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Strategic Pricing when Electricity is Storable*

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Abstract
In this paper, we develop a simplified oligopoly model where hydro generators engage in dynamic *Bertrand* competition. Each player uses a Markov strategy based on the state of water reservoirs at the beginning of each period. The replenishing of water reservoirs, which affects generators' productive capacity, is governed by a stochastic process. Also, a price cap, i.e. a maximum bid allowed, is imposed on the market. We develop valuable insights for regulatory policy in predominantly hydro based electricity markets, including the effects of price caps, the efficiency of dispatch under strategic behavior and the likelihood of collusion.

1. Introduction

Competitive strategies in deregulated electricity markets have become a very active area of research. However, most of the published literature (see, for example, Green and Newbery 1992; Von der Fehr and Harbord 1993; Borenstein and Bushnell 1999; Rudkevich et al. 1998; Borenstein et al. 1999; and Green 1999) examines strategic behavior in a static setting. Bushnell (1998) and Scott and Read (1997) are, to our

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knowledge, among the few works that analyze dynamic strategic behavior in power markets. Both papers use output as their decision variable, and their analyzes offer limited policy guidance or insights into the specific elements that constrain equilibrium prices.

Despite the scant attention, analyzing dynamic pricing behavior is important to understanding markets where hydroelectric (i.e., hydro) power is a significant or dominant source of production. This is essentially the market situation in New Zealand and Norway, as well as several South American countries including Colombia, Argentina, and Chile.[1] In markets dominated by hydro power, generators are capable of shifting productive capacity from one period to another based on whether they retain or release the water needed to produce electric power.

In this paper, we analyze the price-formation process and its policy implications in an infinite-horizon duopoly model where two hydro generators engage in dynamic *Bertrand* competition. Each player uses a Markov strategy based on the state of water reservoirs at the beginning of each period. The replenishing of water reservoirs, which affect generators' productive capacity, is governed by a stochastic process. In addition, a price cap (i.e., a maximum allowable bid) is imposed on the market. A number of valuable insights for regulatory policy are developed from this model.

We begin with price caps, which seem on the surface, well understood. These caps function as a binding price ceiling during conditions where electricity suppliers otherwise would set prices in excess of the specified cap. One might easily believe that these are the only times where price caps constrain prices, since the difficulty of storing electricity inhibits the transfer of electricity supplies from one period to another, and intertemporal substitution among retail electricity consumers appears quite limited.[2]

Despite these preconceptions regarding the effectiveness of price caps, we show that price caps potentially affect the entire equilibrium pricing distribution in electricity markets where hydro power is a major supply source. Hydro generators face an opportunity cost of producing power. The cost of increasing current water usage, thereby expanding current hydro generation, is that less water is available for future electricity production. Hydro generators thus face a dynamic programming problem where their current pricing decision is the control variable influencing future water levels and profits. In this setting, our analysis demonstrates that the imposition or tightening of a price cap affects the opportunity cost of producing electric power in the present period since it constrains future electricity prices. Thus, decreases in a price cap shift the entire pricing distribution downward, including prices in periods where the cap is not ''apparently'' binding. Our results also show that prices in ''competitive'' states (i.e., the states where both producers have available water reserves to produce electricity) decline when the discount factor decreases or the probability of water replenishment increases.

We extend our model in a few notable ways. First, we examine whether our conclusions

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1 See ''Profiles of Power Sector Reform in Latin America'', IADB (1999).

2 Retail customers typically purchase power under fixed-price contracts and do not have metering equipment that reports their electricity usage to electricity providers on a ''real-time'' or frequent basis.

change if hydro generators face competition from thermal (i.e., coal, gas, nuclear) electricity generators. Once again, the results show that the equilibrium price distribution over most states of nature is influenced by the imposed price cap. An exception occurs when the marginal cost of thermal supply is high relative to the opportunity cost of supplying hydro power. In that situation, the price cap only influences equilibrium pricing ‘‘directly’’. That is, it influences prices only during states where one (or both) of the hydro producers faces a depleted water supply and the price cap directly constrains the equilibrium price level. When both hydro producers have available water reserves and thermal suppliers face relatively high marginal costs, price undercutting by the hydro producers eliminates any thermal electricity production. The result of this strategic behavior is that the reliability of the electricity system is compromised; the undercutting behavior leads to a higher probability of system outage (due to inadequate electricity supply as a result of depleted water reservoirs) than if some thermal production had occurred. In this situation, raising the price cap may lead hydro producers to hold onto more of their water reserves when their reservoirs are full. It also may result in equilibrium prices in this state that are below, but proportional to, the price cap.

As a second extension, we explore pricing behavior when the hydro producers face differing circumstances with respect to water replenishment. Again, prices in the ‘‘competitive’’ state are less than, but proportional to, the price cap. In equilibrium, the firm with the lower opportunity cost of providing power supplies the market when both producers have available water reserves. That firm sets its price equal to the higher opportunity cost, where the opportunity cost derives from the value obtained when a firm refrains from using its water in the current period. This withholding behavior increases the likelihood of reaching a ‘‘monopoly’’ state in the future. A monopoly state arises when one producer has a full reservoir and its rival has an empty reservoir.

Interestingly, the hydro producer with the lower probability of reservoir replenishment often has the lower opportunity cost of selling electric power. This implies that the producer with the lower probability of replenishment empties its reservoir first when both firms have full reservoirs. This increases the likelihood of reaching a future state where both producers have empty reservoirs, which imposes a reliability risk on the electric system. Hence, the strategic behavior of hydro producers leads to an outcome opposite to that desired by a benevolent social planner, who would release first the reservoir of the firm with the higher probability of water inflow.

As a third extension, we examine the implications of increased reservoir size. With a larger reservoir, our analysis shows that relatively high prices are observed during states where each firm’s reservoir is partially filled. By holding onto water in these states and allowing a rival to drain its reservoir, a given hydro producer may find itself in position to exercise market power over a longer time period. By contrast, during the state where both reservoirs are full, the likelihood is reduced of reaching a future state where a rival’s reservoir is drained. Consequently, prices are more constrained. In all of these states, however, prices are still proportional to the imposed price cap.

Lastly, our model examines how the likelihood of collusion is affected by the price cap. In order to ensure rational behavior during the ‘‘punishment’’ phase, we do not assume that play reverts to a repeated static Bertrand–Nash equilibrium. Instead, it is assumed that play reverts to the Markov perfect Bertrand–Nash equilibrium. This assumption is made because the hydro generators face a dynamic optimization problem, which includes a positive probability that each generator will find itself in a monopoly position due to limited water availability facing its rival.

Under this punishment assumption, we find that the magnitude of the price cap does not affect whether collusion is sustainable. Given that the collusive price in the competitive state exceeds the price obtained under Bertrand–Nash dynamic competition, the sustainability of collusion depends solely on the discount factor and the stochastic nature of water replenishment. Decreases in the discount factor make collusion less likely, and they also lower the Bertrand–Nash equilibrium price in the competitive state. A decline in the probability of water replenishment lowers the probability of collusion; however, it raises the Bertrand–Nash equilibrium price.

This paper is organized as follows. Section 2 introduces the model, solving for the Markov-perfect equilibrium under Bertrand pricing behavior. Extensions of the model consider the impact of asymmetric competition among the hydro generators, additional competition from thermal production sources, different demand allocation rules when competitors set identical prices, and increases in water reservoir size. Section 3 examines the likelihood of collusion. Section 4 offers concluding remarks.

2. The Model

In this section, we introduce a simple infinite-horizon model of strategic behavior in hydroelectric power markets. Two hydro generators, drawing upon reservoirs of equal size, use water to produce electric power. Water reservoir levels are observed perfectly at the beginning of each period, after which both hydro generators simultaneously set prices for electric power. The market then clears, with consumers purchasing from the low-priced firm. When both firms set the same price, it is assumed initially that one of the two firms supplies the entire market demand, where the probability of serving the market equals one-half for each firm. This assumption produces asymmetries in the available reservoirs of the hydro firms, even under conditions where both firms experience the same realized water inflows each period. Although this assumption aids the exposition, we show later that our results are completely unaffected if it is assumed instead that demand is spread evenly across both firms when both firms set the same price.

