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Research Papers

Integrated optimal structural and vibration control design*

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Abstract An integrated design procedure which is composed of structural design, control design, and actuator locations design is proposed in this paper. First, a composite objective function, formed by a structural and a control objective, is optimized in steady state through the homogenization design method. Then an independent modal space control algorithm (IMSC) is performed on this optimal structure to reduce the dynamic response. Finally, to minimize the control force while still obtaining the same modal response for the controlled modes, the optimal choice for actuator locations is discussed.

1 Introduction

Structural vibration control, which has enormous applications in engineering, is an important consideration in the design of dynamic systems. During the past two decades, substantial effort has been made toward reducing the building cost of a structure, and then most of the modern structures have become much lighter, less stiff, and therefore more vulnerable to unexpected excessive external loads. In general, inherent damping of a flexible structure is very low. Thus, once oscillation has started, it will continue for a period without any large additional energy input. A modern structural control concept is proposed to accomplish the dual purposes of making a structure as light as possible and keeping it away from the risk of external disturbances.

The design of an efficient structural control system is of fundamental interest to both structural and control engineers. Systematic approaches for both structural and control designs are receiving increased applications. However, these design techniques, for the most part, have been applied independently within the entire design process. Traditionally, the structural designer develops his design based on strength and stiffness requirements, and the control designer creates the control algorithm to reduce the dynamic response of a structure. The designer of active controls has little input in the evolution of the basic structural design, and the structural analyst's participation in control design is limited to providing the frequencies and mode shapes. However, there have been strong indications recently that cost as well as response improvement can be realized by designing the structure and controls simultaneously.

In work done by Bendsøe and Rodrigues (1991), Díaz and Kikuchi (1992), Kamat *et al.* (1983), and Rozvany and Zhou (1991), only the structural optimization problems are studied without considering the control system effect. The structure is designed subject to some prescribed stiffness or strength requirements, and the structural engineers have no idea what will happen if they put an actuator in their design. How large will the control force be? How will engineers modify or change the design of the structure? It is not easy to answer these questions directly. In research studied by Balas (1978, 1979), Rofooei and Tadjbakhsh (1993), Soong (1990), and Yang *et al.* (1987), a control system is designed to improve the dynamic response of a given structure. The control engineers must find appropriate locations for actuators and use a lot of control energy to reduce the response of the structure. Furthermore, they do not know how to modify the structure which is suitable for control while still satisfying the structural design criterion (maximum stress or deformation). Thus, a simultaneous integrated design of structure and control system is proposed (see Hale *et al.* 1985; Miller and Shim 1987; Kajiwara *et al.* 1994). These approaches are suitable for small dimensional problems because solving two point boundary value problems or Riccati equations is too expensive. Furthermore, if the final time of our objective function is not infinite, the optimal control gains will not be constant matrices over time, and then we must calculate and store the control gain matrices at every instant of time. The cost of computing and storage will be tremendous. Canfield and Meirovitch (1994) have proposed an objective designed in modal space. Good results are obtained if the structural response can be represented by only a few lowest modes.

The first objective of this paper is to design a structure giving consideration to the control effect, second, to devise a control algorithm to reduce the vibration without excessive stress value, and finally, to find a suitable actuator locations for additional modal control. The optimal structural design is completed through a homogenization design method while the steady state (S.S.) control force is obtained by the displacement feedback law. Control design for transient response is performed in the modal space; both classical and optimal controls are presented. Two structural dynamic responses will be compared. One is the static design structure using the approach proposed by Suzuki and Kikuchi (1991). The other is the controlled structural design presented by Ou and Kikuchi (1996). It is almost impossible to have an efficient structural control design without careful selection for actuator positions. However, this issue is ignored in most integrated structural control design. Some general studies for placement of actuators can be found in the papers by Johnson (1981) and Martin (1978). An approach to selecting an optimal actuator location is proposed as a problem of minimization of the control forces.

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2 Formulation of structural optimization

Shape and topology optimization using the homogenization design method has been quite successful recently (Suzuki and Kikuchi 1991; Díaz and Kikuchi 1992). Its original idea was introduced by Bendsøe and Kikuchi (1988). This method is based on the concept of optimizing the material distribution of infinitely many microscale voids in a perforated structure. The optimality criteria derived from the necessary conditions of minimizing the mean compliance are solved by an optimality criteria (OC) method. For simplicity, we assume that the microstructure is defined by three design variables, $\mathbf{a}, \mathbf{b}$ and $\boldsymbol{\theta}$, where $1 - \mathbf{a}$ and $1 - \mathbf{b}$ are the size of a rectangular hole which is rotated by $\boldsymbol{\theta}$ with respect to $\mathbf{x}_1$ coordinate as shown in Fig. 1.

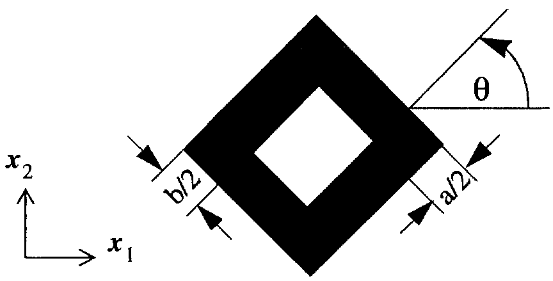

Fig. 1. Design variables of a microstructure

For optimal structural design in steady state, we choose an objective function as the sum of the strain energy and control energy defined as

$$\min_{\mathbf{a},\mathbf{b},\boldsymbol{\theta}} J = \mathbf{f}^t \mathbf{R} \mathbf{f} + \mathbf{d}^t \mathbf{Q} \mathbf{d} \,, \tag{1}$$

$$\text{st} \left\{ \begin{array}{l} \mathbf{K}\mathbf{d} = \mathbf{G}_1 \mathbf{f} + \mathbf{F} \\ P(\mathbf{a}, \mathbf{b}) = 0 \\ \mathbf{L} = (x1, x2) \end{array} \right. \tag{2}$$

