Experiments in Fluids 5, 401–406 (1987)

Experiments in Fluids

Digital interferometry for flow visualization *

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Abstract. Digital holographic interferometry is a hybrid optical-digital technique for determining the phase of an interferogram. This technique improves the accuracy of interferometric measurement of fluid properties and enhances the utility of interferometric flow visualization. Displays of the interferometric phase produce excellent images of weakly refracting two-dimensional flows and can be used to produce integral projection images of three dimensional flows which differ from and complement schlieren and shadowgraph images. The technique is explained herein and examples of its use in both continuous wave and pulsed interferometry are presented.

1 Introduction

Schlieren, shadowgraph and interferometric flow visualization techniques use the integrated effect of a fluid optical property on a beam of light passing through an object to form an image of a flow pattern. These techniques work well for a variety of flows but have limited utility for three-dimensional flows and weakly refracting flows. Schlieren and shadowgraph images are formed by ray bending which is approximately proportional to refractive index gradients and second derivatives of the refractive index, respectively. The former two techniques work well for two-dimensional flows characterized by large refractive index gradients, such as shock patterns. They also have been used to observe complex structures like turbulent jets. Such images show qualitative flow features such as the gross outline of the motion, the fine grained structure of turbulence (Crow & Champagne 1971), and the presence of large, two dimensional structures [e.g. the vortex-like structures of Brown & Roshko (1974)], but conceal other structural aspects of the flow such as the presence of unmixed, entrained fluid inclusions within the flow.

Interferometric methods are based on the phase delay of a plane wave passing through the object and produce an integral projection of the object's refractive index field. However, this phase delay is encoded in a fringe pattern which can be difficult to interpret. The fringes are useful for visualizing objects such as plumes and boundary layers, but become complicated for three-dimensional objects. None of these methods works well for weakly-refracting flows because the gradients involved produce indistinct schlieren and shadowgraph images and yield broad, ambiguous interferometric fringes.

Digital interferometry is a technique by which the interferometric phase delay is determined very accurately at a large number of points in the image. The phase may be displayed as a gray scale, yielding an image that has none of the ambiguities associated with Schlieren and shadowgraph images. Furthermore, the technique may be used to image weakly-refracting flows since direct determination of the phase allows one to enhance the contrast of flow details that can not be resolved by conventional interferometry. Finally, the technique may be used to make distributed measurements of flow quantities.

2 Digital interferometry

Digital interferometry is a recently developed hybrid optical-digital metrology technique combining two exposure holographic interferometry with digital image acquisition and computer processing to determine the interferometric phase directly from a set of image irradiance measurements (Dändliker & Thalmann 1985; Hariharan 1985). This technique is similar to heterodyne holographic interferometry in that both manipulate the interferogram's phase in a known manner to determine its magnitude. The image intensity of a holographic interferogram is given by (Vest 1979):

$$I(x,y) = I_0(x,y)\,\{1 + m(x,y)\cos[\Phi(x,y) + \varphi]\} \qquad (1)$$

where $I_0(x,y)$ is the background intensity, $m(x,y)$ is the fringe contrast and $\Phi(x,y)$ is the interferometric phase. The term φ is the uniform phase bias term which in the

* This work was presented in part at the 1985 Optical Society of America Annual Meeting

case of digital or heterodyne interferometry can be manipulated. The image irradiance is recorded for several different values of this bias phase using a digitizing camera and is stored in computer memory.

The unknown phase Φ can be calculated from the values of the recorded image irradiance distribution and the known values of φ using any one of a number of algebraic relationships reviewed recently by Hariharan (1985). In this study, we recorded n separate irradiance patterns with the reference phase φ evenly distributed from zero to 2π. The phase Φ may then be determined, modulo 2π by:

$$\Phi(x,y) = \tan^{-1} \frac{\sum_{j=1}^{n} I_j(x,y) \sin \varphi_j}{\sum_{j=1}^{n} I_j(x,y) \cos \varphi_j} \tag{2}$$

where

$$I_j(x,y) = I_0(x,y) \{1 + m(x,y) \cos[\Phi(x,y) + \varphi_j]\} \tag{3}$$

and $\varphi_j = \frac{2\pi j}{n}$, n = number of exposures. The inverse tangent function maps the fringe pattern into a linear, discontinuous function. This enables one to eliminate the phase sign ambiguity normally associated with cosinusoidal fringes. The usual fringe counting procedure is replaced by a simple, computational sorting operation. A negative discontinuity indicates an increase in fringe number and a positive discontinuity a decrease. Because the phase is evaluated independently at each point in the image, its determination is unaffected by spatial variation in the background irradiance or fringe contrast. The accuracy of phase determination may be of the order of 1/50–1/100 of a fringe, compared with 1/100–1/1,000 of a fringe for heterodyne interferometry and 1/5–1/10 fringe for conventional interferometry, depending in all cases on the nature of the object being studied (Dändliker & Thalmann 1985). The combined use of computational fringe counting and high resolution image storage devices enables one to resolve complex fringe patterns. Digital interferometry is a convenient, accurate, high-resolution, interferometric technique.

