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Pre-auction Investments By Type-conscious Agents

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Abstract:

This paper examines pre-auction investments made by asymmetric agents that compete for a supply contract from a monopolist principal. Agents are privately aware of their managerial efficiencies which determine how well they can leverage fixed investments to reduce their variable costs for servicing the contract, and they privately choose investment levels prior to the procurement mechanism being declared by the principal. Hence, the distribution of "types" that is standard in the principal-agent literature is, here, endogenously determined by the private actions of the agents. The principal declares a mechanism that is optimal for her, after agents have made their private investment decisions. We show that in equilibrium all optimal investment strategies by competing firms will have the form of investing as if there is no reservation price up to a critical level of managerial type, and investing minimally thereafter. This feature, however, implies that only trivial pure strategy equilibria can exist when the principal has any reasonably competitive alternative for servicing the contract. This is because in these cases an optimal mechanism induces agents to adopt a discontinuous investment strategy which provides the principal an incentive to deviate from the declared mechanism. An intuitive extrapolation of the extant literature to our context (in which agents adopt technologies featuring a fixed-variable cost trade-off) would suggest that we would see "underinvestment," manifesting itself as lower fixed and higher variable cost technologies in the industry. However, this intuition is either sustained trivially or cannot be sustained in pure strategies when the principal has any reasonable outside options for supply. The question of what cost structure we will see in equilibrium in these contexts will require future effort, and a consideration of mixed strategies.

Pre-auction Investments by Type-conscious Agents

1. Introduction

This paper considers the induced level of investment in productive resources made by firms that know they will be competing with other firms for a supply contract from a monopolist buyer. Firms can incur fixed and sunk costs to lower their variable costs of supply, and investments are made prior to the buyer declaring the rules (mechanism) by which the indivisible supply contract will be allocated. Each supplying firm is privately conscious of its own cost structure prior to making its investment decision (that is, they are "type conscious").

The motivating context for this work was a situation in which the capacity of the operating room suite at a large Midwestern hospital had to be increased. As described in Lovejoy and Li (2002) the hospital had two choices for enhancing its capacity. It could build new operating rooms or extend the working hours in the existing rooms incurring costly overtime. The former solution incurred higher fixed costs but lower variable costs, and the latter solution the reverse of that cost structure. What choice should they make if they knew they would be in competition with other hospitals for a large health care contract from a monopoly payer? A simple classroom experiment can reveal the complications inherent in this decision. Imagine assigning to each of a group of individuals (representing firms in a supply pool) a private cost structure, with the range of structures in the population featuring a fixed-variable cost trade-off (those with higher fixed costs will have lower variable costs). Then, the principal declares she will auction off a supply contract to the lowest bidder in an open-cry auction. A rational individual will be willing to bid until the price drops below his variable cost, so that the individual with the lowest variable cost (highest fixed cost) will get the contract for a price equal to the second lowest variable cost in the population. If the range of costs in the room is sufficiently dense, what will happen is that the winning firm will just cover its variable cost, so that all firms end up with net losses essentially equal to their fixed costs. In fact, the winning individual will lose the most money, since he has the highest fixed cost. Now, after the individuals have played this game, tell them they will play again but this time they get to choose their cost structure from a menu of choices that feature a fixed-variable cost trade-off. What will they do? Knowing that all firms ended up with a net loss equal to their fixed cost, it is tempting to conclude that they will all choose a low fixed cost, high variable cost structure. But, if all firms do this wouldn't at least one firm be tempted to defect, anticipating that all competitors feature very high variable costs? Also, the simple open-cry format may not be the optimal mechanism design for the principal, and in particular we made no mention of a reservation price which commonly attends optimal mechanisms.

It is by now well understood how a seller can design an optimal auction mechanism that gives her the highest expected revenues from the sale of an indivisible object to potential buyers whose valuation of the object is their private information (Myerson, 1981; Riley and Samuelson, 1981; Maskin and Riley, 1984). The classical formulation of the auction as a non-cooperative game is facilitated through the notion of a "type", where, following Harsanyi (1967-68), players' uncertainty about another player's valuation is captured as an exogenously specified probability distribution. Much of the extant literature on the theory of auctions relies on this type distribution being readily available. Not much has been written on designing optimal auction mechanisms when buyers may not approach the auction with pre-determined valuation for the object they intend to bid for, but actively "choose their type" before bidding. Such is the case, for example, when defense contractors invest substantial resources in R&D before bidding for a government contract (Lichtenberg, 1986; Rogerson, 1989). Such proactive investments are costly, but could lead to a lower cost structure or a higher probability of success in developing a new technology, thus higher chance of winning the contract.

This paper analyzes the existence and character of pure-strategy equilibria in a game where several supplier (agent) firms can make pre-auction investments to affect their cost structure prior to entering an auction for an indivisible supply contract from a monopoly buyer (principal). The analysis applies in general to principal-agent situations where agents "buy their type" prior to bidding in an auction, principal declares her mechanism after agents' decisions have been made, and the assumptions imposed in our model (which are fairly standard; see below) hold.

The question of pre-auction investments is important because it informs the induced cost structures in an industry. For example, an intuitive extrapolation of the extant literature (see below) suggests that if the principal does not commit to a mechanism prior to the agents making their investment decisions, agents will underinvest. In our context, this would mean that agents incur lower fixed costs and hence exhibit a higher variable cost structure than might be socially or individually (for the principal and agents alike) optimal. For example, if there were no safeguards to support fixed costs in electricity auctions, we would expect no new base load plants to be built and to see all new generating capacity in the form of gas turbines or other low fixed but high variable cost production systems. This intuitive story, however, is too simple. If all competitors had high variable costs, then at least one firm would perceive it advantageous to invest more fixed costs to lower their variable costs, because they would perceive the likelihood of winning the auction to be very high. In that situation, they might approach investing as if they were guaranteed to win, and that may include robust investment in fixed cost assets. What is the nature of an equilibrium in this setting? Here we show that no pure strategy equilibrium will exist in situations where the principal has a meaningful alternative to awarding the contract. That is, unless the cost of alternative outside supply (or the cost of holding onto the contract herself) is so high that it exceeds the worst case among the agents at minimal investment levels, or so low that no supplier has any chance to win, there will be no pure strategy equilibrium.

Existing literature

We largely follow Myerson (1981) in our auction design. Myerson assumes that an agent's value estimate for the object to be auctioned is privately known to the agent, and other players' uncertainty about agent i's valuation can be described by a probability distribution whose density is positive everywhere over a finite interval. This assumption is almost universally employed by most authors in their analysis of optimal auctions (Riley and Samuelson, 1981; Maskin and Riley, 1984), specific auction formats (Vickrey, 1961; Wilson, 1969; Milgrom and Weber, 1982), and most applications thereof. We extend Myerson's analysis to situations where the "type distribution" for each agent i is generated endogenously via a common prior belief on agent's ability, and his conscious "investment" decision.

The existing literature on pre-auction investments was largely motivated by either patent races or by firms competing for government contracts (for example, defense contracts). In both cases, firms can invest at time zero to reap probabilistic rewards (either winning the contract, or cost reductions) at some future time. The focus of the articles is on the level of time-zero investment relative to the social optimal.

Loury (1979) models a patent race in which the first firm to win gets all of the rents in perpetuity from the patent. Firms choose investment levels that beget random times of discovery, and the first discovery time wins. Firms are assumed to be ex ante symmetric, and Loury restricts his attention to symmetric equilibria. He shows that firms over-invest relative to a social optimum because they ignore the externality they place on others by investing. However in Loury (1979) there is no self-interested principal trying to extract rents from the agents. Dasgupta (1990) adds the feature of the self-interested principal, and models firms that can invest in R&D to purchase a random cost outcome with a contingent distribution that is known to all. The firms must choose a level of investment prior to competing for a production contract in an auction mechanism designed by a monopoly buyer. The level of investment and cost outcome are private information, and the suppliers choose investment levels before the principal declares the mechanism design. The suppliers are ex-ante symmetric and Dasgupta focuses on symmetric equilibria (for which the optimal mechanism can take on several standard forms; the author analyzes implementation via a sealed low-bid auction). Since agents must choose levels of investment, which incur sunk costs, prior to the principal declaring a mechanism the suppliers are exposed to opportunistic behavior by the buyer. This results in under-investment by the agents. If the buyer can pre-commit to a mechanism the suppliers' levels of investment increase toward the social optimum. This aligns with known results that pre-commitment in competition can sometimes make things better for everybody. Piccione and Tan (1996) model a situation very similar to that in Dasgupta, focusing on symmetric equilibria and in particular on the relationship between the contingent cost distributions and equilibrium investment levels. They show that for certain forms of this distribution we again have under-investment relative to the social optimum because of the opportunistic behavior of the rent-extracting buyer. Again, pre-commitment by the buyer can mitigate this effect. For other models where the principal commits to a mechanism prior to the agents' investment decisions see Tan (1992), King et al (1992), Bag (1997), Che and Gale (2003) and Arozamena and Cantillon (2004).

Our model considers firms with the following features:

• The principal declares the mechanism after the firms choose investment levels.

• Any exogenous uncertainty in firm "type" is resolved privately prior to firms choosing investment levels. Firm types and investment levels are not publicly observable.

• Firms can be ex-ante asymmetric.

• The mechanism is not fixed, but is also endogenous in that principal invokes an optimal mechanism given her beliefs about the agents' investment strategies.

All of the previous work cited differs from our model in at least one of these dimensions. Yet, ours is a plausible model for some industrial contexts. For example, the usual assumption that supplying firms invest prior to the resolution of cost (or time) uncertainty means that firms must choose levels of investment without knowing their cost structure. This is plausible in a basic R&D context, but established industrial firms making investments in familiar processes are likely to know their cost structure prior to committing those funds. That is, there is private firm-specific information about the managerial type of each firm that is known by the firm at the time of its investment decision. Our model addresses this context, which explains what we mean by firms being "type conscious."

Che & Gale (2003) and King et al (1992) also assume that an agent knows his own type prior to investing. However, in their models an agent's type is publicly observable while his investment level is not. We treat both an agent's type and his investment level as his private information.

In our model the generally held beliefs about a firm's capabilities can be asymmetrical, as can the investment strategies followed by the firms based on their private information about those capabilities. Also, in our model, firms can enter the decision process with different levels of historical investment in place, and disinvestment can be costly, amplifying the asymmetrical nature of the competition.

The current literature focuses on the social efficiency of the equilibrium levels of investment, assuming pure strategy equilibria exist. Our focus is on the existence of these equilibria, and their structure when they exist.

Radner and Rosenthal (1982), Milgrom and Weber (1982) and Athey (2001) also study the existence of pure-strategy equilibria in games of incomplete information. But they consider situations where each player is an agent possessing private information. We examine whether a balance can be reached between a principal and agents who possess private information.

The existing R&D literature offers sufficient conditions for a pure-strategy equilibrium to exist that can omit relevant situations. In Dasgupta (1990) and Piccione and Tan (1996) these conditions are, essentially, that the buyer has no real alternative to the pool of bidders for supply. Technically, in these papers the consequences to the buyer of not granting the contract to any of the bidders is the incurrence of a cost, v_0, that reflects the cost of a fall-back position for outside (or internal) supply. The conditions imposed are that v_0 is higher than the worst possible random outcome at the lowest possible level of investment for the supplying firms. Again, this may be appropriate for basic research contracts to develop new-to-the-world products but can be less realistic in established industries with global supply alternatives outside the preferred bidders in the auction pool, or with internal sourcing options. It is not clear whether pure strategy equilibria can routinely be expected to exist in these latter contexts. We show that the answer is negative. With meaningful outside costs (that is, unless v_0 is so high it is irrelevant or so low it renders the auction meaningless) no pure strategy equilibrium will exist. That is, the sufficient conditions assumed in the existing R&D literature are very close to necessary.