and

$$P(\mathbf{a}, \mathbf{b}) = \sum_{e=1}^{\text{nel}} \int_{\Omega^e} (\mathbf{a}_e + \mathbf{b}_e - \mathbf{a}_e \mathbf{b}_e) \, \mathrm{d}\Omega^e - \Omega^s \,, \tag{3}$$

where $\mathbf{d}$ is an $n \times 1$ displacement vector, $\mathbf{a}, \mathbf{b}$ and $\boldsymbol{\theta}$ are the design vectors, $P(\mathbf{a}, \mathbf{b}) = 0$ is the material resource constraint, $\mathbf{G}_1$ is an $n \times m$ location matrix for the control force and $\mathbf{F}$ is an $n \times 1$ applied external force vector, $\mathbf{f}$ is an $m \times 1$ control force vector, nel is total number of the finite elements, and Ω^e is the element design domain. The magnitudes of the matrices $\mathbf{Q}$ and $\mathbf{R}$ are assigned according to the relative importance of the state variables and the control force in the minimization procedure. By adjusting the relative values of $\mathbf{Q}$ and $\mathbf{R}$, one can synthesize the control to achieve a proper trade off between these two objectives; $\mathbf{L}$ is a constraint for the actuator position specified at coordinate $(x1, x2)$.

Using the displacement closed-loop feedback control we can assume

$$\mathbf{f} = -\mathbf{R}^{-1} \mathbf{G}_1^t \mathbf{d} \,, \tag{4}$$

then the first equation of (2) becomes

$$(\mathbf{K} + \mathbf{G}_1 \mathbf{R}^{-1} \mathbf{G}_1^t) \mathbf{d} = \mathbf{F} \,. \tag{5}$$

Reformulating the equilibrium equation yields

$$\mathbf{K}_2 \mathbf{d} = \mathbf{F} \,, \tag{6}$$

where $\mathbf{K}_2 = \mathbf{K} + \mathbf{G}_1 \mathbf{R}^{-1} \mathbf{G}_1^t$ is the modified stiffness matrix under the control effect. Problem (6) can be solved by the finite element method which is very similar to the standard problem $\mathbf{K}\mathbf{d} = \mathbf{f}$. The eigenvalues of $\mathbf{K}$ are called open-loop eigenvalues, the eigenvalues of $\mathbf{K}_2$ are called the closed-loop eigenvalues. By introducing this feedback control, the control effect will modify the stiffness matrix so that the eigenvalues of the structure are shifted.

The weighting matrix $\mathbf{R}$ is chosen as

$$\mathbf{R} = w / \left(\sum_{i=1}^{\text{neig}} W_i \lambda_i \right) \,, \tag{7}$$

where neig is the number of the critical eigenvalues considered in the weighting, W_i is a weighting constant for each eigenvalue. Thus, the contribution from different eigenvalues can be adjusted by W_i, and the ratio of the control objective is regulated by w. During the iterations of optimization, the order of critical modes may change, and the lowest eigenvalue is not necessarily the dominant one. If we always use the eigenvalues of fixed order to estimate the stiffness of a structure, then we probably cannot obtain the good results that we expect. We thus use the modal energy to extract the critical modes in each iteration, and the stiffness of the structure can be estimated correctly.

3 Independent modal space control

Without considering the structural inherent damping, the second-order modal dynamic equation for the i-th mode is written as

$$\ddot{\eta}_i + \lambda_i \eta_i = \phi_i^t (\mathbf{G}_1 \mathbf{f} + \mathbf{F}) \,, \tag{8}$$

where η_i is the generalized coordinate, $\boldsymbol{\lambda}_i$ is the i-th eigenvalue and ϕ_i the i-th normalized eigenvector. Assume

$$\phi_i^t \mathbf{G}_1 \mathbf{f} = \boldsymbol{\Gamma}_i = -\mathbf{g}_i \eta_i - \mathbf{h}_i \dot{\eta}_i \,, \tag{9}$$

then (8) becomes

$$\ddot{\eta}_i + \mathbf{h}_i \dot{\eta}_i + \boldsymbol{\Lambda}_i \eta_i = \phi_i^t \mathbf{F} \,, \tag{10}$$

where $\boldsymbol{\Gamma}_i$ is the modal control force, $\boldsymbol{\Lambda}_i = \boldsymbol{\lambda}_i + \mathbf{g}_i$ is the closed-loop eigenvalue. If there are r controlled modes, we can write

$$\mathbf{f} = \mathbf{S}^{-1} \hat{\boldsymbol{\Gamma}} \,, \tag{11}$$

where $\mathbf{f}$ is the physical control force, and

$$\hat{\boldsymbol{\Gamma}} = \begin{bmatrix} \boldsymbol{\Gamma}_1 \\ \dots \\ \boldsymbol{\Gamma}_r \end{bmatrix} \,, \tag{12}$$

$$\mathbf{S} = \begin{bmatrix} \phi_1^t \\ \dots \\ \phi_r^t \end{bmatrix} \mathbf{G}_1 \,. \tag{13}$$

The control designer constructs the modal control forces according to the modal response, and transforms them into the physical control forces by (11). Instead of building the control based on the whole system, we develop the control for the critical modes. This is the key idea of IMSC.

4 Time integration scheme

When we apply IMSC to control the structure, some control energy may shift from the controlled modes to the residual modes, which is called control spillover effect. It is impractical to calculate the response for each modal equation to monitor the spillover effect for large dimensional problems. However, we can simply use an accurate time integration scheme to estimate the energy of the system. If both the kinetic and strain energy are limited in a reasonable range, then we can say the spillover effect is not serious. Otherwise, we must eliminate the spillover by either changing the design or including more modes to control.

