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Optimal design of controlled structures*

J.-S. Ou and N. Kikuchi

Applied Mechanics Department, Computational Mechanics Laboratory, University of Michigan, Ann Arbor, MI 48109, USA

Abstract A formulation that finds the optimal design of a controlled structure is proposed. To achieve this goal, a composite objective composed of structural and control objectives is introduced to be optimized, and the effect of the control weighting is examined. A feedback control law is defined before the structural optimization and then the composite objective will only become a function of structural design variables. As a result, optimal structural design and control forces in steady state are obtained.

1 Introduction

Before the notion of structural control was introduced, structural design had to be conservative in practical applications to be safe enough against various uncertainties on material properties, defining loading and support conditions as well as variations in external excitations. During the past two decades, substantial effort has been made toward reducing the building cost of a structure, and then most modern structures have become much lighter, less stiff, and therefore more vulnerable to the unexpected excessive external loads. In general, inherent damping of a structure is very low. Thus, once oscillation has started, it will continue for a period without any large additional energy input. A modern structural control concept is proposed to accomplish the purpose of both making a structure as light as possible and keeping it away from the risk of external disturbances.

The design of an efficient structural control system is of fundamental interest to both structural and control engineers. Systematic approaches for both structural and control systems are receiving increased applications. However, these design techniques, for the most part, have been applied independently within the entire design process. Bendsøe and Rodrigues (1991), Díaz and Kikuchi (1992), Kamat *et al.* (1983), and Rozvany and Zhou (1991) studied only the structural optimization problems without considering the control system effect. The structure is designed subject to some prescribed stiffness or strength requirements, and the structural engineers have no idea what will happen if they put an actuator in their design. How large will the control force be? How can the design of the structure be modified or changed? It is not easy to answer these questions directly. Balas (1978, 1979), Rofooei and Tadjbakhsh (1993), and Yang *et al.* (1987) designed a control system to improve the dynamic response of a present structure. The control engineers must find appropriate locations for actuators, and use a lot of control energy to reduce the response of the structure. They also do not know how to modify the structure which is suitable for control while still satisfying the structural design criterion (maximum stress or deformation). Thus, a simultaneous integrated design of a structure and control system is proposed (see Hale *et al.* 1985; Kajiwara *et al.* 1994; Miller and Shim 1981). These approaches are only suitable for small dimensional problems because solving two-point boundary value problems or the Riccati equations is too expensive. Furthermore, if the final time of our objective function is not infinite, the optimal control gain will not be a constant matrix over time, and then we must calculate and store the control gain matrices at every instant of time. The cost of computing and storage will be tremendous. Other interesting work about structural control design can be found in the papers by Rozvany and Zhou (1992), and Canfield and Meirovitch (1994).

The goal of this paper is to design an optimal controlled structure which possesses a better response than the one without control, and still satisfies the structural design criterion. After the design is finished, the location for the actuator, which is selected by control engineers and offered by structural designers, will be an ideal place to install the control device. Thus the strain energy of the controlled structure is kept low; the structure still meets some structural design requirements, such as von Mises stress and deformation. This paper proposes a new approach which combines the controlled structural optimization with the homogenization design method. By setting up an appropriate composite objective function, the distribution of the material will tend to reduce the response caused by external excitations, and increase the control effect on the structure simultaneously. Three major advantages of our approach will be illustrated in this paper, and specified as follows.

(1) The composite objective function, consisting of a structural and a control objective, becomes a function of design variables when an appropriate feedback law is introduced, and there is no need to solve the huge Riccati equations.

(2) Positions of actuators can be selected before the structural design. If the possible locations for the actuators are very limited or they are constraints to the structural design, then the problem can be formulated as structural optimization under a specific feedback control law with a constraint to the position of the actuator.

(3) The structure is designed with consideration of both the structural and control effect simultaneously, so the response of this kind of structure is much more improved than the one designed without control, and also superior to the one applied by the control force without simultaneous design.

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2 Structural design by a homogenization method

Shape and topology optimization using the homogenization design method has been quite successful recently (see Suzuki and Kikuchi 1991; Ma *et al.* 1995). Its original idea was introduced by Bendsøe and Kikuchi (1988). This method is based on the concept of optimizing the material distribution of infinitely many microscale voids in a perforated structure. The optimiality criteria derived from the necessary conditions of minimizing the mean compliance are solved by an optimality criteria (OC) method.

The homogenization method can be stated in the following. A perforated structure can be characterized by elastic coefficients $E_{ijk\ell}$ and mass density ρ, which are functions of the porosity of a structure. Both of them vary with the changes of the characteristic sizes and shape of the voids, and they must be very small so that it is reasonable to claim that all quantities have two coordinate dependencies. One is for the macroscopic coordinate $\mathbf{x}$, and the other is for the microscopic one $\mathbf{x}/\varepsilon$, where ε is the size of the microstructure to characterize microscale porous media. By using the homogenization theory, the elastic coefficients can be derived by reflecting the properties of the microscopic behaviour without looking at the details of all the material points of the body. Assume that $\Omega \subset \mathbf{R}^3$ and is a perforated porous design domain. The weak formulation of the equilibrium equation is written as

$$\int_\Omega \mathbf{E}^\varepsilon_{ijk\ell} \left(\frac{\partial}{\partial \mathbf{x}_j} \mathbf{u}^\varepsilon_i\right) \frac{\partial \boldsymbol{\nu}_k}{\partial \mathbf{x}_\ell} \mathrm{d}\Omega - \int_\Omega \mathbf{f}^\varepsilon_i \boldsymbol{\nu}_i \,\mathrm{d}\Omega - \int_\Gamma \mathbf{t}_i \boldsymbol{\nu}_i \,\mathrm{d}\Gamma + \int_\Omega \rho^\varepsilon \left(\frac{\partial^2}{\partial t^2}\right) \mathbf{u}^\varepsilon_i \,\mathrm{d}\Omega = 0\,, \tag{1}$$

where $\mathbf{u}^\varepsilon_i$ is the displacement in the structure, $\boldsymbol{\nu}_i$ is the virtual displacement, $\mathbf{f}^\varepsilon_i$ the body force, $\mathbf{t}_i$ the boundary traction on boundary Γ, and

