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Each Non-Zero Convolution Operator on the Entire Functions Admits a Continuous Linear Right Inverse

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Let $A(\mathbb{C}^n)$ denote the Fréchet space of all entire functions on $\mathbb{C}^n$. It is a classical theorem of Ehrenpreis [5] and Malgrange [9] that each non-zero convolution operator T on $A(\mathbb{C}^n)$ is surjective. As one of the main results of the present article, we prove that each of these operators T admits a continuous linear right inverse, which is equivalent to $\ker T$ being complemented in $A(\mathbb{C}^n)$. This extends a theorem of Treves [16] for partial differential operators with constant coefficients to convolution operators. We obtain this as a special case of a result on the complementation of certain closed ideals in weighted algebras $A_p(\mathbb{C}^n)$.

To introduce these algebras, let p be a nonnegative plurisubharmonic function on $\mathbb{C}^n$ which satisfies some mild technical conditions. Then $A_p(\mathbb{C}^n)$ consists of all $f \in A(\mathbb{C}^n)$ which satisfy an estimate $|f| \leqq A e^{Bp}$ for suitable constants A and B depending on f. For $1 \leqq k \leqq n$ and $F=(F_1, \ldots, F_k) \in (A_p(\mathbb{C}^n))^k$ we denote by $I(F)$ the ideal in $A_p(\mathbb{C}^n)$ which is algebraically generated by $F_1, \ldots, F_k$. If $F=(F_1, \ldots, F_k)$ is slowly decreasing in the sense of Berenstein and Taylor [2], then $I(F)$ is closed in $A_p(\mathbb{C}^n)$, where $A_p(\mathbb{C}^n)$ is endowed with its natural inductive limit topology. We prove that for each slowly decreasing ideal $I(F)$ the strong dual $(A_p(\mathbb{C}^n)/I(F))'_b$ of $A_p(\mathbb{C}^n)/I(F)$ is a nuclear Fréchet space with the property (Ω). This property is a linear topological invariant, introduced by Vogt and Wagner [19], to characterize the quotient spaces of s, the space of rapidly decreasing sequences. Our proof of $(A_p(\mathbb{C}^n)/I(F))'_b$ having the property (Ω) is based on the work of Berenstein and Taylor [2], some elementary functional analysis and an appropriate description of $A_p(\mathbb{C}^n)/I(F)$ in terms of Čech cohomology with bounds.

Knowing that $(A_p(\mathbb{C}^n)/I(F))'_b$ has (Ω) for all slowly decreasing ideals $I(F)$, we can use the splitting theorem of Vogt and Wagner [19] together with the main result of our article [13] to prove the following: For each k with $1 \leqq k \leqq n$, each slowly decreasing ideal $I(F_1, \ldots, F_k)$ in $A_p(\mathbb{C}^n)$ is complemented if and only if $A_p(\mathbb{C}^n)'_b$ has the property (DN). The property (DN) was introduced by Vogt [17], to characterize the linear topological subspaces of s. Various characterizations for $A_p(\mathbb{C}^n)'_b$ having (DN) were given in [13]. In particular, it was shown that for radial weight functions p on $\mathbb{C}^n$ the property (DN) of $A_p(\mathbb{C}^n)'_b$ can be characterized in terms of the behaviour of p^{-1}.

For radial weights p on $\mathbb{C}^n$ with $p(2z)=O(p(z))$ Berenstein and Taylor [2] have shown that each non-zero principal ideal in $A_p(\mathbb{C}^n)$ is slowly decreasing. If we assume that for such a weight $A_p(\mathbb{C}^n)'_b$ has (DN), then it follows that each principal ideal $I(F)$ in $A_p(\mathbb{C}^n)$ is complemented. Since $A_p(\mathbb{C}^n)/I(F)$ can be identified with a space $A_p(V)$ of holomorphic functions on the multiplicity variety V of the ideal $I(F)$, we get in this situation the existence of a continuous linear extension operator $E: A_p(V) \to A_p(\mathbb{C}^n)$ for each principal variety V. Moreover, $(A_p(\mathbb{C}^n)/I(F))'_b$ can be identified with $\ker M^t_F$, where M^t_F, where M^t_F denotes the adjoint of the operator of multiplication by F. In many cases M^t_F can be identified with a convolution operator on a certain Fréchet space. Hence our results imply in particular the existence of a continuous linear right inverse for each non-zero convolution operator T on $A(\mathbb{C}^n)$ or on the spaces E^0_s, $s>1$, which were investigated by Martineau [10], where

$$E^0_s := \{f \in A(\mathbb{C}^n) \mid \sup_{z \in \mathbb{C}^n} |f(z)| \exp(-\varepsilon |z|^s) < \infty \text{ for all } \varepsilon > 0\}.$$

Throughout this article, we shall use the standard notation from complex analysis and functional analysis. Besides this, we introduce the following notion which will be used later on.

1. Definition. A function $p: \mathbb{C}^n \to [0, \infty[$ is called a *weight function* if it has the following properties:

(1) p is continuous and plurisubharmonic.

(2) $\log(1+|z|^2) = O(p(z))$.

(3) There exists $C \geqq 1$ such that for all $w \in \mathbb{C}^n$ we have

$$\sup_{|z-w| \leqq 1} p(z) \leqq C(1 + \inf_{|z-w| \leqq 1} p(z)).$$

A weight function p is called radial if $p(z)=p(|z|)$ for all $z \in \mathbb{C}^n$, where $|z| = \left(\sum_{j=1}^n |z_j|^2\right)^{1/2}$.

2. Examples. The following functions p are typical examples of weight functions on $\mathbb{C}^n$:

(1) $p(z) := |z|^\rho$, $\rho > 0$.

(2) $p(z) := (\log(1+|z|^2))^s$, $s > 1$.

(3) $p(z) := \log(1+|z|^2) + |\operatorname{Im} z|$.

(4) $p(z) := |z|^\alpha + |\operatorname{Im} z|^\beta$, $0 < \alpha < \beta$ and $\beta \geqq 1$.

For further examples we refer to Berenstein and Taylor [2, 3] and Meise [11].

