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Notes on Sensitivity and Stability of the Classifications of Returns to Scale in Data Envelopment Analysis: A Comment

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Seiford and Zhu (1999) address the issue of sensitivity of returns to scale (RTS) classifications in data envelopment analysis (DEA). As noted in Jahanshahloo, Lotfi and Zohrehbandian (2004), a number of Theorems (e.g., Theorems 11, 12, 16 and 17) in Seiford and Zhu (1999) may not be true for some decision making units (DMUs). They proposed a remedy for these Theorems. We point out that the issue can be addressed directly by the findings in Seiford and Zhu (1999).

Note that such an issue is caused by DMUs located outside the set T_0 defined on p. 59 of Seiford and Zhu (1999). Based upon T_0, two models, namely models (3) and (9) (Seiford and Zhu, 1999, p. 59 and 64), are established to test for whether a specific DMU_0 is located outside the set T_0.

As pointed out in Seiford and Zhu (1999, p. 61), if model (3) is infeasible, then DMU_0 does not belong to T_0 and the RTS classification should be dealt with Theorem 4 (Seiford and Zhu, 1999, p. 61). If model (9) is infeasible, then DMU_0 does not belong to T_0 and the RTS classification should be dealt with Theorem 10 (Seiford and Zhu, 1999, p. 65). Seiford and Zhu (1999) also indicate that Theorems 4 and 10 are true under the general situation discussed in section 4.

As a result, the issue pointed out in Jahanshahloo, Lotfi and Zohrehbandian (2004) can be addressed directly by the these results in Seiford and Zhu (1999). We should add "model (3) is feasible" as a condition for Theorems 11 and 12 and "model (9) is feasible" for Theorems 16 and 17 in Seiford and Zhu (1999). If model (3) (model (9)) is infeasible, we should use Theorem 4 (Theorem 9) to study the RTS stability. Furthermore, the last sentence of the discussion after the proof of Theorem 12 should read (p. 67): *Therefore if* (3) *is infeasible for* DMU_0, *the RTS stability region is* $\boldsymbol{R}^{IRS} = \{\alpha : 1 < \alpha < \vartheta^*\}$, *where* ϑ^* *is the optimal value to* (2). The last sentence of the discussion after Theorem 17 should read (p. 69): *Therefore if* (3) *is infeasible for* DMU_0, *the RTS stability region is* $\boldsymbol{R}^{DRS} = \{\xi : \theta^* < \xi \leq 1\}$, *where* θ^* *is the optimal value to* (1) *when evaluating* DMU_0.

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Finally, consider the example provided in Jahanshahloo, Lotfi and Zohrehbandian (2004). DMU_2's RTS stability should be analyzed by Theorem 4, because model (3) is infeasible when DMU_2 is under consideration. DMU_5's RTS stability should be analyzed by Theorem 9, because model (9) is infeasible when DMU_2 is under consideration.

References

Jahanshahloo, G. R., F. H. Lotfi and M. Zohrehbandian. (2004). "Notes On Sensitivity and Stability of the Classifications of Returns to Scale in Data Envelopment Analysis." *Journal of Productivity Analysis* (this issue).

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