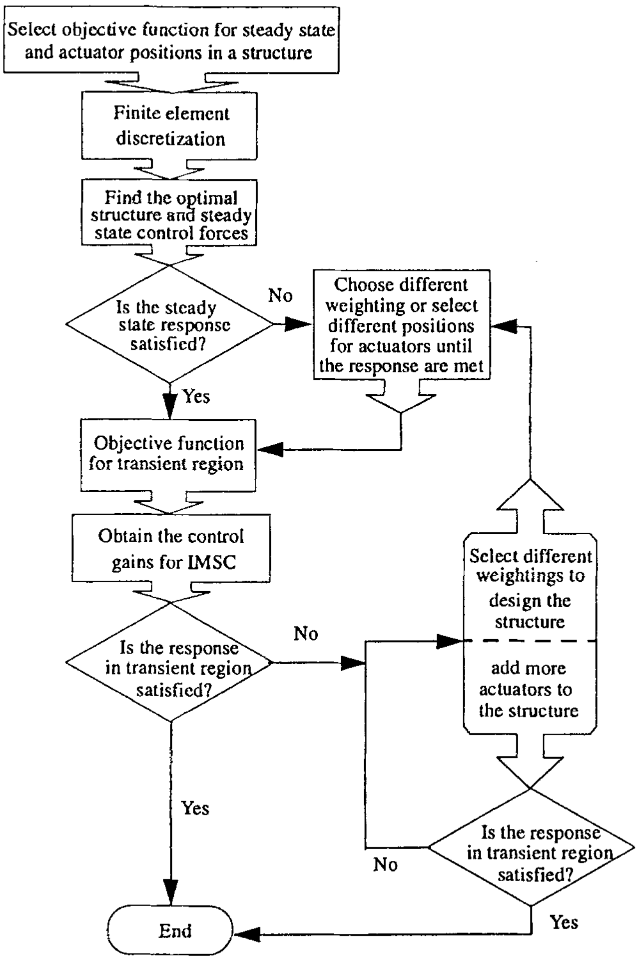

Fig. 2. Flow chart for optimal structural control design

For dynamic problems in time domain analysis, we use a direct integration method to predict the response of a system. The trapezoid rule of the Newmark family is an unconditionally stable method, and it is an appropriate numerical scheme for our problems. The formula of this algorithm is stated as follows. Considering the equation of motion

$$\mathbf{M}\ddot{\mathbf{d}} + \mathbf{C}\dot{\mathbf{d}} + \mathbf{K}\mathbf{d} = \mathbf{F}\,, \tag{14}$$

defining the predictors

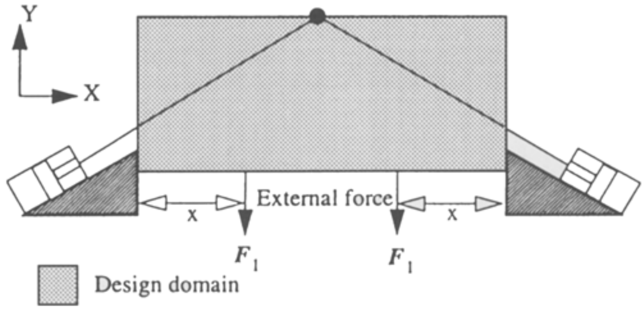

Fig. 3. Initial design domain and control devices

Fig. 4. Optimal structure for static case

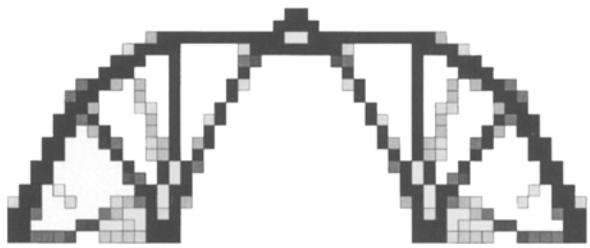

Fig. 5. Optimal structure for controlled structural design

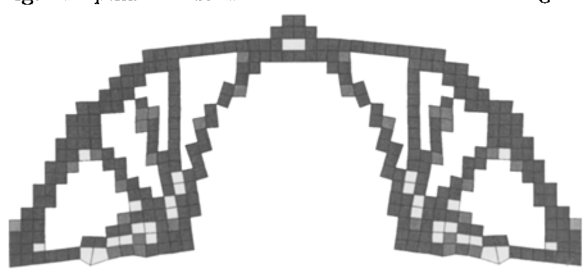

Fig. 6. Critical mode shape for controlled structural design

$$\tilde{\mathbf{d}}_{n+1} = \mathbf{d}_n + \Delta t \boldsymbol{\nu}_n + \frac{\Delta t^2}{2}(1-2\beta)\mathbf{a}_n\,,$$
$$\tilde{\boldsymbol{\nu}}_{n+1} = \boldsymbol{\nu}_n + (1-\gamma)\Delta t \mathbf{a}_n\,, \tag{15}$$

the displacement and velocity can be found from the correctors

$$\mathbf{d}_{n+1} = \tilde{\mathbf{d}}_{n+1} + \beta \Delta t^2 \mathbf{a}_{n+1}\,,$$
$$\boldsymbol{\nu}_{n+1} = \tilde{\boldsymbol{\nu}}_{n+1} + \gamma \Delta t \mathbf{a}_{n+1}\,. \tag{16}$$

The acceleration $\mathbf{a}_{n+1}$ can be determined by

$$(\mathbf{M}+\gamma\Delta t\mathbf{C}+\beta\Delta t^2\mathbf{K})\mathbf{a}_{n+1} = \mathbf{F}_{n+1}-\mathbf{C}\tilde{\boldsymbol{\nu}}_{n+1}-\mathbf{K}\tilde{\mathbf{d}}_{n+1}\,, \tag{17}$$

where $\mathbf{a} = \ddot{\mathbf{d}}$, $\boldsymbol{\nu} = \dot{\mathbf{d}}$, $\beta = 1/4$, and $\gamma = 1/2$.