$$\mathbf{E}^\varepsilon_{ijk\ell} = \begin{cases} \mathbf{E}^0_{ijk\ell} & \text{in solid} \\ 0 & \text{in void} \end{cases} \quad \text{and } \rho^\varepsilon = \begin{cases} \rho^0 & \text{in solid} \\ 0 & \text{in void} \end{cases}, \tag{2}$$

where $\mathbf{E}^0_{ijk\ell}$ and ρ^0 are the elastic coefficients and density of the solid portion. Since $\mathbf{E}^\varepsilon_{ijk\ell}$ depends on the local variable $\mathbf{y} = \mathbf{x}/\varepsilon$, $\mathbf{u}^\varepsilon$ also depends on $\mathbf{x}$ and $\mathbf{y}$. Using the asymtotic expansion of $\mathbf{u}^\varepsilon$ with respect to ε

$$\mathbf{u}^\varepsilon(\mathbf{x},\mathbf{y}) = \mathbf{u}^0(\mathbf{x},\mathbf{y}) + \varepsilon \mathbf{u}^1(\mathbf{x},\mathbf{y}) + \varepsilon^2 \mathbf{u}^2(\mathbf{x},\mathbf{y}) + \dots\,, \tag{3}$$

and substituting the relation

$$\frac{\partial}{\partial x_i} \phi(\mathbf{x}, \mathbf{x}/\varepsilon) = \left(\frac{\partial}{\partial x_i} + \varepsilon^{-1} \frac{\partial}{\partial y_i}\right) \phi(\mathbf{x},\mathbf{y}) \,|_{\mathbf{y}=\mathbf{x}/\varepsilon} \tag{4}$$

into (1), we have the following equations (see Guedes 1990):

$$\int_\Omega \mathbf{E}^H_{ijk\ell} \frac{\partial \mathbf{u}^0_i}{\partial \mathbf{x}_j} \frac{\partial \boldsymbol{\nu}_k}{\partial \mathbf{x}_\ell} \mathrm{d}\Omega - \int_\Omega \mathbf{f}^H_i \boldsymbol{\nu}_i \mathrm{d}\Omega - \int_\Gamma \mathbf{t}_i \boldsymbol{\nu}_i \mathrm{d}\Gamma + \int_\Omega \rho^H \left(\frac{\partial^2}{\partial t^2} \mathbf{u}^0_i\right) \boldsymbol{\nu}_i \mathrm{d}\Omega = 0\,, \tag{5}$$

$$\mathbf{E}^H_{ijk\ell} = \frac{1}{|\mathbf{Y}|} \int_{\mathbf{Y}} \left[\mathbf{E}^\varepsilon_{ijmn} - \mathbf{E}^\varepsilon_{ijmn} \frac{\partial \chi^{(k\ell)}_m}{\partial \mathbf{y}_n}\right] \mathbf{dY}\,,$$

$$\rho^H = \frac{1}{|\mathbf{Y}|} \int_{\mathbf{Y}} \rho^\varepsilon \mathbf{dY}\,, \quad \mathbf{f}^H = \frac{1}{|\mathbf{Y}|} \int_{\mathbf{Y}} \mathbf{f}^\varepsilon \mathbf{dY}\,, \tag{6}$$

after collecting the terms with the same order of ε, and letting $\varepsilon \to 0$, where u^0_i is the component of the average displacements in the microstructure, $\mathbf{E}^H_{ijk\ell}$, ρ^H and $\mathbf{f}^H_i$ are the homogenized elastic coefficients, mass density and body force, respectively, and $\chi^{(k\ell)}_m$ is a proportionality constant similar to the eigenmode due to the unit global strain specified, and is the solution of the microscopic problem that characterizes the micromechanical behaviour of a specific microstructure; $\chi^{(k\ell)}_m$ can be obtained by solving

$$\int_Y \left[\mathbf{E}^\varepsilon_{ijk\ell} - \mathbf{E}^\varepsilon_{ijmn} \frac{\partial \chi^{(k\ell)}_m}{\partial \mathbf{y}_n}\right] \frac{\partial \boldsymbol{\nu}_i}{\partial \mathbf{y}_j} \mathrm{d}\mathbf{Y} = 0\,, \quad \forall \nu \in \mathbf{V_Y}\,, \tag{7}$$

where the space $\mathbf{V_Y}$ is $\mathbf{Y}$-periodical defined on the microstructural domain $\mathbf{Y}$.

For simlicity, we assume the microstructure is defined by three design variables, $\mathbf{a}$, $\mathbf{b}$ and $\boldsymbol{\Theta}$, where $1 - \mathbf{a}$ are the size of a rectangular hole which is rotated by $\boldsymbol{\Theta}$ with respect to the x_1 coordinate as shown in Fig. 1. Using these design variables, three possible design domains are formed; they are the full material domain, full void domain and porous domain (see Fig. 2). The rotated homogenized elasticity tensor can be obtained by

$$\mathbf{E}^G_{ijk\ell} = \mathbf{E}^H_{ijk\ell} \mathbf{R}_{iI} \mathbf{R}_{jJ} \mathbf{R}_{kK} \mathbf{R}_{\ell L}\,,$$

the rotation matrix $\mathbf{R}$ is defined by

$$\mathbf{R}(\Theta) = \begin{bmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{bmatrix}.$$

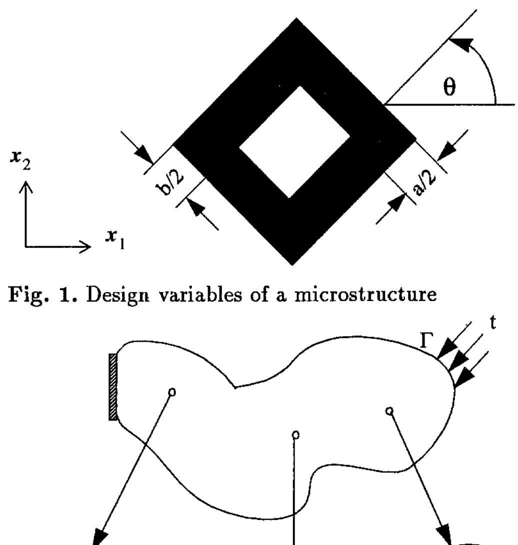


Fig. 1. Design variables of a microstructure

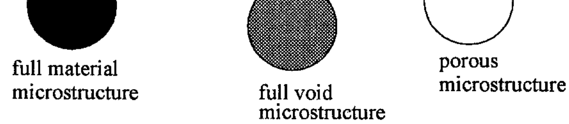


Fig. 2. Three possible microstructures used in homogenization design

Letting $\mathbf{u}^0 = \mathbf{u}$ and considering the static case only, (5) can be written in terms of $\mathbf{E}^G$:

$$\int_{\Omega} \mathbf{E}^G_{ijk\ell} \frac{\partial \mathbf{u}_i \partial \boldsymbol{\nu}_k}{\partial \mathbf{x}_j \partial \mathbf{x}_\ell} \,\mathrm{d}\Omega = \int_{\Omega} \mathbf{f}^H_i \boldsymbol{\nu}_i \,\mathrm{d}\Omega + \int_{\Gamma} \mathbf{t}_i \boldsymbol{\nu}_i \,\mathrm{d}\Gamma \,. \tag{8}$$