For an open set Ω in $\mathbb{C}^n$ we denote by $A(\Omega)$ the algebra of all holomorphic functions on Ω. For each weight functions p on $\mathbb{C}^n$ we define a subalgebra $A_p(\mathbb{C}^n)$ of $A(\mathbb{C}^n)$ in the following way:

3. Definition. For a weight function p on $\mathbb{C}^n$ we put

$$A_p(\mathbb{C}^n) := \{f \in A(\mathbb{C}^n) \mid \text{there exists } k \in \mathbb{N}: \sup_{z \in \mathbb{C}^n} |f(z)| \exp(-kp(z)) < \infty\},$$

and endow $A_p(\mathbb{C}^n)$ with its natural inductive limit topology. Then $A_p(\mathbb{C}^n)$ is a locally convex algebra and a (DFN)-space, i.e. $A_p(\mathbb{C}^n)$ is the strong dual of a nuclear Fréchet space (see e.g. Meise [11], 2.4).

The algebras of type $A_p(\mathbb{C}^n)$ arise at various places in complex analysis and functional analysis. We are particularly interested in certain closed ideals in $A_p(\mathbb{C}^n)$. Therefore we recall some notation from Kelleher and Taylor [7].

4. Definition. a) For an ideal I in $A_p(\mathbb{C}^n)$ we define its *localization* by

$$I_{\text{loc}} := \{f \in A_p(\mathbb{C}^n) \mid [f]_a \in I_a \text{ for all } a \in \mathbb{C}^n\},$$

where I_a denotes the ideal in the local ring $\mathcal{O}_a$ which is generated by the germs $[g]_a$ of all $g \in I$. If $I = I_{\text{loc}}$ then I is called a localized ideal.

b) Let $F = (F_1, \ldots, F_m) \in (A_p(\mathbb{C}^n))^m$ be given. Then we denote by

(α) $I(F)$ the ideal in $A_p(\mathbb{C}^n)$ which is algebraically generated by the functions $F_1, \ldots, F_m$.

(β) $I_{\text{loc}}(F)$ the localization of $I(F)$.

Note that I_{loc} is a closed ideal in $A_p(\mathbb{C}^n)$ which contains I (see Kelleher and Taylor [7]).

From Berenstein and Taylor [2], Def. 5.1, we recall:

5. Definition. Let $F = (F_1, \ldots, F_m) \in (A_p(\mathbb{C}^n))^m$, $1 \leqq m \leqq n$, be given and let $\mathscr{L}$ denote a family of m-dimensional affine subspaces of $\mathbb{C}^n$ with

$$V(F) := \{z \in \mathbb{C}^n \mid F_j(z) = 0,\ 1 \leqq j \leqq m\} \subset \bigcup_{L \in \mathscr{L}} L.$$

(a) F is called *slowly decreasing for* $\mathscr{L}$ if there exist constants $\varepsilon, C, K > 0$ such that

(i) for each $L \in \mathscr{L}$, all components of the set

$$S(F, L, \varepsilon, C) := \{z \in L \mid |F_j(z)| < \varepsilon \exp(-Cp(z)),\ 1 \leqq j \leqq m\}$$

are bounded and

(ii) for each $L \in \mathscr{L}$ and each component S of $S(F, L, \varepsilon, C)$ we have

$$\sup_{z \in S} p(z) \leqq K(1 + \inf_{z \in S} p(z)).$$

b) Let F be slowly decreasing for $\mathscr{L}$, let ε and C be as in (a) and denote by $\mathscr{G}_L$, $L \in \mathscr{L}$, all the componnts of $S(F, L, \varepsilon, C)$. Then for $\eta, D > 0$ and $G \in \mathscr{G}_L$, $L \in \mathscr{L}$, sets of the form

$$\Omega(F, L, \varepsilon, C, \eta, D, G) = \bigcup_{w \in G} \{z \in \mathbb{C}^n \mid |z - w| < \eta \exp(-Dp(w))\}$$

are called *good open sets*. For fixed ε, η, C and D

$$\mathscr{C}(F, \mathscr{L}, \varepsilon, C, \eta, D) := (\Omega(F, L, \varepsilon, C, \eta, D, G))_{G\in\mathscr{G}_L,\, L\in\mathscr{L}}$$

is called a *good family*.

Remark. Assume that $F=(F_1, \dots, F_m)$ is slowly decreasing for $\mathscr{L}$ and let ε and C be as in 5(a). Then we have for each $L\in\mathscr{L}$

$$S(F, L, \varepsilon', C') \subset S(F, L, \varepsilon, C)$$

whenever $0<\varepsilon'<\varepsilon$ and $C'>C$. This implies that for each $L\in\mathscr{L}$ and each $G'\in\mathscr{G}'_L$ there exists a unique $G\in\mathscr{G}_L$ with $G'\subset G$. Hence there exists a map $\rho_L: \mathscr{G}'_L \to \mathscr{G}_L$ satisfying $G'\subset \rho_L(G')$, $L\in\mathscr{L}$. Now let $0<\eta'<\eta$ and $D'>D>0$ be given. Then Definition 5(b) implies that for each $G'\in\mathscr{G}'_L$ we have

$$\Omega(F, L, \varepsilon', C', \eta', D', G') \subset \Omega(F, L, \varepsilon, C, \eta, D, \rho_L(G')).$$

Hence the maps $(\rho_L)_{L\in\mathscr{L}}$ induces a natural refinement map

$$\rho: \mathscr{C}(F, \mathscr{L}, \varepsilon', C', \eta', D') \to C(F, \mathscr{L}, \varepsilon, C, \eta, D)$$

by $\rho(\Omega(F, L, \varepsilon', C', \eta', D', G')) = \Omega(F, L, \varepsilon, C, \eta, D, \rho_L(G'))$.

6. Definition. Let $F=(F_1, \dots, F_m)\in(A_p(\mathbb{C}^n))^m$, $1\leqq m\leqq n$, be slowly decreasing for $\mathscr{L}$.

(a) The family $\mathscr{L}$ is called *almost parallel* if for every good family $\mathscr{C}(F, \mathscr{L}, \varepsilon, C, \eta, D)$ there exists a refinement $\mathscr{C}(F, \mathscr{L}, \varepsilon', C', \eta', D')$ such that for each Ω_0, $\Omega_1\in\mathscr{C}(F, \mathscr{L}, \varepsilon', C', \eta', D')$ with $\Omega_0\cap\Omega_1\neq\emptyset$ we have $\bar{\Omega}_0\cup\bar{\Omega}_1\subset \rho(\Omega_0)\cap\rho(\Omega_1)$, where ρ is the refinement map defined above.