5 Design procedure

A design procedure for a structural control system was proposed by Ou and Kikuchi (1996). Here an organized flow chart for the design procedure is plotted in Fig. 2.

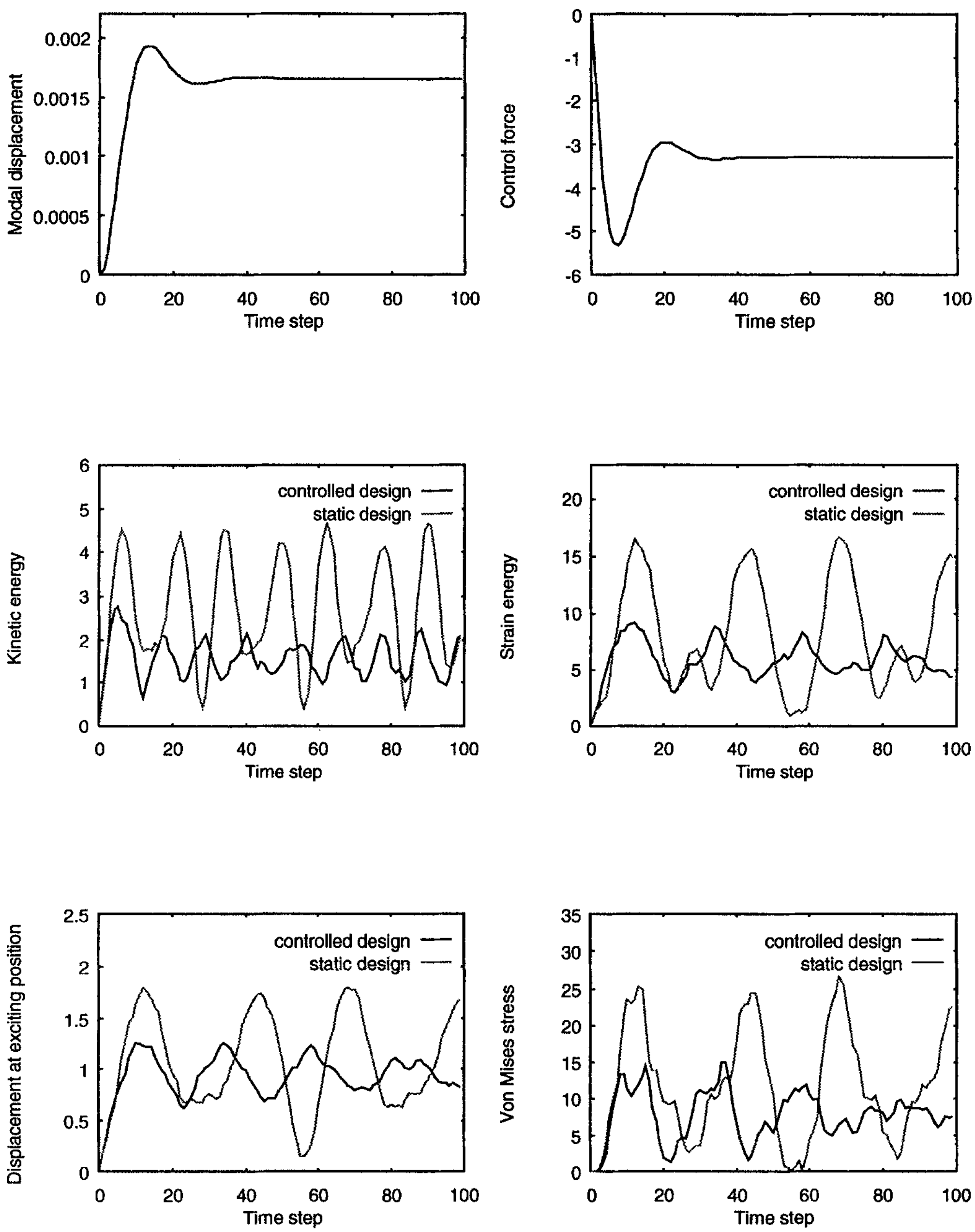

Fig. 7. Dynamic response of Example 1

6 Classical control theory for step response

The transfer function of a second-order differential equation (see Phillips and Harbor) for the i-th mode can be written as

$$G(s) = \frac{\hat{a}\omega_i^2}{s^2 + 2\xi_i\omega_i s + \omega_i^2}, \tag{18}$$

where $\hat{a}$ is a constant, ω_i is the natural frequency, ξ_i is the damping ratio. The percent of overshoot **PO** is derived as

$$\mathbf{PO} = e^{-\xi_i\pi\sqrt{1-\xi_i^2}} \times 100, \tag{19}$$

the settling time of the system is defined by

$$T_s = \frac{k}{\xi_i\omega_i}. \tag{20}$$

Choosing the desired **PO**, the damping ratio can be determined from (19), and usually $k = 4$ is selected. The gain g_i is chosen so that $-g_i\eta_i$ is equal to the steady state control force obtained in the structural design.

6.1 Example 1

Young's modulus = 100 GPa, Poisson's ratio = 0.3, density ρ = 1E-6, design domain: 4 cm by 10 cm, finite element mesh: 20 by 50, volume constraint: 10, location of actuator: **L** = (5.0, 4.0), **PO** = 16.3, time step = 2E-4.

