the value of k. Once the row deviates described in the next section are available, the number of transformations required is commonly less than $k/2$.

7. *The Use of Row Deviates*

A device which is useful in speeding the convergence of the method is the use of row deviates. Any constant may be subtracted from any row without changing the solution since (2.3) is not changed by subtracting a constant from c_{iJ_i} and from c_{ij}. Subtraction of the mean of the row from each element in the row results in row deviates from the mean. Preferably one may use large row deviates defined by

$$C_{ij} = k(c_{ij} - \bar{c}_i) = kc_{ij} - \sum_{j=1}^{k} c_{ij} = kc_{ij} - c_{i.}\,, \tag{7.1}$$

where $c_{i.}$ and $\bar{c}_i$ are, respectively, the sum and mean for row i.

The matrix of row deviates is then treated by the method described above. In the illustration used above the values of $U_i^{(0)}$ and $V_j^{(0)}$ obtained from the C_{ij} matrix are almost adequate for determining the solutions. This is shown in Table 6. Only a slight additional adjustment is necessary in column 4. The advantage of the use of the large row deviate transformation may be seen from the fact that the columns of the C_{ij} matrix are generally uncorrelated or slightly negatively correlated so that large values in one column are not apt to be accompanied by large values in some other column. The values of J_i in Table 6 are identical with those of Table 4.

8. *Solution of a Problem in the Frequency Form*

An illustration is next presented with $k = 5$ and in which it is necessary to analyze subsets of columns even though (large) deviates are used. For this purpose a frequency-form problem which Votaw and Dailey [4, p. 7] have worked with the simplex method is examined. A frequency-form problem results from the grouping of individual categories so that frequencies (f_i) as well as quotas (q_j) appear. The number of personnel categories is n.

The $n = 4$ values of f_i, as well as the $k = 5$ values of q_j, are shown in the first matrix of Table 7. The values of $c_{i.}$ are first computed and then the

TABLE 1

Determination of $v_j^{(0)}$, $u_i^{(0)}$, $J_i^{(0)}$, and u_i

$q_j^{(0)}$	[illegible]-0	0-0	[illegible]-0	[illegible]-0			
$i \backslash q_j$	4	1	4	[illegible]	$u_i^{(0)}$	$J_i^{(0)}$	u_i
1	23	13	15	14	-[illegible]	1	-[illegible]
2	23	17	27*	37	0	3	0
3	30*	20	20	1[illegible]	1	1	-1
4	26	22	22	1[illegible]	-3	1	-[illegible]
5	28	2[illegible]	30*	41*	3	3	3
6	29*	27	20	13	0	1	-2
7	29*	30	36*	3[illegible]	[illegible]	3	9
8	2[illegible]	33	20	22	-[illegible]	1	-[illegible]
9	25	37	33*	3[illegible]	[illegible]	3	[illegible]
10	30*	49*	2[illegible]	26	1	1	1
$v_j^{(0)}$	29	49	27	41	321		
v_j	31	48	27	38			315

TABLE 2

$c_{ij}^{(1)}$ Matrix

$i \backslash q_j$	4	1	4	1
1	0	-30	-5	-21
2	-[illegible]	-32	0	-4
3	0	-30	-8	-26
4	0	-24	-2	-23
5	-4	-27	0	-3
6	0	-22	-7	-28
7	-9	-28	0	-12
8	0	-12	-3	-15
9	-10	-18	0	-9
10	0	-1	-3	-16

TABLE 3

The $t_{ij}^{(1)}$ Matrix with $v_j^{(1)}$, $u_i^{(1)}$, $J_i^{(1)}$, and $q_j^{(1)}$

$q_j^{(1)}$	5-1	0-1	3-1	0-1			
$q_j^{(0)}$	[illegible]-0	0-0	4-0	0-0			
$i \backslash q_j$	4	1	4	1	$u_i^{(1)}$	$J_i^{(0)}$	$J_i^{(1)}$
1	0*	30	5	21	0	1	1
2	6	32	0*	4	0	3	3
3	0*	30	8	26	0	1	1
4	0*	24	2	23	0	1	1
5	4	27	0*	3*	0	3	3,[illegible]
6	0*	22	7	28	0	1	1
7	9	28	0*	12	0	3	3
8	0	12	3	15	0	1	1
9	10	18	0*	9	0	3	3
10	0*	1*	3	16	0	1	1,2
$v_j^{(1)}$	0	1	0	3	4		

TABLE 4

The $t_{ij}^{(2)}$ Matrix with $v_j^{(2)}$, $u_i^{(2)}$, $J_i^{(2)}$, $q_j^{(2)}$, and J_i

