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QCD sum rules on the light cone and B $\to \pi$ form factors

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Abstract. The semileptonic decay B $\to \pi \bar{l} \nu$ is one of the most important reactions for the determination of the CKM matrix element $|V_{ub}|$. However, in order to extract $|V_{ub}|$ from data one needs an accurate theoretical calculation of the hadronic matrix element describing the B to π transition. QCD sum rules, based on operator product expansion on the light cone, provide a reliable approach to achieve this aim. QCD corrections and higher twist contributions can be taken systematically into account.

1. Motivation: CP violation and the CKM matrix

Within the standard model CP violation is parametrized by a complex phase in the CKM-matrix V_{CKM}. The various entries of V_{CKM} are known with different accuracy: the best known matrix elements are V_{ud} and V_{us}: the first one is obtained from the comparison of super-allowed nuclear β-decay with μ-decay, the latter one from the decay K $\to \pi \bar{l} \nu$. Among the matrix elements which are fairly well known are V_{cd} (obtained from single charm production in deep-inelastic νN-scattering and from semileptonic decays of charmed mesons), V_{cs} (which can be obtained from the decay D $\to \bar{K} \bar{l} \nu$) and V_{cb}. The latter one can be measured either inclusively, using B $\to$ D transitions, or exclusively in the semileptonic B $\to$ D transition. Among the matrix elements which are least well known are V_{ub}, V_{td}, V_{ts} and V_{tb}. The matrix elements involving the top quark may be obtained from B_d–$\bar{B}_d$ mixing (V_{td}), B_s–$\bar{B}_s$ mixing (V_{ts}) and single top production (V_{tb}). The remaining one, V_{ub}, can be obtained either from the lepton spectrum in inclusive B $\to X \bar{l} \nu$ decays or from exclusive semileptonic B-decays.

In order to extract the entries of the V_{CKM} matrix from the experimental data one needs certain input information from theory. In many cases the theoretical calculations have to rely on non-perturbative methods for QCD. Among the techniques which have been used are: chiral perturbation theory (for the extraction of V_{us}), heavy quark effective theory (for V_{cb}), lattice calculations (for V_{ub}, V_{cs} and V_{td}), QCD sum rules (for V_{ub} and V_{cs}) as well as quark models (for V_{ub} and V_{cs}). Among these the first four enjoy the property that they are based on first principles, whereas quark models are, as the name already indicates, phenomenological models, whose systematic errors are difficult to quantify.

In this paper we will focus on the extraction of $|V_{ub}|$ from the exclusive semileptonic decay B $\to \pi \bar{l} \nu$, using QCD sum rules techniques. In the next section we explain the basic principles underlying the sum rule technique. QCD sum rules are based on the operator product expansion (OPE). We mention briefly the differences between a short-distance expansion and an expansion on the light cone. In the last section we will focus on the sum rule for the decay B $\to \pi \bar{l} \nu$ and give numerical results.

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2. The QCD sum rule technique

The derivation of a QCD sum rule [1] involves the following six steps.

Step 1: take a correlation function. For example, in order to calculate the B-meson decay constant f_B using QCD sum rules, one would start from the correlation function

$$\Pi(q^2) = \mathrm{i} \int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}qx} \langle 0|T(\bar{\mathrm{u}}(x)\mathrm{i}\gamma_5 \mathrm{b}(x), \bar{\mathrm{b}}(0)\mathrm{i}\gamma_5 \mathrm{u}(0))|0\rangle. \tag{1}$$

Step 2: write a dispersion relation for $\Pi(q^2)$:

$$\Pi(q^2) = \frac{1}{\pi} \int \mathrm{d}s \frac{\mathrm{Im}\,\Pi(s)}{s - q^2} + \text{subtractions}. \tag{2}$$

Step 3: express the absorptive part of $\Pi(q^2)$ in terms of hadronic quantities. In our example above this would involve f_B.

Step 4: calculate the correlation function in the asymptotic region using QCD and OPE.

Step 5: use quark–hadron duality to relate the hadronic representation from step 3 to the absorptive part of the QCD calculation from step 4.

