BOOK REVIEWS

DONALD DAVIDSON, PATRICK SUPPES, AND SIDNEY SIEGEL. *Decision Making, An Experimental Approach.* Stanford: Stanford University Press, 1957. Pp. 121.

The basic problem which the authors have set out to tackle in the studies reported in this book is the separation of the effects of psychological probability and utility in decision making. They have considered this problem both in its theoretical framework and in the experimental verification of the theories set forth. The first chapter gives an introductory discussion of the problems of empirical interpretation of theories of decision making under uncertainty. The second chapter fills almost half of the book and deals with the basic model proposed by the authors. The third chapter reports an experiment which was designed to measure the cardinal utility of nonmonetary outcomes and to use the computed utilities to predict further choices. Two models were compared, one a linear programming model and the other an ordinal model based on straightforward comparisons. The linear programming model turned out to be considerably superior to the other and both were much superior to a random guessing method; moreover, if thresholds are ignored in order to obtain a larger number of predictions the accuracy remains significantly better than chance. The fourth chapter considers the problem of formulating utilities for incomparable outcomes; in contrast with the two preceding chapters the considerations here are entirely axiomatic in character.

The second chapter offers an explicit theory for the explanation of individual decision making under conditions of risk, and reports an experiment designed to test the theory in certain limited situations. The first step in the theory is to construct an event which has a psychological probability of one-half. Next, a set of six outcomes is constructed so as to be equally spaced in utility, and from these a utility function is constructed which is adequate to account for a certain class of preference and indifference relations. The experimental results lead the authors to conclude among other things that (1) the theory provides a practical approach to the problem of resolution of utility and psychological probability in situations involving risk; (2) under suitably controlled conditions certain people make choices among risky alternatives as though they were attempting to maximize expected utility, and (3) for such persons it is possible to construct a utility function unique up to a linear transformation. The point of departure for this model was the work of Mosteller and Nogee (An experimental measurement of utility, *Journal of Political Economy*, 1951, **59**, 371–404); connections with other previous work are traced and a short but well-chosen bibliography is included.

In my opinion this book will take a prominent place in the literature of decision making, but it is also clear that it does not represent the final word on any of the points considered. The expository level of the book is excellent; the prospective reader should be prepared for a reasonable amount of mathematical development of the axiom-definition-theorem type.

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WARREN S. TORGERSON. *Theory and Methods of Scaling.* New York: John Wiley and Sons, 1958. Pp. xiii + 460.

In 1950 the Social Science Research Council appointed a Committee on Scaling Theory and Methods to review the status of scaling procedures in the social sciences. This committee came to the inevitable conclusion that a good survey of the recent and prolific work on scaling procedures was necessary. In 1951, Warren Torgerson, as a Research Associate of the Council, undertook the preparation of a monograph on scaling procedures.

After seven years, two of which constituted a long lost weekend in the Navy, the monograph had become a book and was published.

The result of Torgerson's and the Committee's efforts is a book which will be of considerable influence and of great value to social scientists. It is an excellent summary of the state of the art and theory of measurement in the social sciences, the sort of book of which there is a very real shortage in psychology and the social sciences in general. It was not written as an undergraduate textbook, and probably cannot be used as such. While written primarily as a reference book for technical workers, it does seem possible to use it at the graduate level if a reasonable mathematical background is assumed on the part of the student. I suspect it will be more widely used than was anticipated.

Whatever shortcomings the book has are due more to the state of the art than to incorrect handling of the issues by the author. Torgerson provides an excellent organization of the scaling methods as they are in fact used; the various methods are covered in sufficient detail to enable anybody to use a particular method after a thorough reading of the appropriate section of the book. While extensive mathematical treatment of the various methods is provided, the author does not remain solely at the abstract mathematical level, but introduces the reader to the realities of collecting data and treating them as required by a particular scaling technique. The book is, in other words, a happy combination of erudite sophistication and down-to-earth realism.

The book starts out with the usual introductory and organizing chapters—in this case three of them. The first two chapters cover the nature of measurement, types of measurement, etc. The third chapter organizes measurement as it occurs in psychology into three classes: (1) where subjects are scaled; (2) where the stimuli are scaled, and subject differences are attributed to sampling error; and (3) where both stimuli and subjects are scaled from the same set of data. The first type has not led to any important scaling developments and is largely ignored. The second type, called the *judgment approach*, and the third, called the *response approach*, form the basis for the organization of the rest of the book. The term *response* for the last approach is a little unfortunate, since it does not differentiate that approach from the second, in which responses are also made.

The next seven chapters deal with the judgment methods. Three chapters are devoted to subjective estimates, fractionation, and equisection methods—those methods which involve some appreciation on the part of the subject of numerical values on the subjective continuum under consideration. The next four chapters deal with the discriminative, or differential sensitivity methods—all those methods which are based primarily on the Thurstone models. The last three chapters are concerned with the response methods. One chapter is concerned primarily with the Guttman techniques, one is concerned with Lazarsfeld's latent-structure model, and the last is concerned with the techniques developed by Coombs and his students for dealing with comparative response data.

