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Sales Quota Plans: Mechanisms For Adaptive Learning

MURALI K. MANTRALA
Associate Professor of Marketing, 207 Bryan Hall, University of Florida, Gainesville, FL 32611, E-mail: mantrala@dale.cba.ufl.edu

KALYAN RAMAN
Marketing Department, University of Michigan, Flint, MI 48502

RAMARAO DESIRAJU*
Department of Business Administration, University of Delaware, Newark, DE 19716

Abstract

A vexing problem in managing sales forces occurs when the sales manager does not know the characteristics of a sales territory. In this paper we show how sales quota-bonus plans can be optimally used to learn about unknown territory characteristics over a multiperiod horizon. We demonstrate this process in two scenarios—one in which the sales manager and the rep are symmetrically informed about the territory characteristics, and the other in which the rep is better informed than the manager. Overall our analysis underscores the importance of Bayesian techniques in sales force management.

Key words: Sales force management, Sales quota-bonus plans, Sales response uncertainty, Bayesian learning-by-doing, Adaptive control

Sales force management operates in an uncertain environment. Not only are sales subject to random shocks but also sales managers as well as salespeople are often uncertain about parameters of individual territory sales-to-selling effort response functions. For example, a manager and a salesperson assigned to a new territory may be equally uncertain about this area's base sales level. Such uncertainty about sales response not only hampers the manager's ability to effectively control the salesperson's selling activities but also increases sales planning errors and related costs, *e.g.,* inventory stockout (holding) costs incurred by the regional sales depot when actual sales in a territory are more (less) than planned.

One way by which sales managers can reduce their uncertainty about sales response parameters is to follow an adaptive or "learning-by-doing" approach (*e.g.,* Arrow, 1962; Grossman et al 1977; Cyert and DeGroot, 1987), *i.e.,* systematically vary the sales representative's ('rep's') selling effort, observe the resultant sales and update priors with respect to uncertain response parameters over time. Unfortunately, this is easier said than done in field selling situations where it is usually very costly for a manager to directly monitor and control a rep's selling effort. At best, the manager can attempt to indirectly influence the rep's effort via a sales incentive mechanism such as a commission plan (see *e.g.,* Basu et al, 1985). However, companies usually fix a common commission plan for their sales forces for the length of an accounting year, limiting the usefulness of commission rates as a tool for learning about a particular rep's territory response parameters in, say, a few months. There is, however, a much more flexible mechanism that sales managers can use for adaptive control of specific salespeople's selling efforts. This is a sales quota-bonus system which allows a rep's own territory sales quota to be periodically adjusted by her manager even though the related commission parameters are fixed for the horizon (see *e.g.,* Mantrala, Sinha and Zoltners, Raju and Srinivasan (RS) 1996).

The use of sales quotas as a device for improving knowledge about uncertain sales response parameters over time has been hinted at in text books (*e.g.,* Churchill, Ford and Walker, 1996) but so far, no formal model of this process has been proposed in the literature. In this paper, our objective is to present such a model and methodology for optimally using quotas and discretionary bonuses to learn about an uncertain territory sales characteristic, considering the overall compensation plan and the rep's own effort decisions under this plan. Our work provides practical insights into optimal quota-based adaptive control of selling efforts as well as offers a basic modeling framework for investigating more general problems of multiperiod control of selling effort under uncertainty.

1. Problem setting and model development

The proposed model applies to the following selling situation: Consider a firm which employs a sales force to promote a mature product at a fixed price in a repetitive selling environment (*e.g.,* office supplies), with each salesperson assigned to an independent sales territory and supervised by a district sales manager (DSM). Suppose that the firm uses an individual sales quota-based common commission plan to motivate and control the reps' selling efforts over say, a 12-month accounting year, allowing DSMs to set quotas and discretionary bonuses for individual salespeople period-by-period during the year.