(b) F will be called *slowly decreasing* if there exists an almost parallel family $\mathscr{L}$ of m-dimensional affine subspaces of $\mathbb{C}^n$, such that F is slowly decreasing for $\mathscr{L}$.

Remark. For a comprehensive discussion of the slowly decreasing condition, we refer to the Sects. 6, 7 and 8 of Berenstein and Taylor [2]. In [2], 6.4, they show that slowly decreasing maps $F: \mathscr{C}^n \to \mathscr{C}^m$ are "generic" for $m\geqq n$. In [2], Sect. 8, they explain that it is difficult to find examples of slowly decreasing m-tuples in $A_p(\mathbb{C}^n)$ for $2\leqq m\leqq n-1$.

We are interested in the following linear topological invariant for Fréchet spaces, which was introduced by Vogt and Wagner [19].

7. The Property (Ω). Let E be a Fréchet space and let $(\|\ \|_k)_{k\in\mathbb{N}}$ be a fundamental system of semi-norms for E. For $k\in\mathbb{N}$ put

$$U_k := \{x\in E \mid \|x\|_k < 1\}.$$

E has the property (Ω) if the following holds:

For each $p\in\mathbb{N}$ there exists $q\in\mathbb{N}$ such that for each $k\in\mathbb{N}$ there exist $d>0$ and $C>0$ such that for all $r>0$

$$U_q \subset C r^d U_k + \frac{1}{r} U_p.$$

8. *Remark.* a) It is easy to check that (Ω) is a linear topological invariant which is inherited by quotient spaces. By Vogt and Wagner [19], 1.8, a nuclear Fréchet space E has (Ω) iff E is isomorphic to a quotient space of s, the space of all rapidly decreasing sequences.

b) By Meise and Taylor [13], 1.15, $A_p(\mathbb{C}^n)'_b$, the strong dual of $A_p(\mathbb{C}^n)$, has (Ω) for each weight function p.

9. The Spaces $k^\infty(\mathbb{E}, \alpha)$. Let I be an infinite index set, let $\mathbb{E}=(E_i, \|\ \|_i)_{i\in I}$ be a family of Banach spaces and let $\alpha=(\alpha_i)_{i\in I}$ be a family of positive real numbers which is unbounded. Then we define for $j\in\mathbb{N}$

$$K_j:=\{x\in\prod_{i\in I} E_i \mid |||x|||_j:=\sup_{i\in I}\|x_i\|_i\exp(-j\alpha_i)<\infty\}$$

and put

$$k^\infty(\mathbb{E}, \alpha)=\operatorname*{ind}_{j\to} K_j.$$

10. Proposition. *Let $\mathbb{E}$ and α be as in 9. Then the following holds:*

(a) *A subset B of $k^\infty(\mathbb{E}, \alpha)$ is bounded iff there exists $m\in\mathbb{N}$ with*

$$\sup_{x\in B}|||x|||_m=\sup_{x\in B}\sup_{i\in I}\|x_i\|_i\exp(-m\alpha_i)<\infty.$$

(b) *$k^\infty(\mathbb{E}, \alpha)'_b$, the strong dual of $k^\infty(\mathbb{E}, \alpha)$, is a Fréchet space which has the property (Ω).*

Proof. (a) It is easy to see that $k^\infty(\mathbb{E}, \alpha)$ is an (LB)-space. Hence it suffices to show that each bounded set B in $k^\infty(\mathbb{E}, \alpha)$ satisfies the condition stated in (a). To do this, let B be given. Since each (LB)-space is a (DF)-space, it follows from Köthe [8], §29, 5 (4), that there exist $m\in\mathbb{N}$ and $M>0$ such that B is contained in the closure of the set

$$C(m, M):=\{x\in k^\infty(\mathbb{E}, \alpha) \mid \sup_{i\in I}\|x_i\|_i\exp(-m\alpha_i)\leq M\},$$

where the closure is taken with respect to the inductive limit topology. Since the map $\Phi_i\colon k^\infty(\mathbb{E}, \alpha)\to\mathbb{R}$, $\Phi_i(x):=\|x_i\|_i$, is continuous for each $i\in I$, it follows easily that $C(m, M)$ is in fact closed in $k^\infty(\mathbb{E}, \alpha)$. Hence B is contained in $C(m, M)$, which implies the desired estimate.

(b) Since $k^\infty(\mathbb{E}, \alpha)$ is an (LB)-space, $k^\infty(\mathbb{E}, \alpha)'_b$ is a Fréchet space. In order to see that $k^\infty(\mathbb{E}, \alpha)'_b$ has (Ω) we first give a different representation of $k^\infty(\mathbb{E}, \alpha)$. For $n\in\mathbb{N}$ we put

$$I_n:=\{i\in I \mid \alpha_i\in[n-1, n[\}$$

and

$$F_n:=\{\xi\in\prod_{i\in I_n} E_i \mid \|\xi\|_n:=\sup_{i\in I_n}\|\xi_i\|_i<\infty\}.$$

Then we define $\mathbb{F}:=(F_n, \|\ \|_n)_{n\in\mathbb{N}}$ and $\beta:=(n)_{n\in\mathbb{N}}$. It is easy to check, that the map

$$\Phi\colon k^\infty(\mathbb{E}, \alpha)\to k^\infty(\mathbb{F}, \beta), \qquad \Phi((x_i)_{i\in I}):=((x_i)_{i\in I_n})_{n\in\mathbb{N}},$$

is a linear topological isomorphism.

Now we define

$$\Lambda_\infty(\mathbb{F}', \beta) := \left\{ y \in \prod_{n\in\mathbb{N}} (F_n)'_b \mid \|y\|_k := \sum_{n=1}^{\infty} \|y_n\|'_n \exp(k\beta_n) < \infty \text{ for each } k \in \mathbb{N} \right\},$$

which is a Fréchet space if we endow it with the l.c. topology induced by the norms $(\|\ \|_k)_{k\in\mathbb{N}}$. By part (a) and standard arguments, it follows that the map $\Psi: \Lambda_\infty(\mathbb{F}', \beta) \to k^\infty(\mathbb{F}, \beta)'_b$ defined by

$$\Psi(y)[\xi] := \sum_{n=1}^{\infty} \langle y_n, \xi_n \rangle_{\langle F'_n, F_n \rangle},$$

is a linear topological isomorphism. Since the proof of Vogt and Wagner [19], 1.3, extends to $\Lambda_\infty(\mathbb{F}', \beta)$, it follows that $\Lambda_\infty(\mathbb{F}', \beta)$ and hence $k^\infty(\mathbb{E}, \alpha)'_b$ has the property (Ω).

