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Note on analysis of quartz grain dimensions in foliated greywackes

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With 2 figures and 2 tables

Zusammenfassung

Quarzkorn-Abmessungen aus Grauwacken wurden auf ein internes Referenzsystem (reference aspect ratio, RAR) bezogen und mit Hilfe der linearen Regressionsanalyse der reduzierten Hauptachse (reduced major axis, RMA) ausgewertet. Das verwendete statistische Verfahren unterscheidet im Gegensatz zur Methode der kleinsten Quadrate nicht zwischen abhängigen und unabhängigen Variablen.

Die Anwendung der RMX-Methode in Verbindung mit den RAR-Meßwerten kann sehr hilfreich für Vergleiche innerhalb geschieferter Grauwacken sein, bei denen unterschiedliche Prozesse in Korngrößenbereich wirksam waren wie etwa Drucklösung und Festkörper-Rotation. Die RAR/RMA-Analyse erfaßt Kornregelungen und ist daher auch einsetzbar für die Klassifikation von Schieferungen.

Darüber hinaus wird die Anwendung der RAR/RMA-Analyse für die Bestimmung der Deformation diskutiert. Robin strain-Werte werden mit arithmetischen und harmonischen Mittelwerten der RAR-Analyse verglichen. Es zeigt sich, daß das arithmetische Mittel des RAR ein vernünftiges Maß für die longitudinale Deformation darstellt.

Abstract

Quartz grain dimensions, measured parallel to an internal reference system (reference aspect ratio, RAR), were analyzed using the »reduced major axis« (RMA) linear regression analysis. In contrast to least-squares analysis, this statistical technique does not distinguish between dependent and independent variables.

Application of the RMA analysis in conjunction with RAR values can be most useful for comparisons between foliated greywackes in which different grain scale processes, such as pressure solution and rigid-body rotation, were active. The RAR/RMA analysis reflects grain alignment and is therefore also useful for cleavage classification.

In addition to the above, the application of RAR/RMA analysis for the determination of strain is discussed. Robin strains are compared with the arithmetic and harmonic means from RAR's and it is concluded that the arithmetic mean RAR produces a reasonable estimate of longitudinal strain in these rocks.

Résumé

Les dimensions des grains de quartz, mesurés parallèlement à un système de référence interne (»reference aspect ratio«: RAR) ont été traitées par la méthode de la régression linéaire (»reduced major axis«: RMA). Contrairement à la méthode des moindres carrés, cette technique statistique ne fait pas de distinction entre les variables dépendantes et indépendantes.

L'application de ce type de méthode s'avère très utile à la comparaison de grauwackes schisteuses dans lesquelles les dimensions des grains peuvent être la conséquence de processus différents, tels que la dissolution (pressure solution) et la rotation. L'analyse RAR/RMA traduit l'alignement des grains et peut, de ce fait, être utilisée aussi à l'appréciation du type de schistosité.

D'autre part, l'auteur discute l'application de l'analyse RAR/RMA à la détermination de la déformation finie. Si on compare les déformations »Robin« aux moyennes arithmétique et harmonique des RAR, on peut conclure que les moyennes RAR arithmétiques fournissent une estimation raisonnable de la déformation longitudinale.

Краткое содержание

Помимо интернациональной системы RAR, размеры зерен кварца анализировали с помощью регрессивного анализа «Изменение основных осей» - RMA. В отличие от анализа методом наименьшего квадратов, при названном статистическом методе различия между зависимыми и независимыми переменными не учитываются. Комплексное применение RAR и RMA лучше всего подходит при сравнении рассланцеванных граувакков, в которых находят зерна различной величины в результате воздействия процессов крупного масштаба, как напр.: давление растворов и кручение (ротация) жестких тел. Комплексный анализ с помощью комбинации RAR/RMA отражает выравнивание зерен и поэтому его можно применить и для классификации кливажей.

Помимо выше сказанного дискутируется применение комбинации RAR/RMA и для определения степени деформации. Деформацию Робина сравнили с арифметическим и гармоническим способом на RAR и пришли к выводу, что арифметический способ RAR дает вполне приемлимые результаты при оценке объема деформации.