This method can be used for both real-time (Hariharan et al. 1982) and double-exposure holographic interferometry. Because real-time images show temporal phase variation due to even small perturbations which are inherent in many fluid mechanics experiments, double exposure methods are preferred for most steady flow studies and all unsteady flow studies. The double exposure technique requires two reference waves, one for each exposure (Fig. 1). The first exposure is made without the object (flow field) and the second exposure is made with the object present. The film is developed and the image is reconstructed by illuminating it with both reference waves simultaneously. Primary, conjugate and cross reconstructions are then present (Dändliker et al. 1976). The conjugate and cross reconstructions must be properly compensated for so that their presence does not significantly reduce the accuracy of the technique (Dändliker et al. 1982). The two primary reconstructions overlap to form the desired interferogram with the phase bias term given by:

$$\varphi = k(r_1 - r_2) - k(r_1' - r_2') \tag{4}$$

where $k = \frac{2\pi}{\lambda}$; λ = recording wavelength; r_1, r_2 are the reference source distances during recording, and r_1', r_2' are the reference source distances during reconstruction. The phase bias term φ may be shifted by changing the path length of either reference wave by a small amount, Δr. Therefore the change in the phase bias term is $\Delta\varphi = k\Delta r$.

This technique can be applied to flow visualization or measurement with either plane-wave or diffused object illumination. The plane wave setup (Fig. 2) is preferable when laser power is limited. When a plane object wave is used, the two reference waves must have a wide angular separation in order to eliminate troublesome overlap of the cross reconstructions.

Diffused illumination (Fig. 3) is generally preferable, however, because it allows the two reference waves to have a small angular separation, thereby reducing the errors due to misalignment of the hologram with the two reference waves. Although the cross-reconstructions overlap, they may be almost completely decorrelated if their