From (8), we can solve the displacement field by the homogenized coefficients $\mathbf{E}^G_{ijk\ell}$ which are the functions of $\mathbf{a}, \mathbf{b}$ and $\boldsymbol{\Theta}$.

3 Design procedure

One of the major difficulties in optimization problems together with control is how to find an appropriate objective function. A standard objective function used in optimal control theory is the sum of strain, kinetic and control energy, and is written in the following form:

$$J = \int_0^{t_f} (\mathbf{x}^t \mathbf{Q} \mathbf{x} + \mathbf{f}^t \mathbf{R} \mathbf{f}) \,\mathrm{d}t \,, \tag{9}$$

where $\mathbf{x}$ is a state variable vector, $\mathbf{f}$ is a control force vector, $\mathbf{R}$ is the positive definite weighting matrix for control forces, and Q is the semipositive definite weighting matrix for state variables. One question raised is how to choose the termination time t_f for dynamic problems for optimal control. Suppose t_f is the time the system reaches the steady state. The response before t_f is called transient response region, and after t_f we call it steady state region. If t_f is small, then the objective function (9) emphasizes the transient response, if t_f is large, then (9) emphasizes the steady state response, while for median t_f, the objective function includes both the transient and steady state regions. However, no matter what value of t_f is chosen for the linear time-invariant system, the cost for the solution of Riccati equations is too high for large scale problems, and the situation becomes even worse if t_f is not large, because the calculations in sensitivity for both the structural design variables and optimal gains are incredibly expensive.

An alternative way to deal with a wide range of structural control problems may be the application of a sequential approach of an optimal structural design with consideration of the control effect, and a control algorithm design for transient response by classical or optimal control theory. The procedure can be written in the following steps.

(1) Define the composite objective function in the steady state, i.e. $t_f \to \infty$.
(2) Establish the mathematical formulation for the optimization problem and construct a finite element model for the design domain.
(3) Find the optimal structures and control forces in steady state by solving the minimization of the objective function.
(4) If the steady state response is satisfied, then go to step 6. Otherwise go to step 5.
(5) Choose different weighting matrices for the control objective and/or select other actuators' locations until the steady state response is fulfilled.
(6) Define the design requirements in the transient region for the optimal structures obtained in the steady state.
(7) Find the control gains in the transient region for the obtained optimal structure.
(8) If the response in the transient region is satisfied, then stop; if not go to step 9.
(9) Add more actuators to the optimal structure or go to step 5.

We consider only steps 1 to 5 in this paper, and the other steps will be solved in the future.

4 Formulation of optimization

For optimal structural design in ready state, we choose an objective function as the sum of the strain energy and control energy,

$$\min_{\mathbf{a},\mathbf{b}\boldsymbol{\Theta}} J = \mathbf{f}^t \mathbf{R} \mathbf{f} + \mathbf{d}^t \mathbf{Q} d \,, \tag{10}$$

$$\text{s.t.} \begin{cases} \mathbf{K}d &= \mathbf{G}_1 \mathbf{f} + \mathbf{F} \\ P(\mathbf{a}, \mathbf{b}) &= 0 \\ \mathbf{L} &= (x1, x2) \end{cases} , \tag{11}$$

and

$$P(\mathbf{a}, \mathbf{b}) = \sum_{e=1}^{\text{nel}} \int_{\Omega^e} (a_e + b_e - a_e b_e) \,\mathrm{d}\Omega^e - \Omega^s \,, \tag{12}$$

where $\mathbf{d}$ is a $n \times 1$ displacement vector, $\mathbf{a}, \mathbf{b}$ and Θ are the design variable vectors, $P(\mathbf{a}, \mathbf{b}) = 0$ is the material resource constraint, $\mathbf{G}_1$ is $n \times m$ location matrix for the control force, $\mathbf{F}$ is a $n \times 1$ applied external force vector, $\mathbf{f}$ is a $m \times 1$ control force vector, nel is the total number of the finite elements, and Ω^e is the element design domain. The magnitude of the matrices $\mathbf{Q}$ and $\mathbf{R}$ is assigned according to the relative importance of the state variables and the control forces in the minimization procedure. By adjustng the relative values of $\mathbf{Q}$ and $\mathbf{R}$, one can synthesize the control to achieve a proper trade off between these two objectives, $\mathbf{L}$ is a constraint of the position of the actuator specified at coordinate $(\mathbf{x1}, \mathbf{x2})$.

Once we define the objective function in this form, we must choose the appropriate weighting matrices. Should they be constant matrices of functions of structural parameters? For simplicity, constant weighting matrices (Hale *et al.* 1985; Kajiwara *et al.* 1994) are usually chosen in most works on simultaneous design, while the magnitude of their elements is selected by existing experience. For the weighting matrices that are not constant, in general, one selects $\mathbf{Q} = \mathbf{K}$, then $\mathbf{d}^t \mathbf{Q} \mathbf{d}$ becomes twice the strain energy. How to select $\mathbf{R}$? An improper choice of $\mathbf{R}$ will lead to a design far from our goal. In the homogenization design method, the structure changes dramatically during the optimization iterations, the constant $\mathbf{R}$ is definitely not a good choice if $\mathbf{Q}$ is not a constant matrix. Venkayya and Tichler (1985) mentioned a suitable way of choosing $\mathbf{R}$ by setting $\mathbf{R} = \mathbf{G}_1^t \mathbf{K}^{-1} \mathbf{G}_1$. Unfortunately, we cannot use their choice because the sensitivity of $\mathbf{K}^{-1}$ is almost impossible to obtain for large scale problems. In this work, we select $\mathbf{Q} = \mathbf{K}$, and choose a control weighting which possesses a similar property of $\mathbf{G}_1^t \mathbf{K}^{-1} \mathbf{G}_1$.