$q_j^{(2)}$	4-1	1-0	3-2	0-1				
$q_j^{(1)}$	5-1	0-1	3-1	0-1				
$i \backslash q_j$	4	1	4	1	$u_i^{(2)}$	$J_i^{(1)}$	$J_i^{(2)}$	J_i
1	0	29	5	18	2	1	1	1
2	6	31	0	1		3	3	3
3	0	29	8	23	2	1	1	1
4	0	[illegible]	2	20	2	1	1,3	3
5	4	[illegible]	[illegible]	[illegible]		3,4	3,4	4
6	0	21	7	[illegible]	2	1	1	1
7	9	27	0	9		3	3	3
8	0	11	3	12	2	1	1	1
9	10	17	0	6		3	3	3
10	[illegible]	[illegible]	3	13		1,2	2	2
$v_j^{(2)}$	-2				2			

TABLE 5

Final Matrix

$q_j^{(2)}$	4-1	1-0	3-2	0-1				
$i \backslash q_j$	4	1	4	1	$u_i^{(3)}$	$J_i^{(2)}$	J_i	c_{iJ_i}
1	0	27	3	16		1	1	23
2	8	31	0	1		3	3	27
3	0	27	[illegible]	21		1	1	30
4	[illegible]	21	[illegible]	18		1,3	3	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]		3,4	4	41
6	0	19	5	23		1	1	[illegible]
7	11	27	0	9		3	3	30
8	0	9	1	10		1	1	[illegible]
9	12	17	0	6		3	3	33
10	[illegible]	0	3	13		2	2	[illegible]
$v_j^{(3)}$					315			315

values C_{ij} are recorded in the second matrix. In determining the values $v_j^{(0)}$ consider the frequencies associated with each row. Thus $V_1^{(0)} = -1$, since the 12 + 23 values of -1 in column 1 are more than ample for the quota of 15. The values of $J_i^{(0)}$ are then obtained with the generalized Brogden condition. It is at once apparent that the columnar quotas can be met individually but that there is a deficiency in the subset of columns 1, 2, 5, since the $12 + 23 = 35$ men available cannot fill the $15 + 20 + 12 = 47$ jobs. A transformation is in order.

The values, $U_i^{(0)} = 0$, are computed and then the values $T_{ij}^{(1)}$ appearing in the third matrix. The values of $J_i^{(0)}$ summarize the zero terms. The deficiency in the subset consisting of columns 1, 2, 5 can be met after the matrix is reduced by subtracting some quantity from each of these columns to admit more zeros in the columns. The quantity to be subtracted is the smallest non-zero quantity in the rows not tentatively assigned to columns 1, 2, or 5. This quantity is 1; so $V_1^{(1)} = V_2^{(1)} = V_5^{(1)} = 1$ and, of course, $V_3^{(1)} = V_4^{(1)} = 0$.

The values $J_i^{(1)}$ are then determined. The number of available assignments in each row is so large that assignments satisfying the frequencies and quotas can be met in many different ways.

The additional transformation indicated by the values of $V_j^{(1)}$ is made so that the $T_{ij}^{(2)}$ matrix results. This transformation is not necessary to the solution, since a solution can be obtained from the last column of the third matrix, but the solution may also be obtained by making assignments to the zero terms of the last matrix in any way so as to satisfy the quotas and frequencies.

9. *The Determination of u_i and v_j*

It is now possible to determine the values of u_i and v_j of (2.1). If $t_{ij} = t_{ij}^{(m)}$ represents the final transform, let

$$(9.1)\qquad \begin{aligned} t_{ij} &= 0 \text{ for assigned values,} \\ t_{ij} &\geq 0 \text{ for unassigned values.} \end{aligned}$$

Consider first the case in which the transformations are applied to the c_{ij} matrix without using row deviates. Then

$$(9.2)\qquad t_{ij} = u_i^{(0)} + v_j^{(0)} - c_{ij} - (u_i^{(1)} + v_j^{(1)} + \cdots + u_i^{(m)} + v_j^{(m)}).$$

$$(9.3)\qquad \begin{aligned} u_i &= u_i^{(0)} - u_i^{(1)} - u_i^{(2)} - \cdots - u_i^{(m)}, \\ v_j &= v_j^{(0)} - v_j^{(1)} - v_j^{(2)} - \cdots - v_j^{(m)}. \end{aligned}$$

The values of u_i and v_j for the problem of Table 1 were computed using (9.3) and are shown in the last column and row of Table 1.

The determination of u_i and v_j for problems using the large row deviate transformation is more involved. If the u_i and v_j appropriate to the C_{ij}

TABLE 6

Solution with Large Row Deviates

	c_{ij}					C_{ij}					
					$q_j^{(0)}$	3-1	1-0	5-1	0-0		
i \ q_j	4	1	4	1	$c_{i.}$ \ q_j	4	1	4	1	$U_i^{(0)}$	$J_i^{(0)}$
1	23	13	16	14	66	26*	-14	-2	-10	7	1
2	23	17	27	37	104	-12	-36	4*	44*	5	3
3	30	20	20	16	86	34*	-6	-6	-22	15	1
4	26	22	22	15	85	19*	3	3*	-25	4	3
5	28	25	30	41	124	-12	-24	-4	40	-3	3
6	29	27	20	13	89	27*	19	-9	-37	8	1
7	29	30	36	38	133	-17	-13	11*	19	12	3
8	25	33	20	22	100	0	32	-20	-12	-19	1,3
9	25	37	33	38	133	-33	15	-1*	19	0	3
10	30	49	25	26	130	-10	66*	-30	-26	0	2
v_j	5	16 3/4	0	11	1050	19	63	-1	44	211	