Both firms have a maximum reservoir capacity and maximum output capability equal to one unit. Demand also equals one unit in each period, and is perfectly inelastic.[3] Consequently, each player can satisfy the entire market demand when its reservoir is full.

In strategically setting prices, each player faces incomplete information regarding the size of future water resources for itself and its rival. Water inflows, w, follow a simple binomial process. During each period $w = 1$ with probability q and $w = 0$ with probability

3 Our assumption regarding perfectly inelastic demand is consistent with the operating reality of many real-time wholesale electricity markets. With nearly all retail customers purchasing power under fixed rate contracts, the responsiveness of market demand to short-term changes in wholesale energy prices is extremely limited. For an updated review on this issue, see Train and Selting (2000).

$1 - q$. In other words, either it rains one unit (with probability q), or it does not. Water inflows occur at the end of the period. Initially, it is useful to assume that both players are identical. Hence, the realization of w is the same for both players. Finally, marginal costs are identical for both players and normalized to equal zero.

Ignoring the impact of the pricing decision on future profits, one has a static optimization problem that is easy to solve. When both reservoirs are empty, no production takes place. If one generator has a full reservoir and the other generator has an empty reservoir, the generator with water is effectively a monopolist. It will charge the maximum allowed price, c^*, which is either a reservation price or a price cap set by the regulator. If both reservoirs are full, we have the so-called ''Bertrand paradox'' where each firm charges its marginal cost, zero.

In the dynamic setting, our attention focuses particularly on the state where both reservoirs are full. We refer to this situation as the ''competitive'' state. When both reservoirs are full, a generator setting its price above its rival's price may find itself in a monopoly position in the near future. This occurs because the lower-priced rival will deplete its reservoir, and that reservoir may not refill prior to the next period. An ''opportunity cost'' exists in selling electric power in the current period, since future monopoly opportunities may be forsaken. Consequently, when a price cap binds behavior in the monopoly state, the opportunity cost of selling power in the competitive state is affected necessarily. Under Bertrand competition, this opportunity cost directly determines the equilibrium price in the competitive state. In this fashion, the price cap affects the magnitude of the equilibrium ''competitive'' price, even though that price lies beneath the price cap.

2.1. Solving the Dynamic Game

In our game, the two hydro players meet infinitely often. The different states of nature visited by these firms, as tracked by their reservoir levels, are a function of the pricing strategies followed and the random inflows of water. Pricing stategies and payoffs are ''Markovian,'' in that they depend only on current reservoir levels, and not the history of such levels.

When determining the optimal pricing strategy in each state, we need merely to assess how each firm's pricing decision affects its payoff in the current period, and the probability of reaching a given reservoir state in the next period. The firm's current pricing decision affects the probability of reaching a particular state in the next period because it determines whether a firm depletes its reservoir in the current period in order to produce electric power. Since the game is Markovian and the horizon is infinite, the payoff to each player from attaining a given reservoir state can be expressed succinctly by use of a ''value function.'' The value function represents the present-value payoff to the player for the remainder of the game once a particular state is reached (assuming that optimal behavior occurs). Hence, if we know the current-period payoff from following a particular pricing strategy, and the strategy's impact on the probability of reaching a given state in the next period, we know the impact of the strategy on payoffs throughout the life of the game by virtue of the value function. This is all the information that we need to choose the optimal pricing strategy.

Let $V_{x,y}$ denote firm i's value function for the state (x, y), where $x \in \{0, 1\}$ denotes firm

i's reservoir level, and $y \in \{0, 1\}$ denotes its rival's reservoir level. This value function represents the present value of firm i's payoff under optimal behavior for the remainder of the game once the state (x, y) is reached. Given that the game is infinitely-lived and that firms use Markov strategies, this state-dependent value is independent of time or prior history. Since both players face identical marginal costs, common demand conditions, and the same water-replenishment probabilities, the value functions are symmetric (i.e., $V^a_{x_a, y_b} = V^b_{x_b, y_a} = V_{x,y}$ for firms a and b when $x_a = x_b = x$ and $y_a = y_b = y$).

Consider initially the value for firm i in the state $(0, 0)$. Neither firm has any water in its reservoir. No profit is earned in the current period because no water is available to convert into electricity. By the beginning of the next period, both generators will have replenished reservoirs (of one unit capacity) with probability q, implying that they will receive the value, $V_{1,1}$, for the remainder of the game. The reservoirs will remain empty at the beginning of next period with probability $1 - q$, implying that both generators receive $V_{0,0}$ for the remainder of the game. Letting $\beta \in (0, 1)$ represent the discount factor, firm i's value for the state $(0, 0)$ can be expressed as follows:

$$V_{0,0} = \beta\left[(1-q)V_{0,0} + qV_{1,1}\right]. \tag{2.1}$$

Next, consider the state $(0, 1)$. Firm i's own reservoir is empty while its rival's reservoir is full. The rival exploits its monopoly position, draining its reservoir to sell one unit of electric power at the maximum allowed price c^*. Both reservoirs fill to capacity by the beginning of the next period with probability q, or remain empty with probability $1 - q$. Consequently, firm i's value in this state can be expressed as follows:

$$V_{0,1} = \beta\left[(1-q)V_{0,0} + qV_{1,1}\right]. \tag{2.2}$$

Note that equations (2.1)–(2.2) are identical (i.e., $V_{0,0} = V_{0,1}$). When a firm's own reservoir is currently depleted, that firm's value is independent of its rival's reservoir level. Either its rival's reservoir is also empty; or, it is full, implying that the rival will drain its entire reservoir to exploit its monopoly position. In either case, the rival's reservoir is empty prior to any water replenishment that takes place at the end of the period. Consequently, both firms will have the same reservoir level in the next period. Either both reservoirs are full with probability q or empty with probability $1 - q$.

Next, consider the state $(1, 0)$. Firm i's own reservoir is full, while its rival's reservoir is empty. Acting as a monopolist, firm i drains its reservoir to sell one unit of output at the maximim allowed price c^*. That depletes its reservoir, causing both firms to have empty reservoirs in the next period unless replenishment occurs. Consequently, firm i's value in this state can be expressed as follows:

$$V_{1,0} = c^* + \beta\left[(1-q)V_{0,0} + qV_{1,1}\right]. \tag{2.3}$$

Note that, by subtracting equation (2.1), or (2.2), from equation (2.3), it follows that:

$$V_{1,0} = c^* + V_{0,0} = c^* + V_{0,1}. \tag{2.4}$$

In other words, the value to firm i in the ''monopoly'' state is c^* greater than the value in the ''depleted'' states. The immediate payoff is c^*, and the continuation payment depends on the random evolution of the reservoir levels.

Lastly, consider the ''competitive'' state $(1, 1)$. In contrast to prior states, both firms have full reservoirs and are capable of producing output in the current period. Each firm's response to its rival's pricing strategy determines whether or not it will sell power in the current period.

Consider firm i's optimal response to its rival's price choice $p_{1,1}$. Firm i can bid ε below this price, and capture the entire market. In that case, firm i earns $p_{1,1} - \varepsilon$ in the current period. Its rival will have a full reservoir for the next period; and, firm i's reservoir will replenish at period's end with probability q or remain empty with probability $1 - q$. Consequently, firm i's value from pursuing this strategy is as follows:

$$(p_{1,1} - \varepsilon) + \beta\left[(1 - q)V_{0,1} + qV_{1,1}\right].$$

For sufficiently small values of ε, this expression can be approximated as follows:

$$p_{1,1} + \beta\left[(1 - q)V_{0,1} + qV_{1,1}\right]. \tag{2.5}$$

Alternatively, firm i can bid ε above $p_{1,1}$. Under this strategy, firm i makes no sales in the current period, while its rival serves the entire market and depletes its reservoir. In the next period, firm i finds itself in the monopoly position with probability q, or back at the competitive state with probability $1 - q$. Hence, firm i's value from pursuing this strategy is as follows:

$$\beta\left[(1 - q)V_{1,0} + qV_{1,1}\right]. \tag{2.6}$$

Finally, firm i can match the bid $p_{1,1}$. Since both firms charge the same price, firm i will be chosen to produce electric power for the entire market with probability $1/2$. If firm i is chosen, then the value to firm i is represented by equation (2.5). If firm i's rival is chosen instead, then the value received by firm i is represented by equation (2.6).