In our model, higher levels of investment will generate higher fixed costs, but lower variable costs. This trade-off will be present in any set of undominated technology choices, because nobody would invest in a technology that raises both of these types of costs. This is also consistent with industrial reality, where higher fixed costs should translate into lower variable costs of execution. Mathematically, there is little difference between this interpretation and one in which firms invest a dollar amount in R&D to increase their chances of winning a contract. However, this alternative interpretation has interesting intuitive consequences. The model addresses the equilibrium cost structure one will expect to see among the bidding firms. The study of pre-auction investments that feature fixed-variable cost tradeoffs is not restricted to the R&D literature. Although differing in their focus, Shleifer (1981), Pope (1990), Laffont & Tirole (1986), Kjerstad & Vagstad (2000), Fuloria

& Zenios (2001) all take into consideration fixed-variable cost tradeoffs and discuss how to induce a socially optimal cost-reduction effort, which is a firm's private information, if there are observable costs upon which a regulator relies for the design of a contract or an auction. Newhouse (1996) provides an extensive review of this line of research and its impact on managing health care payments. In our analysis the principal does not observe the firms' costs. That is, our principal's optimal auction design has to be based on anticipation of the firms' investment strategies rather than observation of their costs.

The structure of this paper is as follows. Section 2 describes the basic model. Sections 3 and 4 analyze properties of an optimal mechanism and optimal investment strategies, respectively. Section 5 examines the role of v_0 in relation to the existence of a pure strategy equilibrium. Section 6 concludes the paper.

2. Basic setting

We consider a market where n agents (firms) vie for an indivisible contract through a procurement auction designed by a monopolist principal. All actors are risk neutral. Prior to the auction, agent i chooses his investment level k_i from the interval $[\underline{k}_i, \bar{k}_i]$, incurring fixed cost $g_i(k_i)$. Given that level of investment agent i would incur a variable cost of $V_i(k_i, y_i)$ to service the contract, where $y_i \in [\underline{y}_i, \overline{y}_i]$ is the agent's "managerial type." An agent's managerial type reflects his ability to make efficient use of investments. Agent i knows y_i, but the principal and his competitors do not, and do not observe the investment level k_i. The common belief among these other players is that y_i is distributed as a random variable Y_i with cumulative distribution function H_i and continuous, strictly positive density h_i. The Y_i's are assumed independent.

Throughout the paper we make the following assumptions:

(A1) g_i is non-negative, strictly convex and continuously differentiable in investment level.

(A2) k_i^L is the unique investment level on $[\underline{k}_i, \bar{k}_i]$ that minimizes g_i.

(A3) V_i is non-negative, strictly decreasing, strictly convex and continuously differentiable in investment level.

(A4) V_i is non-negative, strictly increasing and continuous in managerial type.

(A5) $\frac{\partial V_i(k_i, y_i)}{\partial k_i}$ is continuous and non-decreasing in managerial type.

(A6) $k_i^{Qy_i}$ is the unique investment level on $[\underline{k}_i, \bar{k}_i]$ that minimizes $g_i(k_i) + V_i(k_i, y_i)$.

(A7) The principal's outside opportunity cost $v_0 < \infty$, and when she is indifferent between allocating her contract and incurring an outside opportunity cost v_0, she will allocate the contract.

Assumptions A1 to A3 describe the fixed-variable tradeoffs in question. A2 specifies a unique fixed cost minimizing level of investment, beyond which g_i is strictly increasing. If agent i is certain that he will not win the contract he will choose investment level k_i^L to minimize his fixed costs. Intuitively, one can think of agents starting with no capital investments, so $k_i^L = 0$ and fixed costs strictly increase with investment level $k_i \geq k_i^L$. We generalize this to accommodate potential situations that might arise in regulated industries, in which an agent can begin with a non-zero level of investment and incur decommissioning charges to reduce that level, resulting in a $k_i^L > 0$.

Assumption A4 implies that an agent with a smaller managerial type makes more efficient use of investments than does an agent with a larger managerial type. Assumption A5 implies that an increase in managerial type has an adverse impact on the rate at which variable cost declines with investment level. Assumption A6 simply specifies the notation used for the cost minimizing investment level for an agent that is certain he will get the contract. We use the notation $k_i^{Qy_i}$ to signal that this can depend on the size of the contract Q and managerial type y_i. Although, in our indivisible setting we will henceforth refer only to the contract and not to its size Q. The uniqueness of $k_i^{Qy_i}$ follows from the strict convexity assumptions A1 and A3.

Assumption A7 is a technical one that ensures a bounded support for the agents' type distributions, and facilitates more concise proofs without substantive loss to either the results or intuition.

3. Properties of an optimal mechanism

Here we analyze the features of an optimal mechanism for any set of fixed investment strategies by the agents. An investment strategy for an agent is a function from the agent's managerial type y_i to an investment level k_i. We denote by γ_i agent i's investment strategy, and let $\gamma = (\gamma_1, ..., \gamma_n)$ and $\gamma_{-i} = (\gamma_1, ..., \gamma_{i-1}, \gamma_{i+1}, ..., \gamma_n)$.

If agent i adopts investment strategy $\gamma_i(y_i)$ he will have variable cost $v_i = V_i(\gamma_i(y_i), y_i)$ to service the contract. Let F_i be the distribution of v_i, derived from the known distribution H_i of y_i and the investment strategy γ_i. For any fixed set of investment strategies, hence distributions F_i, the principal will want to design an optimal mechanism. The situation is a standard independent private-values auction as analyzed by Myerson (1981). There, however, an agent's "type" is the agent's valuation of the contract v_i and is given exogenously, either as a known number (for agent i) or in distribution (for all other players).

This standard notion of type is appropriate in our setting, as well, once agents have chosen their investment strategies. Once agent i has chosen investment level k_i his variable cost to service the contract is $v_i = V_i(k_i, y_i)$. Clearly, with fixed costs sunk, agent i

would accept the contract for expected payments above v_i, would be indifferent when expected payment is at that value and would reject expected payments below it. So, v_i is the appropriate notion of type relevant to the principal's mechanism design problem, because the mechanism is declared after investments have been made. It is because of this sympathy with the existing literature that we call v_i an agent's "type" without further qualification. In contrast, we will call y_i the agent's "managerial type." Note that once investment strategies have been adopted, the distributions F_i of types are known and well-defined. However, since investment strategies are strategically chosen by agents we cannot automatically assume that they possess sufficient structure to ensure that F_i has a density, or given a density that it is positive. Indeed, we will see below that the type distribution for agents in equilibrium will have gaps of zero probability. In the following we note what can be said about optimal mechanisms with general type distributions, and make an assumption about the class of mechanisms that will be adopted by the principal.

We first introduce notation for the support of F_i and a decomposition of that support into sets with various combinations of Lebesgue (denoted by λ) and probability (denoted by μ_i for F_i) measures. Define

$\underline{v}_i = \sup\{v | F_i(v) = 0\}$

$\overline{v}_i = \inf\{v | F_i(v) = 1\}$

The structure of the problem (and assumption A7) will guarantee that these are bounded, specifically $\underline{v}_i \geq 0$ and $\overline{v}_i < \infty$. Let f_i denote the density for F_i when it exists. Following Bergemann and Pesendorfer (2001) we partition $[\underline{v}_i, \overline{v}_i]$ into the set M_i of probability mass points, a set P_i with positive density and a set O_i with zero density. Specifically, recall that λ refers to Lebesgue measure and μ_i to the probability measure associated with F_i, and define

$M_i = \{v_i \in [\underline{v}_i, \overline{v}_i] | \lambda(v_i) = 0 \text{ but } \mu_i(v_i) > 0\}$

$P_i = \{v_i \in [\underline{v}_i, \overline{v}_i] | f_i(v_i) \text{ exists and is strictly positive }\}$

$O_i =$ the maximal union of open intervals $\cup_j (a_i^j, b_i^j)$ contained in $[\underline{v}_i, \overline{v}_i]$ such that $f_i = 0$ on each interval.

Because F_i is nondecreasing, it is differentiable a.e.-λ (almost everywhere in Lebesgue measure) so $\mu_i(M_i \cup P_i) = 1$. Technically, however, we will include the right hand limits of the intervals in O_i in the support of F_i. That is, define $B_i = \cup\{b_i^j | (a_i^j, b_i^j) \in O_i\}$. If there is a point mass at $v_i \in B_i$ then v_i is already in the set M_i, but if not v_i could be a point where only the right side derivative of F_i exists. Define $\Omega_i = M_i \cup P_i \cup B_i$. Clearly

$\mu_i(\Omega_i) = 1$ and we will use Ω_i as the support of F_i. We will restrict our attention to direct revelation mechanisms (see below), so Ω_i is the set of credible reports from agent i, and $\Pi_i\Omega_i$ is the product space on which the mechanism is defined.

Facing any set of type distributions the principal will declare a mechanism, which is a set of allocation functions p and transfer functions x with the following properties. The principal asks for reports from each agent, and based on these reports the principal allocates the contract to agent i with probability p_i. Also as a function of the vector of reports from the n agents, the principal transfers x_i dollars to agent i. The mechanism is chosen in such a way that the agents will accept the contract if it is offered to them, and be rewarded by an amount equal to the transfer. Hence, p_i and x_i are both functions of the vector of reports from the agents. Let $p = (p_1, p_2, .., p_n)$ and $x = (x_1, x_2, ..., x_n)$. The set of functions (p, x) is a mechanism.

The principal commits to the mechanism (p, x), the agents submit their reports, and then the mechanism is implemented. As in Myerson, we will invoke the Revelation Principle and restrict our attention to feasible direct revelation mechanisms. In such a mechanism an agent cannot be worse off for playing the game ("individual rationality" or IR) and will choose to report his true type to the principal ("incentive compatibility" or IC). By the IC constraints, an agent will reveal v_i truthfully to the principal, so in the set of functions (p, x), p_i and x_i are both functions of the truthful report vector $v = (v_1, v_2, ..., v_n)$. The principal chooses a cost minimizing (p, x) subject to the IR and IC constraints, and such that the p_i are legitimate probabilities (that is, are non-negative and sum to something equal to or less than unity). We next look at the agents' reporting decisions, and then the principal's mechanism design problem.

Agent optimal reporting

For any $v_i \in \Omega_i$ define $G_i(v_i) = E_{v_{-i}} p_i(v_{-i}, v_i)$ (the probability that agent i wins with a report of v_i) and $\overline{x}_i(v_i) = E_{v_{-i}} x_i(v_{-i}, v_i)$ (the expected transfer with a report of v_i). So, agent i's expected utility for being type $v_i \in \Omega_i$ and reporting that he is type $s_i \in \Omega_i$ will be

$$\mathcal{U}_i(s_i|v_i) = -G_i(s_i)v_i + \overline{x}_i(s_i). \tag{1}$$

Facing any mechanism, the utility of agent i will be the best possible expected value over all credible reports, or

$$\mathcal{U}_i(v_i) = sup_{s_i \in \Omega_i} \mathcal{U}_i(s_i|v_i). \tag{2}$$

Since $\mathcal{U}_i(v_i)$ is the supremum of a set of affine functions (with finite slopes) it is convex, absolutely continuous and differentiable a.e.-λ on $[\underline{v}_i, \overline{v}_i]$. If $r_i(v_i)$ attains the supremum at type v_i, then $-G_i(r_i(v_i))$ is a subgradient of the convex function $\mathcal{U}_i(v_i)$ at v_i. Clearly if the IC constraints hold ($\mathcal{U}_i(v_i|v_i) \geq \mathcal{U}_i(s_i|v_i)$ for all $s_i \in \Omega_i$) $r_i(v_i)$ is attained at v_i for all $v_i \in \Omega_i$. We will assume that the suprema in (2) are attained on all of $[\underline{v}_i, \overline{v}_i]$. If not, we could follow the logic in Milgrom (1999) to extend the feasible set of affine functions to its closure, and extend the utility function to this set to guarantee attainment. Practically, all this does is allow an agent to choose a signal that represents getting arbitrarily close to the supremum, attaining the correct value.