11. Corollary. *Let* $\mathbb{E}$ *and* α *be as in 9 and let* X *be a closed linear topological subspace of* $k^\infty(\mathbb{E}, \alpha)$ *for which* X'_b *is a Fréchet space. Then* X'_b *has property* (Ω).

Proof. Let $J: X \to k^\infty(\mathbb{E}, \alpha)$ denote the inclusion map. Then $J^t: k^\infty(\mathbb{E}, \alpha)'_b \to X'_b$ is continuous and surjective by the Hahn-Banach theorem. Hence the open mapping theorem implies that X'_b is a quotient space of $k^\infty(\mathbb{E}, \alpha)'_b$. Thus the result follows from Proposition 10 and the inheritance properties of (Ω).

12. Theorem. *For* $1 \leqq m \leqq n$ *let* $F = (F_1, \ldots, F_m) \in (A_p(\mathbb{C}^n))^m$ *be slowly decreasing. Then* $I_{\text{loc}}(F)$ *coincides with* $I(F)$ *and* $(A_p(\mathbb{C}^n)/I(F))'_b$ *has the property* (Ω).

Proof. By Berenstein and Taylor [2], 5.4(ii), applied with $r=0$, we know that $I_{\text{loc}}(F) = I(F)$. Since F is slowly decreasing for some family $\mathscr{L}$, we can choose ε, $C>0$ as in 5(a) as well as η, $D>0$ such that $\mathscr{C} := \mathscr{C}(F, \mathscr{L}, \varepsilon, C, \eta, D)$ is a good family. For $\Omega \in \mathscr{C}$ we denote by R_Ω the map

$$R_\Omega: A^\infty(\Omega) \to \prod_{a\in\Omega\cap V(F)} \mathcal{O}_a/I_a$$

defined by

$$R_\Omega(f) := ([f]_a + I_a)_{a\in\Omega\cap V(F)}.$$

We remark that I_a is closed in $\mathcal{O}_a$ for the locally convex topology of simple convergence on $\mathcal{O}_a$ (see Hörmander [6], 6.3.5). Hence there exists a locally convex topology τ_Ω on $\prod_{a\in\Omega\cap V(F)} \mathcal{O}_a/I_a$ for which R_Ω is continuous. We put

(1) $E_\Omega = \operatorname{im} R_\Omega$ endowed with the quotient norm $\|\ \|_\Omega$, i.e.

$$\|R_\Omega(g)\| := \inf\{\|f\|_{A^\infty(\Omega)} \mid R_\Omega(f) = R_\Omega(g)\}.$$

Then $(E_\Omega, \|\ \|_\Omega)$ is a Banach space. Next we put

(2) $$\alpha_\Omega := \sup\{p(z) \mid z \in \Omega\}, \qquad \Omega \in \mathscr{C}$$

and we define $\mathbb{E} := (E_\Omega, \|\ \|_\Omega)_{\Omega\in\mathscr{C}}$ and $\alpha := (\alpha_\Omega)_{\Omega\in\mathscr{C}}$.

Now we define

(3) $$\tilde{R}: A_p(\mathbb{C}^n) \to k^\infty(\mathbb{E}, \alpha), \qquad \tilde{R}(f) := (R_\Omega(f \mid \Omega))_{\Omega \in \mathscr{C}}.$$

It is easy to check that $\tilde{R}$ is a continuous linear map.

Obviously we have

(4) $$\ker \tilde{R} = I_{\mathrm{loc}}(F).$$

Hence $\tilde{R}$ induces an injective continuous linear map

(5) $$R: A_p(\mathbb{C}^n)/I_{\mathrm{loc}}(F) \to k^\infty(\mathbb{E}, \alpha).$$

Next put

$$\mathscr{C}_0 := \{(\Omega_0, \Omega_1) \in \mathscr{C} \times \mathscr{C} \mid \Omega_0 \cap \Omega_1 \cap V(F) \neq \varnothing\}$$

and define for $(\Omega_0, \Omega_1) \in \mathscr{C}_0$ the map

$$\Phi_{\Omega_0, \Omega_1}: k^\infty(\mathbb{E}, \alpha) \to \prod_{a \in \Omega_0 \cap \Omega_1 \cap V(F)} \mathcal{O}_a / I_a$$

by

$$\Phi_{\Omega_0, \Omega_1}((x_\Omega)_{\Omega \in \mathscr{C}}) := ((x_{\Omega_0})_a - (x_{\Omega_1})_a)_{a \in \Omega_0 \cap \Omega_1 \cap V(F)}.$$

If we use the same topology as above on $\prod_{a \in \Omega_0 \cap \Omega_1 \cap V(F)} \mathcal{O}_a / I_a$, then $\Phi_{\Omega_0, \Omega_1}$ is continuous, hence $\ker \Phi_{\Omega_0, \Omega_1}$ is closed in $k^\infty(\mathbb{E}, \alpha)$. Now we define

(6) $$X := \bigcap \{\ker \Phi_{\Omega_0, \Omega_1} \mid (\Omega_0, \Omega_1) \in \mathscr{C}_0\}.$$

Then X is a closed linear subspace of $k^\infty(\mathbb{E}, \alpha)$ and it is obvious that im R is contained in X.

We claim that the following holds:

(7) For each $k \in \mathbb{N}$ and each $A > 0$ there exist $B, B' > 0$ such that for each $x \in X$ with $|||x|||_k \leqq A$ there exists $f \in A_p(\mathbb{C}^n)$ with $\sup_{z \in \mathbb{C}^n} |f(z)| e^{-Bp(z)} \leqq B'$ such that $\tilde{R}(f) = x$.