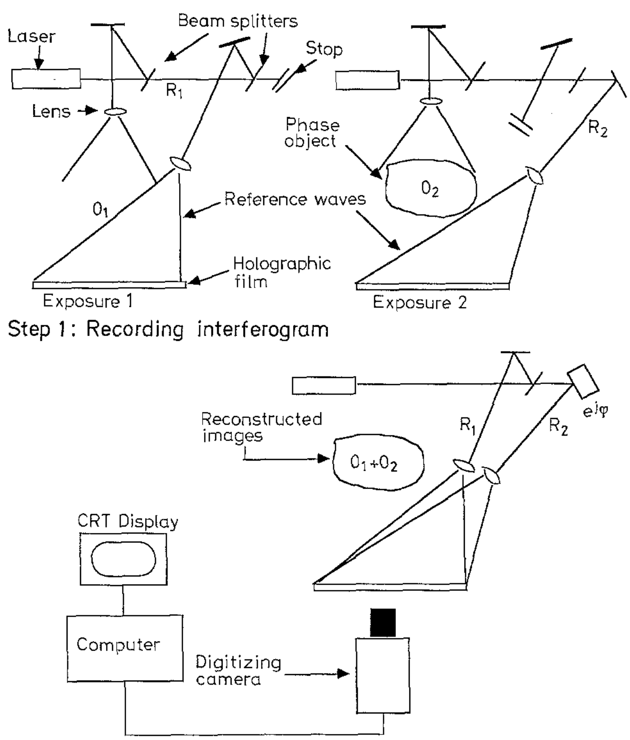

Fig. 1. Schematic setup for digital interferometry of phase objects

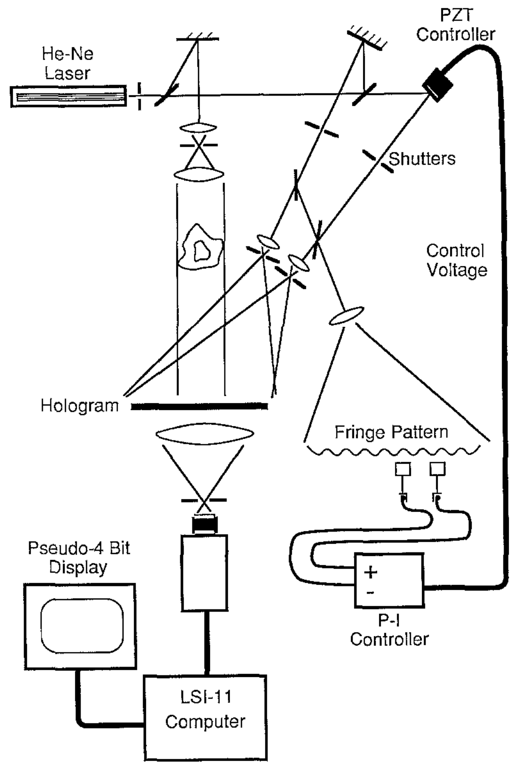

Fig. 2. Experimental setup for plane wave digital interferometry

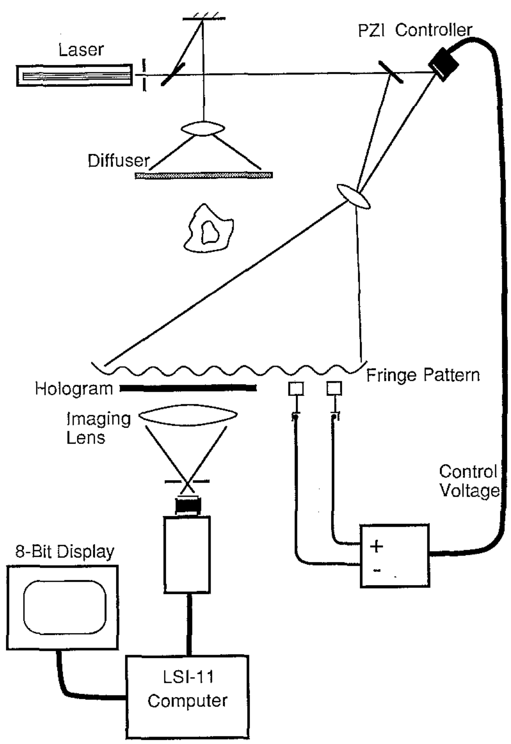

Fig. 3. Experimental setup for diffuse-illumination digital interferometry

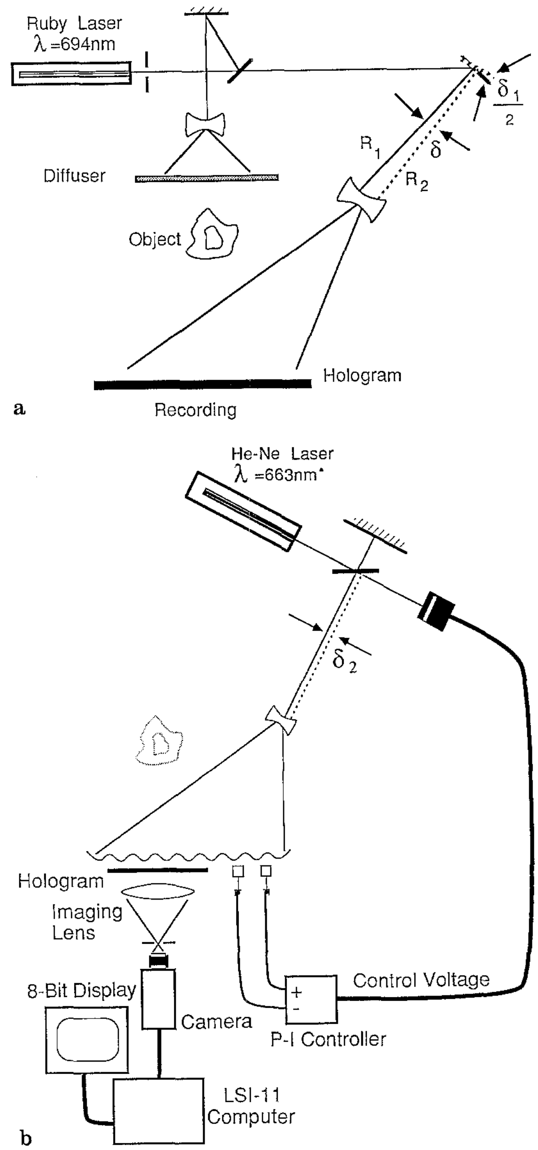

Fig. 4a and b. Setup for **a** recording and **b** processing pulsed-laser digital interferogram

images are displaced in the image plane by an amount greater than the speckle size. This overlap reduces the fringe contrast, but does not significantly reduce the accuracy (Dändliker et al. 1982). The small angular separation of the two reference waves causes a regular, visible fringe pattern to be present on the hologram surface, which causes a periodic error. This effect is minimized by placing a low f-number imaging lens very close to the hologram so that the regular fringe pattern is outside the field of focus.