Step 6: the sum rule can be improved by applying a Borel transformation to both sides.

These steps provide a ‘cooking recipe’ for QCD sum rules. A few comments are in order here. The QCD calculation in step 4 is based on the OPE. A nonlocal composite operator like $j(x)j(0)$ (in the example above we have $j(x) = \bar{u}(x)\mathrm{i}\gamma_5\mathrm{b}(x)$) is expanded into a series of well-defined local operators $\mathcal{O}_n$:

$$j(x)j(0) = \sum_n C_n \mathcal{O}_n. \tag{3}$$

This separates soft and hard physics: the Wilson coefficients C_n contain the information on short-distance physics and can be calculated using perturbation theory. The matrix elements of the local operators $\mathcal{O}_n$ parametrize the long-distance physics. They are universal non-perturbative quantities. There are two versions of the operator product expansion: the short-distance expansion and the expansion on the light cone. In the former one the various local operators are classified by their dimensions, whereas in the latter one the classification goes by twist (dimension minus spin).

The sum rule can be improved by applying the Borel operator to both the hadronic representation and the QCD calculation. The Borel operator is given by

$$\hat{B} = \lim_{Q^2\to\infty, n\to\infty, Q^2/n=M^2} \frac{1}{(n-1)!}(Q^2)^n \left(-\frac{\mathrm{d}}{\mathrm{d}Q^2}\right)^n. \tag{4}$$

The Borel transformation gives rise to an exponential suppression of the higher resonances and the continuum in the hadronic representation. The power corrections in the QCD calculations are suppressed by factors of $(1/M^2)^n$. An upper limit on M^2 is obtained by requiring that the contributions from the higher resonances and the continuum should not be too large. A lower limit on M^2 is obtained by requiring that terms suppressed by powers of $1/M^2$ should be subdominant. It is important to check that this defines a window, in which the final results are insensitive to the variation of the Borel parameter M^2.

3. $\mathrm{B} \to \pi \bar{\mathrm{l}} \nu$ and sum rules on the light cone

The relevant hadronic matrix element is parametrized by two form factors f^+ and f^-:

$$\langle \pi(q)|\bar{\mathrm{u}}\gamma_\mu \mathrm{b}|\mathrm{B}(p+q)\rangle = 2f^+(p^2)q_\mu + (f^+(p^2) + f^-(p^2))p_\mu. \tag{5}$$

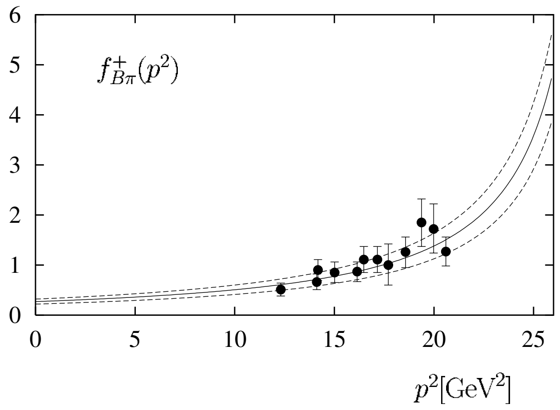

Figure 1. The LCSR prediction [11] for the B $\to$ π form factor $f^+_{\mathrm{B}\pi}(p^2)$ (solid curve) in comparison to lattice results [8]. The estimated theoretical uncertainties are shown by dashed curves.