More precisely, as in previous principal-agent models, *e.g.,* Basu et al. (BLSS) (1985), Lal and Srinivasan (LS) (1993), we view the interaction between the DSM ('principal') and the individual rep ('agent') as a leader-follower game in which the DSM 'leads' by deciding and informing the rep of her quota and bonus at the beginning of each period. Based on this communication, and considering the total compensation plan, the rep puts in her selling effort (unobservable by the DSM). Then, at the end of the period, she is paid for her realized sales output in that period. The DSM is then free to revise the rep's quota and bonus for the next period. Lastly, let us also suppose that at the beginning of each period the DSM must 'order' the shipment of the period's expected sales quantity to be sent to and held by the regional sales depot and there is a service cost if realized sales deviate from expected sales.

Next, we make the following specific assumptions with respect to the salesperson's objective function, the sales response function, the compensation plan and the DSM's objective function.

1.1 Basic model assumptions

Considering an n-period horizon, in any period, $t = 1, \ldots, n$:

A1. The sales rep is risk-neutral with respect to her total income, $\tilde{I}_t$, where $\tilde{I}_t$ is dependent on her sales in period t, which in turn depend on the level of selling effort u_t (*e.g.,* hours) put forth by the rep in period t (see *e.g.,* Rao, 1990). Further, the rep's cost (money equivalent) of effort, $C(u_t)$, is an increasing convex function of effort. Specifically, like Rao (1990) and LS (1993), we assume for the sake of analytical and expositional convenience that,

$$C(u_t) = (c/2)u_t^2, \quad c > 0 \tag{1}$$

and that the rep's net utility in period t, $\tilde{U}_t$, is given by:

$$\tilde{U}_t = \tilde{I}_t - (c/2)u_t^2. \tag{2}$$

A2. The rep's dollar sales in period t, $\tilde{S}_t$, are stochastic, positive, and linearly increase with effort. Specifically, we assume:

$$\tilde{S}_t = \alpha + \beta u_t + \tilde{\epsilon}_t \tag{3}$$

where α is the base sales level in the territory; β denotes the salesperson's effectiveness; and $\{\tilde{\epsilon}_t\}$ is a sequence of inter-temporally independent and unobserved random disturbances. We recognize that it would be more realistic to specify sales increasing with effort at a decreasing rate. However, given that the rep has a convex cost of effort function, the assumption of a linear rather than concave sales-effort response function does not involve much loss in generality as shown by BLSS (1985), LS (1993) and RS (1996).

A3. The random disturbance $\tilde{\epsilon}_t$ in (3) has a normal distribution with zero mean and variance $1/\tau$, known to both the manager and the rep, where τ is the precision. Thus $\tilde{S}_t \sim N(\mu, 1/\tau)$, where $\mu_t = \alpha + \beta u_t$, is the expected sales in period t. We assume that the base sales level α is sufficiently large and the variance of $\tilde{\epsilon}_t$ sufficiently small so that the probability of sales being negative is negligible (see *e.g.,* LS 1993).

A4. The firm sets a sales quota-based compensation plan of the following form for the rep:

$$\tilde{I}_t = A_t + B_1\tilde{S}_t - B_2(\tilde{S}_t - q_t)^2 \tag{4}$$

where q_t denotes the quota, $B_1 > 0$, $B_2 > 0$, are fixed commission parameters with $B_1 \gg B_2$; and A_t denotes a fixed salary plus bonus which can be changed from period to period at the discretion of the DSM. Hereafter, we refer to A_t as simply the 'salary' parameter with the understanding that if there is any change in A_t from one period to the next, it is specifically the discretionary bonus portion of A_t that is being changed.