In the classical context ($M_i = O_i = \emptyset$) we know from Myerson (1981) that the IC and IR constraints hold if and only if

$$\mathcal{U}_i(v_i) = \mathcal{U}_i(\overline{v}_i) + \int_{v_i}^{\overline{v}_i} G_i(s)ds,$$

$G_i(v_i)$ is non-increasing on $[\underline{v}_i, \overline{v}_i]$ and $\mathcal{U}_i(\overline{v}_i) \geq 0$. In our more general context, but following identical logic (also see Milgrom 1999 and Milgrom and Segal 2002), it can be shown that the IC and IR constraints hold if and only if

$$\mathcal{U}_i(v_i) = \mathcal{U}_i(\overline{v}_i) + \int_{v_i}^{\overline{v}_i} G_i(r_i(s))ds \text{ for all } v_i \in \Omega_i, \tag{3}$$

$G_i(r_i(s))$ is non-increasing on $[\underline{v}_i, \overline{v}_i]$ and $\mathcal{U}_i(\overline{v}_i) \geq 0$. Note that evaluating (3) requires that $r_i(v_i)$ be defined for each v_i between $\underline{v}_i$ and $\overline{v}_i$. Intuitively, while an agent cannot credibly claim to be of type $v_i \in (a_i^j, b_i^j) \subseteq O_i$, an agent of type a_i^j can claim to be b_i^j and the principal needs to grant sufficient information rents to dissuade this deceit. Hence, the integral in (3) extends across intervals in O_i as well as intervals in the credible reporting set Ω_i. Without any appeal to optimality on the part of the principal, yet, equation (3) will hold for any agent that faces a known mechanism (p, x), knows his type and chooses a utility maximizing report.

The principal's objective

The principal will incur a cost v_0 if she does not award the contract to any agent, and will transfer in expectation $E_v \sum_{i=1}^n x_i(v)$ to the agents, so the principal wishes to maximize

$$E_v\big[-(1 - \sum_{i=1}^n p_i(v))v_0 - \sum_{i=1}^n x_i(v)\big] = -v_0 + \sum_{i=1}^n E_{v_i} G_i(v_i)v_0 - \sum_{i=1}^n E_{v_i}\overline{x}_i(v_i)$$

subject to $p_i(v) \geq 0$ and $\sum_{i=1}^n p_i(v) \leq 1$ for all v, and the IR and IC constraints. When the IR constraints hold $r_i(v_i) = v_i$ and $\overline{x}_i(v_i) = \mathcal{U}_i(v_i) + G_i(v_i)v_i$ for all $v_i \in \Omega_i$. Using (3) we have that the principal's objective is to maximize

$$-v_0 + \sum_{i=1}^n E_{v_i} G_i(v_i)[v_0 - v_i] - \sum_{i=1}^n E_{v_i}\mathcal{U}_i(v_i)$$

$$= -v_0 - \sum_{i=1}^{n} \mathcal{U}_i(\overline{v}_i) + \sum_{i=1}^{n} E_{v_i} G_i(v_i)\big[v_0 - v_i\big] - \sum_{i=1}^{n} E_{v_i} \int_{v_i}^{\overline{v}_i} G_i(r_i(s))ds \tag{4}$$

subject to the constraints that for each agent i, $G_i(r_i(s))$ is non-increasing on $[\underline{v}_i, \overline{v}_i]$, $p_i(v) \geq 0$ and $\sum_{i=1}^{n} p_i(v) \leq 1$ for all v. v_0 is a constant and following classical logic the principal will set the transfers such that $\mathcal{U}_i(\overline{v}_i) = 0$ for all agents. Following the logic in Bergemann and Pesendorfer (2001) we note that $r_i(v_i)$ for $v_i \in O_i$ impacts the principal's objective only through the integral in (4), so with G_i non-increasing the principal will want to maximize r_i if possible. That is, the principal will design a mechanism such that $r_i(v_i) = b_i^j$ for all $v_i \in (a_i^j, b_i^j) \subseteq O_i$. Using this, and the usual analysis (c.f Myerson 1981 for the general logic, and Begemann and Pesendorfor 2001 for its application to this problem type in an auction rather than procurement setting) we get that the principal's objective is to maximize

$$\sum_{i=1}^{n} \int_{v_i \in P_i} G_i(v_i)\big[v_0 - v_i - \frac{F_i(v_i)}{f_i(v_i)}\big] f_i(v_i)dv_i]$$

$$+\sum_{i=1}^{n} \sum_{v_i \in M_i} G_i(v_i)(v_0 - v_i)\pi_i(v_i) - \sum_{i=1}^{n} \sum_{b_i^j(O_i)} G_i(b_i^j)F_i(b_i^j)(b_i^j - a_i^j) \tag{5}$$

(where $\pi_i(v_i)$ is the probability mass at v_i and the final sum is over all upper endpoints of the closure of intervals in O_i) subject to the constraints that for each agent i, $G_i(v_i)$ is non-increasing on Ω_i, $p_i(v) \geq 0$ and $\sum_{i=1}^{n} p_i(v) \leq 1$ for all v . The principal then sets the transfers to satisfy

$$\overline{x}_i(v_i) = G_i(v_i)v_i + \int_{s \in P_i, s \geq v_i} G_i(s)ds + \sum_{b_i^j(O_i), b_i^j \geq v_i} G_i(b_i^j)(b_i^j - a_i^j). \tag{6}$$

We remark, as in Bergemann and Pesendorfer, that this expression reduces to the correct known expressions (c.f. Myerson 1981 and Lovejoy 2006) for the principal's objective function in the cases with positive density everywhere ($M_i = O_i = \emptyset$) and with discrete type spaces ($P_i = \emptyset$ but $M_i \neq \emptyset$). In the former case (5) reduces to

$$E_v \sum_{i=1}^{n} p_i(v)\big[v_0 - v_i - \frac{F_i(v_i)}{f_i(v_i)}\big]$$

and in the latter case with $\Omega_i = \{v_i^1, v_i^2, ..., v_i^m\}$ and when $v_i = v_i^j$, define $\Delta v_i = v_i^j - v_i^{j-1}$, (5) reduces to

$$E_v \sum_{i=1}^{n} p_i(v)\big[v_0 - v_i - \frac{F_i(v_i)}{\pi_i(v_i)}\Delta v_i\big].$$

Much of the insight into optimal mechanisms and the intuitive appeal of the related literature stems from the linear form of the principal's objective in p, and the notion of a virtual cost, exhibited by these two special cases. Indeed, when $f_i > 0$ everywhere Myerson (1981) shows that even in cases where $v_i - \frac{F_i(v_i)}{f_i(v_i)}$ is not monotone, the problem can be transformed into one in which there exists a nondecreasing function $\overline{c}_i(v_i)$ such that the principal's objective is to maximize

$$E_v \sum_{i=1}^{n} p_i(v)\left[v_0 - \overline{c}_i(v_i)\right] \tag{7}$$

subject to $p_i(v) \geq 0$ and $\sum_{i=1}^{n} p_i(v) \leq 1$ for each v. Naturally, for any report vector v, setting $p_i(v) = 0$ if the bracketed term is less than zero for all i, and otherwise distributing $p_i(v)$ over the agents with the minimum $\overline{c}_i(v_i)$ will satisfy the monotonicity requirement on G_i and hence be an optimal allocation. This in turn defines an optimal transfer, which is any transfer such that $\overline{x}_i(v) = G_i(v_i)v_i + \int_{v_i}^{\overline{v}_i} G_i(s)ds$.

Unfortunately, such a reduction is not available for the general form (5). When there exist intervals $(a_i^j, b_i^j) \in O_i$ with no point mass at b_i^j, there is no easy way to separate p out to generate a linear form with its attendant insights. Here we will assume that the principal restricts her choice of mechanism to the class $\mathcal{C}$ of mechanisms with allocations generated as in (7) for some functions $\overline{c}_i(v_i)$ non-decreasing on $[\underline{v}_i, \overline{v}_i]$. This class contains all standard mechanisms as well as all published mechanisms known to the authors. We now show that for any absolutely continuous type distributions the principal can get arbitrarily close to optimality by choosing an allocation in this class. Since we later show that all equilibrium type distributions will be absolutely continuous, this result will suffice.

Proposition 1: For any absolutely continuous μ and $\epsilon > 0$, there exists a feasible mechanism (p, x) such that $p \in \mathcal{C}$ and the principal's expected utility is within ϵ of its optimal value.

From the proof of Proposition 1, the mechanism implemented by the principal is optimal for a type distribution with positive density everywhere on $(\underline{v}_i, \overline{v}_i)$, and arbitrarily close to the true absolutely continuous μ. For any type vector v, the allocation p is chosen to maximize (7) such that $p_i(v_{-i}, v_i)$ and $G_i(v_i)$ are non-increasing on $[\underline{v}_i, \overline{v}_i]$ and the transfers are set such that

$$\overline{x}_i = G_i(v_i)v_i + \int_{v_i}^{\overline{v}_i} G_i(s)ds \tag{8}$$

for $v_i \in [\underline{v}_i, \overline{v}_i]$. As shown in the proof, such a mechanism can be chosen to get arbitrarily close to the principal's optimal utility facing μ. The structure inherent in (7) and (8) will be used in what follows.

4. Properties of optimal investment strategies

In this section we investigate how agents will invest when facing the mechanism (p, x) described above. Define $\hat{\nu}_i = inf\{v_i \in [\underline{v}_i, \overline{v}_i] | G_i(v_i) = 0\}$. If $G_i(v_i) > 0$ for all $v_i \in [\underline{v}_i, \overline{v}_i)$ define $\hat{\nu}_i = \overline{v}_i$. If $G_i(v_i) = 0$ for all $v_i \in (\underline{v}_i, \overline{v}_i]$ define $\hat{\nu}_i = \underline{v}_i$. Because G_i is non-increasing once it hits zero it never recovers, so $G_i(v_i) = 0$ for $v_i > \hat{\nu}_i$. $\hat{\nu}_i$ is agent i's "reservation price" because agent i cannot win with any bid higher than $\hat{\nu}_i$. If an agent wins the contract, he will be compensated for his variable costs plus earn an information rent equal to $\int_{v_i}^{\overline{v}_i} G_i(s)ds = \int_{v_i}^{\hat{\nu}_i} G_i(s)ds$. As is usual in these contexts, any agent reporting his maximal feasible variable cost $\overline{v}_i$ (or, indeed, any variable cost $v_i \geq \hat{\nu}_i$) will earn no information rent.

From the form of the optimal transfers, agent i will be compensated for his expected variable costs plus an information rent equal to $\int_{V_i(k_i,y_i)}^{\hat{\nu}_i} G_i(z)dz$. So agent i with managerial type y_i who invests at level k_i will enjoy an expected profit of

$$\phi_i(k_i, y_i) = -g_i(k_i) + \int_{V_i(k_i,y_i)}^{\overline{v}_i} G_i(z)dz = -g_i(k_i) + \int_{V_i(k_i,y_i)}^{\hat{\nu}_i} G_i(z)dz.$$

We note that while an agent will consider this total expected profit when choosing an investment level k_i, the principal will ignore the sunk costs $g_i(k_i)$ when designing her mechanism. This is a consequence of the mechanism being declared after agents have committed to their investment levels.

With a mechanism and γ_{-i} fixed, agent i will choose an investment level $k \in [\underline{k}_i, \overline{k}_i]$ to maximize $\phi_i(k, y_i)$. For $y_i \in [\underline{y}_i, \overline{y}_i]$, define

$$\Pi_i(y_i) = max_{k \in [\underline{k}_i, \overline{k}_i]} \phi_i(k, y_i)$$

and

$$\Gamma_i(y_i) = \{k \in [\underline{k}_i, \overline{k}_i] : \Pi_i(y_i) = \phi_i(k, y_i)\}.$$

That is, $\Pi_i(y_i)$ is the maximal expected profit for agent i with managerial type y_i, and $\Gamma_i(y_i)$ is the set of maximizing investment levels at y_i. Define Γ_i as the set of optimal investment strategies for agent i, that is the set of functions $\gamma_i : [\underline{y}_i, \overline{y}_i] \to [\underline{k}_i, \overline{k}_i]$ with $\gamma_i(y_i) \in \Gamma_i(y_i)$ for all y_i. The following lemma shows that the optimal investment functions will inherit some continuity properties.