There is no need to go into the specific content of each chapter. Each deals with its subject matter in a detailed and thorough way. The only chapter which I wish had not been included is that which introduces the differential sensitivity methods, and treats briefly and lightly of the traditional psychophysical methods. In the rest of the book if Torgerson covered a subject at all, he covered it thoroughly. In this one chapter, however, the coverage of the psychophysical procedures is entirely inadequate, and it would have been more in keeping with the rest of the book not to discuss the subject at all.

My major over-all reactions to the book were concerned less with what Torgerson wrote than with the state of measurement theory and practice in psychology today. For example, Torgerson sets up quite clearly the different types of data matrix which are used in scaling work, and differentiates methods on the basis of the nature of the data. He makes clear the kinds of assumptions which are made with regard to both the stimulus and the subject variables in such matrices, and thus provides a more fundamental look at the over-all picture than is customary. However, I found a desire to go back a step further, and to note that all basic sets of data involve three variables—stimulus, subject, and *response*—and that all of them have certain relations to the underlying true continuum. Just as each stimulus, or each subject, can be located on the continuum, so can each response—and there can be interactions between all three variables. It is not really necessary to assume anything fixed by the response, even a comparative response, and a truly general model for measurement would include solutions for the response as well as for stimuli and subjects. It is usual, of course, to have stimuli and subjects be orthogonal in experiments, while neither will normally be orthogonal to the response variable. It is quite possible, however, to give each subject a response and then tell him to find the stimulus which satisfies this response—just as is done with some fractionation procedures and with the equisection procedure. Any truly fundamental model for measurement must be able to deal with the relations between all three variables and the underlying continuum.

Actually, of course, we run into the reality of the lack of degrees of freedom for complete solutions, as Torgerson so often points out. And in fact the practical solutions with which we must deal often are such that an equivalent solution could have been obtained with simplifying assumptions other than those actually made. Using assumptions about the values or distributions of any of the three basic variables, we can solve for values of the other one or two. A change in the variables about which the assumptions are made will change the solutions as well. In other words, we cannot create more knowledge than the data give us; we can simply assign the knowledge to different variables.

In this frame of mind, I have one last reaction to report. It is that we have more sophistication about the nature of mathematical models of measurement than we do about experimental techniques for validating the models. Elegant scales can be constructed, but only after we have made enough assumptions to reduce the number of parameters to the number of available independent observations. Any verification of the assumptions requires the availability of more degrees of freedom, and experimental techniques must be devised which provide these degrees of freedom in a form appropriate to the assumptions being made. The mathematical models tell us only what *can* be so; better experimental techniques are necessary to tell us whether it *is* so.

These reactions are not intended as criticisms of the book, but rather as compliments to it. The book presents the whole range of material in sufficiently compact form that one is forced to try to get an overview. This fact, plus the over-all excellence of the presentation, will stimulate new and good research. I am tempted to say—and so I will say—that this book is a milestone.

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D. A. S. FRASER. *Nonparametric Methods in Statistics.* New York: John Wiley and Sons, 1957. Pp. x + 299.

Nonparametric Methods in Statistics (*NMS*) is an advanced work in statistical theory. *NMS* consists of two parts: an introduction to recent developments in the Neyman-Pearson tradition in statistical inference (Chapters 1 and 2) and an application of these developments to nonparametric statistics (Chapters 3, 4, 5, and 7). In addition, Chapter 6 is a survey of limit theorems useful in nonparametric theory.

The mathematical background necessary for comfortably reading *NMS* is a year's course in function theory. With less than advanced calculus the statements of many of the definitions and theorems are hardly intelligible. Mood (*Introduction to the Theory of Statistics*, McGraw-Hill, 1950) or preferably Cramér (*Mathematical Methods of Statistics*, Princeton University Press, 1946) are reasonable prerequisites in statistical theory. With-out this background, *NMS* can possibly be used as a reference book for the theory. Siegel's book (*Nonparametric Statistics*, McGraw-Hill, 1956), at the other extreme, is nearly devoid of theory and mathematical content.

Besides the standard Neyman-Pearson optimum properties (e.g., most powerful, unbiased, and consistent), sufficiency, invariance, and completeness are stressed. These ideas are developed extensively and used in finding good nonparametric procedures—tests of hypotheses, point estimates, and tolerance intervals. Sufficiency and invariance have a strong intuitive appeal as criteria for optimality. Completeness is a mathematical condition that is useful when available. To illustrate these ideas consider the following problem. It is assumed that $X_1, \cdots, X_m, Y_1, \cdots, Y_n$ are mutually independent random variables. Assume all of the X's have a common distribution and all of the Y's have a common distribution (not necessarily the same as that of the X's). How should one estimate $\Pr(X_i < Y_j)$? ($\Pr(X_i < Y_j)$ appears in the study of the Wilcoxon two-sample procedure.)