The assumed form of this compensation plan deserves further comment. First, note that the sales-based incentive pay increases with realized sales and is concave over the range $0 < S \leq S_m = B_1 / 2B_2 + q$, and then declines for values of $S > S_m$. However, considering the salesperson's cost associated with selling effort, it will never be optimal for her to choose to produce $S > S_m$. Effectively, therefore, the relevant sales range to which the incentive plan applies is $0 < S \leq S_m$ and over this range the plan approximates regressive or linear commission plans with *earnings caps* often observed in practice (e.g, Churchill, Ford and Walker, 1996). (About half the responding firms in one sales force compensation survey reported using salary plus commission plans with earnings caps, Barnes 1986.) Next, the quadratic term in (4) implies that a penalty is being imposed on the salesperson for positive as well as negative deviations of realized sales from the assigned quota. Similar to earnings caps, the concept of a penalty for overfulfillment of sales quotas is not uncommon among companies with limited supply capacity who want to fully utilize planned capacity but not oversell, because of the associated service problems and costs, e.g, White (1984), Gonik (1978), Mantrala and Raman (1990). More recently, Hauser, Simester and Wernerfelt (1996) have reported that they have found support for such marketing incentive systems in many industries and specifically propose a quadratic 'target-value incentive system' for coordinating internal supplier-customer dyads.

A5. The firm's profits $\tilde{\Pi}_t$ derived from the salesperson-territory in period t are defined as contribution dollar revenues less payout to the rep less the service cost incurred by the firm if realized sales deviate from the expected sales μ_t for that period. More specifically,

$$\tilde{\Pi}_t = g\tilde{S}_t - \tilde{I}_t - \omega(\tilde{S}_t - \mu_t)^2 \tag{5}$$

where g is a constant product gross margin per unit expressed as a fraction of the fixed sales price, and $\omega > 0$ is a constant service cost per unit squared deviation of realized sales from expected sales in period t. A quadratic loss function such as that in (5) is a common assumption in stochastic control theory models of the firm, *e.g.,* Cyert and DeGroot (1987) and consistent with the assumption of a quadratic penalty term in (4). Lastly, we assume $g > B_1$ and ω is of the same order of magnitude as B_2.

2. Static analysis

In this section, we derive the optimal single-period decision rules of the rep and the DSM and characterize their equilibrium behavior, assuming both have perfect information about the parameters of the response function. This analysis will serve as a benchmark for the multiperiod scenarios involving an uncertain sales parameter that we study in the next section. For expositional convenience, we drop the time subscript in the following analysis.

2.1. Salesperson's problem

Given the set of assumptions A1 to A4, upon being informed of her quota and salary for the period, the rep's objective is to choose the level of effort which maximizes her net expected utility, given by:

$$E[\tilde{U}] = E[\tilde{I}] - \left(\frac{c}{2}\right)u^2 = A + B_1(\alpha + \beta u) - B_2\left[(\alpha + \beta u - q)^2 + \frac{1}{\tau}\right] - \left(\frac{c}{2}\right)u^2. \quad (6)$$

The rep's choice of effort level which maximizes (6) is then:

$$u^* = \frac{K}{\beta}[B_1 - 2B_2(\alpha - q)] \quad (7)$$

where $K = 1 \big/ \{(c/\beta^2) + 2B_2\}$. Then, substituting (7) in (3), we obtain:

$$E[\tilde{S}(u^*)] = \mu^* = \alpha + K[B_1 - 2B_2(\alpha - q)]. \quad (8)$$

Given (7) and (8), the rep's maximized expected utility, *E[Ũ*]*, is:

$$E[\tilde{U}^*] = A + B_1\mu^* - B_2[(\mu^* - q)^2 + 1/\tau] - (c/2)u^{*2}. \quad (9)$$