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Introduction

In a paper on mica beards in foliated greywackes, the author used detrital quartz grain dimensions to characterize slate microtextures from three distinct geographic regions (van der Pluijm 1984). Grains were measured parallel to a fixed, internal reference system with the following properties: axis **a** lies in the cleavage plane and is perpendicular to the fold axis; axis **b** is parallel to the fold axis; axis **c** is perpendicular to the cleavage plane. The (**ab**)-plane in this reference system is therefore parallel to the cleavage, and the (**ac**)-plane is the fold profile plane. These axes are geometric axes and should not be confused with kinematic axes which are associated with specific models of fold development (e.g. Hobbs et al., 1976, p. 193).

The geometric reference system is particularly useful in folded and cleaved rocks because the orientation of the fold axis and the cleavage plane define the orientation of the three orthogonal geometric axes in any sample. Consequently, rocks from different localities, possibly with different deformation histories, can be compared using this internal reference system.

Measurements of grain dimensions as outlined above, which will be referred to as the reference aspect ratio (RAR), produce values that generally do not correspond with minimum and maximum dimensions of elongate grains (the true aspect ratio; TAR). The difference is a function of the angle between grain elongation direction and the cleavage plane (i.e. (**ab**)-plane), and may be considerable; this relationship is schematically illustrated in Figure 1. In this paper, the reference aspect ratio is defined as the dimensions of a section through a grain measured parallel to the geometric axes **a**, **b**, **c**; in the (**ac**)-plane **a**/**c** is calculated; in the (**bc**)-plane, **b**/**c**.

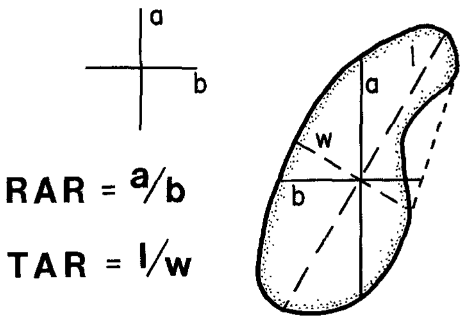

Fig. 1. Grain dimensions measured as maximum length over width (TAR: true aspect ratio) and parallel to a reference system a, b (RAR: reference aspect ratio). Shapes of irregular grains are redefined by straight lines (dashed).

In this short paper the statistical approach to the data sets given in van der Pluijm (1984), and its possible application for cleavage morphological classification and strain analysis will be discussed. It is not intended here to present a thorough treatment of statistical techniques; for this the reader is referred to existing textbooks on statistical methods (e.g. Davis, 1986).

Statistics and grain dimensions

Probably the most commonly applied method to investigate the relationship between two variables is a least-squares analysis. In van der Pluijm (1984) a least-squares-through-the-origin method was used to compare data sets from three different slates on the basis of line slope. The main characteristic of the least-squares method for the purpose of our discussion is that one variable is considered dependent (Y) and the other independent (X). Thus, in order to analyze the relationship between two variables, the dependent variable will have to be defined.

In slates, the long and short dimensions of elongate detrital grains may be (1) a primary feature, and/or (2) a result of grain shape modification by (a) mass transfer processes (e.g. pressure solution) or (b) crystal plasticity (intracrystalline deformation). Crystal plasticity is a relatively unimportant strain producing mechanism in these low grade rocks (<350°C). Mass transfer processes, on the other hand, are more important especially in the presence of a rock fluid, and result in preferential removal and deposition of material (e.g. Engelder & Marshak, 1985). Mass transfer processes in foliated greywackes change both TAR and RAR; TAR may increase or decrease, depending upon the original grain shape and orientation, while RAR will generally increase as the cleavage planes become the dissolution surface. In addition to or alternative to grain dissolution, rigid-body rotation of grains may occur (particulate flow; Borradaile, 1981). Consequently, the original dimensions of grains (TAR) remain unmodified during cleavage formation; however, as with solution processes, the RAR will generally increase since elongate grains tend to rotate their long axes into the cleavage ((**ab**)-) plane.