Using the displacement feedback closed-loop control we can assume $\mathbf{f} = -\mathbf{R}^{-1} \mathbf{G}_1^t d$, then the first equation of (11) becomes

$$(\mathbf{K} + \mathbf{G}_1 \mathbf{R}^{-1} \mathbf{G}_1^t) \mathbf{d} = \mathbf{F} \,. \tag{13}$$

Reformulating the equilibrium equation yields

$$\mathbf{K}_2 \mathbf{d} = \mathbf{F} \,, \tag{14}$$

where $\mathbf{K}_2 = \mathbf{K} + \mathbf{G}_1\mathbf{R}^{-1}\mathbf{G}_1^t$ is the modified stiffness matrix under the control effect. Problem (14) can be solved by the finite element method, and it is very similar to the standard problem $\mathbf{Kd} = \mathbf{f}$. The eigenvalues of $\mathbf{K}$ are called open-loop eigenvalues, the eigenvlues of $\mathbf{K}_2$ are called the closed-loop eigenvalues. By introducing this feedback control, the control effect will modify the stiffness matrix so that the eigenvalues of the structure are shifted.

An eigenvalue is a good choice for defining a global control weighting, because its sensitivity can be easily computed even for large scale problems. We may choose

$$\mathbf{R} = \frac{\mathbf{w}}{\lambda_1}, \tag{15}$$

where λ_1 is the smallest eigenvalue and $\mathbf{w}$ is a normalized weighting constant with respect to the smallest eigenvalue. Traditionally, control system design is performed after the structural design is obtained, but it may give control designers few choices of actuators' positions, and then a lot of control energy must be put in to stabilize the structure. If locations of the actuators are the constraints for the designers, then how does the structural designer formulate the problem, so they can still accomplish the optimal design. If we use (10), the control system designers can select appropriate actuators' positions before the structural design (usually the choice depends on the amount of control energy consumption, easiness of installing control devices), and we do expect that a structure designed by this formulation will offer us a good transient response control.

5 Sensitivity analysis and updating scheme

Taking the derivative of (10),

$$\frac{\partial J}{\partial \mathbf{a_e}} = \mathbf{F}^t \frac{\partial \mathbf{d}}{\partial \mathbf{a_e}}, \tag{16}$$

and substituting $\frac{\partial \mathbf{d}}{\partial \mathbf{a_e}} = -\mathbf{K}_2^{-1}\frac{\partial \mathbf{K}_2}{\partial \mathbf{a_e}}\mathbf{d}$ and $\frac{\partial \mathbf{K}_2}{\partial \mathbf{a_e}} = \frac{\partial \mathbf{K}}{\partial \mathbf{a_e}} + \frac{1}{\mathbf{w}}\mathbf{G}_1\frac{\partial \lambda_1}{\partial \mathbf{a_e}}\mathbf{G}_1^t$ into (16), we have

$$\frac{\partial J}{\partial \mathbf{a_e}} = -\left(\mathbf{d}^t\frac{\partial \mathbf{K}}{\partial \mathbf{a_e}}\mathbf{d} + \frac{1}{\mathbf{w}}\mathbf{d}^t\mathbf{G}\frac{\partial \lambda_1}{\partial \mathbf{a_e}}\mathbf{G}_1^t\mathbf{d}\right). \tag{17}$$

The sensitivity of the stiffness matrix can be found by knowing the sensitivity of elasticity constants and shape functions used in the finite element analysis. The sensitivity of the eigenvalue (see Haftka *et al.* 1990) is represented by

$$\frac{\partial \lambda_1}{\partial \mathbf{a_e}} = \phi_1^t\left(\frac{\partial \mathbf{K}}{\partial \mathbf{a_e}} - \lambda_1\frac{\partial \mathbf{M}}{\partial \mathbf{a_e}}\right)\phi_1 =$$

$$\phi_{1e}^t\left(\frac{\partial \mathbf{k_e}}{\partial \mathbf{a_e}} - \lambda_1\frac{\partial \mathbf{m}_e}{\partial \mathbf{a_e}}\right)\phi_{1e}, \tag{18}$$

where ϕ_1 is the normalized eigenvector w.r.t. the mass matrix corresponding to the first eigenvalue, $\mathbf{k_e}$ is the element stiffness matrix, $\mathbf{m_e}$ is the element mass matrix, and ϕ_{1e} is the first normalized eigenvector in element e. Thus, (18) can be calculated in the element level, and the computing cost is not so high even for large scale problems.

6 An example of the difficulty in applying the control system to a structure designed by structural engineers

In this paper, all problems use the properties of material corresponding to Young's modulus = 100 GPa, Poisson's ratio = 0.3 and density $\rho = 1e-6$.

- Problem specifications (example 1): design domain 4 cm by 10 cm; finite element mesh 28 by 70; volume constraint 10; location of actuator $\mathbf{L} = (2, 5.875)$; boundary conditions are shown in Fig. 3

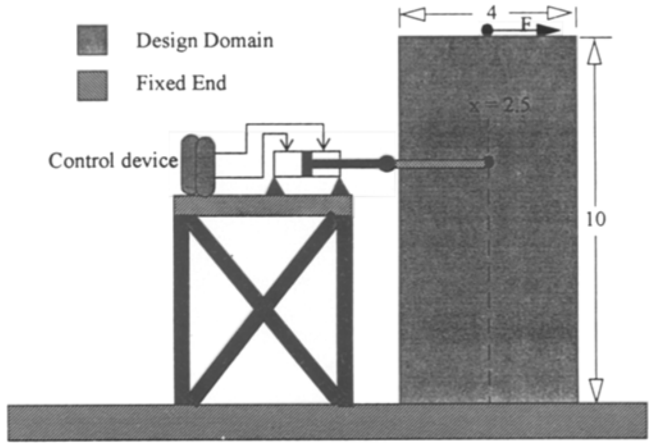
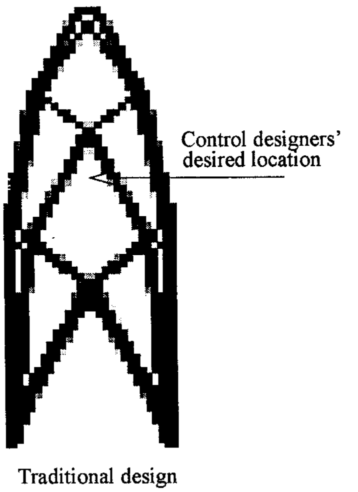


