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Fine tuning and six-dimensional gauged $N = (1, 0)$ supergravity vacua

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Abstract

We find a new family of supersymmetric vacuum solutions in the six-dimensional chiral gauged $N = (1, 0)$ supergravity theory. They are generically of the form $\mathrm{AdS}_3 \times S^3$, where the 3-sphere is squashed homogeneously along its Hopf fibres. The squashing is freely adjustable, corresponding to changing the 3-form charge, and the solution is supersymmetric for all squashings. In a limit where the length of the Hopf fibres goes to zero, one recovers, after a compensating rescaling of the fibre coordinate, a solution that is locally the same as the well-known $(\mathrm{Minkowski})_4 \times S^2$ vacuum of this theory. It can now be viewed as a fine tuning of the new more general family. The traditional 'cosmological constant problem' is replaced in this theory by the problem of why the four-dimensional $(\mathrm{Minkowski})_4 \times S^2$ vacuum should be selected over other members of the equally supersymmetric $\mathrm{AdS}_3 \times S^3$ family. We also obtain a family of dyonic string solutions in the gauged $N = (1, 0)$ theory, whose near-horizon limits approach the AdS_3 times squashed S^3 solutions.

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1. Introduction

Six-dimensional chiral supergravities with eight real supersymmetries admitting spontaneous compactifications to four-dimensional Minkowski spacetimes have received both early [1–5] and more recent [6–8] attention as possible scenarios for achieving a naturally small cosmological constant. The starting point for a minimal model of this type is a six-dimensional chiral $N = (1, 0)$ Einstein–Maxwell supergravity with an exponential potential for the dilaton, which results from the gauging of a $U(1)$ R-symmetry. In the absence of branes, compactification to four dimensions proceeds by turning on a monopole configuration on a 2-sphere, resulting in a geometry of the form (Minkowski)$_4 \times S^2$ [1].

In general, achieving a small four-dimensional cosmological constant requires a fine tuning to balance the bulk vacuum energy with the monopole flux[4]. However, it has been argued [2–4] that a form of self-tuning occurs naturally in the gauged chiral $N = (1, 0)$ supergravity of [1]. The bosonic sector of this theory includes a 2-form potential and a dilaton, in addition to the metric and a Maxwell field. Self-tuning in this theory occurs because the cosmological constant is replaced by an exponential potential for the dilaton field. In this case, the only possibility for a maximally symmetric solution turns out to be (Minkowski)$_4 \times S^2$ [1], thus selecting a Minkowski vacuum without the need for any apparent fine tuning. The vacuum yields $N = 1$ supergravity in four dimensions with chiral matter [1].

In the framework of [6–8], this basic scenario is extended by introducing 3-branes into the (Minkowski)$_4 \times S^2$ background, giving rise to (American) 'football-shaped' extra dimensions. Being co-dimension two objects, the only effect of the 3-branes on the background is to introduce deficit angles on the S^2. In particular, the four-dimensional cosmological constant remains zero (or at least small), provided it is initially vanishing in the absence of branes. It is remarkable that this is true regardless of whether supersymmetry is preserved or broken on the 3-branes themselves. Thus we could be living on a braneworld without supersymmetry and yet still experience a naturally small cosmological constant. Essentially what happens in these models is that the cosmological constant problem is transformed into that of finding a naturally stable bulk solution admitting a flat Minkowski background.

Since the (Minkowski)$_4 \times S^2$ background is the starting point for the braneworld models with football-shaped extra dimensions, it is important to address the argued uniqueness of this background in the context of the $N = (1, 0)$ gauged supergravity theory. In this paper, we show that there is in fact a more general class of $N = 1$ supersymmetric vacuum solutions. We find a family of solutions of the form $\mathrm{AdS}_3 \times S^3$, where S^3 is the 3-sphere with a homogeneously squashed metric. There is a non-trivial parameter in the family of solutions, which allows the length L of $U(1)$ fibres in the description of S^3 as the Hopf bundle over S^2 to be adjusted freely, relative to the size of the S^2 base. In the singular limit where the length L goes to zero one can recover, after making an appropriate rescaling of the fibre coordinate, a solution that is locally the same as the previous (Minkowski)$_4 \times S^2$ solution found in [1]. The new feature in the more general family of $\mathrm{AdS}_3 \times S^3$ solutions is that in addition to the 2-form 'monopole flux', there are also electric and magnetic charges carried by the 3-form field.

In the light of our new solutions, all of which are supersymmetric, it can be argued that the ostensible absence of fine tuning in the (Minkowski)$_4 \times S^2$ solution of [1] is somewhat deceptive. In fact there *is* a fine tuning, in that it is the special case of our new supersymmetric vacua in which the electric and magnetic 3-form charges are set to zero. The familiar notion of fine tuning a maximally symmetric vacuum solution to have vanishing cosmological constant is now replaced by a rather less familiar type of fine tuning. If the 3-form charges are allowed to become non-zero not only does one lose the feature of a vanishing cosmological constant, but one also loses a dimension. Namely, one of the three spatial dimensions of the previous (Minkowski)$_4$ vacuum acquires a non-trivial 'twist' and becomes the Hopf fibre coordinate of a 3-sphere, whose S^2 base formed the original internal space in the Minkowski vacuum.

The celebrated 'cosmological constant problem' is thus replaced, in this theory, by the 'Hopf fibration problem'.