If we neglect lepton masses only the form factor f^+ gives a contribution to the decay width. In order to obtain f^+ from QCD sum rules we start from the correlation function

$$\begin{aligned} F_\mu(p,q) &= \mathrm{i}\int \mathrm{d}^4x\, \mathrm{e}^{\mathrm{i}px}\langle\pi(q)|T(\bar{\mathrm{u}}(x)\gamma_\mu \mathrm{b}(x), \bar{\mathrm{b}}(0)\mathrm{i}m_\mathrm{b}\gamma_5\,\mathrm{d}(0))|0\rangle \\ &= F(p^2,(p+q)^2)q_\mu + \tilde{F}(p^2,(p+q)^2)p_\mu. \end{aligned} \tag{6}$$

It can be shown [2] that a short-distance expansion is only useful in the soft-pion limit ($q \to 0$). A better approach is provided by the expansion around the light cone $x^2 = 0$ with operators of increasing twist. In our case the leading twist-2 contribution is given by

$$\langle\pi(q)|\bar{\mathrm{u}}(x)\gamma_\mu\gamma_5\,\mathrm{d}(0)|0\rangle = -\mathrm{i}q_\mu f_\pi \int_0^1 \mathrm{d}u\,\mathrm{e}^{\mathrm{i}uqx}\varphi_\pi(\mathrm{u}). \tag{7}$$

$\varphi_\pi(\mathrm{u})$ is known as the twist-2 pion light-cone wavefunction. The correlation function F can now be written as a convolution of the pion wavefunction with a hard-scattering amplitude $T(p^2,(p+q)^2,\mathrm{u})$:

$$F(p^2,(p+q)^2) = -f_\pi \int_0^1 \mathrm{d}u\,\varphi_\pi(\mathrm{u})T(p^2,(p+q)^2,\mathrm{u}). \tag{8}$$

The hard-scattering amplitude T can be calculated within perturbation theory, whereas the light-cone wavefunction contains the non-perturbative information. At next-to-leading order (NLO) both the hard-scattering amplitude T and the wavefunction φ_π depend on a factorization scale μ. The evolution of φ_π with this scale can be calculated perturbatively [3]. The Brodsky–Lepage evolution kernel for the pion wavefunction is analogous to the Altarelli–Parisi kernel describing the evolution of parton densities. It turns out that it is convenient to express the wavefunction in terms of Gegenbauer polynomials. To leading order these Gegenbauer polynomials are eigenfunctions of the evolution kernel. However this is no longer true at NLO and mixing effects have to be taken into account. At present, the twist-2 contributions have been calculated to NLO [4–6], and twist 3 and twist 4 to leading order [7]. Figure 1 shows the result for the B $\to$ π form factor obtained from the light-cone sum rule in comparison to lattice results. The theoretical error is estimated from a variation of the Borel parameter, the b-quark mass, the threshold parameter s_0, the quark condensate density and the normalization scale. Furthermore, our insufficient knowledge of the shape of the light-cone wavefunctions, unknown higher-order perturbative corrections or higher twist effects contribute as well to the theoretical error. Our final result is fitted to the parametrization [9]

$$f^+(p^2) = \frac{f^+(0)}{(1 - \frac{p^2}{m_{B^*}^2})(1 - \alpha \frac{p^2}{m_{B^*}^2})} \tag{9}$$

with

$$f^+(0) = 0.27 \pm 0.05, \qquad \alpha = 0.35 \pm 0.01. \tag{10}$$

The form factor $f^+(p^2)$ enters the partial decay width

$$\frac{d\Gamma}{dp^2} = \frac{G_F^2 |V_{ub}|^2}{24\pi^3} (E_\pi^2 - m_\pi^2)^{3/2} (f^+(p^2))^2. \tag{11}$$

Integration yields

$$\Gamma(B^0 \to \pi^- e^+ \nu_e) = (6.7 \pm 2.8) |V_{ub}|^2 \text{ ps}^{-1}. \tag{12}$$

Comparing this with the experimental value [10]

$$\Gamma(B^0 \to \pi^- e^+ \nu_e) = (1.15 \pm 0.34) \times 10^{-4} \text{ ps}^{-1} \tag{13}$$

one obtains [11]

$$|V_{ub}| = 0.0041 \pm 0.0005 \pm 0.0006, \tag{14}$$

where the first error corresponds to the current experimental uncertainty and the second error to the estimated theoretical uncertainty. One may expect more precise data from the forecoming experiments of Babar and Belle. On the theoretical side, an improved knowledge of the pion light-cone wavefunction and perturbative corrections to twist 3 would reduce the theoretical error.

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