From (7) we get that the range of R equals X. Moreover, (7) and Proposition 10(a) imply that $R^{-1}(B)$ is bounded for each bounded set B in $k^\infty(\mathbb{E}, \alpha)$. Since $A_p(\mathbb{C}^n)/I_{\mathrm{loc}}(F)$ is a (DFN)-space, this implies by the Baernstein-Lemma [1], that R is a topological homomorphism. Hence $A_p(\mathbb{C}^n)/I_{\mathrm{loc}}(F)$ is isomorphic to X. Since $(A_p(\mathbb{C}^n)/I_{\mathrm{loc}}(F))'_b$ is a nuclear Fréchet space, it follows from Corollary 11 that $(A_p(\mathbb{C}^n)/I_{\mathrm{loc}}(F))'_b$ has (Ω). Since $I_{\mathrm{loc}}(F) = I(F)$, as we remarked above, the proof of the theorem is complete, if we show that (7) holds.

To prove (7), let $x \in X$ with $|||x|||_k \leqq A$ be given. Then note that by Definition 5 and 1(3) there exists $L \geqq 1$ such that

$$\sup_{z \in \Omega} p(z) \leqq L(1 + \inf_{z \in \Omega} p(z)) \quad \text{for each } \Omega \in \mathscr{C}.$$

Hence the definition of the spaces $(E_\Omega, \| \ \|_\Omega)$, $\Omega \in \mathscr{C}$, imply that for each $\Omega \in \mathscr{C}$ there exists $f_\Omega \in A^\infty(\Omega)$ with $R_\Omega(f_\Omega) = x_\Omega$ and

(8) $$|f_\Omega(z)| \leqq 2A \exp(kL + kLp(z)) \quad \text{for all } z \in \Omega.$$

Moreover, for $(\Omega_0, \Omega_1) \in \mathscr{C}_0$ we have that $f_{\Omega_0 | \Omega_0 \cap \Omega_1} - f_{\Omega_1 | \Omega_0 \cap \Omega_1}$ is in the ideal generated by $F_1, \ldots, F_m$ in $A(\Omega_0 \cap \Omega_1)$, since x is in X. Now we are in the situation, where we can apply the arguments out of the last part of the proof of Berenstein and Taylor [2], Theorem 5.6. Using their notation, we put $\gamma := (f_\Omega)_{\Omega \in \mathscr{C}}$. Then $\gamma \in \mathscr{A}_0^0(\mathscr{C})$ and we have that $\omega := \delta\gamma \in \mathscr{A}_0^1(\mathscr{C})$ has the same properties which are explained on p. 224 of [2]. Since $\delta\omega = 0$, there exists a good refinement $\mathscr{C}'$ of $\mathscr{C}$ and $\eta \in \mathscr{A}_1^1(\mathscr{C}')$ with $P(\eta) = \rho(\omega)$ and $\delta\eta = 0$ by Berenstein and Taylor [2], Theorem 5.4. Then there exists a good refinement $\mathscr{C}''$ of $\mathscr{C}'$ and $\theta \in \mathscr{A}_1^0(\mathscr{C}'')$ with $\delta\theta = \rho(\eta)$.

Now we define

$$\tilde{\gamma} := \rho(\gamma) - P(\theta).$$

Then $\delta(\tilde{\gamma}) = \rho(\omega) - \delta P(\theta) = 0$. Hence $\tilde{\gamma}$ defines an analytic function on $\bigcup_{\Omega \in \mathscr{C}''} \Omega =: W$. Since there exist $\varepsilon', C' > 0$ such that $W \supset S(F; \varepsilon', C')$ it follows from Berenstein and Taylor [2], Theorem 2.2, that there exist $f \in A_p(\mathbb{C}^n)$ with $\tilde{R}(f) = x$.

Now observe that the changes in the bounds for η and θ which are caused by the application of Berenstein and Taylor [2], Theorem 5.4, depend only on the previous bounds and $\mathscr{C}'$ resp. $\mathscr{C}''$ and not on the individual functions and that the same applies to Berenstein and Taylor [2], Theorem 2.2. Hence we have shown that (7) holds, which completes the proof.

The dependences of the bounds which we have quoted above, are not all stated explicitly in Berenstein and Taylor [2]. Therefore we include a more detailed proof of (7) in the case of a principal ideal, which is the most relevant case for applications.

Proof of (7) *in the Case* $F = F_1$. Let $x \in X$ with $|||x|||_k \leq A$ be given. Then we proved already that we can find $f_\Omega \in A^\infty(\Omega)$, $\Omega \in \mathscr{C}$, which satisfy (8). Now, for each $(\Omega_0, \Omega_1) \in \mathscr{C}_0$ we can find $h_{\Omega_0, \Omega_1} \in A^\infty(\Omega_0 \cap \Omega_1)$ satisfying

$$f_{\Omega_0}(z) - f_{\Omega_1}(z) = h_{\Omega_0, \Omega_1}(z)\, F(z) \quad \text{for all } z \in \Omega_0 \cap \Omega_1. \tag{9}$$

Moreover, (9) implies that the family $(h_{\Omega_0, \Omega_1})_{(\Omega_0, \Omega_1) \in \mathscr{C}_0}$ is a cocycle. To get estimates for this cocycle out of the estimates (8) we use Definition 5(b) and 1(3) together with Berenstein and Taylor [2], 3.1, to find a good refinement $\mathscr{C}'$ of $\mathscr{C}$ such that there exist positive constants A_1 and B_1, depending only on $A, k, L, F, \mathscr{C}$ and $\mathscr{C}'$ but not on $(f_\Omega)_{\Omega \in \mathscr{C}'}$ such that the cocycle $(H_{\Omega'_0, \Omega'_1})_{(\Omega'_0, \Omega'_1) \in \mathscr{C}'_0}$,

$$H_{\Omega'_0, \Omega'_1} := h_{\rho(\Omega'_0), \rho(\Omega'_1)} | \Omega'_0 \cap \Omega'_1, \tag{10}$$

satisfies the estimates

$$|H_{\Omega'_0, \Omega'_1}(z)| \leq A_1 \exp(B_1\, p(z)) \quad \text{for all } z \in \Omega'_0 \cap \Omega'_1,\ (\Omega'_0, \Omega'_1) \in \mathscr{C}'_0. \tag{11}$$