Breuckmann and Thieme (1985) used this arrangement to correct for errors due to wavelength shift when performing digital interferometry when the recording and reconstruction lasers had different wavelengths (see Fig. 4a and b). In this case, the bias phase is given by:

$$\varphi = (k_1 r_1 - k_2 r_1') - (k_1 r_2 - k_2 r_2') + k_2 \Delta r_2'$$

where $k_1 = \frac{2\pi}{\lambda_1}$, $k_2 = \frac{2\pi}{\lambda_2}$, λ_1 = recording wavelength, λ_2 = reconstruction wavelength, or

$$\Delta\varphi = k_1 (r_1 - r_2) - k_2 (r_1' - r_2') + k_2 \Delta r_2'.$$

For reference beams separated by a small angle δ whose bisector forms an angle θ with the hologram plane:

$$r_1 - r_2 = r \cos\theta \sin\delta.$$

Thus:

$$\Delta\varphi = 2\pi r \left(\frac{\cos\theta \sin\delta_1}{\lambda_1} - \frac{\cos\theta \sin\delta_2}{\lambda_2}\right) + k_2 \Delta r_2'.$$

The terms of the form $\frac{\cos\theta \sin\delta}{\lambda}$ can be shown to be the frequency of the fringe pattern formed by the two reference waves in the hologram plane. Therefore, by setting δ_2 such that the fringe spacing of the reconstruction reference waves is equal to a pattern that would be formed by the two recording reference waves, the chromatic errors in the phase are eliminated. This allows the interferometric phase to be evaluated in the usual way.

3 Experimental setup

A digital interferometer with a plane object wave is shown in Fig. 2. The two reference waves are separated by about 20°. The hologram was held and developed in a real-time liquid gate to reduce alignment errors. A second set of experiments was made with diffused illumination (Fig. 4) using a pulsed ruby laser for recording and a He-Ne laser for reconstruction in a manner described by Breuckmann and Thieme (1985). The second recording reference wave was derived in this case by tilting a mirror in the original reference beam by about 0.5°. The hologram is reconstructed with a He-Ne laser using a Michelson interferometer to create the desired fringe pattern. In both cases the phase shifting is done by translating a mirror in one of the reference beams a fraction of a wavelength. The mirror is mounted on a piezoelectric cell, to which a control voltage is applied. In both cases, the two reference waves are brought together to form a fringe pattern which serves as a measure of their mutual phase difference. A pair of photodiodes are placed in the fringe pattern and the error signal between them is used with a proportional-integral