Fig. 3. Design domain and boundary conditions for example 1

- Traditional approach: structural designers ignore the specifications imposed by control designers. They use mean compliance as the objective function, and obtain the optimal structural design shown in Fig. 4. When control designers receive this design layout, there is no material distributed at their desired location for the actuator. Even if some material distribute at $\mathbf{L}$, the controllability of this system is still unknown. There is no guarantee that the control device can transfer their energy to reduce the structural deformation. If the structure does not perform well under the control force, then the control designers must either change the specification of the location of the actuator or request to design the structure again. Changing the location of the actuator involves the building of a foundation for the control device, the design of a mechanism, power supply for hydraulic motors, and so on. If they decide to design the structure again, how much material should the structural designer put at the location of the actuator, and how can they reinforce this location? Neither situation is easy to solve.

- Current approach: by using (10), (11) and (12), the optimal structure distributes some material at the location desired by the control designer, see Fig. 4. The structural designers construct an optimal structure while still making this position available for the control designers to install an actuator to control the response of the optimal structure. The current approach obtains a satisfactory design much easier than the traditional sequential structural control design. The objective function and control force converges smoothly as shown in Fig. 5. This example demonstrates the advantage of selecting the positions of the actuator by the control designers before the structural design is done.



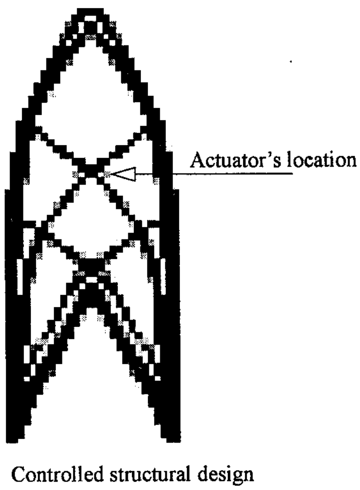


Fig. 4. Optimal structures for example 1

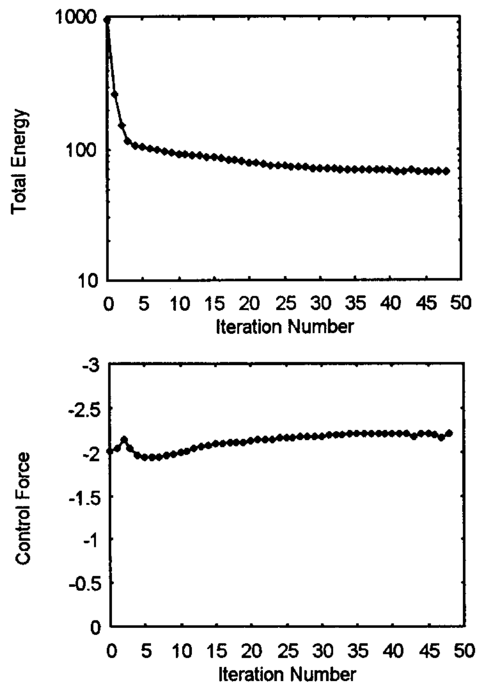


Fig. 5. Convergent results in example 1

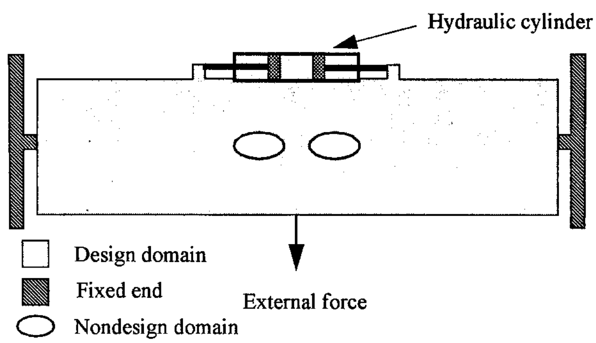


Fig. 6. Initial design domain and a control device for example 2

7 Effect of the control weighting constant w

- Problem description (example 2): design domain 2 cm by 8 cm; nondesign domain, two elliptic holes; finite element mesh 40 by 160; volume constraint 4; locations of actuator $\mathbf{L}_1 = (2, 2.7)$, $\mathbf{L}_2 = (2, 5.3)$; assumption the weight of control device is neglected
- Design purpose: an initial porous structure fixed at two middle ends, and subjected to a downward force, see Fig. 6. The control designers know that the largest deformation will be close to the top or bottom of the structure. Assuming that the bottom of the structure does not have enough space for adding extra equipment, the control designers choose the top part of the structure as a desired location for the actuator. We attempt to find out how the weighting constant affects the design.
- Discussion: in this example, the control device is mounted on the top of the structure and is used to act at two locations to control the deformation of the structure. The two control forces have the same magnitude, but with opposite directions. The displacements (or strain) at these two locations affect the reaction of the control device. Positive strain causes the control system to generate compressive forces, contrariwise, negative strain invokes tensile forces.

To investigate the effect of the weighting value on the structural design, five different $\mathbf{w}$ are chosen for design. These weighting values and major computed data for the final structures are listed in Table 1.

Some of the optimal structures corresponding to these weightings are shown in Fig. 8. It is noted that when $\mathbf{w}$ decreases, more material is distributed at the positions where the control is applied, i.e. the distribution of material tends to produce a structure easier for control while still keeping reducing the structural deformation. A short conclusion from this example is summarized as below.