Next we choose a good refinement $\mathscr{C}''$ of $\mathscr{C}'$ and numbers $\delta_1 > 0$ and $E_1 > 0$ so that for each $\Omega \in \mathscr{C}''$ and each $z \in \Omega$ we have

$$\{w \in \mathbb{C}^n \mid |w - z| < \delta_1 \exp(-E_1\, p(z))\} \subset \rho'(\Omega),$$

where $\rho' : \mathscr{C}'' \to \mathscr{C}'$ denotes the refinement map. Then choose a good refinement $\mathscr{C}'''$ of $\mathscr{C}''$ and $0 < \delta_2 < \delta_1$ and $E_2 > E_1$ so that for each $\Omega \in \mathscr{C}'''$ and each $z \in \Omega$

we have

$$\{w\in\mathbb{C}^n \mid |w-z|<\delta_2 \exp(-E_2\, p(z))\}\subset\rho''(\Omega).$$

Then note that by Berenstein and Taylor [2], p. 217, there exist positive numbers ε_1 and C_1 with

$$\bigcup\{\Omega \mid \Omega\in\mathscr{C}'''\}\supset S(F,\varepsilon_1,C_1).$$

By the proof of Berenstein and Taylor [2], 2.2, there exist $0<\varepsilon_2<\varepsilon_1$ and $C_2>C_1$ as well as $0<\delta_3<\delta_2$ and $E_3>E_2$ such that for each $z\in S(F,\varepsilon_2,C_2)$ we have

$$\{w\in\mathbb{C}^n \mid |w-z|<\delta_3\cdot\exp(-E_3\, p(z))\}\subset S(F,\varepsilon_1,C_1).$$

Now we put $\Omega_*:=\mathbb{C}^n\setminus\overline{S(F,\varepsilon_2,C_2)}$ and define the coverings $\tilde{\mathscr{C}}$, $\tilde{\mathscr{C}}'$, $\tilde{\mathscr{C}}''$ and $\tilde{\mathscr{C}}'''$ of $\mathbb{C}^n$ by adding Ω_* to the families $\mathscr{C}$, $\mathscr{C}'$, $\mathscr{C}''$ and $\mathscr{C}'''$. Then the corresponding refinement maps have obvious extensions, again denoted by ρ, ρ', ρ''.

Next we define $f_{\Omega_*}\in A^\infty(\Omega_*)$ by $f_{\Omega_*}\equiv 0$. If we define the cocycle H as before, but with $\mathscr{C}'$ replaced by $\tilde{\mathscr{C}}'$, then H satisfies the estimates (11) with other constants A_1 and B_1.

Now observe that the choices above imply the existence of positive numbers $0<\delta<1$ and $E>0$ such that we can find a globally finite cover of $\mathbb{C}^n$ by open cubes $(Q_j)_{j\in\mathbb{N}}$ for which diam Q_j is approximately $\delta\cdot\exp(-E p_j)$ for $p_j=\sup_{z\in Q_j} p(z)$, and such that for each $j\in\mathbb{N}$ we can choose $\Omega(j)\in\tilde{\mathscr{C}}''$ such that $Q_j\subset\rho'(\Omega(j))$. Moreover, we can choose $\varphi_j\in\mathscr{D}(Q_j)$ with $\sum_{j=1}^{\infty}\varphi_j\equiv 1$ on $\mathbb{C}^n$ and

(12) $$|\bar\partial\varphi_j(z)|\leqq A_2\exp(B_2\, p(z)) \quad \text{for all } z\in\mathbb{C}^n \text{ and all } j\in\mathbb{N},$$

where A_2 and B_2 are suitable positive constants, depending on δ, E and n.

Next we define for each $\Omega\in\tilde{\mathscr{C}}''$

$$h_\Omega:=\sum_{j=1}^{\infty}\varphi_j H_{\rho'(\Omega),\,\rho'(\Omega(j))}\,|\,\Omega.$$

Then it is easy to see that h_Ω is in $C^\infty(\Omega)$ and that the cocycle property of H implies for each $(\Omega_1,\Omega_2)\in\tilde{\mathscr{C}}_0''$

(13) $$h_{\Omega_1}|\Omega_1\cap\Omega_2-h_{\Omega_2}|\,\Omega_1\cap\Omega_2=H_{\rho'(\Omega_1),\,\rho'(\Omega_2)}\,|\,\Omega_1\cap\Omega_2$$

and

(14) $$\bar\partial h_{\Omega_1}|\Omega_1\cap\Omega_2=\bar\partial h_{\Omega_2}|\,\Omega_1\cap\Omega_2.$$

From (14) we see that there exists $u\in C^\infty_{(0,1)}(\mathbb{C}^n)$ with $u\,|\,\Omega=\bar\partial h_\Omega$ for each $\Omega\in\tilde{\mathscr{C}}''$. Since the partition of unity $(\varphi_j)_{j\in\mathbb{N}}$ is globally finite, (11) and (12) (for $\tilde{\mathscr{C}}_0'$) imply the existence of positive numbers A_3, B_3, depending only on A, k, L, F, $\mathscr{C}$, $\mathscr{C}'$, n, δ and E such that

(15) $$|u(z)|\leqq A_3\exp(B_3\, p(z)) \quad \text{for all } z\in\mathbb{C}^n.$$

Since $\bar{\partial} u | \Omega = \bar{\partial} \bar{\partial} h_\Omega = 0$, we get from Hörmander [6], 4.4.2, the existence of $H \in C^\infty(\mathbb{C}^n)$ with $\bar{\partial} H = u$ and

(16) $$\int |H(z)|^2 \exp(-2(B_3 p(z) + (n+1) \log(1+|z|^2)))\, d\lambda \leqq A_3.$$

Then $b_\Omega := h_\Omega - H|_\Omega$ is in $A(\Omega)$ for each $\Omega \in \mathscr{C}''$. By (13) we have for all $(\Omega_1, \Omega_2) \in \tilde{\mathscr{C}}''_0$

(17) $$b_{\Omega_1} | \Omega_1 \cap \Omega_2 - b_{\Omega_2} | \Omega_1 \cap \Omega_2 = H_{\rho'(\Omega_1), \rho'(\Omega_2)} | \Omega_1 \cap \Omega_2.$$