The optimal designs corresponding to the static problem and the controlled structural design are shown in Figs. 4 and 5. These two designs were obtained by Ou and Kikuchi (1996), and the authors concluded that a control force cannot be applied at an arbitrary position because of a very large maximum stress generated inside the structure at that time, i.e. the static design applied by a control force lost the qualification to be studied further there. Thus, two cases for step response are studied in this example. Case 1: static design, using the approach in the papers by Suzuki and Kikuchi (1991), is subject to external forces only. Case 2: the controlled structural design, obtained in the papers by Ou and Kikuchi (1996), is subject to external forces and classical control algorithm. The critical mode for Case 2 is the fourth mode, which is plotted in Fig. 6; it is also the only controlled mode. The dynamic response for these two cases is shown in Fig. 7. The von Mises stress is evaluated at the location where the maximum stress occurred in S.S. in the structure. From these charts, we can see that the controlled modal displacements achieve the S.S. smoothly without large overshoot. The response of the static design is almost twice that of the controlled structural design under active control. Our current approach achieve the purpose of making a structure possess good performance in S.S. and transient region.

7 Optimal independent modal space control

In classical optimal control theory, we must solve the Riccati equations for the whole system. These equations need to be solved backward and implemented forward in time. Nevertheless, it is impractical to do so for large dimensional problems based on present computing ability. The IMSC was introduced to solve n 2 by 2 Riccati equations instead of solving the $2n$ by $2n$ Riccati equations. The computing cost is dramatically reduced, and if only a few modes are dominant then control of these critical modes will be sufficient to control the whole structure.

The formulation of IMSC, derived by Meirovitch and Baruh (1980), associated with the distributed-parameter system can be written briefly as follows. The modal equation for the system is

$$\ddot{\eta}_r + \omega_r^2 \eta_r = \hat{\Gamma}_r(t)\,, \quad r = 1, 2, \ldots \tag{21}$$

Let $\nu_r(t)$ be defined as $\dot{\eta}_r(t) = \omega_r \nu_r(t)$, then introduce the associated modal state vector $\mathbf{w}_r(t)$ and the associated control vector $\mathbf{W}_r(t)$ in the form

$$\mathbf{w}_r(t) = [\eta_r \nu_r]^t\,, \quad \mathbf{W}_r(t) = \begin{bmatrix} 0 & \dfrac{\hat{\Gamma}_r(t)}{\omega_r} \end{bmatrix}^t\,, \tag{22}$$

$$\mathbf{A}_r = \begin{bmatrix} 0 & \omega_r \\ -\omega_r & 0 \end{bmatrix}\,, \tag{23}$$

the modal state equation is

$$\dot{\mathbf{w}}_r(t) = \mathbf{A}_r \mathbf{w}_r(t) + \mathbf{W}_r(t)\,. \tag{24}$$

Assume $\mathbf{W}_r$ depends on $\mathbf{w}_r$ alone, i.e.

$$\mathbf{W}_r = \mathbf{W}_r(\mathbf{w}_r)\,, \tag{25}$$

the modal control $\mathbf{W}_r$ is designed independently of any modal state vector other than $\mathbf{w}_r$. The optimal control force can be determined from the optimal control theory. The objective function is

$$\mathbf{J} = \sum_{r=1}^{\mathrm{nc}}\,, \tag{26}$$

where nc is the number of controlled modes, and

$$\mathbf{J}_r = \int_0^{t_f} (\mathbf{w}_r^t \mathbf{Q}_r \mathbf{w}_r + \mathbf{W}_r^t \mathbf{R}_r \mathbf{W}_r) \mathrm{d}t\,. \tag{27}$$

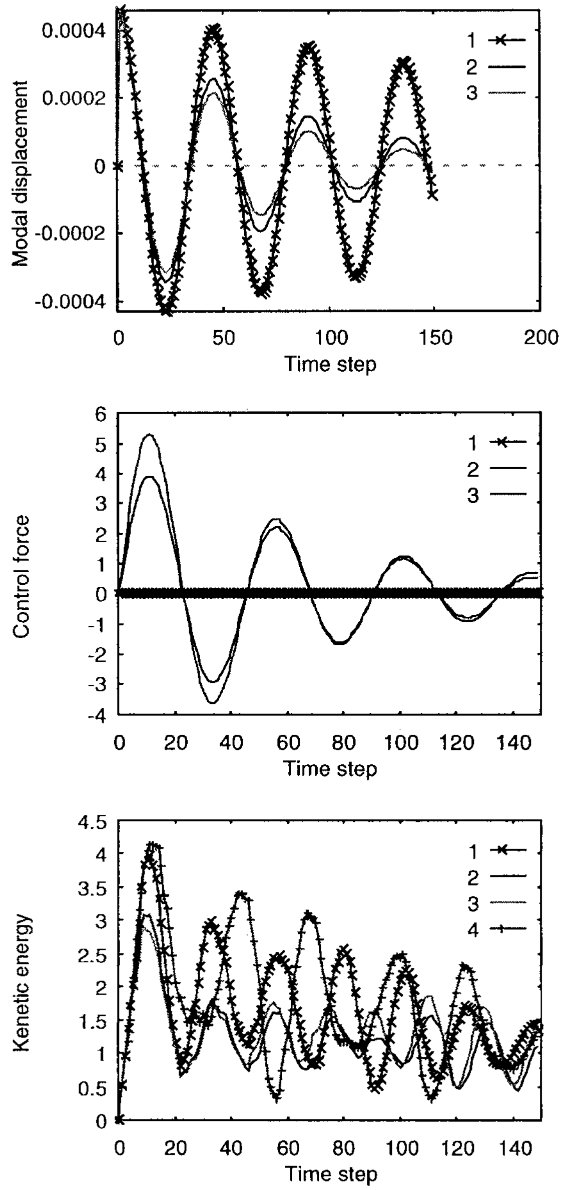

Fig. 8. Response of Example 2

Minimization of $\mathbf{J}_r$, yields

$$\mathbf{W}_r(t) = -\mathbf{R}_r^{-1} \mathbf{P}(t) \mathbf{w}_r(t)\,, \tag{28}$$

let $\mathbf{Q}_r = \omega_r^2 \mathbf{I}$, and $\mathbf{R}_r$ can be represented by