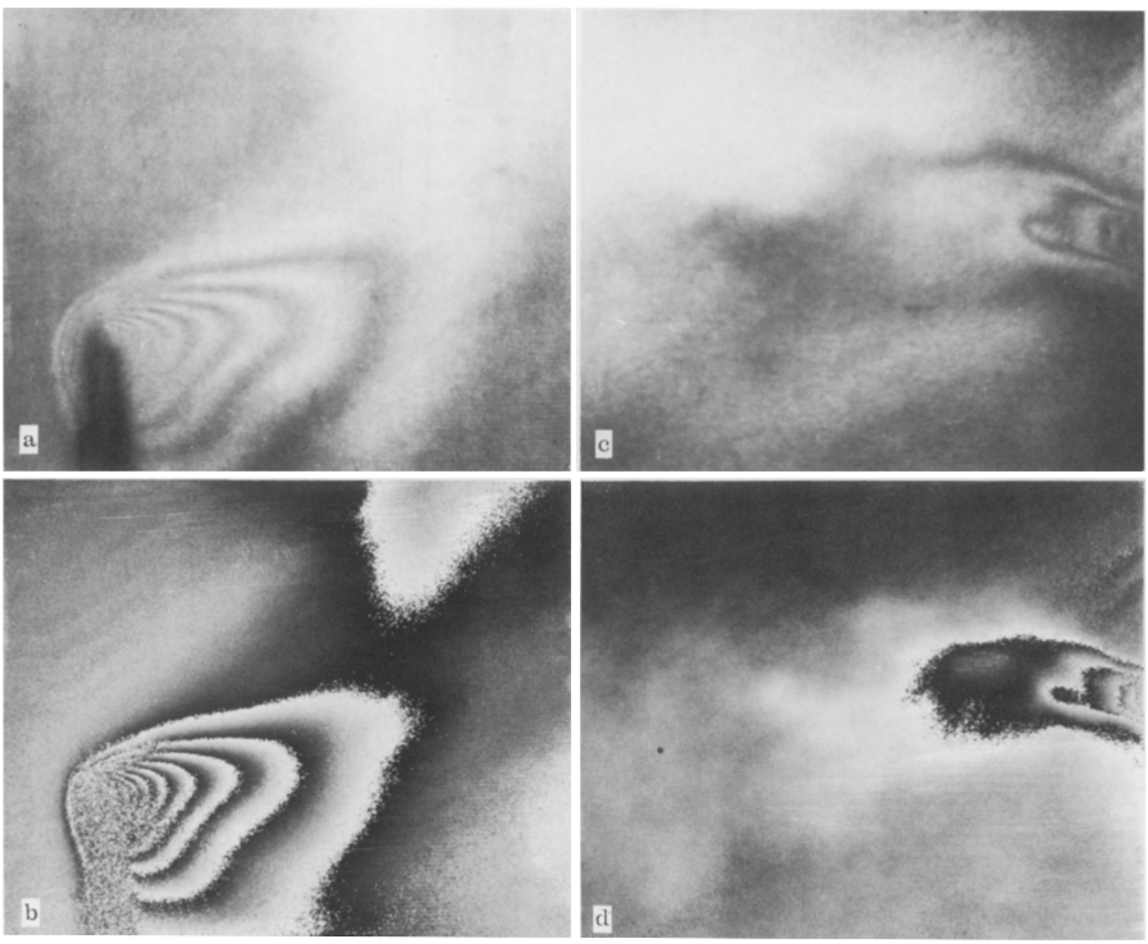

Fig. 5. a Interferogram of strong plume from a heated wire in a cross flow, **b** phase display of interferogram in **a**, **c** interferogram of weakly-heated wire in a cross-flow, **d** phase display of interferogram in **c**

controller to generate the control voltage for the piezoelectric cell to maintain a stable fringe pattern (and therefore phase shift) in the presence of electrical, mechanical and thermal perturbations. The plane wave interferographic images were made with a 128 by 128 pixel array CID camera while the diffuse-illumination images were made with a video camera in conjunction with a 256 by 384 pixel video frame store. Each pixel has 255 gray levels. Image computations were done on an LSI-11 based computer, which is interfaced with the camera.

4 Results

The plane-wave interferometer was used to visualize the flow around a cylindrical heated wire. Figure 5a shows the digitized interferogram of a strong, laminar plume while Fig. 5b shows the gray scale display of the phase, modulo 2π. The conversion of the fringe pattern to a linear discontinuous function is evident. Figure 5c shows the digitized interferogram of a slightly heated wire in a cross-flow. This interferogram consists of a few narrowly spaced fringes near the wire and a single broad, indistinct fringe in the wake. The phase display of this interferogram (Fig. 5d) has a greatly enhanced contrast that clearly shows thermal variation in the wake, which is not apparent in the interferogram.

The diffuse-illumination interferometer was used to visualize a nominally axisymmetric, turbulent, helium jet injected in still air. The interferogram shown in Fig. 6a is complicated and difficult to interpret especially near the jet center where the fringes become broad and indistinct. The phase display (Fig. 6b) is an improvement, producing a contour map of the phase. Using a sorting procedure based on the nature of discontinuities in the phase display, we can compute the total phase at each point in the image. Displaying this total phase as a gray scale (normalized to 255 gray levels) yields the image in Fig. 6c. This image represents a true integral projection of the density of the jet. Close examination of this image reveals two interesting features. The first is the axial distribution of areas of high concentration of helium. The second is the presence of large inclusions of unmixed ambient fluid near the center line of the jet. Although the implications of these structures will not be discussed here, it is interesting to compare the absolute phase plot to the schlieren images of a similar jet made by Crow and Champagne (1971) and the laser-induced fluorescence images of Dimotakis et al. (1983). The schlieren pictures show an apparent fine-grained "surface" of the jet with little indication of the internal structure. The integral phase plot is similar to the centerline laser fluorescence pictures, in that both indicate areas of high jet fluid concentration and large inclusions of unmixed ambient fluid. This is somewhat surprising because the integration would tend to average out the effects of locally strong variation in the concentration field. Further study of such images could be used to examine the symmetry properties of the jets and other mixing phenomena.