Next we define $a_\omega := b_{\rho''(\omega)} | \omega$ and $f_\omega := f_{\rho \circ \rho' \circ \rho''(\omega)}$ for $\omega \in \mathscr{C}'''$. By (17), (10) and (9) we get for all $(\omega_1, \omega_2) \in \tilde{\mathscr{C}}'''_0$

$$a_{\omega_1} | \omega_1 \cap \omega_2 - a_{\omega_2} | \omega_1 \cap \omega_2 = \left. \frac{f_{\omega_1} - f_{\omega_2}}{F} \right| \omega_1 \cap \omega_2,$$

and consequently

$$(f_{\omega_1} - a_{\omega_1} F) | \omega_1 \cap \omega_2 = (f_{\omega_2} - a_{\omega_2} F) | \omega_1 \cap \omega_2.$$

This implies the existence of $g \in A(\mathbb{C}^n)$ satisfying

(18) $$g | \omega = f_\omega - a_\omega F \quad \text{for all } \omega \in \mathscr{C}'''.$$

Next we observe that our choices imply by (16) and standard arguments, that there exist positive constants A_4 and B_4 depending on A_3, B_3, p, n and $\mathscr{C}'''$ such that

(19) $$|a_\omega(z)| \leqq A_4 \exp(B_4 p(z)) \quad \text{for all } z \in \omega, \text{ all } \omega \in \tilde{\mathscr{C}}'''.$$

From (19) and (8) we finally get A_5 and B_5 depending only on A, k, L, F, p, n, δ, E, $\mathscr{C}'$, $\mathscr{C}''$ and $\mathscr{C}'''$ but not on x such that

(20) $$|g(z)| \leqq A_5 \exp(B_5 p(z)) \quad \text{for all } z \in \mathbb{C}^n.$$

Hence g is in $A_p(\mathbb{C}^n)$. By (18) we have $\tilde{R}(g) = x$, which completes the proof of (7) in the particular case.

In order to show that Theorem 12 implies that in certain algebras $A_p(\mathbb{C}^n)$ all slowly decreasing ideals are complemented, we recall the definition of the linear topological invariant (DN) which has been introduced by Vogt [17].

13. The Property (*DN*). Let E be a metrizable locally convex space with a fundamental system $(\| \ \|_k)_{k \in \mathbb{N}}$ of semi-norms. *E has the property* (DN) if the following holds:

(DN) There exists $m \in \mathbb{N}$ such that for each $k \in \mathbb{N}$ there exists $n \in \mathbb{N}$ and $C > 0$ with $\| \ \|_k^2 \leqq C \| \ \|_m \| \ \|_n$.

It is easy to check that $\| \ \|_m$ is in fact a norm on E and that (DN) is a linear topological invariant which is inherited by linear topological subspaces. The significance of the property (DN) was first observed in Vogt [17], 1.3.

14. Theorem. *Let p be a weight function on $\mathbb{C}^n$ which satisfies* $\log(1+|z|^2) = o(p(z))$. *Then the following assertions are equivalent:*

(1) $A_p(\mathbb{C}^n)'_b$ *has* (DN).

(2) *For each* $k \in \mathbb{N}$ *with* $1 \leqq k \leqq n$ *and each slowly decreasing k-tuple* $F=(F_1, \ldots, F_k) \in (A_p(\mathbb{C}^n))^k$ *the ideal* $I(F)$ *is complemented in* $A_p(\mathbb{C}^n)$.

If, in addition, p is radial, i.e. $p(z)=p(|z|)$, *then* (1) *and* (2) *are also equivalent to*

(3) *For each* $C>1$ *there exist* $R_0>0$ *and* $0<\delta<1$ *such that for each* $R \geqq R_0$ *we have* $p^{-1}(CR)\,p^{-1}(\delta R) \leqq (p^{-1}(R))^2$.

Proof. (1)⇒(2): Fix $k \in \mathbb{N}$ with $1 \leqq k \leqq n$ and let $F=(F_1, \ldots, F_k) \in (A_p(\mathbb{C}^n))^k$ be slowly decreasing. Then we have noted in Theorem 12 that $I(F)$ is closed in $A_p(\mathbb{C}^n)$. Hence the continuous linear map

$$M\colon (A_p(\mathbb{C}^n))^k \to A_p(\mathbb{C}^n), \qquad M(g_1, \ldots, g_k) := \sum_{j=1}^{k} g_j F_j,$$

has closed range $I(F)$. Since $A_p(\mathbb{C}^n)$ is a (DFN)-space, this implies that $I(F)$ is a quotient space of $(A_p(\mathbb{C}^n))^k$. Consequently, $I(F)'_b$ is isomorphic to a subspace of $(A_p(\mathbb{C}^n)^k)'_b \simeq (A_p(\mathbb{C}^n)'_b)^k$. Since $A_p(\mathbb{C}^n)'_b$ has (DN) by (1), this implies that $I(F)'_b$ has (DN).

Now we look at the exact sequence

$$0 \to I(F) \to A_p(\mathbb{C}^n) \xrightarrow{R} A_p(\mathbb{C}^n)/I(F) \to 0, \tag{4}$$

where R denotes the quotient map. Dualizing (4) we get the exact sequence of nuclear Fréchet spaces

$$0 \to (A_p(\mathbb{C}^n)/I(F))'_b \to A_p(\mathbb{C}^n)'_b \to I(F)'_b \to 0. \tag{5}$$

We proved already that $I(F)'_b$ has (DN). By Theorem 12, $(A_p(\mathbb{C}^n)/I(F))'_b$ has (Ω). Hence we can apply the splitting theorem of Vogt and Wagner [19], 1.4 (see also Vogt [18], 2.2), to get the splitting of the sequence (5). Obviously, this implies that the exact sequence (4) splits. Hence $I(F)$ is complemented in $A_p(\mathbb{C}^n)$.

(2)⇒(1): This is an immediate consequence of [13], Theorem 2.17.

(1)⇔(3): For radial weight functions p, this follows from [13], Theorem 2.17 and Proposition 3.1. A more direct proof is given in Meise, Momm and Taylor [12], 2.11, 2.12. This proof uses a sequence space representation of $A_p(\mathbb{C}^n)'_b$ to evaluate Vogt [17], 2.3.

Remark. We remark that in [13], Theorem 2.17, we have proved various properties related with a weight function p on $\mathbb{C}^n$ to be equivalent to $A_p(\mathbb{C}^n)'_b$ having (DN).