$$\mathbf{R}_r = \begin{bmatrix} \infty & \\ & \mathbf{R}_{\eta r} \end{bmatrix}\,, \tag{29}$$

where $\mathbf{P}(t)$ is the Riccati matrix which can be obtained by solving 2×2 Ricatti equations. When $t_f \to \infty$, the optimal generalized modal control gains are

$$\mathrm{g}_r = -\omega_r \left(\omega_r - \sqrt{\omega_r^2 + \tilde{\mathbf{R}}_{\eta r}^{-1}} \right)\,, \tag{30}$$

$$\mathbf{h}_r = \left(2\mathrm{g}_r + \tilde{\mathbf{R}}_{\eta r}^{-1} \right)^{1/2}\,, \tag{31}$$

where $\mathbf{R}_r = \omega_r^2 \tilde{\mathbf{R}}_r$, g_r is a gain for modal stiffness and $\mathbf{h}_r$ is a gain for modal damping. Substituting (30) and (31) into (9), we can obtain the optimal control force.

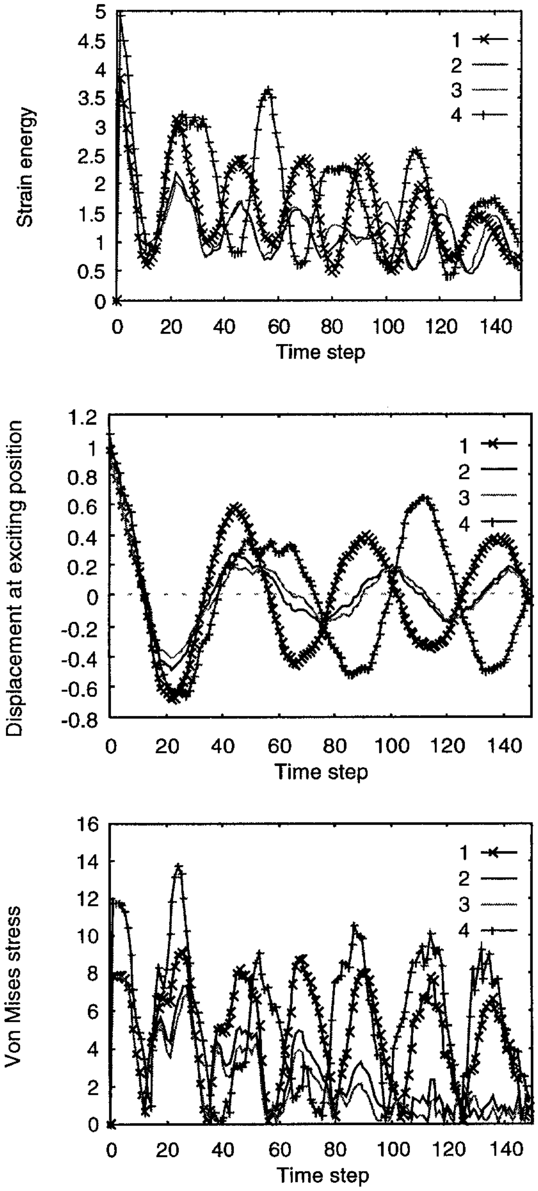

Fig. 9. Response of Example 2

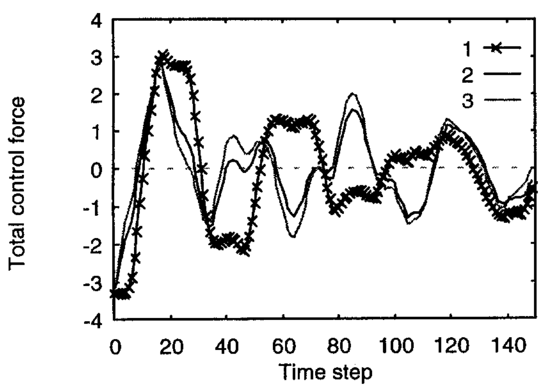

Fig. 10. Total control force for Example 2

7.1 Example 2

Two structures used in Example 1 are subject to the initial deformation due to the same external forces. Four cases are investigated in this example. Cases 1, 2 and 3 are the response of a controlled structural design (Fig. 5); Case 1 is under the state feedback control law (4); in addition to the state feedback control, optimal control is applied to Cases 2 and 3, but Case 2 has smaller weighting $\tilde{\mathbf{R}}_{\eta r}^{-1}$ than Case 3. Case 4 is the response for the static design (Fig. 4) without control. From Figs. 8 and 9, we see the response in Case 4 is the worst, and larger weighting $\tilde{\mathbf{R}}_{\eta r}^{-1}$ corresponding to Case 3 gives us the best performance. We can conclude that controlled structural design under optimal control provides the best response; the response of the controlled structural design without optimal control is still much better than the static design. Superposition of these two control forces can provide the total control force, see Fig. 10. The optimal control does not increase the total control force compared to the state feedback control, while it has much better performance. This tells us that how important it is to allocate control forces appropriately.