The above discussion suggests an equilibrium pricing strategy. The equilibrium price $p^*_{1,1}$ should leave each player indifferent between trying to capture the market in the current period, or waiting to produce electric power in the future. If the value from producing and selling at the rival's price offer in the current period equals the value from withholding production in the current period (and possibly receiving a monopoly price in the next period), then there is no incentive to undercut the rival's price offer. Thus, the equilibrium price $p^*_{1,1}$ must equate the values in equations (2.5)–(2.6):

$$p^*_{1,1} + \beta\left[(1 - q)V_{0,1} + qV_{1,1}\right] = \beta\left[(1 - q)V_{1,0} + qV_{1,1}\right]. \tag{2.7}$$

Rearranging equation (2.7), we obtain:

$$p^*_{1,1} = \beta(1-q)(V_{1,0} - V_{0,1}).$$

Recognizing that $V_{1,0} - V_{0,1} = c^*$ from equation (2.4), we derive the following equation:

$$p^*_{1,1} = \beta(1-q)c^*. \tag{2.8}$$

This equation leads directly to the following proposition:

Proposition 1: *The (Markov-perfect) Bertrand–Nash equilibrium price in the competitive state equals* $\beta(1-q)c^*$. *The equilibrium price is increasing in the price cap* (c^*) *and the discount factor* (β), *and decreasing in the probability of water inflow* (q).

Intuitively, the ''opportunity cost'' of selling electric power in the current period in the competitive state depends on the additional value that firm i receives in future periods if it refrains from selling power currently. That additional value equals the probability of reaching the monopoly state in the next period, multiplied by the present value of next period's monopoly profits. The probability of reaching the monopoly state in the next period equals $1-q$, which represents the probability that one's rival will not have its reservoir replenished. The present value of monopoly profits equals the monopoly price, c^*, which is constrained by the price cap, multiplied by the discount factor, β. Thus, the additional value in future periods from refraining to sell electric power in the current period equals $\beta(1-q)c^*$. This expression determines the equilibrium price in the competitive state. When the regulator raises (or lowers) the price cap, c^*, the equilibrium price in the competitive state increases (or decreases) proportionately.

Finally, note that as $q \to 1$, the equilibrium price in the competitive state approaches zero. Thus, as $q \to 1$, the equilibrium price in the dynamic game approaches the equilibrium price from the one-period static game where both firms have full reservoirs. This makes sense in that reservoirs are full every period when $q = 1$, implying that the dynamic game can be treated as a series of one-shot static games.

2.2. A Different ''Tie-Breaking'' Procedure

Let us assume now that whenever both players bid the same price, the quantity of electric power demanded is split equally between the two players. That situation is relevant only to the competitive state where both firms have nonempty reservoirs. When both firms set the same price in that state, each will sell 1/2 unit of output. This, in turn, implies that the state $(1/2, 1/2)$ would be reached in the next period with probability $(1-q)$. In the $(1/2, 1/2)$ state, the dominant strategy for each firm is to set the price c^* and sell 1/2 unit of output. This results in an empty reservoir, which is then replenished with probability q. Accordingly, the value for the state $(1/2, 1/2)$ is as follows:

$$V_{\frac{1}{2},\frac{1}{2}} = \frac{c^*}{2} + \beta\left[(1-q)V_{0,0} + qV_{1,1}\right]. \tag{2.9}$$

Armed with this value, we now consider the equilibrium pricing strategy for the competitive state, $(1,1)$. First, there should be no gains from undercutting a rival that is selling at the equilibrium price $\tilde{p}_{1,1}$. This requires that the equilibrium price must satisfy the following condition:

$$\tilde{p}_{1,1} - \varepsilon + \beta\left[(1-q)V_{0,1} + qV_{1,1}\right] \leq \frac{\tilde{p}_{1,1}}{2} + \beta\left[(1-q)V_{\frac{1}{2},\frac{1}{2}} + qV_{1,1}\right]$$
$$\text{or} \tag{2.10}$$
$$\frac{\tilde{p}_{1,1}}{2} - \varepsilon \leq \beta\left[(1-q)(V_{\frac{1}{2},\frac{1}{2}} - V_{0,1})\right].$$

The expression to the left of the first inequality represents the value received from undercutting the rival's price, while the expression to the right represents the value received from matching that price. Recognizing that $(V_{\frac{1}{2},\frac{1}{2}} - V_{0,1}) = c^*/2$ (see equations (2.2)–(2.9)), the above equation simplifies as follows:

$$\tilde{p}_{1,1} \leq \beta(1-q)c^* + 2\varepsilon. \tag{2.11}$$

Note that this condition must hold for any $\varepsilon > 0$. Thus $\tilde{p}_{1,1} \leq \beta(1-q)c^*$. For $\tilde{p}_{1,1}$ to represent an equilibrium price, there also must be no gains from selling above that price. To meet this requirement, the following condition must be satisfied:

$$\beta\left[(1-q)V_{1,0} + qV_{1,1}\right] \leq \frac{\tilde{p}_{1,1}}{2} + \beta\left[(1-q)V_{\frac{1}{2},\frac{1}{2}} + qV_{1,1}\right]$$
$$\text{or} \tag{2.12}$$
$$\beta\left[(1-q)(V_{1,0} - V_{\frac{1}{2},\frac{1}{2}})\right] \leq \frac{\tilde{p}_{1,1}}{2}.$$

The expression to the left of the first inequality represents the value received from selling above the rival's price. The firm behaving in this fashion acts as a monopolist in the next period with probability $1-q$, or finds itself in the competitive state with probability q. The right-hand expression represents the value received from matching the rival's price. Given that $(V_{1,0} - V_{\frac{1}{2},\frac{1}{2}}) = c^*/2$ (see equations (2.3)–(2.9)), the above expression simplifies as follows:

$$\beta(1-q)c^* \leq \tilde{p}_{1,1}. \tag{2.13}$$

Together equations (2.11) and (2.13) cannot hold unless $\tilde{p}_{1,1} = \beta(1-q)c^* = p^*_{1,1}$. Thus, the equilibrium price in the competitive state is not sensititive to the assumption concerning how demand is allocated when both firms set the same price.

2.3. Adding Thermal Production

In this section, we examine whether the above equilibrium still arises when thermal (i.e., coal, gas, nuclear-powered) electricity production offers competition to hydro production in the market. This situation reflects supply conditions in the electricity markets in certain South American countries (e.g., Colombia, Argentina, Venezuela, Brazil, and Chile) and in Nordpool (Sweden and Norway).

We assume that thermal production has a deterministic capacity of one unit and incurs the marginal cost c_T. Without loss of generality, one can view thermal production as being offered by either a price-taking fringe or a single Bertrand-playing producer. To facilitate comparison with our prior setting, we assume that demand equals two units, so that excess capacity remains at one unit. Hence, when both hydro firms have full reservoirs, one firm can refrain from using its reservoir. If its rival's reservoir does not replenish, the ''refraining'' hydro firm earns c^* in the next period because only that firm and the (capacity-constrained) thermal suppliers can meet market demand in that period. When faced with this potential ''refraining'' strategy, the opportunity cost of selling electric power in the current period in the ''competitive'' state equals $p^*_{1,1} = \beta(1-q)c^*$, as demonstrated by our prior analysis. Of course, this opportunity cost is proportional to the regulator's imposed price cap.

However, both hydro producers must consider another strategy in the presence of thermal production. Both producers can sell one unit of output in the ''competitive'' state if they choose to undercut the thermal producers' marginal cost, c_T. Since, absent this possibility, the value of holding onto water equals $p^*_{1,1} = \beta(1-q)c^*$, undercutting becomes an equilibrium strategy whenever $p^*_{1,1} < c_T$. In equilibrium, both hydro producers set price equal to c_T (or $c_T - \varepsilon$), and serve the entire market demand of two units.