Lemma 1: For all agents i

a) $\Pi_i(y_i)$ is continuous in y_i on $[\underline{y}_i, \overline{y}_i]$.

b) $\Gamma_i(y_i)$ is an upper semi-continuous point-to-set map on $[\underline{y}_i, \overline{y}_i]$. In particular, for all $y \in [\underline{y}_i, \overline{y}_i]$ and all $\epsilon > 0$, there exists a $\delta > 0$ such that $y' \in [\underline{y}_i, \overline{y}_i]$ and $|y' - y| < \delta$ will imply that every element of $\Gamma_i(y')$ is within ϵ of some element of $\Gamma_i(y)$.

c) If $\Gamma_i(y)$ is single-valued at y then any optimal investment function γ_i is continuous at y. That is, if $\Gamma_i(y) = \{k\}$ a singleton, then for any sequence $y^j \to y$ in $[\underline{y}_i, \overline{y}_i]$ and any $\gamma_i \in \Gamma_i$, we will have $\gamma_i(y^j) \to \gamma_i(y) = k$. $\diamond$

Recall k_i^L minimizes agent i's fixed costs. Lemma 2 below shows that any optimal investment strategy $\gamma_i \in \Gamma_i$ will be either k_i^L everywhere, or will begin at some investment level strictly above k_i^L at $\underline{y}_i$ and descend until the first time it hits k_i^L, and then it will stay at k_i^L for all greater y_i up to the upper limit $\overline{y}_i$. Also, a positive investment level above the minimal k_i^L is associated with a positive probability of winning the auction. $V_i(\gamma_i(y_i), y_i)$ is strictly increasing in y_i, and is either $\geq \hat{\nu}_i$ everywhere (in which case $\gamma_i(y_i) = k_i^L$ everywhere), or begins at $V_i(\gamma_i(\underline{y}_i), \underline{y}_i) < \hat{\nu}_i$ (and $\gamma_i(\underline{y}_i) > k_i^L$) and then as managerial type y_i increases the variable cost $V_i(\gamma_i(y_i), y_i)$ increases up to $\hat{\nu}_i$ at which point the minimal investment k_i^L is optimal there and for all greater y_i up to $\overline{y}_i$. Parts (a) through (d) establish the characteristics of the relevant functions that will generate monotone optimal policies using lattice programming (c.f. Topkis 1978) technology. To briefly review this, we say that $\phi(k, y)$ has "antitone differences" in k and y if for all $k > k'$, $\phi(k, y) - \phi(k', y)$ is non-increasing in y. That is, the marginal improvement for an additional unit of investment in k declines as managerial type y increases. Under these conditions, we expect the optimal level of investment to be lower at higher y values (as shown in part (h) below). A "descending" point-to-set map is a generalization of the notion of a non-increasing function. We say $\Gamma(y)$ is descending if for all $y' < y$, $k' \in \Gamma(y')$ and $k \in \Gamma(y)$ will imply $Min\{k', k\} \in \Gamma(y')$ and $Max\{k', k\} \in \Gamma(y)$. Parts (e) through (j) below employ these conditions and results to establish the form that all optimal investment strategies must take.

Lemma 2: For all agents i and all $\gamma_i \in \Gamma_i$

a) $\phi_i(k_i, y_i)$ is strictly decreasing in y_i whenever $V_i(k_i, y_i) < \hat{\nu}_i$.

b) $\phi_i(k_i, y_i)$ has antitone differences in k_i and y_i everywhere on $[\underline{k}_i, \overline{k}_i] \times [\underline{y}_i, \overline{y}_i]$, and strictly antitone differences whenever $V_i(k_i, y_i) < \hat{\nu}_i$.

c) $\gamma_i(y_i) \geq k_i^L$ always.

d) $\Gamma_i(y_i)$ is a descending point-to-set map.

e) If $V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i$ then $\gamma_i(y_i) > k_i^L$. In particular if $V_i(k_i^L, y_i) < \hat{\nu}_i$ then $\gamma_i(y_i) > k_i^L$.

f) $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ if and only if $\gamma_i(y_i) = k_i^L$.

g) If for any y_i', $\gamma_i(y_i') = k_i^L$, then $\gamma_i(y_i) = k_i^L$ for all $y_i \geq y_i'$.

h) $\gamma_i(y_i)$ is non-increasing in y_i.

i) $V_i(\gamma_i(y_i), y_i)$ is strictly increasing in y_i.

j) $\gamma_i(\overline{y}_i) = k_i^L$ uniquely and $\overline{v}_i = V_i(\gamma(\overline{y}_i), \overline{y}_i) = V_i(k_i^L, \overline{y}_i)$. ⋄

As Lemma 2 shows, any optimal investment strategy by any agent will yield a variable cost strictly increasing in managerial type y_i. As long as that variable cost is less than $\hat{\nu}_i$ then the optimal investment level will be strictly greater than k_i^L. However, if the variable cost ever hits $\hat{\nu}_i$ then the optimal investment level will drop to k_i^L and stay there for all higher managerial types. For any $\gamma_i \in \Gamma_i$ define $\psi_i^\gamma \equiv sup\{[\underline{y}_i, \bar{y}_i] \mid V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i\}$ where $\psi_i^\gamma \equiv \underline{y}$ if $V_i(\gamma_i(\underline{y}_i), \underline{y}_i) \geq \hat{\nu}_i$. Then ψ_i^γ is the switch point between these two policy regimes. We will suppress the superscript γ when there is no confusion.

Lemma 3: For any $\gamma_i \in \Gamma_i$

a) $\gamma_i(y_i) > k_i^L$ for $y_i < \psi_i$.

b) $\gamma_i(y_i) = k_i^L$ for $y_i > \psi_i$.

c) If $\psi_i < \bar{y}_i$ then $\hat{\nu}_i \leq V_i(k_i^L, \psi_i)$. ⋄

In summary, an optimal investment strategy by any agent will feature a variable cost that is strictly increasing in managerial type y_i. The optimal investment level will be non-increasing in y_i but strictly greater than k_i^L for $y_i < \psi_i$, which is the cutoff beyond which the agent's variable cost exceeds his reservation price. For $y_i > \psi_i$, the agent will minimize his fixed costs with investment level k_i^L. A representative situation is shown graphically in Figure 1, which will be more completely described below. It is clear from Lemma 3 that if the limits exist

$$lim_{y_i \uparrow \psi_i} V_i(\gamma_i(y_i), y_i) \leq \hat{\nu}_i \leq lim_{y_i \downarrow \psi_i} V_i(\gamma_i(y_i), y_i) = V_i(k_i^L, \psi_i).$$

That is, in Figure 1 the reservation price must be somewhere in the gap indicated by the dotted line. There may be a discontinuity in the optimal investment function at ψ_i, and this point is determined by the agent's reservation price, itself of a function of the principal's outside opportunity cost v_0. The next section explores more completely the influence of this outside cost.

5. The role of the opportunity cost v_0

We now explicitly consider the role of the principal's opportunity to service the contract at cost v_0. This will affect the optimal allocation chosen by the principal by affecting the reservation price on each agent, and therefore will affect the point at which agent i cannot hope to win the contract (and as a result will choose the minimal level of investment k_i^L). We first consider the effect of v_0 on optimal allocations, holding the agents' investment strategies fixed (Lemma 4 below) and then the effect of v_0 on the optimal allocation and investment strategies in equilibrium (Lemmas 5 and 6 and Proposition 2).

As described in section 3, for any fixed set of investment strategies and therefore type distributions, there will exist for each agent i a nondecreasing function $\bar{c}_i(v_i)$ of v_i (agent i's virtual cost) such that given any report vector $v = (v_1, ..., v_n)$ the principal will choose an allocation p to maximize (7) subject to p being nonnegative and $\sum_{i=1}^n p_i \leq 1$.

We focus on the interplay of γ and p only, because given p the set of optimal transfers x is preordained as usual. It is clear from (7) that the principal will put maximal allocation on agents with minimal virtual cost. For any report vector v, define $\bar{c}_{min}$ to be the minimum $\bar{c}_i(v_i)$ over all agents i, and define M to be the set of agents with that minimal virtual cost, that is $M = \{i | \bar{c}_i(v_i) = \bar{c}_{min}\}$. When $v_0 < \bar{c}_{min}$ the set of optimal allocations is a singleton, the zero vector. When $\bar{c}_{min} \leq v_0$ the set is the nonnegative p such that $\sum_{i \in M} p_i = 1$. The following Lemma follows directly from this. For any fixed investment strategies by agents, let $P^{v_0}(v)$ denote the set of optimal allocations at report vector v and let $G_i^{v_0}(s)$ denote $G_i(s) = E_{v_{-i}} p_i(v_{-i}, s)$ when $p \in P^{v_0}$. The following lemma shows that the sets P^{v_0} and functions $G_i(v_i)$ are essentially nondecreasing (componentwise and pointwise, respectively) in v_0. The more complicated presentation is the result of potential non-uniqueness issues for these sets and functions.

Lemma 4: For any $v_0' < v_0$ and report vector v:

a) For any $p' \in P^{v_0'}(v)$ there exists a $p \in P^{v_0}(v)$ with $p_i' \leq p_i$ for all i.

b) For any $p \in P^{v_0}(v)$ there exists a $p' \in P^{v_0'}(v)$ with $p_i' \leq p_i$ for all i.

c) For any $G_i^{v_0'}(s)$, there exists a $G_i^{v_0}(s)$ such that $G_i^{v_0'}(s) \leq G_i^{v_0}(s)$ for all s.

d) For any $G_i^{v_0}(s)$, there exists a $G_i^{v_0'}(s)$ such that $G_i^{v_0'}(s) \leq G_i^{v_0}(s)$ for all s. $\diamond$

The outside cost v_0 has a discontinuous effect on the set of optimal allocations at any report vector v. As long as v_0 is high enough that the principal wishes to allocate the contract, further increases in v_0 do not affect the optimal allocations. That is, once v_0 exceeds the minimum virtual cost among the agents, the same set of allocations will be optimal for all higher v_0's. However, when v_0 is reduced below the minimal virtual cost, non-allocation is the unique optimum. The intuitive result is that for any agent i and bid vector v, the probability of winning the contract is nondecreasing in v_0.

We now consider the equilibria that can be obtained at different levels of outside opportunity cost v_0. For any distribution of managerial types and outside cost $v_0 \leq \infty$, let (γ, p, x) denote an equilibrium solution. That is, if the agents use investment strategies γ then (p, x) is an optimal mechanism for the principal, and if the principal invokes (p, x) the agents will optimally invest at the levels determined by γ. Define $(\gamma^\infty, p^\infty, x^\infty)$ analogously for $v_0 = \infty$ (technically, we use $v_0 = \infty$ to denote an opportunity cost arbitrarily higher than the level at which the contract is awarded with probability one, practically in this case the principal has no real alternative options for supply).

Because once v_0 is "high enough" the optimal allocations are independent of v_0, the equilibrium solutions will be also. Lemma 5 shows that in equilibrium any investment strategy $\gamma_i \in \Gamma_i$ will either match an equilibrium strategy with $v_0 = \infty$, or will be k_i^L and in fact will switch from the former to the latter at the point that $V_i(\gamma_i(y_i), y_i)$ hits $\hat{\nu}_i$ (at $y_i = \psi_i^\gamma$). So, at first agents will invest as if the principal has no outside opportunities, but then at a critical managerial type they will drop to the minimal level of investment. This is because agents know their managerial type y_i and their variable cost $V_i(\gamma_i(y_i), y_i)$ exactly, so as long as $V_i < \hat{\nu}_i$ they only need to worry about winning against the other agents, which is precisely the situation they are in when $v_0 = \infty$. Once y_i hits ψ_i^γ ($V_i(\gamma_i(y_i), y_i)$ hits $\hat{\nu}_i$) the agent gets no rents but will pay her fixed costs, so will minimize those. Lemma 5 shows this formally.