It is clear that the (temporal) order in which the observations are made is irrelevant and attention can be restricted to $X_{(1)}, \cdots, X_{(m)}, Y_{(1)}, \cdots, Y_{(n)}$, where $X_{(1)}$ is the smallest of $X_1, \cdots, X_m$; $X_{(2)}$ is the second smallest, etc. In short, the order statistics form a sufficient statistic for the problem. The parameter of interest, $\Pr(X_i < X_j)$, will have the same value whether the original random variables are used or whether any monotone increasing function (e.g., exponential of the random variable) is used. Therefore it is reasonable to restrict attention to those estimators that will not change when an arbitrary monotone transformation of the observations is applied. This is the principle of invariance. For this problem, invariance implies that the estimator must be a function of the ranks. Completeness implies that there is a unique unbiased estimator (and hence minimal variance unbiased estimator) which is a function of the invariant sufficient statistic (the ranks). It is the number of pairs (X_i, Y_j), where $X_i < Y_j$, divided by mn.

The approach of the above paragraph is applied to many estimating and testing experiments which arise in practice, e.g., (a) making inferences from a random sample about the location parameter of a distribution; (b) testing the null hypothesis that c samples come from the same distribution against the alternative that the c samples come from distributions differing in location only; (c) making inferences about the amount of dependence in a bivariate distribution function, i.e., testing for independence and estimating correlation; and (d) constructing tolerance sets from univariate and multivariate data.

The orientation towards Neyman-Pearson theory and linear models (analysis of variance, etc.) explains the lack of emphasis on tests of goodness of fit. In keeping with the theoretical orientation, many mathematical examples are given, and, on the other hand, applied examples and tables of distributions are not given.

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PHILIP J. MCCARTHY. *Introduction to Statistical Reasoning.* New York: McGraw-Hill, 1957. Pp. xiii + 402.

The author states his aims clearly in the Preface: ". . . a one-semester, nonmathematical course in statistics in which the instructor wishes to present a careful introduction to statistical reasoning. . . . A first course should emphasize the concepts of statistical reasoning rather than attempt to cover the wide variety of techniques. . . . Illustrative material should be drawn from investigations that are as significant as possible, and . . . has been chosen to range broadly over the social sciences. A very brief account of research problems usually accompanies each illustrative example and any student . . . may expect to improve his insight into the problems of research methodology in the social sciences. . . . This selection of illustrative material from the social sciences has also influenced to some extent the topics discussed in the book."

I agree with these aims and judge that the author has met them very well indeed; hence, I recommend this excellent book for a one-semester, nonmathematical, introductory statistical course in the social sciences. It excels in the choice of exercises and illustrations drawn from important research publications in the social sciences, has many good examples and fine figures, tables, and charts to illustrate the important problems.

How does this book differ from some of its better competitors? First, it emphasizes, as the title states, statistical reasoning and inference rather than statistical techniques and manipulation. Rather than trying to provide a reference book on a large variety of statistical techniques, the author concentrates on a thorough presentation of the nature of statistical inference. He does this briefly in two chapters which contain the two most useful statistical methods for social scientists: Chapter 8, The Binomial Probability Model and Statistical Inference; and Chapter 9, Drawing Statistical Inferences from the Arithmetic Mean of a Large Sample.

Second, the statistical problems and examples are not alienated from their origins and their destinations. The problems and examples are established firmly in the substantive problems from which they arise, and in the problems of collection and processing which precede the data. The relation of sample to population is often and well developed. The meaning of statistical tools and of statistical inference is constantly emphasized; that statistics and probability statements are guides to action and to decisions is the spirit that pervades the presentation (although there is no formal presentation of statistical decision function theory). Chapter 2, The Components of Statistical Investigation, presents this approach early and well.

Third, the writing is rigorous and precise. The presentation is not mathematical and does not require a mathematical background, but it does demand close attention and careful reading. The style is clear, precise, and to the point, and will aid the student through the hard thinking that rigorous statistical inference demands.

Chapters 3 through 7 deal well with the necessary introductory topics of distributions, their locations and spread, and the elements of probability. Chapter 10 gives in 30 pages an excellent, lucid, and penetrating presentation of Elements of Sample Design. Each of the last two chapters presents an important technique for social scientists: Chapter 11, Chi-Square Procedures for Qualitative Data, and Chapter 12, The Linear Association of Two Quantitative Variables.

Some instructors will prefer to add one or two additional techniques to complete the course; for economists, perhaps time series and indexes; for psychologists, perhaps the difference of two correlated means and the elements of experimental design. The student will have to read his assignments thoroughly and sometimes repeatedly. The instructor will have to explain the finer points at greater length. But perhaps that is not too much to ask of a scientific subject in a scientific age. I think this is a good book.

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