(1) When the structure changes during the iterations, the eigenvalue also changes, and the magnitude of the control force will adjust according to the magnitude of the eigenvalue. This example shows that the eigenvalue is an appropriate weighting, and can be used as an information to estimate the control gain for the controlled structure.
(2) Both the strain energy and total energy have the same trend, i.e. as the weightng $\mathbf{w}$ increases, strain energy and total cost also increase, but the control force decreases as

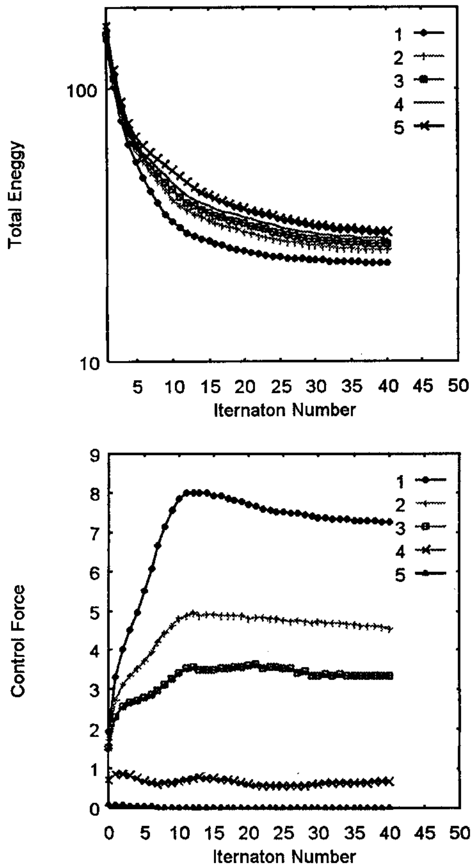


Fig. 7. Convergent results in example 2

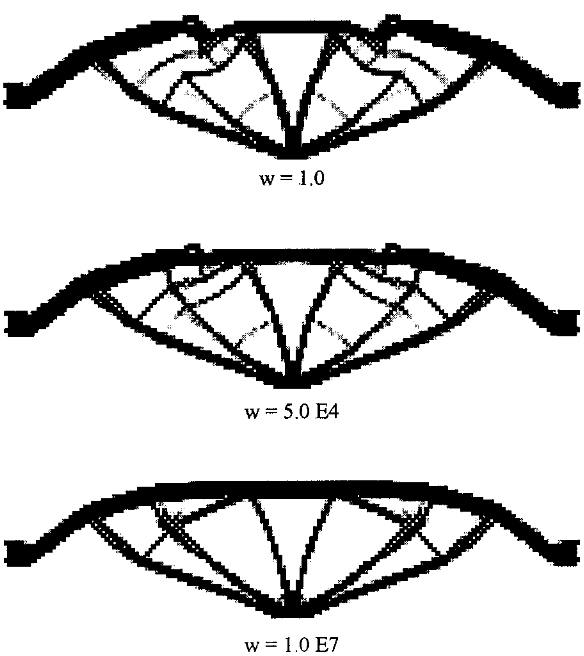


Fig. 8. Optimal structures in example 2

the weighting w increases (Fig. 7). This means that large control force will further decrease the objective value of the structure. Thus, the smaller the objective we want, the larger the control forces we need.

(3) The matrix $\mathbf{G}_1\mathbf{R}^{-1}\mathbf{G}_1^t$ in (13) is, in general, semipositive, so the control effect will only increase the eigenvalue of the structure under a suitable control design algorithm. Thus, the closed-loop eigenvalues are always larger than the open-loop eigenvalues.

Table 1. Major results for different weightings

Case	Case 1	Case 2	Case 3	Case 4	Case 5
Weighting value	$w = 1$	$w = 5e4$	$w = 1e5$	$w = 2e5$	$w = 1e7$
1st open-loop eigenvalue	$4.77E5$	$6.86E5$	$6.99E5$	$7.03E5$	$4.44E5$
1st close-loop eigenvalue	$8.96E5$	$8.42E5$	$7.98E5$	$7.60E5$	$4.47E5$
Steady state control force	7.27	4.54	3.34	2.22	$2.06E-2$
Final objective function	$2.29E1$	$2.55E1$	$2.68E1$	$2.81E1$	$2.97E1$

8 Multiple eigenvalues and dynamic control weighting

8.1 Formulation

In previous examples, only the lowest eigenvalue as a control weighting was considered. Does this weighting always give a good result? To answer this question, a more general form, which includes multiple eigenvalues and a dynamic weighting, are presented. A conclusion will be made after the comparison of some numerical data. First let us consider the control weighting with multiple eigenvalues of the form

$$\mathbf{R} = \mathbf{w}/\left(\sum_{i=1}^{\text{neig}} W_i\lambda_i\right), \tag{19}$$

where neig is the number of eigenvalues considered in the weighting, W_i is a weighting constant for each eigenvalue. Thus, the contributions from different eigenvalues can be adjusted by W_i and the ratio of the control objective can be regulated by $\mathbf{w}$.

During the iterations of optimization, the order of critical modes may change, and the lowest eigenvalue is not necessarily the dominant one. If we always use the fixed eigenvalues to estimate the stiffness of a structure, then we probably cannot obtain the good results that we expect. A dynamic weighting that uses the modal energy to extract the critical modes is introduced. The form of the dynamic weighting is the same as (19); the difference is that neig is the number of critical modes, and λ_i is the critical eigenvalue.

8.2 Comparisons of different control weightings

- Problem specification (example 3): design domain 4 cm by 10 cm; finite element mesh 20 by 50; volume constraint 10; location of actuator $\mathbf{L} = (4.0, 5.0)$
- Discussion: the initial design domain is rectangular, two downward external forces act at the bottom, and one cable connected two hydraulic cylinders offers the vertical control force, see Fig. 9. The four combinations of weighting constants and one dynamic weighting shown in Table 2 are used to investigate their effects on structural design.

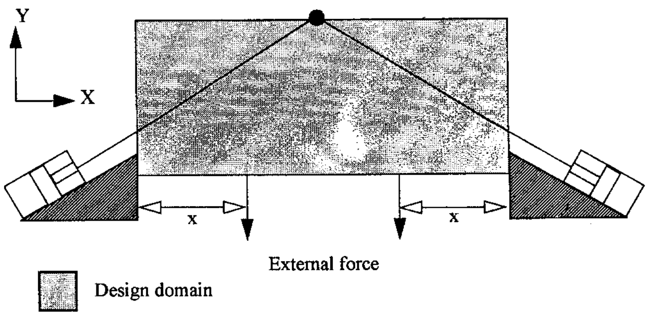


Fig. 9. Initial structural control configuration for example 3

To make a fair comparison, we choose different weighting constants **w**, so that the final control forces for all cases are almost equal. The optimal designs corresponding to these data are shown in Fig. 10. The comparison of the Skyline size, objective value, strain energy, and displacement at the exciting position are plotted in Fig. 11. The Skyline size is the storage size of double precision real value for stiffness matrix. While convergence of the layout is achieved, more and more elements become void, so its size decreases accordingly. Thus, a smaller Skyline size means that the structure can be easily manufactured.