2.2. DSM's problem

Following the standard agency-theoretic framework, with B_1 and B_2 taken to be fixed, the DSM's problem is to choose A and q to maximize the firm's expected profits subject to meeting the rep's minimum expected utility requirement (or individual rationality constraint), and taking into account her optimal effort decision rule, $u^*(q)$. Let M denote the known minimum expected utility for the rep to stay with the firm, *i.e.,* the DSM must meet the condition:

$$E[\tilde{U}^*] \geq M. \quad (10)$$

From (5), it follows that the firm's expected profits when the rep applies effort $u^*(q)$ are:

$$E[\tilde{\Pi}] = g\mu^* - \{A + B_1\mu^* - B_2[(\mu^* - q)^2 + 1/\tau]\} - \frac{\omega}{\tau}. \quad (11)$$

Now, notice that on the one hand, the firm's expected profits decline as A increases; on the other hand, we can see from (7) that the rep's optimal effort is unaffected by A. Therefore, the optimum value of A would be such that (10) holds as equality, *i.e.,*

$$A^* = M - B_1\mu^* + B_2[(\mu^* - q)^2 + 1/\tau] + (c/2)u^{*2}. \tag{12}$$

Next, substituting (12) into (11) and simplifying reduces the DSM's problem to:

$$\underset{q}{\text{Max}}\ E[\tilde{\Pi}] = g\mu^* - M - (c/2)u^{*2} - \frac{\omega}{\tau} \tag{13}$$

where u^* and μ^* are given by (7) and (8) respectively. The solution to (13) is then:

$$q^* = \left(\frac{g\beta^2}{2B_2Kc}\right) - \left(\frac{B_1}{2B_2}\right) + \alpha = \frac{g - B_1}{2B_2} + \frac{g\beta^2}{c} + \alpha. \tag{14}$$

Thus, we obtain an explicit analytical solution to the optimal sales quota problem. Previously, RS (1996) have inferred the influence of rep and territory characteristics on the optimal quota using numerical optimization techniques. Equation (14), however, shows exactly how the optimal quota depends on the rep and territory-specific characteristics (β, c, and α) as well as the commission plan and firm-specific parameters (B_1, B_2, and g). In particular, note that other things being equal, a higher base sales level and selling effectiveness should lead to a higher sales quota for the rep.

Upon substituting (14) into (7) we obtain the rep's optimal effort, u^{**}, in equilibrium:

$$u^{**} = \frac{g\beta}{c}. \tag{15}$$

That is, the rep's optimal effort is independent of the base sales level. This result highlights the role of optimal quotas in adjusting for heterogeneity in salespeople's territory base sales levels (see also RS 1996). Next, given that the DSM sets the rep's salary according to (12), and takes into account (15), the firm's expected profits (13) at equilibrium are:

$$E[\tilde{\Pi}^{**}] = g\alpha + \frac{g^2\beta^2}{2c} - M - \frac{\omega}{\tau}. \tag{16}$$

Note that the firm's equilibrium expected profits are higher in a rep-territory where α and β are higher and the variance associated with the sales output is lower.

3. Multiperiod learning using sales quotas

In this section, we provide two examples of how the sales quota-setting process can be utilized to gainfully learn about an uncertain parameter of a territory's sales response function over a multiperiod horizon, $t = 1,\ldots,n$.

3.1. Example 1

In this illustration, we retain all the basic model assumptions of Section 2 and consider the following scenario: Suppose that at the beginning of the horizon, a veteran sales rep of known selling effectiveness has just been assigned to a vacant sales territory where recently there have been some competitive shifts impacting α, the base level of sales of the firm's product in the territory. Thus, both the DSM and rep are uncertain about the new value of α but suppose that the DSM has some prior information about its probability distribution which he shares with the rep. Specifically, let this prior information be represented by a normal distribution with mean α_1 and known precision h_1. Next, based on this prior information, the DSM sets the rep's quota and salary for period 1, the rep puts in her optimal effort and is paid for her realized output at the end of the period. Then, at the beginning of the second period, the DSM can revise his previous information about α based on the observed first period sales output of the rep. This process can continue over the entire horizon. We assume that any such learning by the DSM is always shared with the rep, *i.e.,* the DSM and rep have and utilize symmetric information about α in every period. Given this setting, we now analyze the rep's and DSM's optimal decisions.