Lemma 5:

If an equilibrium exists at any $v_0 \leq \infty$, then for every equilibrium (γ, p, x) there exists an equilibrium $(\gamma^\infty, p^\infty, x^\infty)$ with $v_0 = \infty$ such that for all agents i:

a) $\gamma_i^\infty(y_i) = \gamma_i(y_i)$ when $y_i < \psi_i^\gamma$.

b) $G_i^\infty(v_i) = G_i(v_i)$ if $v_i < \hat{\nu}_i$. $\diamond$

The situation as proved in Lemma 3 and Lemma 5 is illustrated in Figure 1. Optimal investment strategies γ_i will have a simple structure, being equal to either γ_i^∞ or k_i^L, and will jump from one to the other at $\hat{\nu}_i$. In equilibrium the variable cost $V_i(\gamma_i(y_i), y_i)$ as a function of managerial type y_i is strictly increasing, but may exhibit a jump discontinuity as agents jump from a robust investment level to k_i^L at ψ_i. If this is the case, the investment strategy γ_i generates a set of variable costs between the left limit as $y \uparrow \psi_i$ and the right limit as $y \downarrow \psi_i$ that cannot occur with positive probability. Unfortunately, this situation is not sustainable in equilibrium.

The next lemma shows that when gaps like this exist, the principal is always better off setting $\hat{\nu}_i$ at least as low as the left side limit. But, with $\hat{\nu}_i$ at that level any "competitive" agent (any agent that could win at $y = \psi_i$ against other agents, but cannot win against

v_0) would defect to k_i^L strictly prior to ψ_i, contradicting the optimality of γ. This is because the information rents decrease to zero as y increases to ψ_i but the agent's fixed costs are bounded away from the minimum. At some point close to ψ_i the agent will not be covering his fixed cost and will defect to k_i^L. In Figure 2 this point is shown as ψ_i'. But, then note that $\hat{\nu}_i$ is now strictly in the middle between the variable costs at the new left and right side limits at ψ_i'. The principal with $\hat{\nu}_i$ as shown is always better off lowering it to $\hat{\nu}_i'$. So, the principal will lower $\hat{\nu}_i$ to $\hat{\nu}_i'$ and the agent will defect even earlier, etc. This process continues until defection at the lower limit $\underline{y}_i$ is reached. This is made formal in the part (b) of the following lemma, which ties the existence of a discontinuity in V_i to a discontinuity in G_i.

Lemma 6:

a) In any equilibrium for all agents i, $\hat{\nu}_i \leq \limsup_{y \uparrow \psi_i^\gamma} V_i(\gamma_i(y), y)$.

b) No pure strategy equilibrium can exist in which $G_i(v_i)$ is discontinuous at $\hat{\nu}_i \in (\underline{v}_i, \overline{v}_i)$ for any i. $\diamond$

We are now ready to show that no equilibrium can exist in which the opportunity cost v_0 plays a meaningful role. That is, if v_0 is so low that $\hat{\nu}_i = \underline{v}_i$ then agent i will always invest minimally and will be indifferent to participating in the auction. If this is true for all agents, the auction is essentially meaningless. If $\hat{\nu}_i = \overline{v}_i$ for all agents i then pure-strategy equilibria can exist but the outside opportunity cost might as well be infinite. These features are intuitively clear. The following proposition essentially extends the latter logic to $\hat{\nu}_i > \underline{v}_i$. That is, we show that any pure strategy equilibrium in which any agent has $\hat{\nu}_i \in (\underline{v}_i, \overline{v}_i)$ must be almost everywhere identical to one with $v_0 = \infty$, or else it cannot exist. Intuitively, we have either a meaningless auction or one in which v_0 has no impact in that it might as well be infinite.

Proposition 2: For any pure strategy equilibrium (γ, p, x) and any agent i with $\hat{\nu}_i \in (\underline{v}_i, \overline{v}_i)$, there will exist an equilibrium $(\gamma^\infty, p^\infty, x^\infty)$ with $v_0 = \infty$ such that

a) $\hat{\nu}_i = \hat{\nu}_i^\infty$

b) $\gamma_i(y_i) = \gamma_i^\infty(y_i)$ almost everywhere on $[\underline{y}_i, \overline{y}_i]$

c) $G_i(v_i) = G_i^\infty(v_i)$ almost everywhere on $[\underline{v}_i, \overline{v}_i]$.

Proposition 2 is easy to interpret in the often-assumed context of symmetric investment strategies yielding type distributions that feature the standard assumptions of regularity (f exists and is positive everywhere, and $v_i + F_i(v_i)/f_i(v_i)$ is strictly increasing). Then, $\overline{c}_i(v_i) = v_i + F_i(v_i)/f_i(v_i)$, $\overline{c}_i(\underline{v}_i) = \underline{v}_i$ and $\overline{c}_i(\overline{v}_i) = \overline{v}_i + 1/f_i(\overline{v}_i) \geq \overline{v}_i$. In this case $v_0 > \underline{v}_i$

will imply $\hat{\nu}_i > \underline{v}_i$ and $v_0 < \overline{v}_i$ will imply $v_0 < \overline{c}_i(\overline{v}_i)$ so that $\hat{\nu}_i < \overline{v}_i$. But, in the symmetric case $\hat{\nu}_i^\infty = \overline{v}_i$ always, so no pure strategy equilibrium can exist with $v_0 \in (\underline{v}_i, \overline{v}_i)$. This leaves only extreme possibilities. If $v_0 < \underline{v}_i$ for all agents, then the principal is better off denying the contract regardless of reports, and knowing this all agents will invest at the minimal level k_i^L. We then obtain a pure strategy equilibrium but the auction is meaningless. Any higher, more meaningful v_0 will provide incentives for at least one agent to deviate from his minimal level of investment and disrupt the equilibrium. The only other hope for a pure strategy equilibrium is for $v_0 \geq \overline{v}_i = V_i(k_i^L, \overline{y}_i)$, the worst possible managerial type at minimal investment and therefore the highest possible variable cost attainable by any combination of action and managerial type. That is, with the common assumptions of symmetry and regularity, the sufficient conditions identified by Dasgupta for the existence of an equilibrium (v_0 greater than the worst possible agent cost) are also (essentially) necessary. If $v_0 < \overline{v}_i$, either pure strategy equilibria do not exist or the equilibrium is trivial and the auction meaningless.

While the assumptions of symmetry and regularity tie our analysis to familiar concepts in the literature, these features cannot be expected to hold routinely for problems of this type. However, the intuition remains intact. When agents are asymmetric, $\hat{\nu}_i^\infty = \overline{v}_i$ is not automatic because the support for one agents' cost distribution may lie completely below another's. However, there is always one agent with $\overline{c}_i(\overline{v}_i) \leq \overline{c}_j(\overline{v}_j)$ for all $j \neq i$. Such an agent has $\hat{\nu}_i^\infty = \overline{v}_i$, and any outside opportunity cost $\underline{v}_i < v_0 < \overline{c}_i(\overline{v}_i)$ disallows pure strategy equilibria (since all it takes to disrupt equilibria is one agent defecting).

So, if the principal has any (even moderately) competitive external or internal alternative for supply outside the pool of bidders, the only equilibrium cost structures we can see among the bidders are all firms investing at minimal levels (low fixed, high variable costs), or a mixture of investments reflecting the non-existence of pure strategies.

6. Conclusion

This paper examines pre-auction investments made by asymmetric, type-conscious firms (agents) that compete for a supply contract from a monopolist buyer (principal). Agents are privately aware of their managerial types prior to choosing their investment levels, removing all uncertainty (for them, but not their competitors) about the cost consequences of their investment. Increased fixed investments reduce their variable cost to service the contract, and hence their valuation of the contract. So, the distribution of "types" that is standard in the literature is, here, endogenously determined by the private actions of the agents. The principal declares a mechanism that is optimal for her, after agents have made their private investment decisions.

We show that in equilibrium all optimal investment strategies by competing firms will have the form of investing as if there is no alternative source of supply beyond the pool of bidders (that is, the firms are competing only with each other) up to a critical level of managerial type, and investing minimally thereafter. So, all optimal investment strategies for games with any outside opportunity cost can be constructed knowing only the optimal strategy for an infinite outside cost and the reservation prices.

This structure, however, implies that only trivial pure strategy equilibria can exist when the principal has a reasonably competitive alernative to awarding the contract. For example, if the outside opportunity cost is so low that no bidding firm can be competitive under any circumstances, then a trivial pure strategy equilibrium will exist with all agents adopting a minimal fixed cost (maximal variable cost) technology. But, in those cases, an auction is not required or meaningful. If the principal's outside opportunity cost is so high that she must assign the contract to some agent regardless of their investment levels or managerial efficiencies, then a pure strategy equilibrium can exist. But, for situations where the outside opportunity is meaningfully competitive, no pure strategy equilibrium can exist and we expect a mixture of investment strategies.

Returning to our motivating context of capacity expansion in a medical system, we have only anecdotal observations to offer as points of comparison with this theory. We do see mixtures of expansion strategies across medical systems, but we cannot make a clear assignment of cause to this result. While there are some powerful national payers approximating our powerful principal, there are also less powerful local payers that can use excess hospital capacity that give the health systems some bargaining power. Also, the medical system contracting office that we interacted with did not describe the negotiations as one in which either party sought to behave in a unilaterally optimal fashion, attempting to extract maximal rents from their opponents. Rather, issues of fairness, trust and reciprocity were evident in our conversations, suggesting that issues of relationship and repeated interaction were relevant to them. Also, the medical industry in general is evolving from a relatively protected environment to one in which intense cost competition will be more prevalent. Our model may better reflect the future of this industry than the status quo. Finally, our model shares with all models based on Bayes Nash equilibria the assumption of common beliefs about managerial types, which may not reflect reality. Still, returning to the simple experiment described in the introduction, we see that the results proved here have intuitive appeal that might suggest robustness to these issues. When firms can incur a fixed cost to reduce their variable costs, they will do so only if the information rents will cover that investment. This means that whatever they perceive the highest competitive variable cost to be, they will cease to invest in cost reductions strictly below that level. So, any attempt by the principal to reduce the information rents that she pays will result in a reaction by the agents that lowers the fixed costs and increases the variable costs in the bidding pool. But, this can only be taken so far. Eventually, when all agents exhibit very high variable costs at least one will perceive an opportunity to invest, lower his variable costs, and win both the contract and higher profits. If the principal anticipates this and tries again to extract some of that rent, the process begins anew.

An intuitive extrapolation of the extant literature to our context (in which agents adopt technologies featuring a fixed-variable cost trade-off) would suggest that we would see "underinvestment," manifesting itself as lower fixed and higher variable cost technologies in the industry. However, this intuition is either sustained trivially or cannot be sustained in pure strategies when the principal has any reasonable outside options for supply. The question of what cost structure we will see in equilibrium in these contexts will require future efforts, and a consideration of mixed strategies.

Appendix:

Proposition 1: For any absolutely continuous μ and $\epsilon > 0$, there exists a feasible mechanism (p, x) such that $p \in \mathcal{C}$ and the principal's expected utility is within ϵ of its optimal value.