	$T_{ij}^{(1)}$							
$q_j^{(1)}$	3-1	1-0	4-2	0-1				
$q_j^{(0)}$	3-1	1-0	5-1	0-0				
i \ q_j	4	1	4	1	$U_i^{(1)}$	$J_i^{(0)}$	$J_i^{(1)}$	J_i
1	0*	87	8	61		1	1	1
2	36	107	0*	5		3	3	3
3	0*	87	20	81		1	1	1
4	4	67	0*	73		3	3	3
5	28	87	0*	1*		3	3,4	4
6	0*	55	16	89		1	1	1
7	48	91	0*	37		3	3	3
8	0*	15	0*	37		1,3	1,3	1
9	52	51	0*	25		3	3	3
10	29	0*	29	70		2	2	2
$V_j^{(1)}$				1	1			
V_j	19	66	-1	43				

TABLE 7

Solution of a Problem in the Frequency Form

	c_{ij}					
f_i \ q_j	15	20	25	28	12	c_i
12	7	8	6	7	8	36
30	6	5	8	6	7	32
35	5	4	5	7	6	27
23	8	9	8	7	9	41
						3280

	C_{ij}						
f_i \ q_j	15	20	25	28	12	$U_i^{(0)}$	$J_i^{(0)}$
12	-1*	4*	-6	-1	4*	0	1,2,5
30	-2	-7	8*	-2	3	0	3
35	-2	-7	-2	8*	3	0	4
23	-1*	4*	-1	-6	4*	0	1,2,5
$V_j^{(0)}$	-1	4	8	8	4	537	1,2,5

	$T_{ij}^{(1)}$							
f_i \ q_j	15	20	25	28	12	$U_i^{(1)}$	$J_i^{(0)}$	$J_i(1)$
12	0	0	14	9	0	-1	1,2,5	1,2,5
30	1	11	0	10	1		3	1,3,5
35	1	11	10	0	1		4	1,4,5
23	0	0	9	14	0	-1	1,2,5	1,2,5
$V_j^{(1)}$	1	1			1	12	1,2,5	

	$T_{ij}^{(2)}$						
f_i \ q_j	15	20	25	28	12		$J_i^{(1)}$
12	0	0	15	10	0		1,2,5
30	0	10	0	10	0		1,3,5
35	0	10	10	0	0		1,4,5
23	0	0	10	15	0		1,2,5
						761	

matrix are U_i and V_j, a set of non-negative values of v_j can be determined from

$$v_j = (V_j - V_{js})/k,$$

where V_{js} is the smallest V_j. Thus in Table 6, the values of V_j are 19, 66, -1, 43; so the values of v_j are 5, 16 3/4, 0, 11. Again, in Table 7, the values of V_j are -2, 3, 8, 8, 3; so the values of v_j are 0, 1, 2, 2, 1. Other sets of v_j can be obtained by adding constants.

10. *The Determination of the Assignment Sum*

The assignment sum can be determined by applying the assignments for each row to the original c_{ij} matrix. This is illustrated in Table 5; the values of c_{iJ_i} are listed for each i, and the sum is 315 units. An alternative method is based on the formula

$$(10.1) \qquad \sum c_{iJ_i} = (\sum u_i^{(0)} + \sum q_j v_j^{(0)}) - (\sum u_i^{(1)} + \sum q_j v_j^{(1)}) - \cdots - (\sum u_i^{(m)} + \sum q_j v_j^{(m)}).$$

The values in parentheses are given in the lower right corner of the respective matrices. If a problem in the frequency form is used, the values of $\sum u_i^{(\ )}$ are replaced by $\sum f_i u_i^{(\ )}$. If large row deviates are used, the appropriate formula is

$$(10.2) \qquad \sum c_{iJ_i} = \frac{1}{k} \{ \sum c_{i.} + (\sum U_i^{(0)} + \sum q_j V_j^{(0)}) - (\sum U_i^{(1)} + \sum q_j V_j^{(1)}) - \cdots - (\sum U_i^{(m)} + \sum q_j V_j^{(m)}) \}.$$

Thus in Table 6,

$$\sum c_{iJ_i} = \frac{1050 + 211 - 1}{4} = 315 \text{ units}.$$

11. *Interpretation of the Method*

In a sense the detailed method of optimal regions is more than a detailed form of the method of optimal regions. For the former, specific rules are given for determining the successive increments to the v_j. It is essentially a method of reduced matrices in which an original matrix is transformed to a reduced matrix from which the assignment can be determined from the zero terms. The method is especially effective, particularly when using large row deviates, in solving non-trivial personnel assignment problems with a small number of positions.

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Original manuscript received 3/18/55

Revised manuscript received 10/10/56