Table 2. Weighting constant used in example 3

	Case 1	Case 2	Case 3	Case 4	Case 5
W_1	1	0	0	0.333	dynamic
W_2	0	1	0	0.333	dynamic
W_3	0	0	1	0.333	dynamic

Two fundamental modes are very important during this structural optimization; one vibrates mainly in the Y-direction (this is the critical mode) and the other vibrates mainly in the X-direction. In each iteration, the order of these two modes may change, and we cannot guarantee that the lowest mode is the critical one. In cases 1 and 2, the critical mode shifts between the first and the second during the iterations, and the performance of these two structures is moderate. Case 3 uses the third eigenvalue as control weighting, and the structure cannot converge well because the third mode never becomes critical during the iterations. The same situation occurs in case 4, although we use the first three eigenvalues as a weighting, the critical mode is always the first one in this problem, and that is why we cannot obtain good results. The previous results motivate us to extract the critical mode by calculating the modal energy, and always use the critical eigenvalue as the control weighting, which we call dynamic weighting. The results show that case 5 has the best performance in all properties that we evaluate. Thus we conclude that the dynamic weighting gives us the best design, and we will have this weighting for the rest of this paper in controlled structural design.

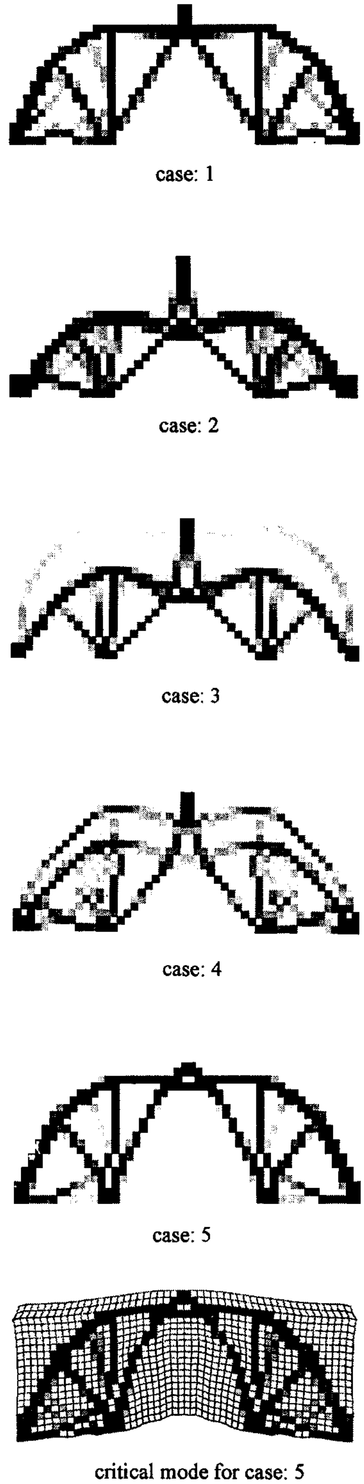


Fig. 10. Optimal design for example 3

9 An evaluation of present method

To demonstrate the perfomance of controlled structural design, we must compare our current approach with other traditional designs. We still use the same design arrangement as the example in Section 8.2.

Three different design problems are specified as follows (example 4):

- problem 1: controlled structural design;
- problem 2: static design (Suzuki and Kikuchi 1991);
- problem 3: static design obtained in problem 2, and application of the same control force obtained in problem 1 to this design.