Proof: For ease of notation, define

$$\mathcal{U}_0^\mu(G) = \sum_{i=1}^n E_{v_i}^\mu \left[G_i(v_i)(v_0 - v_i) - \int_{v_i}^{\overline{v}_i} G_i(r_i(s))ds\right].$$

The principal's objective is to maximize $\mathcal{U}_0^\mu(G)$ over G generated by μ-feasible allocations. An allocation p is μ-feasible if for all v, $p_i(v) \geq 0$ and $\sum_{i=1}^n p_i(v) \leq 1$, and $G_i^{\mu,p}(v_i) := E_{v_{-i}}^\mu p_i(v_{-i}, r_i(v_i))$ is non-increasing in v_i on $[\underline{v}_i, \overline{v}_i]$. In fact, an optimal allocation p can be selected such that $p_i(v_{-i}, r_i(v_i))$ is nonincreasing in v_i on $[\underline{v}_i, \overline{v}_i]$ and therefore continuous a.e.-λ. Any μ-feasible allocation has a preordained transfer x from (6). If we endow the space of functions $G_i : [\underline{v}_i, \overline{v}_i] \to R$ with the metric $d(G, \tilde{G}) = \sup_{v \in [\underline{v}_i, \overline{v}_i]} |G_i(v) - \tilde{G}_i(v)|$ then $\mathcal{U}_0^\mu(G)$ is continuous in G.

For any μ we can define a sequence of measures μ_n each with strictly positive density everywhere on $[\underline{v}_i, \overline{v}_i]$ such that $\mu_n \Rightarrow \mu$ (weak convergence). This means that for any G non-increasing on $[\underline{v}_i, \overline{v}_i]$ we have $\mathcal{U}_0^{\mu_n}(G) \to \mathcal{U}_0^\mu(G)$ since μ is absolutely continuous. Since the μ_n have strictly positive densities on $[\underline{v}_i, \overline{v}_i]$, from Myerson's (1981) classical results we can choose a μ_n-optimal allocation p_n in the class $\mathcal{C}$. Further, we can choose $p_n \in \mathcal{C}$ to be μ-feasible. This is because from (7) we can generate p_n pointwise for each v to satisfy $p_{n,i}(v) \geq 0$ and $\sum_{i=1}^n p_{n,i}(v) \leq 1$, and for all v_{-i}, $p_{n,i}(v_{-i}, v_i)$ is non-increasing in v_i. This last feature guarantees that $G_i^{\mu,p_n}(v)$ is non-increasing on $[\underline{v}_i, \overline{v}_i]$, and hence p_n

is μ-feasible. Hence, the proof is complete if we can show that by choosing n sufficiently large and choosing $p_n \in \mathcal{C}$ μ_n-optimal and μ-feasible, we can make $\mathcal{U}_0^{\mu}(G^{\mu_n,p_n}) - \mathcal{U}_0^{\mu}(G^{\mu,p})$ arbitrarily small. Note that

$$\mathcal{U}_0^{\mu}(G^{\mu_n,p_n}) - \mathcal{U}_0^{\mu}(G^{\mu,p})$$
$$= \mathcal{U}_0^{\mu}(G^{\mu_n,p_n}) - \mathcal{U}_0^{\mu_n}(G^{\mu_n,p_n}) \qquad (a)$$
$$+ \mathcal{U}_0^{\mu_n}(G^{\mu_n,p_n}) - \mathcal{U}_0^{\mu_n}(G^{\mu,p}) \qquad (b)$$
$$+ \mathcal{U}_0^{\mu_n}(G^{\mu,p}) - \mathcal{U}_0^{\mu}(G^{\mu,p}). \qquad (c)$$

Since G^{μ_n,p_n} and $G^{\mu,p}$ are non-increasing (and therefore continuous a.e.-λ), we can make (a) and (c) arbitrarily small (by weak convergence) by choosing n sufficiently large. To complete the proof we need to show that we can make $\mathcal{U}_0^{\mu_n}(G^{\mu_n,p_n}) - \mathcal{U}_0^{\mu_n}(G^{\mu,p})$ arbitrarily small. This would follow from the continuity of $\mathcal{U}_0$ in G if we can show that for all $\epsilon > 0$ there exists an N_ϵ such that $n \geq N_\epsilon$ will imply $d(G^{\mu_n,\tilde{p}}, G^{\mu,p}) < \epsilon$ for some μ_n-feasible $\tilde{p}$. This will suffice, because then by the μ_n-optimality of p_n we have

$$\mathcal{U}_0^{\mu_n}(G^{\mu_n,p_n}) - \mathcal{U}_0^{\mu_n}(G^{\mu,p}) \geq \mathcal{U}_0^{\mu_n}(G^{\mu_n,\tilde{p}}) - \mathcal{U}_0^{\mu_n}(G^{\mu,p}) \geq -\epsilon$$

which would provide the missing part (b).

To show the existence of such a $\tilde{p}$, note that $G^{\mu_n,p} \to G^{\mu,p}$ by weak convergence and the assumption that p is continuous a.e.-λ. So, for any $\epsilon > 0$ there exists an N_ϵ such that $n \geq N_\epsilon$ implies that $d(G^{\mu_n,p}, G^{\mu,p}) < \epsilon$. For that n $G^{\mu_n,p}$ is everywhere within ϵ of a non-increasing function, so we can construct another non-increasing function $\tilde{G} \leq G^{\mu_n,p}$ such that $d(\tilde{G}, G^{\mu,p}) < \epsilon$. Further, we can generate $\tilde{G}$ from an allocation $\tilde{p}$ that we can construct by lowering (never raising) $p_i(v)$ at select vectors v. Further, this can be done while maintaining $\tilde{p}_i \geq 0$ because only $p_i > 0$ require address. So, $\tilde{G}_i(v) = G_i^{\mu_n,\tilde{p}}(v)$ is non-increasing on $[\underline{v}_i, \overline{v}_i]$. That is $\tilde{p}$ is μ_n-feasible. This completes the argument. $\diamond$

Lemma 1: For all agents i

a) $\Pi_i(y_i)$ is continuous in y_i on $[\underline{y}_i, \overline{y}_i]$.

b) $\Gamma_i(y_i)$ is an upper semi-continuous point-to-set map on $[\underline{y}_i, \overline{y}_i]$. In particular, for all $y \in [\underline{y}_i, \overline{y}_i]$ and all $\epsilon > 0$, there exists a $\delta > 0$ such that $y' \in [\underline{y}_i, \overline{y}_i]$ and $|y' - y| < \delta$ will imply that every element of $\Gamma_i(y')$ is within ϵ of some element of $\Gamma_i(y)$.

c) If $\Gamma_i(y)$ is single-valued at y then any optimal investment function γ_i is continuous at y. That is, if $\Gamma_i(y) = \{k\}$ a singleton, then for any sequence $y^j \to y$ in $[\underline{y}_i, \overline{y}_i]$ and any $\gamma_i \in \Gamma_i$, we will have $\gamma_i(y^j) \to \gamma_i(y) = k$.

Proof: Parts (a) and (b) follow from Berge's (1997) "Maximum Theorem" in chapter 6 of that text, and the definition of an upper semi-continuous map given there. Part (c) follows from (b) because if $\Gamma_i(y) = \{k\}$ a singleton, then for any sequence $y^j \to y$ in $[\underline{y}_i, \overline{y}_i]$ and any sequence k^j such that $k^j \in \Gamma_i(y^j)$ for all j, we will have $k^j \to k$. $\diamond$

Lemma 2: For all agents i and all $\gamma_i \in \Gamma_i$

a) $\phi_i(k_i, y_i)$ is strictly decreasing in y_i whenever $V_i(k_i, y_i) < \hat{\nu}_i$.

b) $\phi_i(k_i, y_i)$ has antitone differences in k_i and y_i everywhere on $[\underline{k}_i, \overline{k}_i] \times [\underline{y}_i, \overline{y}_i]$, and strictly antitone differences whenever $V_i(k_i, y_i) < \hat{\nu}_i$.

c) $\gamma_i(y_i) \geq k_i^L$ always.

d) $\Gamma_i(y_i)$ is a descending point-to-set map.

e) If $V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i$ then $\gamma_i(y_i) > k_i^L$. In particular if $V_i(k_i^L, y_i) < \hat{\nu}_i$ then $\gamma_i(y_i) > k_i^L$.

f) $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ if and only if $\gamma_i(y_i) = k_i^L$.

g) If for any y_i', $\gamma_i(y_i') = k_i^L$, then $\gamma_i(y_i) = k_i^L$ for all $y_i \geq y_i'$.

h) $\gamma_i(y_i)$ is non-increasing in y_i.

i) $V_i(\gamma_i(y_i), y_i)$ is strictly increasing in y_i.

j) $\gamma_i(\overline{y}_i) = k_i^L$ uniquely and $\overline{v}_i = V_i(\gamma(\overline{y}_i), \overline{y}_i) = V_i(k_i^L, \overline{y}_i)$.

Proof:

a) Part (a) follows from the definition of $\phi_i(k_i, y_i)$ and the facts that $V_i(k_i, y_i)$ is strictly increasing in y_i and $G_i(v_i) > 0$ for $v_i < \hat{\nu}_i$.

b) Let $y_i' < y_i$. We want to show that $\phi_i(k_i, y_i) - \phi_i(k_i, y_i')$ is non-increasing in k_i and strictly decreasing if $\hat{\nu}_i > V_i(k_i, y_i) > V_i(k_i, y_i')$. If $\hat{\nu}_i > V_i(k_i, y_i) > V_i(k_i, y_i')$ then $\phi_i(k_i, y_i) - \phi_i(k_i, y_i') = -\int_{V_i(k_i, y_i')}^{V_i(k_i, y_i)} G_i(z)dz$ so we want to show that the negative of this, or $\int_{V_i(k_i, y_i')}^{V_i(k_i, y_i)} G_i(z)dz$, is strictly increasing in k_i. We know this expression is continuous and differentiable almost everywhere, so to show weak (strict) monotonicity it suffices to show the derivative is non-negative (strictly positive) at its points of differentiability, because the remaining points have measure zero. At any point of differentiability

$$\frac{\partial}{\partial k_i} \int_{V_i(k_i, y_i')}^{V_i(k_i, y_i)} G_i(z)dz = G_i(V_i(k_i, y_i)) \frac{\partial V_i(k_i, y_i)}{\partial k_i} - G_i(V_i(k_i, y_i')) \frac{\partial V_i(k_i, y_i')}{\partial k_i}$$

$$\geq G_i(V_i(k_i, y_i')) \frac{\partial V_i(k_i, y_i)}{\partial k_i} - G_i(V_i(k_i, y_i')) \frac{\partial V_i(k_i, y_i')}{\partial k_i}$$

because $G_i(V_i)$ is non-increasing in y_i and $\partial V_i / \partial k_i < 0$ by assumption A3. But this equals $G_i(V_i(k_i, y_i'))[\frac{\partial V_i(k_i, y_i)}{\partial k_i} - \frac{\partial V_i(k_i, y_i')}{\partial k_i}]$ which is > 0 by assumption A5, so $\phi_i(k_i, y_i)$

has strictly antitone differences as long as $V_i(k_i, y_i) < \hat{\nu}_i$. To complete part (b) we need to show antitone differences when $V_i(k_i, y_i) \geq \hat{\nu}_i$. If $V_i(k_i, y_i) > V_i(k_i, y_i') \geq \hat{\nu}_i$ then $\phi_i(k_i, y_i) - \phi_i(k_i, y_i') = 0$ a constant. If $V_i(k_i, y_i) \geq \hat{\nu}_i > V_i(k_i, y_i')$ then $\phi_i(k_i, y_i) - \phi_i(k_i, y_i') = -\int_{V_i(k_i,y_i')}^{\hat{\nu}_i} G_i(z)dz$ which is strictly decreasing in k_i because V_i is strictly decreasing in k_i and $G_i > 0$.

c) It is clear that $\phi_i(k, y_i) = -g_i(k) + \int_{V_i(k,y_i)}^{\hat{\nu}_i} G_i(z)dz$ is strictly increasing in k when $k < k_i^L$, so any optimal k will be greater than or equal to k_i^L.

d) Part (d) follows from part (b) and known results (c.f. Topkis, 1978, Theorem 6.1) for monotone optimal policies.

e) Suppose $V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i$, then $G_i(V_i(\gamma_i(y_i), y_i) > 0$ and at $\gamma_i(y_i) = k_i^L$ we have

$$\frac{\partial}{\partial k_i}\{-g_i(k_i) + \int_{V_i(k_i,y_i)}^{\hat{\nu}_i} G_i(z)dz\} = -G_i(V_i(k_i^L, y_i))\frac{\partial V_i(k_i, y_i)}{\partial k_i} > 0.$$