If $p^*_{1,1} \geq c_T$ then the hydro producers gain more by holding onto water rather than undercutting the thermal producers' marginal cost. Consequently, an equilibrium arises where the hydro firms set their price equal to $p^*_{1,1}$, and one unit of thermal production is supplied to the market. Once again, the market price in the ''competitive'' state is proportional to the price cap. Due to the ''opportunity cost'' of selling hydro power, the thermal producers supply power to the market even though they face higher marginal costs than the hydro producers.

Note that the regulator's setting of the price cap determines if $p^*_{1,1} \leq c_T$. Interestingly, when the price cap is set sufficiently low that this condition holds, the reliability of the electric system is compromised because hydro production replaces thermal production. As a consequence of having both hydro firms produce electricity with their water reserves, instead of having one firm holding onto its reserves, the likelihood increases of reaching a future state where water reserves are insufficient to allow electricity production to meet market demand. In essence, if the regulator chooses too low of a price cap, a situation can arise where traditional ''baseload'' (e.g., coal, nuclear) production sources are undercut in their pricing due to the strategic behavior of hydro competitors. This alters the traditional role of hydro firms, which often supplied reserve energy to electricity markets in the past regulatory environment.

2.4. Differing Inflow Patterns

Let us now return to our original setting and relax the symmetry assumption in inflow patterns. That is, we will now assume that the water inflow patterns differ for each player. The probability of water inflow equals q_a for player a and q_b for player b. For player a, the value for the states $(0,0)$, $(0,1)$, and $(1,0)$ can be expressed as follows:

$$V^a_{0,0} = \beta[(1-q_a)(1-q_b)V^a_{0,0} + (1-q_a)\cdot q_b V^a_{0,1} + (1-q_b)\cdot q_a V^a_{1,0} + q_a q_b V^a_{1,1}], \quad (2.14)$$

$$V^a_{0,1} = \beta[(1-q_a)(1-q_b)V^a_{0,0} + (1-q_a)\cdot q_b V^a_{0,1} + (1-q_b)\cdot q_a V^a_{1,0} + q_a q_b V^a_{1,1}], \quad (2.15)$$

$$\begin{aligned} V^a_{1,0} = c^* + \beta[&(1-q_a)(1-q_b)V^a_{0,0} + (1-q_a)\cdot q_b V^a_{0,1} \\ &+ (1-q_b)\cdot q_a V^a_{1,0} + q_a q_b V^a_{1,1}]. \end{aligned} \quad (2.16)$$

The value for these states are analogous for player b, except that the probabilities q_a and q_b are interchanged.

Let us now examine behavior in the competitive state, $(1,1)$. Define the price level, p^*_a, where player a is indifferent between selling or not selling its output in the current period. This price level satisfies the following condition:

$$p^*_a + \beta\left[(1-q_a)V^a_{0,1} + q_a V^a_{1,1}\right] = \beta\left[(1-q_b)V^a_{1,0} + q_b V^a_{1,1}\right].$$

In the above equation, the left side of the inequality represents the value to firm a from selling one unit of output at the ''indifference price,'' recognizing that firm a will be able to replenish its reservoir with probability q_a. If firm b sells one unit of output at this price, implying that firm a holds on to its full reservoir, firm a will become a monopolist in the next period with probability $1 - q_b$, or face the competitive state with probability q_b. Noting from equations (2.15) and (2.16) that $V^a_{1,0} - V^a_{0,1} = c^*$, the ''indifference price'' satisfies the following condition:

$$p^*_a = \beta(1-q_b)c^* + \beta(q_a - q_b)(V^a_{0,1} - V^a_{1,1}). \quad (2.17)$$

By analogous reasoning, player b's ''indifference price'' satisfies the following condition,

$$p^*_b + \beta\left[(1-q_b)V^b_{0,1} + q_b V^b_{1,1}\right] = \beta\left[(1-q_a)V^b_{1,0} + q_a V^b_{1,1}\right]$$

which simplifies to

$$p^*_b = \beta(1-q_a)c^* + \beta(q_b - q_a)(V^b_{0,1} - V^b_{1,1}). \quad (2.18)$$

As shown in Appendix 1, equations (2.17) and (2.18) further simplify as follows:

$$p_a^* = \frac{\beta(1-q_b)c^*(1-\beta q_a+\beta q_a^2)}{(1-\beta q_b+\beta q_b q_a)}, \tag{2.19}$$

$$p_b^* = \frac{\beta(1-q_a)c^*(1-\beta q_b+\beta q_b^2)}{(1-\beta q_a+\beta q_a q_b)}. \tag{2.20}$$

Hence,

$$\frac{p_a^*}{p_b^*} = \frac{(1-q_b)}{(1-q_a)} \cdot \frac{(1-\beta q_a+\beta q_a^2)}{(1-\beta q_b+\beta q_b^2)} \cdot \frac{(1-\beta q_a+\beta q_a q_b)}{(1-\beta q_b+\beta q_a q_b)}.$$

Based on the above analysis, it is straightforward to determine the Bertrand–Nash equilibrium for the competitive state. The opportunity cost of selling power in the current period is the foregone value from selling power during a later period as a monopolist. The ''indifference price'' essentially represents this opportunity cost. Consequently, the player with the lower ''indifference price'' gains less from waiting to sell power in the future. That firm is willing to undercut its rival's price as long as that price exceeds its own indifference price. Since no firm is willing to sell power in the current period at a price below its indifference price, a Bertrand–Nash equilibrium exists where both firms set price equal to the higher of the two indifference prices. The firm with the lower indifference price sells one unit of electric power at the higher indifference price, while its rival does not sell any power in the current period.[4]

This equilibrium is equivalent to the outcome of a static Bertrand duopoly game where the two firms have differing marginal costs. In that situation, an equilibrium exists where both firms set price equal to the marginal cost of the high-cost firm, but all output is sold by the low-cost firm. In this dynamic Bertrand situation, the two firms have different opportunity costs from selling power due to different ''continuation values'' based on differences in their water inflow probabilities. We summarize this discussion below:

Proposition 2: *Let q_a and q_b represent, respectively, the probability of water inflow for firms a and b. Define the ''indifference price'' of firms a and b, respectively, by equations (2.19) and (2.20). In the competitive state, a (Markov-perfect) Bertrand–Nash equilibrium arises where both firms set their price at the higher ''indifference price.'' The firm with the lower indifference price sells one unit of output, while the firm with the higher indifference price does not sell any output.*

We now explore equations (2.19) and (2.20) in further detail. One might naturally think that the firm with the higher probability of water inflow has the higher opportunity cost of selling power in the current period. That firm stands the greatest probability of becoming a monopolist in the next period if it refrains from selling power since it faces a rival with a

4 If $q_a = q_b = q$, then the value functions for each state are the same for each player (i.e., $V^a = V^b$). Consequently, both firms have the same indifference price, $p_a^* = p_b^* = \beta(1-q)c^*$ (see equations (2.17) and (2.18)), which also is the equilibrium price in the competitive state.

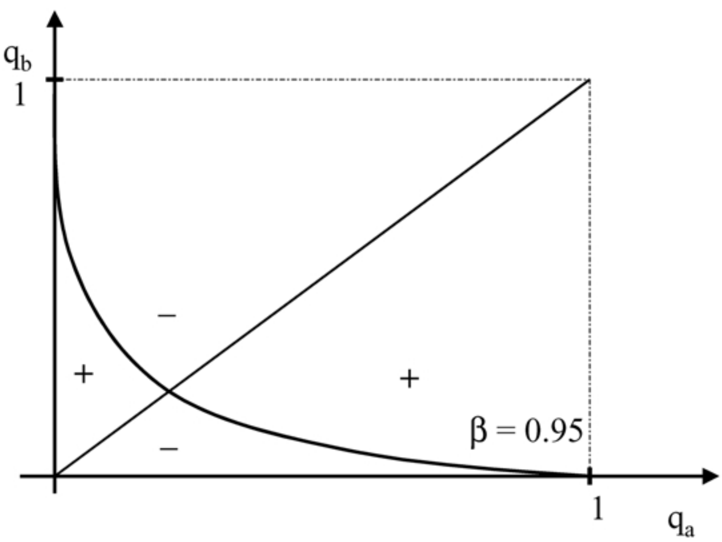

Figure 2.1. A comparison of opportunity costs.