The rep's problem: The rep's effort decision problem from period $t = 1$ onwards can be described as follows: First, note that as (a) the rep always utilizes the same prior distribution of α as the DSM, and (b) conditional on this prior distribution, the DSM sets the rep's salary and quota in a manner that yields exactly her minimum expected utility M in each period, the rep's expected utility in any period $t + 1$ is independent of her realized sales in period t. Consequently, her selling effort in period t will be the same whether she is farsighted or myopic (see *e.g.,* Dearden and Lilien, 1990). Thus, following the development in section 2.1, the rep's optimal effort in any period $t = 1,\ldots,n$ is:

$$u_t^* = \frac{K}{\beta}[B_1 - 2B_2(\hat{\alpha}_t - q_t)] \tag{17}$$

where $\hat{\alpha}_t$ is the expected value of α and q_t is the quota set for the rep at the beginning of period t.

Given (17), the DSM's expected sales in period t are then:

$$\mu_t(u_t^*(\hat{\alpha}_t)) = \hat{\alpha}_t K_1 + K[B_1 + 2B_2 q_t] \tag{18}$$

where we have set $K_1 = (1 - 2B_2K)$ for expositional convenience.

The DSM's problem: Similar to the development in section 2.2, as the leader in this principal-agent game, the DSM takes the rep's optimal effort decision rule (17) into account, and ensures that she receives her minimum expected utility M in each period t by setting the per period salary, A_t, according to the rule:

$$A_t^* = M - B_1\mu_t(u_t^*(\hat{\alpha}_t)) + B_2\left[(\mu_t(u_t^*(\hat{\alpha}_t)) - q_t)^2 + \left(\frac{1}{h_t} + \frac{1}{\tau}\right)\right] + (c/2)u_t^*(\hat{\alpha}_t)^2 \tag{19}$$

with u_t^* and $\mu(u_t^*)$ given by (17) and (18) respectively. Given this policy with respect to A_t, the firm's expected profits in any period t are:

$$E[\tilde{\Pi}_t] = g\mu_t(u_t^*(\hat{\alpha}_t)) - M - (c/2)u_t^*(\hat{\alpha}_t)^2 - \omega\left(\frac{1}{h_t} + \frac{1}{\tau}\right). \tag{20}$$

The DSM's remaining decision is to determine the optimal quotas q_t, $t = 1,\ldots,n$. Before analyzing this dynamic problem, however, we consider how the DSM can learn about α.

Bayesian learning process: The DSM can form his estimate of $\hat{\alpha}_t$ for period t by combining his previous estimate $\hat{\alpha}_{t-1}$ with the information on realized sales he receives at the end of period $t - 1$. Given that the rep puts in effort according to equation (17), the random sales output, given by (3), in period $t - 1$ depends on the rep's quota as follows:

$$\tilde{S}_{t-1} = \alpha + K[B_1 - 2B_2(\hat{\alpha}_{t-1} - q_{t-1})] + \tilde{\epsilon}_{t-1}. \tag{21}$$

Equation (21) is in effect the estimating equation that the DSM can use to update his estimate of α. First, let us rewrite (21) as:

$$\tilde{y}_{t-1} = \tilde{S}_{t-1} - K[B_1 - 2B_2\hat{\alpha}_{t-1} + 2B_2q_{t-1}] = \alpha + \tilde{\epsilon}_{t-1}. \tag{22}$$