If k_i^L is not a point of differentiability for the bracketed expression, we can take a right side limit and achieve the same result that the principal is strictly better off increasing k_i from k_i^L. So k_i^L cannot be optimal and $\gamma_i(y_i) > k_i^L$. The particular case of $V_i(k_i^L, y_i) < \hat{\nu}_i$ yields, because $\gamma_i(y_i) \geq k_i^L$ always by part (c), $V_i(\gamma_i(y_i), y_i) \leq V_i(k_i^L, y_i) < \hat{\nu}_i$ so $\gamma_i(y_i) > k_i^L$.

f) Part (e) showed that $V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i$ implies $\gamma_i(y_i) > k_i^L$, and the inverse is $\gamma_i(y_i) = k_i^L$ implies $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$. So we need to show that $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ implies $\gamma_i(y_i) = k_i^L$. Suppose $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ then $V_i(k_i^L, y_i) \geq V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ because $\gamma_i(y_i) \geq k_i^L$ always. So, $\gamma_i(y_i) > k_i^L$ implies $-g_i(\gamma_i(y_i)) + \int_{V_i(\gamma_i(y_i),y_i)}^{\hat{\nu}_i} G_i(z)dz = -g_i(\gamma_i(y_i)) < -g_i(k_i^L) \leq -g_i(k_i^L) + \int_{V_i(k_i^L,y_i)}^{\hat{\nu}_i} G_i(z)dz$ contradicting the optimality of $\gamma_i(y_i)$.

g) Suppose $y_i > y_i'$, $\gamma_i(y_i') = k_i^L$ and $\gamma_i(y_i) > k_i^L$. From the optimality of k_i^L at y_i' we know that

$-g_i(k_i^L) + \int_{V_i(k_i^L,y_i')}^{\hat{\nu}_i} G_i(z)dz = -g_i(k_i^L)$ (because $V_i(k_i^L, y_i') \geq \hat{\nu}_i$ and part (e))

$\geq -g_i(k_i) + \int_{V_i(k_i,y_i')}^{\hat{\nu}_i} G_i(z)dz$ for all k_i. Now, for $y_i > y_i'$ we have if $\gamma_i(y_i) > k_i^L$ then

$-g_i(\gamma_i(y_i)) + \int_{V_i(\gamma_i(y_i),y_i)}^{\hat{\nu}_i} G_i(z)dz \geq -g_i(k_i^L) + \int_{V_i(k_i^L,y_i)}^{\hat{\nu}_i} G_i(z)dz = -g_i(k_i^L)$ (because $V_i(k_i^L, y_i) > V_i(k_i^L, y_i') \geq \hat{\nu}_i$). So, together we have that for all k_i,

$-g_i(k_i) + \int_{V_i(k_i,y_i')}^{\hat{\nu}_i} G_i(z)dz \leq -g_i(k_i^L) \leq -g_i(\gamma_i(y_i)) + \int_{V_i(\gamma_i(y_i),y_i)}^{\hat{\nu}_i} G_i(z)dz.$

Let $k_i = \gamma_i(y_i)$ and this implies that $\int_{V_i(\gamma_i(y_i),y_i')}^{\hat{\nu}_i} G_i(z)dz \leq \int_{V_i(\gamma_i(y_i),y_i)}^{\hat{\nu}_i} G_i(z)dz.$

But since $V_i(k_i, y_i)$ is strictly increasing in y_i, this is impossible unless $G_i(V_i(\gamma_i(y_i), y_i') = 0$, which since G_i is non-increasing implies $G_i(V_i(\gamma_i(y_i), y_i)) = 0$. That is, $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ and $\gamma_i(y_i) = k_i^L$ by part (f), contradicting the assumption that $\gamma_i(y_i) > k_i^L$.

h) The monotonicity result follows from Topkis (1978) Theorem 6.3 in the region over which $\phi_i(k_i, y_i)$ has strictly antitone differences, which by part (b) is where $V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i$. If $V_i(\gamma_i(y_i), y_i) \geq \hat{\nu}_i$ then $\gamma_i(y_i) = k_i^L$ on $[y_i, \overline{y}_i]$ by part (f). Since $\gamma_i(y_i) \geq k_i^L$ always by part (c), γ_i is non-increasing in y_i.

i) Since γ_i is non-increasing and $V_i(k_i, y_i)$ is strictly decreasing in k_i and strictly increasing in y_i, $V_i(\gamma_i(y_i), y_i)$ is strictly increasing in y_i.

j) For any set of fixed strategies by agents, the principal will perceive the upper limit of the support of agent i's type to be $\overline{v}_i = V_i(\gamma_i(\overline{y}_i), \overline{y}_i)$, and an optimal mechanism will leave no information rents for such an agent. That is, even if the agent gets the contract, he will be just compensated for variable costs and no more, leaving his final profit equal to $-g_i(\gamma_i(\overline{y}_i))$. So, $\gamma_i(\overline{y}_i)$ must equal k_i^L, the unique minimizer of g_i. As a result, $\overline{v}_i = V_i(k_i^L, \overline{y}_i)$. $\diamond$

Lemma 3: For any $\gamma_i \in \Gamma_i$

a) $\gamma_i(y_i) > k_i^L$ for $y_i < \psi_i$.

b) $\gamma_i(y_i) = k_i^L$ for $y_i > \psi_i$.

c) If $\psi_i < \bar{y}$ then $\hat{\nu}_i \leq V_i(k_i^L, \psi_i)$.

Proof: Parts (a) and (b) follow from the definition of ψ_i and Lemma 2 parts (e) and (f). We prove part (c) by contradiction. Suppose $\psi_i < \overline{y}_i$ and $V_i(k_i^L, \psi_i) < \hat{\nu}_i$. Then, by the continuity of V_i in y_i there would exist a $y_i > \psi_i$ such that $V_i(k_i^L, y_i) < \hat{\nu}_i$, but then $V_i(\gamma_i(y_i), y_i) \leq V_i(k_i^L, \psi_i) < \hat{\nu}_i$ contradicting the definition of ψ_i. $\diamond$

Lemma 4: For any fixed investment strategies by agents, let $P^{v_0}(v)$ denote the set of optimal allocations at report vector v. For any $v_0' < v_0$ and v:

a) For any $p' \in P^{v_0'}(v)$ there exists a $p \in P^{v_0}(v)$ with $p_i' \leq p_i$ for all i.

b) For any $p \in P^{v_0}(v)$ there exists a $p' \in P^{v_0'}(v)$ with $p_i' \leq p_i$ for all i.

c) For any $G_i^{v_0'}(s)$, there exists a $G_i^{v_0}(s)$ such that $G_i^{v_0'}(s) \leq G_i^{v_0}(s)$ for all s.

d) For any $G_i^{v_0}(s)$, there exists a $G_i^{v_0'}(s)$ such that $G_i^{v_0'}(s) \leq G_i^{v_0}(s)$ for all s.

Proof: Given any report vector v the principal will choose an allocation p to maximize

$$\sum_{i=1}^{n} p_i[v_0 - \overline{c}_i(v_i)]$$

subject to the constraints that $p_i \geq 0$ and $\sum_{i=1}^{n} p_i = 1$. Define $\overline{c}_{min}$ to be the minimum $\overline{c}_i(v_i)$ over all agents i, and define M to be the set of agents with that minimal virtual cost, that is $M = \{i|\overline{c}_i(v_i) = \overline{c}_{min}\}$. When $v_0 < \overline{c}_{min}$ the set of optimal allocations is a singleton, the zero vector. When $v_0 \geq \overline{c}_{min}$ the set of optimal allocations is the set of all nonnegative p such that $\sum_{i \in M} p_i = 1$. (a) through (d) all follow from this. $\diamond$

Lemma 5:

If an equilibrium exists at any $v_0 \leq \infty$, then for every equilibrium (γ, p, x) there exists an equilibrium $(\gamma^\infty, p^\infty, x^\infty)$ with $v_0 = \infty$ such that for all agents i:

a) $\gamma_i^\infty(y_i) = \gamma_i(y_i)$ when $y_i < \psi_i^\gamma$.

b) $G_i^\infty(v_i) = G_i(v_i)$ if $v_i < \hat{\nu}_i$.

Proof: Define $\tilde{v}_i = inf\{v_i|\overline{c}_i(v_i) > v_0\}$. For any equilibrium (γ, p, x), we first show that there exists an optimal allocation responding to γ, but with $v_0 = \infty$, such that $G_i^\infty(v_i) = G_i(v_i)$ when $v_i < \tilde{v}_i$ for all agents i. We then show that any agent i facing such a G_i^∞ can optimally choose an investment strategy γ_i^∞ satisfying part (a).

If $\overline{c}_i(\overline{v}_i) \leq v_0$ define $\tilde{v}_i = \overline{v}_i$. With just one agent we would have $\tilde{v}_i = \hat{\nu}_i$. But, with multiple asymmetric agents it is possible that a bid v_i by agent i satisfies $\overline{c}_i(v_i) < v_0$ yet $p_i(v_{-i}, v_i) = 0$ for all v_{-i} that can occur with positive probability. This would occur, for example, if v_i is higher than $\overline{v}_j$ for some agent j.

Let γ be from the equilibrium (γ, p, x), and G_i generated from p as usual. For that fixed γ, we know from Lemma 4 that there exists an allocation response G_i^∞ (when $v_0 = \infty$) with $G_i^\infty \geq G_i$ everywhere. We now show we can choose among these an allocation such that $G_i^\infty(v_i) = G_i(v_i)$ when $v_i < \tilde{v}_i$. Since $\hat{\nu}_i \leq \tilde{v}_i$, it suffices to show this for $v_i < \tilde{v}_i$. For any report vector v define $\overline{c}_{min}$ and M as in Lemma 4. $\overline{c}_i(v_i) \geq \overline{c}_{min}$ always. If $\overline{c}_i(v_i) > \overline{c}_{min}$ then $p_i = 0$ is optimal and will remain optimal as v_0 increases to infinity. If $\overline{c}_i(v_i) = \overline{c}_{min}$ and $v_i < \tilde{v}_i$ (implying $\overline{c}_i(v_i) \leq v_0$) then the set of optimal allocations is the set of nonnegative p vectors such that $\sum_{i \in M} p_i = 1$. This set remains constant as v_0 increases to infinity. So for all agents i and report vectors v with $v_i < \tilde{v}_i$, and all optimal allocations $p(v)$ there exists an allocation optimal at $v_0 = \infty$ with $p_i^\infty(v) = p_i(v)$. Clearly, in this case $G_i^\infty(v_i) = G_i(v_i)$ as well. So, with γ fixed there exists an optimal response by the principal (but with $v_0 = \infty$) that generates G_i^∞ with $G_i^\infty(v_i) = G_i(v_i)$ whenever $v_i < \hat{\nu}_i$.

We now show that facing any such G_i^∞, agent i can optimally invest in $k = \gamma_i^\infty(y_i)$ whenever $y_i < \psi_i^\gamma$, which will complete the proof. Denote

$$\phi_i^\infty(k, y_i) = -g_i(k) + \int_{V_i(k,y_i)}^{\overline{v}_i} G_i^\infty(s)ds$$

and let ϕ_i denote this expression with G_i instead of G_i^∞. First, note that $k = \gamma_i(y_i)$ maximizes $\phi_i(k, y_i)$ over $k \in [\underline{k}_i, \overline{k}_i]$, and almost everywhere

$$\frac{\partial \phi_i}{\partial k} = -\frac{\partial g_i}{\partial k} - G_i(V_i(k, y_i))\frac{\partial V_i}{\partial k}.$$

Since $\partial V_i/\partial k < 0$, the right hand side of this equation is nondecreasing in G_i. But we have chosen $G_i^\infty \geq G_i$, so almost everywhere

$$\frac{\partial}{\partial k}\left[\phi_i^\infty(k, y_i) - \phi_i(k, y_i)\right] \geq 0.$$

But, this means that $\phi_i^\infty(k, y_i) - \phi_i(k, y_i)$ is nondecreasing in k ($\phi_i^\infty(k, y) - \phi_i(k, y)$ is continuous and the measure zero points of nondifferentiability do not contribute to the integral) so from Topkis (1978) Lemma 6.1 for all k optimal with G_i there exists a k^∞ optimal with G_i^∞ such that $k^\infty \geq k$. We will now show that in addition we can choose such a k^∞ such that $k^\infty = k$ whenever $V_i(k, y_i) \leq \hat{\nu}_i$.