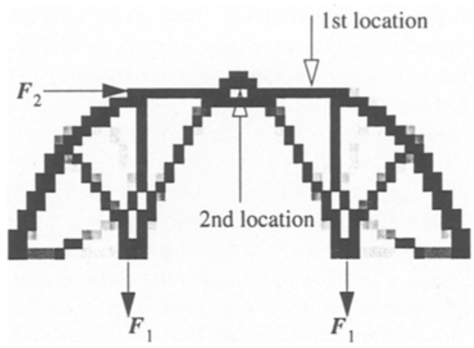

Fig. 11. Structure and external forces in Example 3

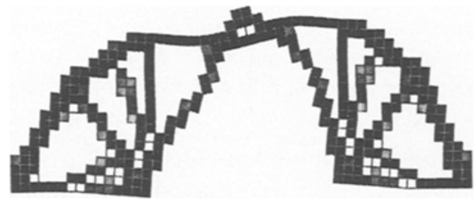

Fig. 12. First mode shape of the controlled structure

8 Optimal design for actuator locations

When the structure is subject to another external force after the structure is installed, changing it to a totally new structure is probably too expensive; we can simply find other locations for additional actuators to control the exciting modes. The placement of actuators for IMSC was first studied by Baruh and Meirovitch (1981), who derived the controlled and residual modes. They minimized the control spillover into the first residual mode and concluded that the work done on the controlled modes does not depend on the actuator locations. The best actuator locations are the nodes of the first residual mode. Although the work on the controlled modes is independent of actuator locations, the work tranferred to the whole structure does rely on them. Moreover, the control force is very large while the work is not so big in some situations; then the work is probably not an appropriate measure to control system design. Putting actuators at the nodes of the first residual mode can avoid spillover to that mode only.

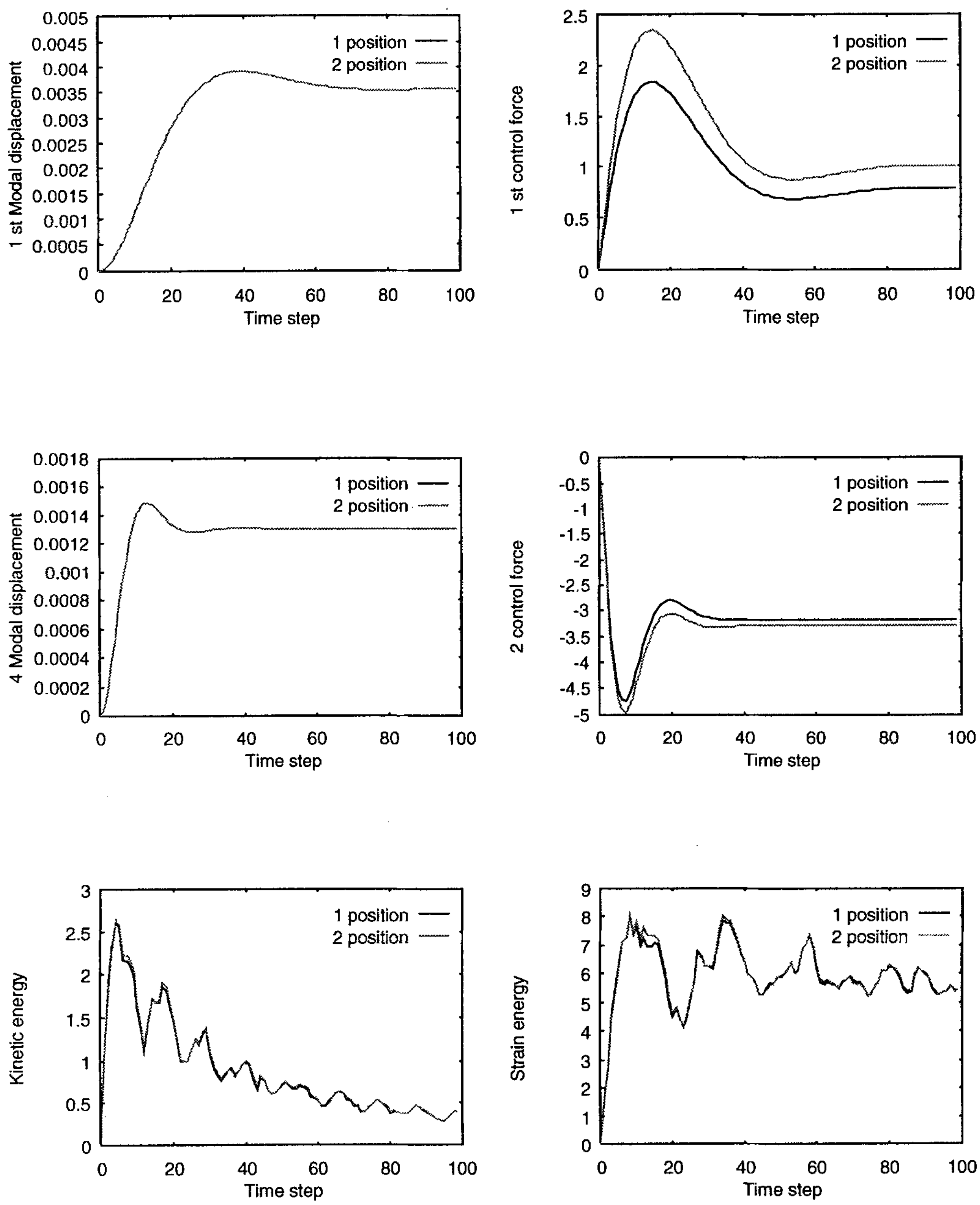

Fig. 13. Comparison of response for Example 3

It does not guarantee good controllability (smaller control force). Another criterion for deciding the optimal actuator placement was introduced by Lindberg and Longman (1984). Here we use their formulation in a more specific and practical way by choosing locations that offer good controllability without excessive spillover. The criterion is defined as

$$\mathrm{CS} = \int_0^{t_f} \mathbf{f}^t \mathbf{f} \mathrm{d}t\,, \tag{32}$$

which is the product of the control forces, and is an appropriate measure if the magnitude of the control force is more considered than the work on the controlled modes. Furthermore, if we want the controlled modes to posses the same response under different choices of actuator locations, we can form a minimization problem after substituting (11) into the above equation,

$$\min_{\mathbf{S}} \ \mathrm{CS} = \int_0^{t_f} (\mathbf{S}^{-1}\hat{\mathbf{r}})^t \mathbf{S}^{-1}\hat{\mathbf{r}} \mathrm{d}t\,. \tag{33}$$

If the location selected according to (33) generates too much spillover, we must choose another location again until the requirements are satisfied.