Thus, a new observation y_{t-1} is available to the DSM at the end of period $t - 1$ which can be combined with $\hat{\alpha}_{t-1}$ to obtain $\hat{\alpha}_t$. Then, for any given value of α, $\tilde{y}_{t-1}$ is normally distributed with mean α and precision τ. Suppose now that at the beginning of period $t - 1$, the prior distribution of α is $N(\hat{\alpha}_{t-1}, 1/h_{t-1})$. It follows by a standard application of Baye's theorem, that the posterior distribution of α (or its prior distribution at the beginning of period t) will again be normal with mean $\hat{\alpha}_t$ and precision h_t given by:

$$\hat{\alpha}_t = \frac{h_{t-1}\hat{\alpha}_{t-1} + y_{t-1}\tau}{h_{t-1} + \tau}, \tag{23}$$

$$h_t = h_{t-1} + \tau. \tag{24}$$

Therefore, recalling our assumption that the DSM's prior distribution of α at the beginning of period 1 is $N(\alpha_1, 1/h_1)$, equations (23) and (24) define how knowledge about α will be updated period-by-period for $t = 2,\ldots,n$.

Dynamic optimization problem of the DSM: Equations (23) and (24) imply an intertemporal linkage between the DSM's quota decision in period $t - 1$ which influences y_{t-1} and its decision in period t which depends on $\hat{\alpha}_t$. Therefore, the farsighted DSM's problem is to determine an optimal sequential quota decision rule which maximizes $E[\tilde{\Pi}_1 + \tilde{\Pi}_2 + \ldots + \tilde{\Pi}_n]$, *i.e.,* the expected total profit over some n-period horizon where $E[\tilde{\Pi}_t]$ is given by (20). The optimal sequential rule of this type can be found by applying the method of *backward induction*.

Suppose the first $n - 1$ periods of the process are over and the DSM must now set the final quota q_n. The DSM will choose q_n simply to maximize $E[\tilde{\Pi}_n]$. It follows from (14) that:

$$q_n^* = \frac{g - B_1}{2B_2} + \frac{g\beta^2}{c} + \hat{\alpha}_n \tag{25}$$

where $\hat{\alpha}_n$ is the estimate of the base sales level going into period n. Then, the firm's maximized expected profits $E[\tilde{\Pi}_n^*]$ are (see equation 16):

$$E[\tilde{\Pi}_n^*] = g\hat{\alpha}_n + \frac{g^2\beta^2}{2c} - M - \omega\left(\frac{1}{h_n} + \frac{1}{\tau}\right). \tag{26}$$

We now move back one period and consider the DSM's choice of q_{n-1} given that the first $n - 2$ periods are over. The DSM must then solve the problem:

$$\underset{q_{n-1}}{\text{Max}}\ E[\tilde{\Pi}_{n-1}] + E[\tilde{\Pi}_n^*] \tag{27}$$

where it is assumed that the optimal quota will be set in period n. However, at the beginning of period $n - 1$, the DSM does not know the realized value of $\tilde{S}_{n-1}$ which enters the determination of $\hat{\alpha}_n$. Therefore, in solving (27), the DSM must take an expectation over $E[\tilde{\Pi}_n^*]$ which depends on $\tilde{S}_{n-1}$, *i.e.,*

$$E[E[\tilde{\Pi}_n^*]] = E\left[\frac{g[h_{n-1}\hat{\alpha}_{n-1} + \{\tilde{S}_{n-1} - K(B_1 - 2B_2\hat{\alpha}_{n-1} + 2B_2 q_{n-1})\}\tau]}{h_{n-1} + \tau}\right] + \frac{g^2\beta^2}{2c} - M - \omega\left(\frac{1}{h_n} + \frac{1}{\tau}\right). \tag{28}$$