15. Corollary. *Let p be a radial weight function on* $\mathbb{C}^n$ *with* $p(2z)=O(p(z))$ *and* $\log(1+|z|^2)=o(p(z))$ *which satisfies condition* (3) *of Theorem* 14. *Then every principal ideal I in* $A_p(\mathbb{C}^n)$ *is complemented.*

Proof. Theorem 7.1 of Berenstein and Taylor [2] implies that each non-zero principal ideal I in $A_p(\mathbb{C}^n)$ is slowly decreasing. Hence the corollary follows from Theorem 14.

Remark. Corollary 15 extends a result of Djakov and Mityagin [4] to principal ideals which are not necessarily generated by a polynomial. Moreover, it extends Meise [11], Theorem 4.7, and Taylor [15], Theorem 5.1, to several variables.

16. Examples. (a) It is easy to check that the following radial weight functions on $\mathbb{C}^n$ satisfy condition (3) of Theorem 14.

(1) $p(z)=|z|^\rho(\log(1+|z|^2))^\sigma$, $\rho>0$, $0\leqq\sigma<\infty$.

(2) $p(z)=\exp(|z|^\alpha)$, $0<\alpha\leqq 1$.

(3) $p(z)=\exp((\log(1+|z|^2))^\alpha)$, $0<\alpha<1$.

(4) Any radial weight function p with $p(2z)=O(p(z))$ for which there exists $A\geqq 1$ with $2p(z)\leqq p(Az)+A$ for all $z\in\mathbb{C}^n$.

(b) For the weight functions $p(z)=(\log(1+|z|^2))^s$, $s>1$ the space $A_p(\mathbb{C}^n)'_b$ does not have (DN). In fact, each infinite codimensional slowly decreasing ideal $I(F_1, \ldots, F_n)$ in $A_p(\mathbb{C}^n)$ is not complemented (see Meise [11], 2.13(2) and 4.9).

From Corollary 15 we can easily get results on the existence of a continuous linear right inverse for all non-zero convolution operators on certain weighted Fréchet spaces of entire functions. To state these results, we recall that for an increasing convex function $q\colon [0, \infty[\to[0, \infty[$, its Young conjugate $q^*\colon [0, \infty[\to[0, \infty]$ is defined by

$$q^*(x)=\sup\{xy-q(y)\mid y\geqq 0\}.$$

17. Theorem. *Let $p\colon [0, \infty[\to[0, \infty[$ be an increasing convex function for which $\tilde{p}\colon z\mapsto p(|z|)$ is a weight function on $\mathbb{C}^n$ which satisfies the hypotheses of Corollary 15. Put $q=p^*$ and define*

$$A_q^0(\mathbb{C}^n):=\left\{f\in A(\mathbb{C}^n)\mid \sup_{z\in\mathbb{C}^n}|f(z)|\exp\left(-\varepsilon q\left(\frac{|z|}{\varepsilon}\right)\right)<\infty \text{ for all } \varepsilon>0\right\}.$$

Then every non-zero convolution operator T on $A_q^0(\mathbb{C}^n)$ admits a continuous linear right inverse.

Proof. By Taylor [14], the Fourier-Borel transform $\mathscr{F}$ gives a linear topological isomorphism between $A_q^0(\mathbb{C}^n)'_b$ and $A_{\tilde{p}}(\mathbb{C}^n)$. Moreover, it is well-known that for each non-zero convolution operator T on $A_q^0(\mathbb{C}^n)$ the exact sequence

$$0\to\ker T\to A_q^0(\mathbb{C}^n)\xrightarrow{T} A_q^0(\mathbb{C}^n)\to 0$$

splits if and only if the principal ideal $\mathscr{F}(T)\,A_{\tilde{p}}(\mathbb{C}^n)$ is complemented in $A_{\tilde{p}}(\mathbb{C}^n)$. Hence the result follows from Corollary 15.

As a particular case of Theorem 17 we note the following corollary which covers all convolution operators on some classical spaces which have been investigated by Ehrenpreis [5], Malgrange [9] and Martineau [10].

18. Corollary. *Every non-zero convolution operator T on $A(\mathbb{C}^n)$ or on $E_s^0(\mathbb{C}^n)$, $s>1$, admits a continuous linear right inverse, where*

$$E_s^0(\mathbb{C}^n)=\{f\in A(\mathbb{C}^n)\mid \sup_{z\in\mathbb{C}^n}|f(z)|\exp(-\varepsilon|z|^s)<\infty \text{ for all } \varepsilon>0\}.$$

Remark. a) Corollary 18 implies in particular that every non-zero linear partial differential operator $P(D)$ with constant coefficients on $A(\mathbb{C}^n)$ admits a continuous linear right inverse. This however, was shown already by Treves [16], Theorem 9.7 and its corollary.

b) Corollary 18 extends Taylor [15], Theorem 5.1, and Meise [11], 5.7 to the case $n > 1$.

In order to state a further corollary of Theorem 14, let p be a weight function on $\mathbb{C}^n$, let I be a closed ideal in $A_p(\mathbb{C}^n)$ and let $V(I)$ denote the multiplicity variety of I (see Berenstein and Taylor [2], Sect. 3). Then one wants to identify $A_p(\mathbb{C}^n)/I$ with a certain space $A_p(V(I))$ of holomorphic functions on $V(I)$. If $V(I)$ is a strongly interpolating complex submanifold of $\mathbb{C}^n$ (see [13], 2.16 and Berenstein and Taylor [3], Theorem 1), then

$$A_p(V(I)) = \{f \in A(V(I)) \mid \sup_{z \in V(I)} |f(z)| \exp(-Ap(z)) < \infty \text{ for some } A > 0\}.$$

However, in the general case, the definition of $A_p(V(I))$ is more involved, as Berenstein and Taylor [2], 3.5 and 3.6 show. Using this definition of $A_p(V(I))$, we get immediately from Theorem 14.

19. Corollary. *Let p be a weight function on $\mathbb{C}^n$ for which $A_p(\mathbb{C}^n)'_b$ has (DN) and let $F = (F_1, \ldots, F_k) \in (A_p(\mathbb{C}^n))^k$ be slowly decreasing. Then there exists a continuous linear extension operator*

$$E: A_p(V(I(F))) \to A_p(\mathbb{C}^n).$$

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