lower probability of water inflow. However, according to equations (2.19) and (2.20), the firm with the higher probability of water inflow does not necessarily have the higher opportunity cost. In figures 2.1 and 2.2, a comparison of ''indifference prices'' p_a^* and p_b^* is presented under different assumed discount factors. A curved line denotes those

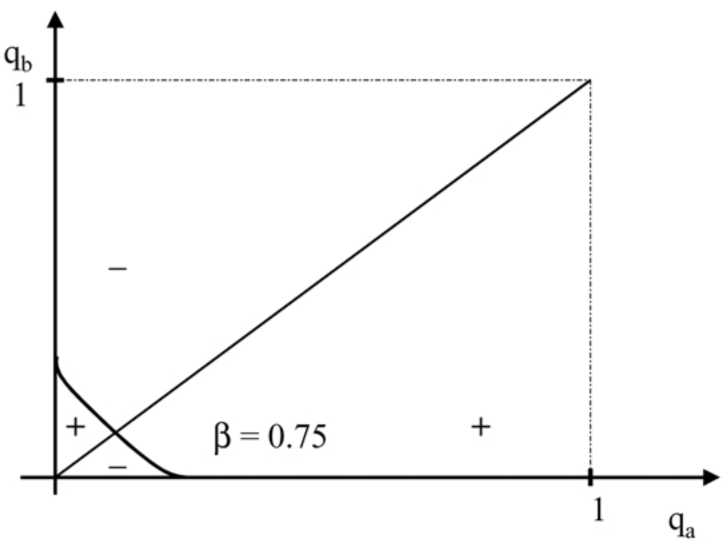

Figure 2.2. Comparison with lower discount factor.

combinations of water inflow probabilities (q_a, q_b) where the indifference prices of both firms are the same (i.e., $p_a^* = p_b^*$). In addition, as is apparent from equations (2.17) and (2.18), the indifference prices are the same whenever $q_a = q_b$ (as represented by the 45-degree line in both figures). Thus, the curved line and the 45-degree line divide the figures into four areas. We designate with a ''plus'' sign those combinations of water inflow probabilities (q_a, q_b) that yield a higher indifference price for firm a (i.e., $p_a^* > p_b^*$). By contrast, a ''minus'' sign denotes those combination of inflow probabilities that yield a higher indifference price for firm b (i.e., $p_a^* < p_b^*$).

Note that as the discount factor decreases, the curved line representing $p_a^* = p_b^*$ moves downwards, eventually leaving the non-negative quadrant. Thus, for sufficiently low discount factors, the 45-degree line becomes the only relevant boundary. Under these circumstances $p_a^* \leq (>) p_b^*$ if $q_b^* \geq (<) q_a^*$.

The intuition for the above results is as follows. With low discount factors (i.e., high discount rates), there is less value in holding onto water for the firm with the lower probability of replenishing its reservoir. Not only is that firm less likely to attain a monopoly state if it refrains from using its water (because it faces a rival with a higher probability of water inflow), future monopoly rents are discounted substantially. With a high discount factor (i.e., low discount rate), the situation is not as simple because future profits take on as much importance as current profits as the discount factor approaches one. In this context, consider first the case when both firms face relatively low probabilities of water inflow. On the one hand, there is little difference in the present value of a monopoly position attained in the next period or in subsequent periods. However, the firm with a lower probability of water replenishment faces the prospect of a longer interval with an empty reservoir if it releases its stored water. Due to this effect, the firm with a lower probability of water replenishment may have a higher ''indifference'' price. As both probabilities of inflow increase, the prospect of a longer sustained interval with an empty reservoir is no longer as relevant. In this situation, the ''monopoly'' effect becomes more important. Since the firm with the lower probability of inflow has the lower probability of earning monopoly rents in the future, it again has the lower ''indifference'' price.

The prospect of the firm with the lower probability of inflow undercutting the one with the higher probability of inflow, which appears to be the more common outcome based on the above analysis, poses an interesting public policy dilemma. Appendix 2 shows that, under conditions where the firm with the lower probability of water inflow spills its water to produce electricity in the competitive state, it takes fewer periods on average to reach a depleted-reservoir state (i.e., a $(0, 0)$ state) than if the firm with the higher probability of water inflow had depleted its reservoir instead. In this sense, the strategic behavior of hydro producers leads to an outcome opposite to that desired by a benevolent social planner who is focused on ensuring system reliability. The social planner would release first the reservoir of the firm with the highest probability of water inflow. We summarize this discussion below:

Remark: *Let q_a and q_b represent, respectively, the probability of water inflow for firms a and b, and equations (2.19) and (2.20) represent their associated ''indifference prices,'' p_a^* and p_b^*. Whenever the firm with the higher probability of water inflow has the higher* *indifference price, strategic behavior in a (Markov-perfect) Bertrand–Nash equilibrium leads to a greater likelihood of a generation shortfall (and system outages) than would arise if a central planner acted to ensure reliability.*

2.5. The Effects of Higher Reservoir Capacity

To gain some intuition regarding how increases in reservoir capacity affect equilibrium prices, we return to our original assumptions, except that reservoir capacity is increased from one unit to two units. Consequently, the states of nature are expanded to include the following: $(0,2)$, $(2,0)$, $(1,2)$, $(2,1)$, and $(2,2)$. In this situation, an equilibrium set of prices is as follows (see Appendix 3 for proof):

$$p^*_{1,2} = p^*_{2,1} = \beta\alpha c^*$$

$$p^*_{1,1} = \beta(1-q)c^* + \beta q p^*_{1,2}$$

$$p^*_{2,2} = \beta(1-q)p^*_{1,2}$$

$$p^*_{1,0} = p^*_{2,0} = c^*, \text{where:}$$

$$\alpha = \frac{(1-q)[1+\beta(1-q)]}{2-\beta q-\beta^2 q(1-q)} \in [0,1).$$

Increases in reservoir size may increase the opportunity cost of supplying power because they provide producers with opportunities for exercising substantial market power over a longer period of time. By refraining from selling output in the $(1,1)$ state, a producer either becomes a monopolist in the next period, or it attains the $(2,1)$ state in that period. If it reaches the $(2,1)$ state, the producer faces a particularly high opportunity cost of selling power, since it could act as a monopolist for two consecutive periods if its rival sells power and depletes its reservoir.

Note that the price in the $(2,2)$ state is less than the price in the $(1,1)$ state. Clearly, in the $(2,2)$ state, it is impossible for a producer to attain a monopoly position in the next period merely by refraining from the sale of power in the current period. Since the producer that refrains from selling power may become a monopolist in the next period in the $(1,1)$ state, the opportunity cost of supplying power is less in the $(2,2)$ state than the $(1,1)$ state. This leads to relatively lower price in that period. The value of holding power in the $(2,2)$ state is equivalent to the probability-weighted present value of attaining the $(2,1)$ state instead of the $(1,2)$ state in the next period. That difference in value can be expressed as $\beta(1-q)(V_{2,1}-V_{1,2}) = \beta(1-q)p^*_{1,2}$. The value of holding power in the $(1,1)$ state equals the present value of attaining the $(2,1)$ state instead of the $(1,2)$ state with probability q, plus the present value of reaching the $(1,0)$ state instead of the $(0,1)$ state with probability $(1-q)$. In total, this value equals $\beta(1-q)c^* + \beta q p^*_{1,2}$.

3. Collusion

In this section, we study the conditions under which collusive pricing arrangements can be enforced via ''trigger''-like strategies, in which play reverts to Bertrand pricing once a deviation from the collusive pricing arrangement is detected. However, the stochastic nature of water replenishment implies that a firm deviating from a collusive agreement (and its rival) can find itself potentially in a future monopoly position. Hence, it would be improper to describe the punishment phase as a series of static Bertrand–Nash equilibrium outcomes. Consequently, we assume that deviation from the agreement leads to the resumption of the dynamic (Markov-perfect) Bertrand–Nash equilibrium described previously.