Recall $E[\tilde{S}_{n-1}]$ is given by (18) with $t = n - 1$. Substituting this expectation into (28) and simplifying, we obtain:

$$E[E[\tilde{\Pi}_n^*]] = g\hat{\alpha}_{n-1} + \frac{g^2\beta^2}{2c} - M - \omega\left(\frac{1}{h_n} + \frac{1}{\tau}\right) \tag{29}$$

at the beginning of period $n - 1$. That is, the expected maximized profits in period n do not depend on the value of q_{n-1} chosen by the DSM in period $n - 1$. Therefore, the DSM can choose q_{n-1} simply to maximize $E[\tilde{\Pi}_{n-1}]$ *i.e.,* follow the myopically optimal decision rule. Further, this argument can be carried through successively earlier stages by induction and in short, the optimal sequential decision procedure is the myopic procedure whereby the DSM sets his optimal quota for any period t according to (25). The DSM, of course, updates his estimate of α according to (23) before setting the quota. So effectively,

$$q_t^* = \frac{g - B_1}{2B_2} + \frac{g\beta^2}{c} + \left\{\frac{h_{t-1}\hat{\alpha}_{t-1} + y_{t-1}\tau}{h_{t-1} + \tau}\right\} \tag{30}$$

i.e., under learning, the optimal quota in period t is linearly related to realized sales in period $t - 1$.

What is the gain from learning? Suppose that the DSM has only a two-period horizon. It follows from the above analysis that with learning absent, *i.e.,* if the DSM's sequential quota decision rule is based simply on the prior information at the beginning of period 1, the two period cumulative expected profits are:

$$E[\tilde{\Pi}_1 + \tilde{\Pi}_2]_{LA} = 2\left\{g\hat{\alpha}_1 + \frac{g^2\beta^2}{2c} - M - \omega\left(\frac{1}{h_1} + \frac{1}{\tau}\right)\right\}.$$

On the other hand, with learning present, the two period total expected profits are:

$$E[\tilde{\Pi}_1 + \tilde{\Pi}_2]_{LP} = 2\left\{g\hat{\alpha}_1 + \frac{g^2\beta^2}{2c} - M - \frac{\omega}{\tau}\right\} - \omega\left(\frac{1}{h_1} + \frac{1}{h_1 + \tau}\right).$$

Therefore, the improvement in expected profits due to learning is:

$$\Delta\Pi_L = \omega\left(\frac{1}{h_1\left(1 + \frac{h_1}{\tau}\right)}\right). \tag{31}$$

Thus, learning about the base sales level becomes more valuable as (a) the service cost per unit forecasting error increases; (b) the variance of the environmental disturbance term decreases, *i.e.,* a high degree of volatility in the external environment interferes with gainful learning about α.

3.2. Example 2

In the previous example we assumed that the DSM and the rep have symmetric knowledge about α over the horizon. In this scenario, Equations (24) and (31) indicate that the precision and value of learning are unaffected by the salesperson-specific characteristics, *i.e.,* the parameters β and c, in the problem. Can these characteristics have any bearning on quota-based learning about the base sales level in a territory? To briefly provide some insights into this issue, we now consider a modification of Example 1. Specifically, suppose the DSM is uncertain about the base sales level in the sales territory, again perhaps due to some recent competitive shifts in the local environment, but believes the rep is aware of the new value of α and acts accordingly, *i.e.,* puts in effort according to equation (8). Given his service costs, the DSM would like to know the precise value of α but may not wish to directly ask the rep fearing the possibility that the latter may understate the true base sales level. The DSM's problem could be overcome by designing an appropriate delegation scheme such as a menu of contracts (see *e.g.,* Lal and Staelin 1986, Rao 1990, Desiraju and Moorthy 1996) to motivate as well as elicit the true value of α from the rep in this environment of asymmetric information. However, in general, this type of solution is complicated and expensive to design and implement, especially within a short accounting horizon. An alternative approach would be for the DSM to attempt to privately learn about α via the sales quota-setting and Bayesian updating process we have described in Example 1.