In addition to the slope of least-squares lines, the arithmetic mean RAR for grains was used in van der Pluijm (1984) to characterize the different samples. However, a mean value does not adequately describe the variation in variables. Furthermore, because detrital grains generally were not necessarily spheroids,

	RMA: Y = A + BX				reference aspect ratio		corr coef	
	A_1	B_1 (±SD)	A_2	B_2 (±SD)	a/c (±SD)	b/c (±SD)	r_1	r_2
CANADA	1.86	1.49±0.17	3.42	1.48±0.09	1.97±1.02	2.22±1.16	0.59	0.90
SCOTLAND	9.75	1.74±0.19	3.23	1.81±0.25	3.30±1.25	2.40±0.95	0.62	0.28
AUSTRALIA	6.00	1.22±0.15	5.03	1.29±0.18	1.81±1.08	1.81±0.92	0.46	0.16

A_1, B_1, r_1 in (ac)-plane, i.e. fold profile plane
A_2, B_2, r_2 in (bc)-plane, i.e.//fold axis and ⊥ cleavage plane
SD is standard deviation
number of measurements in each plane is 50

Tab. 1. Reduced major axes, arithmetic mean reference aspect ratios and correlation coefficients in the (**ac**)- and (**bc**)-planes of the Canadian, Scottish and Australian samples.

nor were they subjected to the same deformation history in each greywacke sample, the dependence of one variable on the other cannot readily be established. This, in turn, makes the least-squares linear regression analysis also unsuitable. Therefore, in order to adequately test the relationship between grain dimensions in samples, a regression method that examines for a relationship without regarding one variable as a function of the other is necessary. A suitable method was presented in Till (1974; see also Davis, 1986): the »reduced major axis« (RMA) analysis. A line is computed by minimizing the standard deviations (SD's) of the two variables. As a consequence, the RMA-line bisects the two regression lines calculated for variable X on variable Y and vice versa. The RMA-line is defined as:

$$Y = A + BX;\ B = SD_Y/SD_X,\ A = \bar{Y} - B\bar{X}$$

In Till (1974) and Davis (1986) equations to calculate standard deviations for A and B, and significance tests are given.

RMA computations

The RMA analysis has been applied to the data sets from the three greywackes described in van der Pluijm (1984). In Table 1 the results from the RMA analysis and the arithmetic mean RAR's are listed for the (**ac**)-plane (subscript 1) and the (**bc**)-plane (subscript 2).

In contrast to the values for the mean aspect ratios, the slopes of the two RMA lines for the (**ac**)- and (**bc**)-planes in each sample are indistinguishable. For example, in the Canadian greywacke values of the RMA-slopes in the (**ac**)- and (**bc**)-plane are very similar (1.49 and 1.48, respectively), which is in good agreement with the observed circular outline of grains in the cleavage plane; the calculated ratio of **a**/**b** from these RMA slopes is 1.01 (Table 2). Similar results were obtained from the Australian sample (**a**/**b** ratio is 0.95) and the Scottish sample (**a**/**b** ratio is 0.96; Tables 1 and 2). In the latter sample we find that the long dimension of a grain always lies in the cleavage plane; in other words, RAR equals TAR.

A trend of increasing grain flattening can be seen from the Australian sample to the Scottish sample in both the RMA slopes and the mean RAR's in the (**ac**)- and (**bc**)-planes. Note that this variation is more pronounced in the RMA values. For the (**ac**)-plane, the slopes for all three samples are significantly different at $\alpha = 0.05$: for the (**bc**)-plane, only the Scottish and Australian samples are significantly different at $\alpha = 0.05$. It should be noted that for these calculations only the original data sets shown in van der Pluijm (1984) were used; when the number of measurements are doubled, the standard deviations become less than 0.10, and the RMA slopes in the (**bc**)-plane are also significantly different for $\alpha = 0.05$. The mean aspect ratios, however, remain statistically not significant at this level.

	X/Y (Robin)	a/b (RAR)	a/b (RMA)
CANADA	0.86	0.89/0.82	1.01
SCOTLAND	1.38	1.38/1.39	0.96
AUSTRALIA	0.97	1.00/0.96	0.95

Tab. 2. Axial ratio of the strain ellipse (Robin method), reference aspect ratio (arithmetic/harmonic means), and reduced major axis slope ratio for the three samples discussed in the text.