5 Applications

These examples point out the potential utility of digital interferometry for flow visualization and measurement. It greatly enhances the image contrast of interferograms of weakly refracting flows, removes the sign ambiguity associated with conventional interferograms, and produces images that display rather subtly flow features. These features could allow unique applications of interferometry

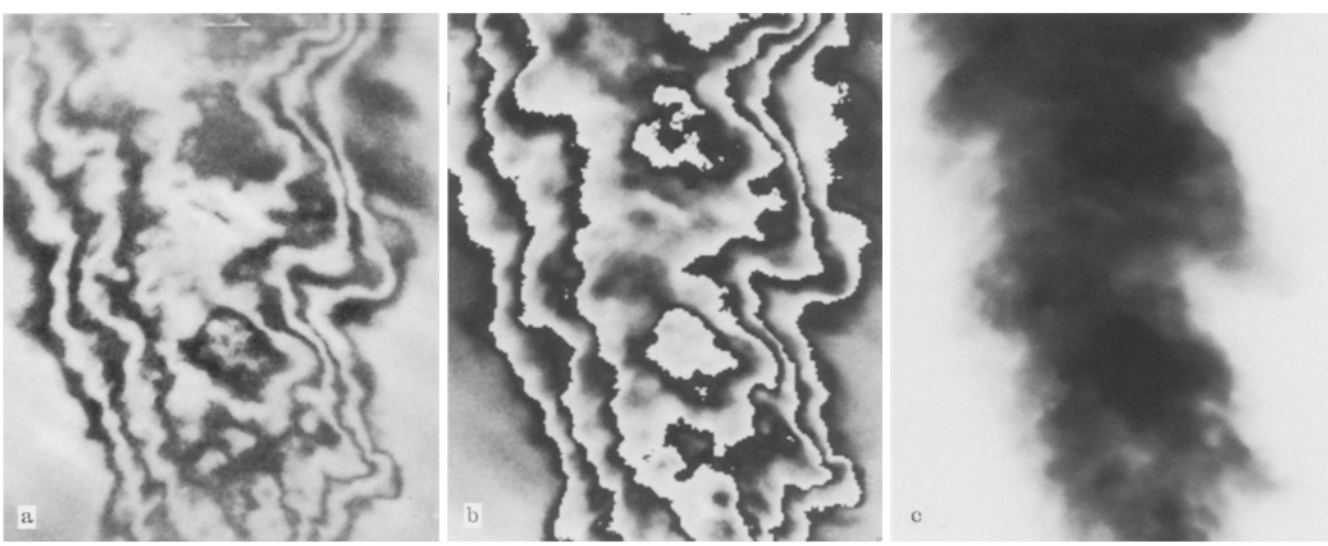

Fig. 6. **a** Pulsed laser interferogram of a round, helium jet ($Re = 2{,}500$), **b** phase display of **a**, **c** gray scale display of absolute phase of **a**

to flow visualization, such as using small temperature variations as tracers of vorticity in non-buoyant, two-dimensional flows and generation of stereoscopic images of three-dimensional scalar fields. Furthermore, the technique's accuracy, high spatial resolution, and facility of operation indicate potential for making spatially distributed scalar measurements.

Two-dimensional scalar fields may be evaluated directly from the phase, the experimental geometry and the constitutive relation of the refractive index to the scalar quantity of interest (e.g. the Gladstone-Dale relation). Cross-sections of three-dimensional fields may be evaluated using optical tomography, a technique for recovering the cross-sectional distributions of a field from its integral projections (Sweeney & Vest 1973; Vest 1979). A large number of contiguous cross-sections could be obtained from a set of projection images, allowing three dimensional maps of the field to be calculated. This would allow investigation of the structure of the scalar distribution. We have a system for doing this underdevelopment.

The technique also has the potential for determining the concentration in multicomponent mixtures through the use of simultaneous interferograms with several different wavelengths. Since each component has a different specific refractivity at each wavelength, the phase measurement in each wavelength is a linear combination of the concentration of each constituent, implying a unique relation between the phase measurements and the constituent concentrations. The technique's sensitivity and its ability to register efficiently the various interferograms could make such a system practical.

Acknowledgements

This work was sponsored by the Office of Army Research.

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Received November 10, 1986