We return to our original model set-up, where each firm's reservoir capacity is one, and both firms realize the same outcome in terms of water inflow. In our model, the collusive agreement governs pricing behavior in the ''competitive'' state, $(1, 1)$. The equilibrium price in the monopoly state remains at the price cap. Under collusive behavior, the players agree to always bid at $\tilde{p}_{1,1}$ when they both have full reservoirs. Further, the collusive price exceeds the price attained under Markov-perfect Bertrand Nash behavior (i.e., $\tilde{p}_{1,1} > p^*_{1,1}$), but it does not exceed the price cap.[5] Under the collusive ''trigger'' strategies, any undercutting of the collusive price would lead to a punishment phase where pricing behavior shifts to the dynamic (Markov-perfect) Bertrand–Nash equilibrium for the remainder of the game. Subject to these conditions, the following result holds:

Proposition 3: *A collusive agreement is sustainable for any price $\tilde{p}_{1,1}$, such that $c^* \geq \tilde{p}_{1,1} > p^*_{1,1}$, if and only if $\beta \geq \frac{1}{1+q}$. Thus, a decline in the discount factor or the probability of water inflow reduces the likelihood of collusion.*

Proof: Let one player deviate from the collusive agreement. This occurs when the player slightly undercuts the collusive price $\tilde{p}_{1,1}$ and sells its entire output in the $(1, 1)$ state. This deviation does not increase the player's profit if the following condition is satisfied,

$$\tilde{p}_{1,1} + \beta[(1-q)V_{0,1} + qV_{1,1}] \leq \tilde{V}_{1,1} \tag{3.1}$$

where '' $\sim$ '' denotes prices and payoffs under collusion. By deviating, the cheating firm receives approximately $\tilde{p}_{1,1}$ in the current period, but its expected future value depends on Bertrand–Nash equilibrium behavior. Given that both firms have the same realization of water inflows, the cheating firm will attain the Bertrand–Nash values, $V_{0,1}$ with probability $1-q$ and $V_{1,1}$ with probability q, in the next period. That is captured by the terms on the left-hand side of the above inequality. As represented on the right-hand side, the firm would receive the value, $\tilde{V}_{1,1}$, if it conforms with the collusive agreement.

For the states $(0, 0)$, $(0, 1)$, $(1, 0)$, the value functions under collusion satisfy relationships analogous to equations (2.1)–(2.3). Hence $\tilde{V}_{1,0} - \tilde{V}_{0,1} = c^*$. However, in

5 The price cap may, in fact, serve as a focal point for collusive behavior.

the $(1,1)$ state where reservoirs are full, the collusive value, $\tilde{V}_{1,1}$, satisfies the following condition:

$$\tilde{V}_{1,1} = \frac{\tilde{p}_{1,1}}{2} + \beta\left[\frac{1}{2}(1-q)\tilde{V}_{0,1} + \frac{1}{2}(1-q)\tilde{V}_{1,0} + q\tilde{V}_{1,1}\right]$$
$$= \frac{\tilde{p}_{1,1} + p^*_{1,1}}{2} + \beta[(1-q)\tilde{V}_{0,1} + q\tilde{V}_{1,1}]. \tag{3.2}$$

The right-hand side of the first equality represents the value derived from splitting the market in the current period. We again assume (without loss of generality) that each producer is chosen to supply the market with probability 1/2 in the event that both firms set the same price. Thus, if no water inflow occurs, a given firm will be a monopolist in the next period with probability 1/2, or hold an empty reservoir with probability 1/2. Otherwise, both firms have full reservoirs in the next period. The second equality is derived by making the substitutions $\tilde{V}_{1,0} - \tilde{V}_{0,1} = c^*$ and $p^*_{1,1} = \beta(1-q)c^*$ into the right-hand side of the first equality.

Substituting equation (3.2) into the inequality in (3.1), deviating from the collusive agreement will not increase profits if the following condition holds:

$$\tilde{p}_{1,1} + \beta[(1-q)V_{0,1} + qV_{1,1}] \leq \frac{\tilde{p}_{1,1} + p^*_{1,1}}{2} + \beta[(1-q)\tilde{V}_{0,1} + q\tilde{V}_{1,1}]$$

or

$$\frac{\tilde{p}_{1,1} - p^*_{1,1}}{2} \leq \beta[(1-q)(\tilde{V}_{0,1} - V_{0,1}) + q(\tilde{V}_{1,1} - V_{1,1})]. \tag{3.3}$$

In Appendix 4, we show that the term $\beta[(1-q)(\tilde{V}_{0,1} - V_{0,1}) + q(\tilde{V}_{1,1} - V_{1,1})]$ can be expressed as follows:

$$\beta[(1-q)(\tilde{V}_{0,1} - V_{0,1}) + q(\tilde{V}_{1,1} - V_{1,1})] = \left\{\frac{1}{|D|}[\beta^2(1-q)q + \beta q(1-\beta(1-q))]\right\} \cdot \frac{\tilde{p}_{1,1} - p^*_{1,1}}{2}$$

where,

$$|D| = 1 - \beta.$$

Inserting the above expression for $\beta[(1-q)(\tilde{V}_{0,1} - V_{0,1}) + q(\tilde{V}_{1,1} - V_{1,1})]$ into inequality (3.3), we derive the following result:

$$\frac{\tilde{p}_{1,1} - p^*_{1,1}}{2} \leq \left\{ \frac{1}{|D|} [\beta^2(1-q)q + \beta q(1 - \beta(1-q))] \right\} \cdot \frac{\tilde{p}_{1,1} - p^*_{1,1}}{2}$$

or

$$1 \leq \left\{ \frac{1}{|D|} [\beta^2(1-q)q + \beta q(1 - \beta(1-q))] \right\}.$$

Multiplying both sides of the above inequality by $|D|$, and recognizing that $|D| = 1 - \beta > 0$, the following inequality arises:

$$(1-\beta) \leq [\beta^2(1-q)q + \beta q(1 - \beta(1-q))]$$

or

$$\beta \geq \frac{1}{1+q}.$$

If the above inequality is satisfied, then it is unprofitable to defect from any collusive price, $\tilde{p}_{1,1} > p^*_{1,1}$. If this condition is not satisfied, then collusion cannot be sustained. ■

Since it is absent from the expression in Proposition 3, the price cap itself has no impact on whether collusion is sustainable. However, the cap does constrain the range of collusive prices, and it may serve as a focal point for reaching a collusive agreement.

According to Proposition 3, the range of discount rates that can sustain collusion becomes smaller as the probability of water inflow declines. In that sense, collusion becomes less likely as q declines in magnitude. When there is a lower probability of water inflow, there is greater incentive to defect from the collusive agreement, since there is a reduced probability of reaching the state $(1, 1)$, within a given period of time. A defector from the collusive agreement undercuts its rival's price during this state in order to attain a greater expected share of the market. Once defection arises, punishment can occur only when both firms again have full reservoirs. Hence, a defector from the collusive agreement forsakes less profits in the future as the probability declines of reaching the state $(1, 1)$. As this probability falls, punishment occurs less frequently, and defection becomes more attractive.

Hence, a decline in the probability of water inflow reduces the likelihood of collusion, but it increases the equilibrium price under (Markov-perfect) Bertrand competition (see Proposition 1). Decreases in the discount factor reduce the likelihood of collusion and the equilibrium price under Bertrand competition.

4. Conclusions

The presence of dynamic strategic behavior in electric power markets has attracted limited attention. However, it may be a key issue in the many markets affected by hydroelectric power production, since hydro producers can refrain from selling power in the present in order to add productive capacity in the future. This paper attempts to identify and evaluate some of the key economic factors governing pricing decisions in markets with hydro producers, as it examines equilibrium behavior under dynamic Bertrand competition and analyzes the likelihood of collusion.

Our findings show that price caps play a significant role in disciplining prices in electric power markets with significant hydro production, even under conditions where the equilibrium price is below the price cap. Price caps affect current pricing decisions, since they potentially constrain the prices attained in the future when a hydro producer sacrifices current production in favor of increased future production. In this fashion, price caps affect the opportunity cost of selling power in the present. As a result, imposing (or reducing) a price cap may depress the entire price distribution in electric power markets. However, when hydro producers compete with thermal producers, lowering the price cap may compromise system reliability by inducing hydro producers to undercut the prices of thermal competitors. This strategic behavior increases the probability that inadequate generation capacity arises in the future as a result of depleted water reservoirs.