Discussion and conclusions

In order to investigate detrital grain dimensions in foliated greywackes, RMA analysis is a more useful approach than least-squares analysis or mean aspect ratio calculation. In some cases it holds that grain dimensions measured perpendicular to the cleavage reflect the amount of preferential solution (e.g. Scotland sample); this dimension is then the dependent variable, and the dimension parallel to cleavage the independent. However, when detrital grains rotate during deformation their shapes are not affected and neither grain dimension is dependent on the other. Because both preferential solution and grain rotation generally occur in natural foliated greywackes (e.g. KNIPE, 1981) and it is difficult to evaluate the contribution of these mechanisms to the formation of the microtexture, it is generally not possible to determine the dependent variable.

This problem is avoided when RMA analysis is applied to the data sets. Similar to least-squares analysis a linear relationship between variables is calculated, however, without choosing the dependent variable. This is particularly useful for comparative rock studies in which (a) different deformation mechanisms were active and (b) the relative importance of these mechanisms varies.

Morphological cleavage classification

A potential application of the RMA method is to classify cleavages on the basis of RMA slope. Various rock cleavage classification schemes have been proposed; for example, POWELL (1979) and BORRADAILE et al. (1982) give a scheme that distinguishes between spaced and continuous cleavage. The boundary between both is drawn at 1 mm average spacing of cleavage surfaces (BORRADAILE et al., 1982, fig. 1.1). As was shown above, the slope of the RMA-line produces a numerical value that reflects grain alignment in cleaved rocks (see POWELL, 1979, fig. 3) and is independent of the mechanism resposible for this texture. Preliminary measurements in planes perpendicular to the fold axis (i.e. (**ac**)-plane) in rocks with spaced and continuous cleavages indicate that the boundary between both types can be drawn at **a**/**c** = 1.50. Thus, in addition to cleavage spacing, the RMA slope in the fold profile plane provides a rapidly determined numerical value for cleavage classification.

Strain analysis

Measurement of grain dimensions parallel to the axes of an internal reference system (RAR's), can produce a reliable estimate of longitudinal strain in these rocks. A method has been proposed by ROBIN (1977) that calculates the axial ratio of the strain ellipse from the logarithmic average of grain dimensions in two orthogonal directions. Some recent studies have successfully used this method and demonstrated its validity (e.g., VAN BERKEL et al. 1986; BABAIE, 1986).

Results of the application of the Robin method to the three samples described above are listed in Table 2 (X/Y (Robin)); in Figure 2 these values are plotted in a modified Flinn diagram. For this comparison, the (**a**/**b**)-plane (cleavage) is assumed to be within a few degrees of parallelism with the X/Y plane of the strain ellipsoid (e.g. HOBBS et al., 1976, p. 233–246). Only the strain ratio from the Scottish sample differs significantly from 1.00, and indicates >35 % greater extension in the **a** direction. From the arithmetic and harmonic mean RAR's a similar strain axial ratio is obtained (**a**/**b**, Figure 2), and they may therefore be used as a first approximate of longitudinal strain. In contrast to the results that were obtained by LISLE (1977; see also RAMSAY & HUBER, 1983, p. 80), the harmonic mean is not necessarily closer to the »true« value of strain than the arithmetic mean. Because of the relatively small number of measurements that were used in the above determinations (n = 50), a more rigorous discussion of strain will not be undertaken here.

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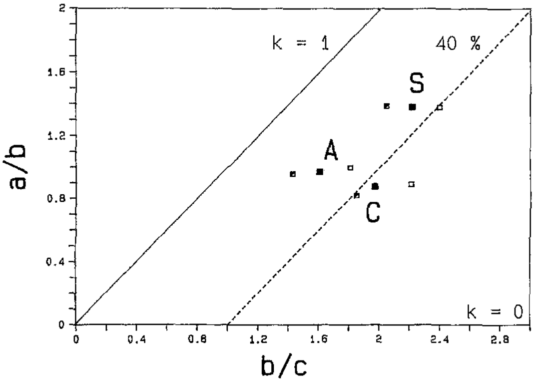

Fig. 2. Harmonic (◩) and arithmetic (□) mean reference aspe ratios and Robin strain ratios (■) in a modified Flinn diagram. The dashed line represent 40% volume loss when plane strain conditions are maintained. A – Australian sampl C – Canadian sample; S – Scottish sample.

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