Three situations for each problem are considered by changing

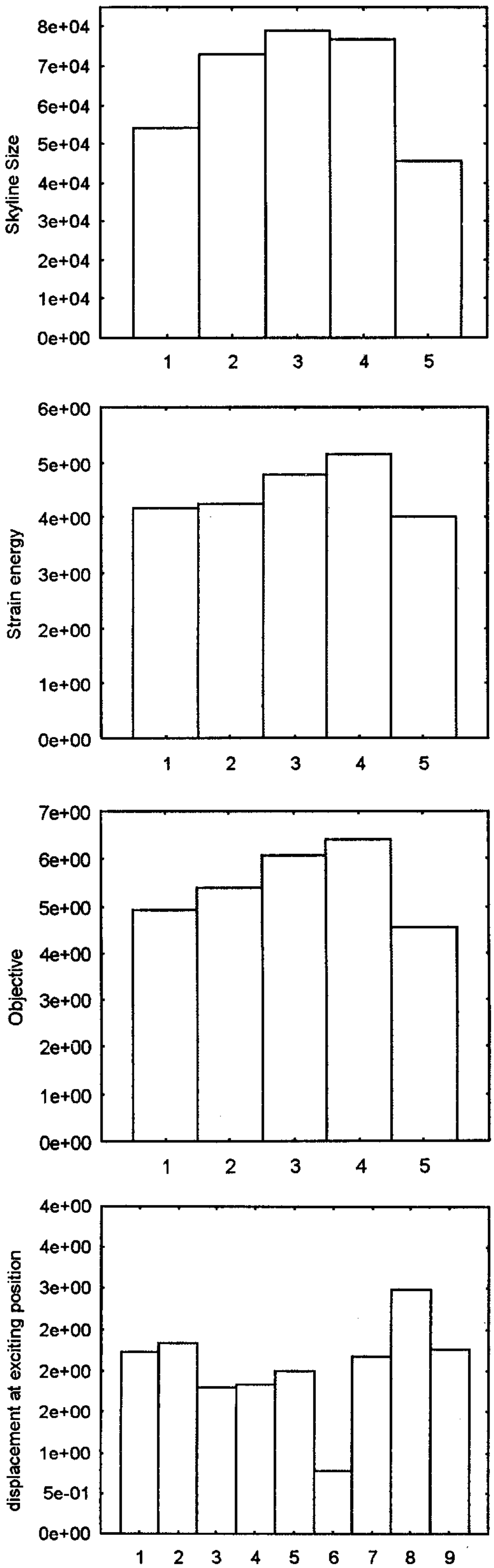


Fig. 11. Evaluation for four different properties in example 3

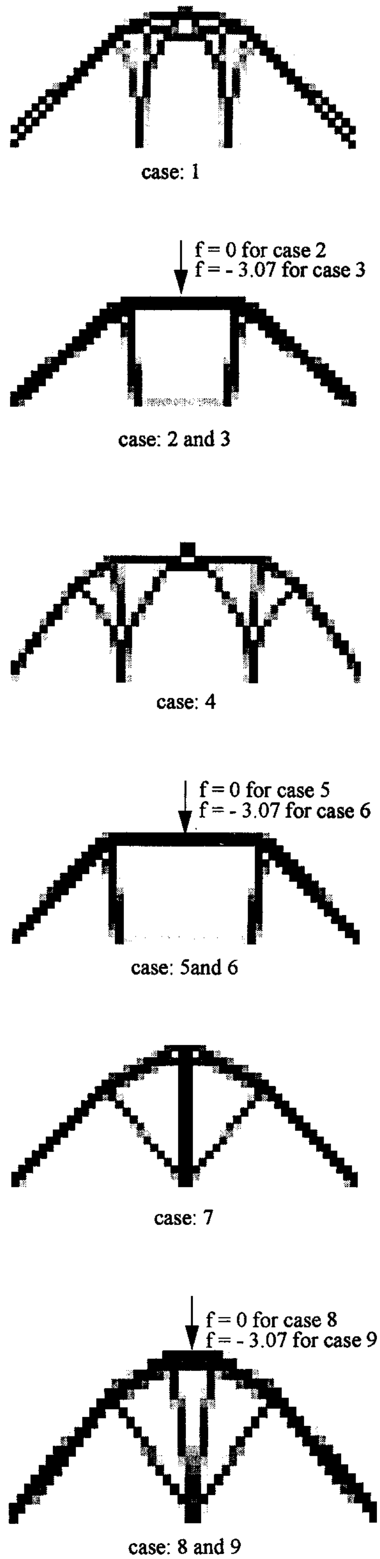


Fig. 12. Optimal design for example 4

the location of the external forces x; they are $x_1 = 3.6$, $x_2 = 3.0$ and $x_3 = 4.8$, respectively. Thus, nine cases in three groups will be investigated, see Table 3.

Cases 1, 4 and 7 are controlled structural design, cases 2, 5 and 8 are static design, and cases 3, 6 and 9 are static design acted on by the control force obtained in the con-

Table 3. Data for nine cases

	Case	x	Control force
Group 1	1, 2, 3	3.6	−3.07
Group 2	4, 5, 6	3.0	−3.26
Group 3	7, 8, 9	4.8	−3.50

trolled structural design. The optimal designs corresponding to these cases are plotted in Fig. 12. Cases 3, 6 and 9 have the same structure as cases 2, 5 and 8, but with the control force applied. The comparisons will be made inside each group. In group 1, case 3 has the best response in displacement, but it sacrifices too much stress response. It is obvious that case 1 is the best design in this group. The same situation occurs in group 2, the von Mises stress in case 6 is almost twice that of case 4. Thus, case 4 is the best design in group 2. In group 3, the controlled structural design has a very similar design to the static design, but case 7 shows superior performance in every response we evaluate as compared to the other two. The conclusion is that the controlled structural design is the best of these three design problems, and contains the potential suitable for dynamic response control.

From this example, we know that if we want to use an actuator to reduce the response of a structure, we cannot simly apply the control force anywhere with arbitrary magnitude to a structure designed by structural engineers. Both the external forces and the control effect must be considered at the same time. Our current approach, controlled structural design, offers a good response in strain energy and displacement at critical positions without having excessive von Mises stress.

10 Conclusion

Control designers can assign the locations of actuators before structural design, while structural designers treat this requirement as a constraint, and finally the simultaneous design for the structures and control forces in the steady state is obtained. Thus, control designers can offer some information to structural designers before the design, and the design procedure for the structure and the control system is not totally separate. The lowest eigenvalue is chosen as the control weighting to attain the purpose of minimizing the composite objective, and the example in Section 7 shows that this weighting can be used successfully. Then a dynamic control weighting is proposed, and it gives the best performance from the numerical results. Finally, comparisons were made between the controlled structural and traditional designs; the performance of present design is really superior to the traditional ones. The approach proposed in this paper already set up an optimal structural design with consideration of the control effect. A transient response control design, our study in the future, will be implemented on this structure.

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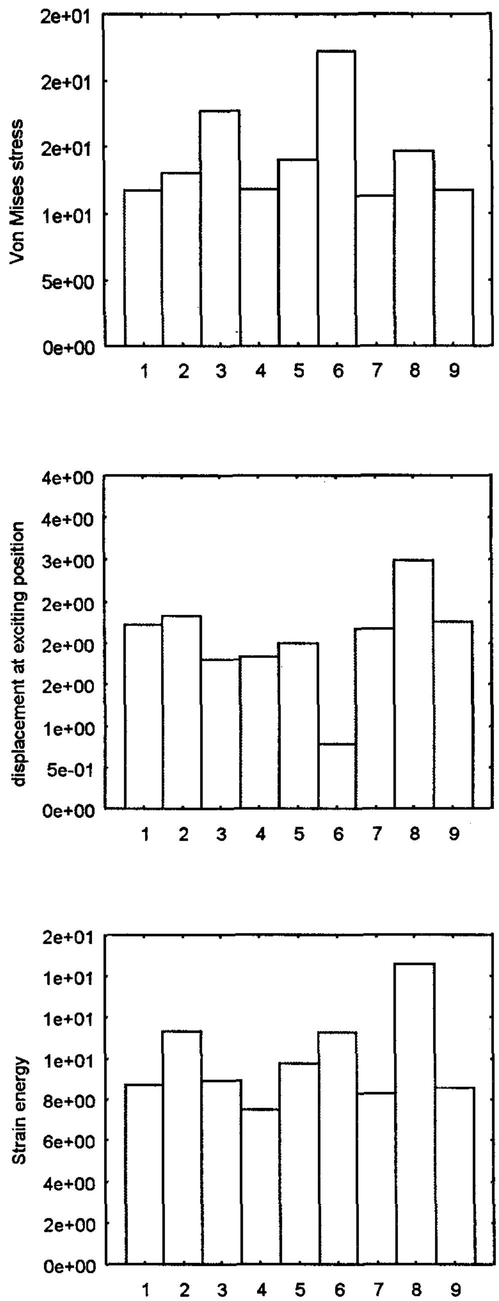

Fig. 13. Comparison of three properties for example 4

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