Under ''competitive'' conditions where hydro producers have full reservoirs and ample capability for producing power, the probability of future water inflows is a key element in price determination. When rivals face a low probability of replenishing their reservoirs, a firm with a full water reservoir has a high probability of reaching a position of substantial market power in the future if it presently refrains from supplying electric power and allows its rivals to consume their reservoirs. Hydro producers thus face a higher opportunity cost of selling power, and seek higher current prices, if water replenishment is less probable. However, a reduced likelihood of water inflows may impede collusive behavior because punishment becomes more difficult.

If hydro producers face differing replenishment rates, the producer with the higher probability of water replenishment often faces a higher opportunity cost of supplying electric power when hydro producers have full reservoirs. This producer faces a higher probability than its rivals of attaining a position of significant market power in the future if it holds onto its water reserves. As a result, price undercutting frequently occurs by a producer with a relatively low probability of water replenishment, leading that producer to drain its water reserves. Strategic behavior under these circumstances compromises system reliability because the likelihood of reaching a state of inadequate generation capacity would be reduced if the producer with the higher probability of water replenishment released its water instead.

Finally, our model, which focused on a duopoly situation involving hydro producers and a particular type of stochastic process involving water replenishment, could be extended to a larger number of hydro players and more general stochastic behavior. With multiple hydro players, the opportunity cost of holding onto water when one's reservoir is full, as opposed to depleting one's reservoir and supplying electric power in the current period, is still governed by the same considerations as in our simpler model. This opportunity cost still depends on the probability that a given firm can reach a future state where it has significant market power due to the depletion of its rivals' water reserves, along with the payoff associated with that state. Since a price cap affects the payoff in the state where a firm has significant market power due to its rivals' depleted reservoirs, it also affects prices in states where multiple firms have full reservoirs and must decide how to price their power. Thus, an extension of our model to include more players is likely to change our results quantitatively but not qualitatively.

5. Appendix

5.1. Appendix 1: Determining ''Indifference Prices''

Firm *a* could refrain from selling output in the competitive state, implying that firm *b* supplies the market. Under this behavior, firm *a*'s value in the competitive state is expressed as follows:

$$V^a_{1,1} = \beta\left[(1-q_b)V^a_{1,0} + q_b V^a_{1,1}\right].$$

Given that $V^a_{1,0} - V^a_{0,1} = c^*$, we obtain:

$$V^a_{1,1} = \beta(1-q_b)c^* + \beta[(1-q_b)V^a_{0,1} + q_b V^a_{1,1}]. \tag{5.1}$$

Moreover,

$$\begin{aligned} V^a_{0,1} &= \beta[(1-q_a)V^a_{0,1} + (1-q_b)\cdot q_a(c^* + V^a_{0,1}) + q_a q_b V^a_{1,1}], \\ V^a_{0,1} &= \beta(1-q_b)q_a c^* + \beta[(1-q_a q_b)V^a_{0,1} + q_a q_b V^a_{1,1}]. \end{aligned} \tag{5.2}$$

Subtracting equation (5.1) from equation (5.2), we obtain:

$$\begin{aligned} V^a_{0,1} - V^a_{1,1} &= \beta(1-q_b)q_a c^* - \beta(1-q_b)c^* + \beta q_b(1-q_a)[V^a_{0,1} - V^a_{1,1}], \\ V^a_{0,1} - V^a_{1,1} &= \frac{-\beta(1-q_b)(1-q_a)c^*}{1-\beta q_b(1-q_a)}. \end{aligned} \tag{5.3}$$

From equation (2.17) in the text, firm *a*'s ''indifference price'' must satisfy the following condition:

$$p^*_a = \beta(1-q_b)c^* + \beta(q_a - q_b)(V^a_{0,1} - V^a_{1,1}).$$

Substituting equation (5.3) into the above equation, the ''indifference'' price is expressed as follows:

$$p^*_a = \beta(1-q_b)c^* - \beta(q_a - q_b)\frac{\beta(1-q_a)(1-q_b)c^*}{1-\beta q_b(1-q_a)}.$$

With further manipulation, the above equation simplifies to the following:

$$p^*_a = \frac{\beta(1-q_b)c^*(1-\beta q_a + \beta q_a^2)}{(1-\beta q_b + \beta q_b q_a)}.$$

Using analogous reasoning, firm b's ''indifference price'' is as follows:

$$p_b^* = \frac{\beta(1-q_a)c^*(1-\beta q_b+\beta q_b^2)}{(1-\beta q_a+\beta q_a q_b)}.$$

The ratio of the two prices can be expressed as:

$$\frac{p_a^*}{p_b^*} = \frac{(1-q_b)}{(1-q_a)} \cdot \frac{(1-\beta q_a+\beta q_a q_b)}{(1-\beta q_b+\beta q_b q_a)} \cdot \frac{(1-\beta q_a+\beta q_a^2)}{(1-\beta q_b+\beta q_b^2)}.$$

5.2. Appendix 2: Examining Outage Risk

Assume that $q_a > q_b$. Let $N_{i,j}$ denote the expected number of transitions it takes to reach state $(0,0)$ from state (i,j). If player b undercuts player a's ''indifference price'' when both reservoirs are full (and thus releases its water to produce electricity), we can express $N_{1,1}$ as follows:

$$N_{1,1} = 1 + (1-q_b)N_{1,0} + q_b N_{1,1}. \tag{5.4}$$

Furthermore, $N_{1,0}$ and $N_{0,1}$ can be expressed as follows:

$$\begin{aligned} N_{1,0} &= 1 + (1-q_b)q_a N_{1,0} + (1-q_a)q_b N_{0,1} + q_a q_b N_{1,1},\\ N_{0,1} &= 1 + (1-q_b)q_a N_{1,0} + (1-q_a)q_b N_{0,1} + q_a q_b N_{1,1}. \end{aligned} \tag{5.5}$$

Noting that $N_{1,0} = N_{0,1}$, the system (5.4) and (5.5) can be simplified as follows:

$$\begin{aligned} N_{1,1} &= \frac{1}{1-q_b} + N_{1,0},\\ N_{1,0} &= \alpha + \gamma \cdot N_{1,1}, \end{aligned} \tag{5.6}$$

where

$$\alpha = \frac{1}{1-q_a-q_b+2q_a q_b} \quad \text{and} \quad \gamma = \frac{q_a q_b}{1-q_a-q_b+2q_a q_b}.$$

Now, let $\tilde{N}_{i,j}$ denote the expected number of transitions it takes to reach state $(0,0)$ from state (i,j), assuming instead that player a undercuts player b's ''indifference price'' when both reservoirs are full. Using an analogous argument to that above, we obtain:

$$\begin{aligned} \tilde{N}_{1,1} &= \frac{1}{1-q_a} + \tilde{N}_{1,0},\\ \tilde{N}_{1,0} &= \alpha + \gamma \cdot \tilde{N}_{1,1}. \end{aligned} \tag{5.7}$$

Hence, from (5.6) and (5.7), it follows that

$$\tilde{N}_{1,1} - N_{1,1} = \frac{1}{1-q_a} - \frac{1}{1-q_b} + \tilde{N}_{1,0} - N_{1,0},$$

or equivalently,:

$$\tilde{N}_{1,1} - N_{1,1} = (1-y)\frac{1}{\gamma}\left(\frac{1}{1-q_a} - \frac{1}{1-q_b}\right).$$

Since $1 > \gamma > 0$ and $q_a > q_b$, it holds that $\tilde{N}_{1,1} > N_{1,1}$. In words, if the firm with the higher probability of water inflow releases its water when both reservoirs are full, a higher number of periods is required on average to reach the fully depleted state (relative to the situation where the firm with the lower probability of inflow releases its water instead).