For any agent i and y_i, $k^\infty = \gamma_i^\infty(y_i)$ must maximize $\phi_i^\infty(k, y_i)$ over $k \in [\underline{k}_i, \overline{k}_i]$. Call this problem I. Define problem II to be to choose k^∞ subject to the additional constraint that $V_i(k^\infty, y_i) \leq \hat{\nu}_i$. Let $F(I)$ denote the set of feasible solutions to I and $S^*(I)$ the set of optimal solutions to I, with $F(II)$ and $S^*(II)$ defined analogously for problem II. We know (c.f. Lovejoy 2006 Lemma 1) that if $S^*(I) \cap F(II) \neq \emptyset$ then $S^*(II) = S^*(I) \cap F(II)$. Now, consider any k optimal in $\phi_i(k, y_i)$ such that $V_i(k, y_i) \leq \hat{\nu}_i$. We have chosen a k^∞ optimal in $\phi_i^\infty(k, y_i)$ with $k^\infty \geq k$, which means $V_i(k^\infty, y_i) \leq V_i(k, y_i) \leq \hat{\nu}_i$, that is $S^*(I) \cap F(II)$ is not empty. So any optimal solution to II is also optimal in I. But, problem II is to choose k subject to $k \in [\underline{k}_i, \overline{k}_i]$ and $V_i(k, y_i) \leq \hat{\nu}_i$ to maximize

$\phi_i^\infty(k, y_i) = -g_i(k) + \int_{V_i(k,y_i)}^{\overline{v}_i} G_i^\infty(s)ds$

$= -g_i(k) + \int_{V_i(k,y_i)}^{\hat{\nu}_i} G_i^\infty(s)ds + \int_{\hat{\nu}_i}^{\overline{v}_i} G_i^\infty(s)ds$

$= -g_i(k) + \int_{V_i(k,y_i)}^{\hat{\nu}_i} G_i(s)ds + \text{constant}$

$= \phi_i(k, y_i) + \text{constant}$

where we have used the fact that $G_i^\infty(v_i) = G_i(v_i)$ when $v_i < \hat{\nu}_i$ and $G_i(v_i) = 0$ when $v_i > \hat{v}_i$. Note that $\gamma_i(y_i)$ maximizes this expression in an unconstrained fashion, so if $\gamma_i(y_i)$ is feasible in II (that is, if $V_i(\gamma_i(y_i), y_i) \leq \hat{\nu}_i$) it is optimal in II, so $\gamma_i(y_i)$ is also optimal in problem I. This implies $\gamma_i^\infty(y_i) = \gamma_i(y_i)$ and completes the proof. $\diamond$

Lemma 6:

a) In any equilibrium for all agents i, $\hat{\nu}_i \leq \limsup_{y \uparrow \psi_i^\gamma} V_i(\gamma_i(y), y)$.

b) No pure strategy equilibrium can exist in which $G_i(v_i)$ is discontinuous at $\hat{\nu}_i \in (\underline{v}_i, \overline{v}_i)$ for any i.

Proof: (a) For ease of notation define

$\underline{L}^\gamma = \liminf_{y \downarrow \psi_i^\gamma} V_i(\gamma_i(y), y)$ and $\overline{L}^\gamma = \limsup_{y \uparrow \psi_i^\gamma} V_i(\gamma_i(y), y)$.

Because $V_i(\gamma_i(y_i), y_i)$ is strictly increasing and y_i has a continuous density, the set $\{y | \overline{L}^\gamma \leq V_i(\gamma_i(y), y) \leq \underline{L}^\gamma\}$ has (λ and μ) measure zero. But by the definition of ψ_i^γ we know that $y_i > \psi_i^\gamma$ will imply $V_i(\gamma_i(y), y) > \hat{\nu}_i$ so $\underline{L}^\gamma \geq \hat{\nu}_i$ and the set $\{y | \overline{L}^\gamma \leq V_i(\gamma_i(y), y) \leq \hat{\nu}_i\} \subseteq \{y | \overline{L}^\gamma \leq V_i(\gamma_i(y), y) \leq \underline{L}^\gamma\}$, so the former set has μ-measure zero. It follows that $\hat{\nu}_i > \overline{L}^\gamma$ cannot be optimal for the principal, because with any such $\hat{\nu}_i$ she would be strictly better off lowering it to $\overline{L}^\gamma$. By doing so, she would not change any allocations except on a set of reported values that occurs with probability zero, but she would transfer a strictly lower amount to agent i. This is because the information rent to agent i at y is

$$\int_{V_i(\gamma_i(y), y)}^{\hat{\nu}_i} G_i(s) ds$$

and by definition $G_i(s) > 0$ for $s < \hat{\nu}_i$.

b) $G_i(v_i)$ discontinuous at $\hat{\nu}_i \in (\underline{v}_i, \overline{v}_i)$ means that there exists an $\epsilon > 0$ such that $G_i(v_i) > \epsilon$ for all $v_i < \hat{\nu}_i$. $G_i(v_i)$ bounded away from zero means that an optimal investment level is bounded away from k_i^L. This is because at y_i agent i will choose $k \in [\underline{k}_i, \overline{k}_i]$ to maximize $\phi_i(k, y_i)$ and almost everywhere

$$\frac{\partial \phi_i(k, y_i)}{\partial k} = -\frac{\partial g_i(k)}{\partial k} - G_i(V_i(k, y_i)) \frac{\partial V_i(k, y_i)}{\partial k}.$$

From Assumptions A3 and A5, we know that

$$\frac{\partial V_i(k, y_i)}{\partial k} \leq \frac{\partial V_i(\overline{k}_i, y_i)}{\partial k} \leq \frac{\partial V_i(\overline{k}_i, \overline{y}_i)}{\partial k} < 0.$$

So for any optimal k such that $V_i(k, y_i) < \hat{\nu}_i$, $k = k_i^L$ is not an option (Lemma 2e) and in fact there will exist a fixed $\chi > 0$ such that a necessary condition for optimality is that $\frac{\partial g_i(k)}{\partial k} > \chi$, implying the existence of a $\delta > 0$ such that an optimal k with $V_i(k, y_i) < \hat{\nu}_i$ must satisfy $k > k_i^L + \delta$.

Now, for any $y_i < \psi_i^\gamma$ we will have $V_i(\gamma_i(y_i), y_i) < \hat{\nu}_i$ and $\gamma_i(y_i) > k_i^L + \delta$. We know from part (a) that $\hat{\nu}_i \leq \overline{L}^\gamma$. So for any sequence $y_n \uparrow \psi_i^\gamma$ we have

$$\limsup_{n \to \infty} \Big\{ -g_i(\gamma_i(y_n)) + \int_{V_i(\gamma_i(y_n), y_n)}^{\hat{\nu}_i} G_i(s) ds \Big\} \leq -g_i(k_i^L + \delta) < -g_i(k_i^L).$$

But this means that there exists a $y_i < \psi_i^\gamma$ such that $\gamma_i(y_i) > k_i^L$ is dominated by k_i^L, and so γ_i cannot be an equilibrium investment strategy for agent i. $\diamond$

Proposition 2: For any pure strategy equilibrium (γ, p, x) and any agent i with $\hat{\nu}_i \in (\underline{v}_i, \overline{v}_i)$, there will exist an equilibrium $(\gamma^\infty, p^\infty, x^\infty)$ with $v_0 = \infty$ such that

a) $\hat{\nu}_i = \hat{\nu}_i^\infty$

b) $\gamma_i(y_i) = \gamma_i^\infty(y_i)$ almost everywhere on $[\underline{y}_i, \overline{y}_i]$

c) $G_i(v_i) = G_i^\infty(v_i)$ almost everywhere on $[\underline{v}_i, \overline{v}_i]$.

Proof: From Lemma 5 we know that under the conditions stated there will exist an equilibrium $(\gamma^\infty, p^\infty, x^\infty)$ such that $G_i^\infty(v_i) = G_i(v_i)$ for $v_i < \hat{\nu}_i$. Hence $G_i^\infty(v_i) > 0$ for $v_i < \hat{\nu}_i$ and hence $\hat{\nu}_i^\infty \geq \hat{\nu}_i$. In fact, we must have $\hat{\nu}_i^\infty = \hat{\nu}_i$, because if $\hat{\nu}_i^\infty > \hat{\nu}_i$ we would have $G_i(v_i) = G_i^\infty(v_i) \geq G_i^\infty(\hat{\nu}_i) > 0$ for all $v_i < \hat{\nu}_i$ (Lemma 5b). But this means that $G_i(v_i)$ is discontinuous at $\hat{\nu}_i$, so (γ, p, x) cannot be an equilibrium (Lemma 6b). So, $\hat{\nu}_i^\infty = \hat{\nu}_i$ which implies that $G_i^\infty(v_i) = G_i(v_i)$ for $v_i < \hat{\nu}_i$ and $G_i^\infty(v_i) = G_i(v_i) = 0$ for $v_i > \hat{\nu}_i$ so the only place these can differ is at $\hat{\nu}_i$. Likewise, $\gamma_i(y_i) = \gamma_i^\infty(y_i)$ for $y < \psi_i^\gamma$ and $\gamma_i(y_i) = \gamma_i^\infty(y_i) = k_i^L$ for $y_i > \psi_i^\infty$, so these two can differ only at ψ_i^∞. $\diamond$

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Figure 1

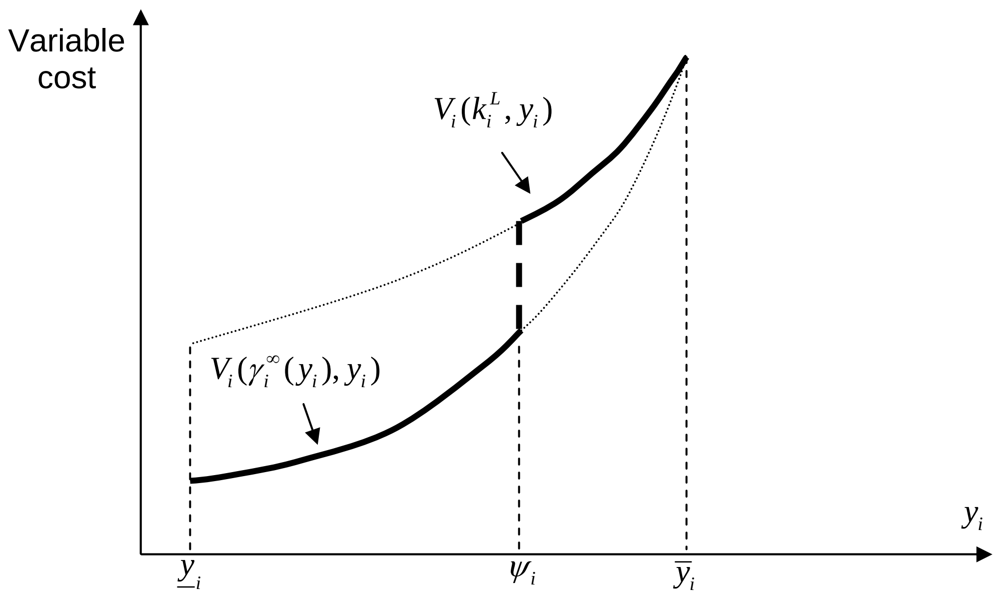

Figure 2