8.1 Example 3

We use the structural and control system designed in Fig. 5, and assume that the structure is subject to another horizontal external step force $\mathbf{F}_2$, which excites mainly the first mode, see Fig. 11. The location for the fourth modal control was already decided in Example 1. We want to choose the best location, which minimizes **CS**, for an additional actuator to control the first mode without excessive spillover. We assume that this additional actuator must be installed at $\mathbf{y} = 4.25$ and calculate **CS** along $\mathbf{y} = 4.25$. We find that location 1 gives us the minimum control energy. To make a comparison, we also investigate the response caused by placing the actuator at location 2. The repsonse corresponding to these two cases is plotted in Fig. 13. The modal displacements for these two cases are the same, which is exactly we want. The kinetic and strain energy response is almost the same, but the physical control forces are smaller when we put the actuator at location 1. This confirms our previous statement that the same modal response can be attained by choosing different actuator locations; of course, we select the one which offers the smallest control force without excessive spillover according to (33).

9 Conclusions

A new integrated design procedure for a structural control system is presented in this paper. The optimal design of a controlled structure and S.S. control forces was achieved through the homogenization method and displacement feedback law. The optimal structure obtained satisfies structural design requirements without possessing large stress value in S.S.; the classical and optimal controls were applied to reduce the structural vibration within a reasonable span of time. The results show that the dynamic response of the controlled structural design with active control is superior to the traditional static design. Thus, active control can remove the energy from the structure effectively if it is carried out appropriately. The procedure of finding the actuator locations for a given structure, suggested in this paper, can offer the same modal response with minimum control forces while the spillover is kept low.

Acknowledgements

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References

Balas, M.J. 1978: Feedback control of flexible systems. *IEEE Trans. Auto. Control* Vol. **AC-23**, 673-679

Balas, M.J. 1979: Direct velocitiy feedback control of large space structures. *J. Guid., Control & Dyn.* **2**, 252-253

Baruh, H.; Meirovitch, L. 1981: On the placement of actuators in the control of distributed-parameter systems. *AIAA/ASME/ASCE/AHS, 22nd Structures, Structural Dynamics and Materials Conf.*, pp. 611-620

Bendsøe, M.P.; Kikuchi, N. 1988: Generating optimal topologies in structural design using homogenization method. *Comp. Meth. Appl. Mech. Engrg.* **71**, 197-224

Bendsøe, M.P.; Rodrigues, H.C. 1991: Integrated topology and boundary shape optimization of 2-D solids. *Comp. Appl. Mech. Engrg.* **87**, 15-34

Díaz, A.R.; Kikuchi, N. 1992: Solution to shape and topology eigenvalue optimization problems using homogenization method. *Int. J. Num. Meth. Engrg.* **35**, 1487-1502

Hale, A.L.; Lisowiki, R.J.; Dahl, W.E. 1985: Optimal simultaneous structural and control design of maneuvering flexible spacecraft. *J. Guid., Control & Dyn.* **8**, 86-93

Johnson, T.L. 1981: Principles of sensor and actuator location in distributed systems. *Proc. Int. Symp. on Engineering Science and Mechanics*, Tainan, Taiwan, **50**, pp. 1-14

Kajiwara, I.; Tsujioka, K.; Nagamatsu, A. 1994: Approach for simultaneous optimization of a structure and control system. *AIAA J.* **32**, 866-873

Kamat, M.P.; Venkayya, V.B.; Khot, N.S. 1983: Optimization with frequency constraints limitations. *J. Sound & Vib.* **91**, 147-154

Lindberg, R.E.; Longman, R.W. 1984: On the number and placement of actuators for independent modal space control. *J. Guid., Control & Dyn.* **7**, 215-221

Martin, J.C.E. 1978: Optimal allocation of actuators for distributed-parameter system. *ASME J. Dyn. Systems, Measurement and Control* **100**, 227-228

Meirovitch, L.; Baruh, H. 1980: Control of self-adjoint distributed-parameter systems. *J. Guid., Control & Dyn.* **5**, 60-66

Miller, D.F.; Shim, J. 1987: Gradient-based combined structural and control optimization. *J. Guid., Control & Dyn.* **10**, 291-298

Ou, J.-S.; Kikuchi, N. 1996: Optimal design of controlled structures. *Struct. Optim.* **11**, 19-28

Phillips, C.L.; Harbor, R.D. 1987: *Feedback control systems.* Prentice Hall

Rofooei, F.R.; Tadjbakhsh, I.G. 1993: Optimal control of structures with acceleration, velocity, and displacement feedback. *J. Eng. Mech.* **119**, 1993-2011

Rozvany, G.I.N.; Zhou, M. 1991: The COC algorithm. Part I: cross-section optimization or sizing; Part II: topological, geometrical and generalized shape optimization. *Comp. Meth. Appl. Mech. Engrg.* **89**, 281-336

Rozvany, G.I.N.; Zhou, M. 1992: Extensions of new discretized optimality criteria to structures with passive control. *Proc. 4th AIAA/USAF/NASA/OAI Symp. on Multidisciplinary Analysis and Optimization* (held in Cleveland, OH), pp. 288-297. Washington D.C.: AIAA

Soong, T.T. 1990: *Active structural control: theory and practice.* New York: Wiley

Suzuki, K.; Kikuchi, N. 1991: A homogenization method for shape and topology optimization. *Comp. Meth. Appl. Mech. Engrg.* **93**, 291-318

Venkayya, V.B.; Tichler, V.A. 1985: Frequency control and its effect on the dynamic response of flexible structures. *AIAA J.* **23**, 1768-1774

Yang, J.N.; Akbarpour, A.; Ghaemmagham, P. 1987: New optimal control algorithms for structural control. *ASCE J. Eng. Mech.* **113**, 1369-1386

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