5.3. Appendix 3: Determining Equilibrium Prices with a Larger Reservoir

Assume that the maximum reservoir size is now two units. A (Markov-perfect) Bertrand-Nash equilibrium satisfies the following conditions for each state:

$$p^*_{1,2} = p^*_{2,1} = \beta\alpha \cdot c^*$$

$$p^*_{1,1} = \beta(1-q)c^* + \beta q p^*_{1,2}$$

$$p^*_{2,2} = \beta(1-q)p^*_{1,2}$$

$$p^*_{1,0} = p^*_{2,0} = c^*$$

where

$$\alpha = \frac{(1-q)[1+\beta(1-q)]}{2-\beta q-\beta^2 q(1-q)} \in [0,1].$$

Proof: When reservoir levels are symmetric, the sufficient conditions to be indifferent between releasing water and not doing so are:

$$p^*_{1,1} + \beta[(1-q)V_{0,1} + qV_{1,2}] = \beta[(1-q)V_{1,0} + qV_{2,1}],$$

$$p^*_{2,2} + \beta[(1-q)V_{1,2} + qV_{2,2}] = \beta[(1-q)V_{2,1} + qV_{2,2}].$$

These conditions can be rearranged as follows:

$$p^*_{1,1} = \beta(1-q)(V_{1,0} - V_{0,1}) + \beta q(V_{2,1} - V_{1,2}),$$

$$p^*_{2,2} = \beta(1-q)(V_{2,1} - V_{1,2}).$$

The value functions for the states $(1, 0)$, and $(0, 1)$ are:

$$V_{1,0} = c^* + \beta[(1-q)V_{0,0} + qV_{1,1}],$$
$$V_{0,1} = 0 + \beta[(1-q)V_{0,0} + qV_{1,1}].$$

Hence, $V_{1,0} - V_{0,1} = c^*$. Similarly, for states $(2, 0)$ and $(0, 2)$ the value functions can be expressed as follows:

$$V_{2,0} = c^* + \beta[(1-q)V_{1,0} + qV_{2,1}],$$
$$V_{0,2} = 0 + \beta[(1-q)V_{0,1} + qV_{1,2}].$$

Hence,

$$V_{2,0} - V_{0,2} = c^* + \beta(1-q)(V_{1,0} - V_{0,1}) + \beta q(V_{2,1} - V_{1,2}),$$
$$V_{2,0} - V_{0,2} = c^* + \beta(1-q)c^* + \beta q(V_{2,1} - V_{1,2}). \tag{5.8}$$

Now, when states $(1, 2)$ and $(2, 1)$ are reached, players should be indifferent between releasing water (receiving a payment of $\tilde{p}_{1,2}$ and $\tilde{p}_{2,1}$ respectively) and withholding:

$$\beta[(1-q)V_{1,1} + qV_{2,2}] = \tilde{p}_{1,2} + \beta[(1-q)V_{0,2} + qV_{1,2}]. \tag{5.9}$$

and

$$\tilde{p}_{2,1} + \beta[(1-q)V_{1,1} + qV_{2,2}] = \beta[(1-q)V_{2,0} + qV_{2,1}]. \tag{5.10}$$

We posit that $\tilde{p}_{1,2} = \tilde{p}_{2,1}$ and have players bid $p^*_{2,1} = p^*_{1,2} = \tilde{p}_{1,2} = \tilde{p}_{2,1}$ so that $V_{2,1} - V_{1,2} = p^*_{2,1}$. In words, under this equilibrium, the value of stored water in intermediate states only depends on the total amount of water available and not on the distribution of it. Substituting $V_{2,1} - V_{1,2} = p^*_{2,1}$ into (5.8) we get:

$$V_{2,0} = V_{0,2} + c^* + \beta(1-q)c^* + \beta q \cdot p^*_{2,1}. \tag{5.11}$$

Moreover, substituting (5.11) and then (5.9) into (5.10), we get:

$$p^*_{2,1} + \beta[(1-q)V_{1,1} + qV_{2,2}] = \beta[(1-q)(V_{0,2} + c^* + \beta(1-q)c^* + \beta q \cdot p^*_{2,1}) + qV_{2,1}],$$
$$p^*_{2,1} + p^*_{1,2} + \beta[(1-q)V_{0,2} + qV_{1,2}] = \beta[(1-q)(V_{0,2} + c^* + \beta(1-q)c^* + \beta q \cdot p^*_{2,1}) + qV_{2,1}],$$
$$(1 - \beta q - \beta^2 q(1-q)) \cdot p^*_{2,1} + p^*_{1,2} = \beta(1-q)[1 + \beta(1-q)]c^*.$$

Finally since $p^*_{1,2} = p^*_{2,1}$ we have:

$$p^*_{2,1} = p^*_{1,2} = \frac{\beta(1-q)[1+\beta(1-q)]c^*}{2-\beta q - \beta^2 q(1-q)}.$$

We finally notice that

$$\alpha = \frac{(1-q)[1+\beta(1-q)]}{2-\beta q-\beta^2 q(1-q)} \in [0,1]. \qquad \blacksquare$$

5.4. Appendix 4: Simplifying the term, $\beta[(1-q)(\tilde{V}_{0,1}-V_{0,1})+q(\tilde{V}_{1,1}-V_{1,1})]$

We now analyze further the term, $\beta[(1-q)(\tilde{V}_{0,1}-V_{0,1})+q(\tilde{V}_{1,1}-V_{1,1})]$. Given that $V_{0,1}=\beta[(1-q)V_{0,0}+qV_{1,1}]$ (see equation (2.2)), and that $V_{1,1}=p^*_{1,1}+\beta[(1-q)V_{0,1}+qV_{1,1}]$ (see equation (2.7)), we can express $V_{0,1}$ and $V_{1,1}$ as follows (using the fact that $V_{0,1}=V_{0,0}$):

$$\begin{bmatrix} 1-\beta(1-q) & -\beta q \\ -\beta(1-q) & 1-\beta q \end{bmatrix} \begin{bmatrix} V_{0,1} \\ V_{1,1} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{p^*_{1,1}}{2}+\frac{p^*_{1,1}}{2} \end{bmatrix}.$$

Since $\tilde{V}_{0,1}$ satisfies an expression analogous to equation (2.2), and since $\tilde{V}_{1,1}$ satisfies equation (3.2), we can express $\tilde{V}_{0,1}$ and $\tilde{V}_{1,1}$ as follows:

$$\begin{bmatrix} 1-\beta(1-q) & -\beta q \\ -\beta(1-q) & 1-\beta q \end{bmatrix} \begin{bmatrix} \tilde{V}_{0,1} \\ \tilde{V}_{1,1} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\tilde{p}_{1,1}}{2}+\frac{p^*_{1,1}}{2} \end{bmatrix}.$$

Subtracting the first set of equations from the second set, we obtain the following:

$$\begin{bmatrix} 1-\beta(1-q) & -\beta q \\ -\beta(1-q) & 1-\beta q \end{bmatrix} \begin{bmatrix} \tilde{V}_{0,1}-V_{0,1} \\ \tilde{V}_{1,1}-V_{1,1} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{\tilde{p}_{1,1}-p^*_{1,1}}{2} \end{bmatrix}.$$

Multiplying both sides of the above equation by the inverse of the first matrix, and then premultiplying both sides by $[\beta(1-q)\ \beta q]$, the following result arises:

$$\begin{aligned} &\beta[(1-q)(\tilde{V}_{0,1}-V_{0,1})+q(\tilde{V}_{1,1}-V_{1,1})] \\ &\quad = [\,\beta(1-q)\ \beta q\,] \begin{bmatrix} 1-\beta(1-q) & -\beta q \\ -\beta(1-q) & 1-\beta q \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{\tilde{p}_{1,1}-p^*_{1,1}}{2} \end{bmatrix}. \end{aligned} \tag{5.12}$$

By solving the right-hand of equation (5.12), the following is obtained:

$$\beta[(1-q)(\tilde{V}_{0,1}-V_{0,1})+q(\tilde{V}_{1,1}-V_{1,1})] = \left\{\frac{1}{|D|}[\beta^2(1-q)q+\beta q(1-\beta(1-q))]\right\} \cdot \frac{\tilde{p}_{1,1}-p^*_{1,1}}{2}$$

where,

$$|D| = [(1-\beta(1-q))(1-\beta q)-\beta^2 q(1-q)] = 1-\beta.$$

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