Thus, assuming the rep behaves according to equation (7), it follows from (21) that the random sales realization in any period $t - 1$ is now given by:

$$\tilde{S}_{t-1} = \alpha K_1 + K[B_1 + 2B_2 q_{t-1}] + \tilde{\epsilon}_{t-1}. \tag{32}$$

That is, the estimating equation that the DSM can use to update his estimate of α would be:

$$\tilde{y}_{t-1} = \frac{\tilde{S}_{t-1} - K[B_1 + 2B_2 q_{t-1}]}{K_1} = \alpha + \frac{\tilde{\epsilon}_{t-1}}{K_1}. \tag{33}$$

Then continuing with the assumption that the DSM's prior distribution of α at the beginning of period $t - 1$ is normal with mean $\hat{\alpha}_{t-1}$ and precision h_{t-1}, it follows that the Bayesian updating rules are now:

$$\hat{\alpha}_t = \frac{h_{t-1}\hat{\alpha}_{t-1} + y_{t-1}\tau K_1^2}{h_{t-1} + \tau K_1^2}, \tag{34}$$

$$h_t = h_{t-1} + \tau K_1^2. \tag{35}$$

Notice that the increment in the precision of the DSM's prior in each period is now τK_1^2, where $K_1 = 1/\left(1 + 2B_2\beta^2 / c\right)$.

Next, assuming the rep's minimum expected utility is met via her base salary and discretionary bonus adjustments, the firm's expected profits in any period t are as given in (20). Under these assumptions, it can again be shown that the dynamically optimal solution is the period-by-period quota-setting rule, *e.g.,* (25) with Bayesian updating of information about α according to (34). However, now, the gain from learning about α depends on τK_1^2.

Thus compared with the situation in Example 1, we see that learning by the DSM is more effective, *i.e.,* faster and more precise when the rep is informed about α. As regards its value, considering total expected profits over a two-period horizon, with and without learning, the gain in total expected profits with learning is now given by:

$$\Delta = \omega\left(\frac{\tau K_1^2}{h_1+\tau K_1^2}\right) \tag{36}$$

It follows that: (i) $\partial\Delta / \partial c > 0$, and (ii) $\partial\Delta / \partial\beta < 0$. Intuitively, as c increases, the rep's disutility for effort increases, and her effort decreases. Now, sales are a result of both the rep's effort and α, *e.g.,* (32). Therefore, when c is large, observed sales are more heavily impacted by the magnitude of α than the effort of the rep, and thereby are more informative about α. Similar arguments explain the sign of the partial derivative with respect to β.

4. Conclusion

In this paper we have addressed the longstanding problem of setting a sales quota for a rep in a common selling situation and derived explicit analytical insights into how the quota should be related to territory, salesperson, incentive plan and firm-specific characteristics. Further, by applying a Bayesian adaptive control methodology, we have shown how quotas can be systematically utilized not only to motivate salespeople but also to generate valuable information about an uncertain territory sales characteristic. We believe this methodology offers not only conceptual benefits to researchers but also is consistent with the way things are done in practice and therefore, readily implementable.

Future research can address several outstanding issues. First, analysis of the case when there is uncertainty about the rep's ability and/or other salesperson characteristics would be valuable. Our preliminary analysis of these problems indicate that optimal quotas will not be intertemporally independent as in the examples provided in this paper. That is, there is scope for experimentation with quotas to expedite learning (*e.g.,* Grossman et al 1977). Second, we briefly examined the case in which the rep is better informed about the territory and how that improved the firm's gain from learning. Given this insight, the effect of such information asymmetry on the rep's response and optimal quotas over time needs to be addressed more carefully. Third, we focused on a repetitive selling environment with a stationary sales response function. Extending the research to environments with time-varying parameters would be useful. Lastly, it would be worthwhile to do a simulation study of the evolution of parameter estimates, optimal quotas and expected profits under different rep-territory conditions using Bayesian updating rules.

In conclusion, effective sales management requires precise knowledge about sales response parameters. Bayesian adaptive control methods provide a natural way of obtaining precise information. However, implementing these control methods needs a careful understanding of the issues involved in controlling sales forces. Our analysis above has highlighted some of the main issues involved and we hope it will spark more research on this topic.

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