[4] For example see [9], where six-dimensional Einstein–Maxwell gravity with an arbitrary cosmological constant is compactified to $M_4 \times S^2$, where M_4 is Minkowski, de Sitter or anti-de Sitter spacetime, depending on the fine tuning of the S^2 monopole charge relative to the 6D cosmological constant.

After constructing the family of AdS_3 times squashed S^3 solutions we then show that they can be viewed as the near-horizon limits of a family of dyonic strings in the six-dimensional gauged $N = (1, 0)$ supergravity. These solutions can be viewed as generalizations of the usual dyonic strings of the ungauged theory [10]. As in those examples, the dyonic strings that we find here preserve one quarter of the original six-dimensional supersymmetry. As one reaches the AdS_3 times squashed S^3 near-horizon limit, the supersymmetry fraction increases from one quarter to one half in the standard way.

2. The six-dimensional $N = (1, 0)$ gauged supergravity

A large class of six-dimensional $N = (1, 0)$ gauged supergravities has been constructed [11–15]. In this paper, we shall be focusing on the simplest example, for which the field content comprises a graviton multiplet with bosonic fields $\left(g_{MN}, B^+_{MN}\right)$ and a chiral (complex) gravitino superpartner ψ_M; a tensor multiplet with bosonic field B^-_{MN} and a chiral spin-1/2 superpartner χ; and a vector multiplet with bosonic field A_M and a chiral superpartner λ [1].

The bosonic sector of the six-dimensional $N = (1, 0)$ gauged supergravity is described by the Lagrangian

$$\mathcal{L} = R{*}\mathbb{1} - \tfrac{1}{4}{*}\,\mathrm{d}\phi \wedge \mathrm{d}\phi - \tfrac{1}{2}\,\mathrm{e}^{\phi}\,{*}H_{(3)} \wedge H_{(3)} - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}\phi}\,{*}F_{(2)} \wedge F_{(2)} - 8g^2\,\mathrm{e}^{-\frac{1}{2}\phi}\,\mathbb{1}, \tag{1}$$

where $F_{(2)} = \mathrm{d}A_{(1)}$, $H_{(3)} = \mathrm{d}B_{(2)} + \frac{1}{2}F_{(2)} \wedge A_{(1)}$, and g is the gauge-coupling constant. This leads to the bosonic equations of motion

$$\begin{aligned}
R_{MN} &= \tfrac{1}{4}\partial_M\phi\partial_N\phi + \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}\phi}\left(F^2_{MN} - \tfrac{1}{8}F^2 g_{MN}\right) + \tfrac{1}{4}\,\mathrm{e}^{\phi}\left(H^2_{MN} - \tfrac{1}{6}H^2 g_{MN}\right) + 2g^2\,\mathrm{e}^{-\frac{1}{2}\phi} g_{MN},\\
\Box\phi &= \tfrac{1}{4}\,\mathrm{e}^{\frac{1}{2}\phi}F^2 + \tfrac{1}{6}\,\mathrm{e}^{\phi}H^2 - 8g^2\,\mathrm{e}^{-\frac{1}{2}\phi},\\
\mathrm{d}\left(\mathrm{e}^{\frac{1}{2}\phi}\,{*}F_{(2)}\right) &= \mathrm{e}^{\phi}{*}H_{(3)} \wedge F_{(2)},\\
\mathrm{d}(\mathrm{e}^{\phi}\,{*}H_{(3)}) &= 0.
\end{aligned} \tag{2}$$

The transformation rules for the fermionic fields are given by[5]

$$\begin{aligned}
\delta\psi_M &\equiv \tilde{D}_M\epsilon = \left[D_M + \tfrac{1}{48}\,\mathrm{e}^{\frac{1}{2}\phi}H^+_{NPQ}\Gamma^{NPQ}\Gamma_M\right]\epsilon,\\
\delta\chi &\equiv -\tfrac{1}{4}\Delta_\phi\epsilon = -\tfrac{1}{4}\left[\Gamma^M\partial_M\phi - \tfrac{1}{6}\,\mathrm{e}^{\frac{1}{2}\phi}H^-_{MNP}\Gamma^{MNP}\right]\epsilon,\\
\delta\lambda &\equiv \tfrac{1}{4\sqrt{2}}\Delta_F\epsilon = \tfrac{1}{4\sqrt{2}}\left[\mathrm{e}^{\frac{1}{4}\phi}F_{MN}\Gamma^{MN} - 8\mathrm{i}g\,\mathrm{e}^{-\frac{1}{4}\phi}\right]\epsilon,
\end{aligned} \tag{3}$$

where D_M is the gauge-covariant derivative, $D_M\epsilon \equiv (\nabla_M - \mathrm{i}gA_M)\epsilon$. Note that the $\pm$ superscripts appearing on the 3-form H_{MNP} in these expressions are redundant, since the chirality of ϵ already implies projections onto the self-dual or anti-self-dual parts, but we include them for convenience, to emphasize which projection occurs in which transformation rule.

3. Vacuum solution

In this section, we show that the $N = (1, 0)$ gauged supergravity theory admits a one-parameter family of supersymmetric solutions that generically are of the form $\mathrm{AdS}_3 \times S^3$. The non-trivial parameter in the solution characterizes the degree of 'squashing' of the internal 3-sphere, which is viewed as the Hopf bundle over S^2. For a particular limiting value of the parameter, in which the length of the $U(1)$ Hopf fibres tends to zero, the solution locally

[5] Note that, compared to [1], we have chosen units in which the gravitational coupling constant $\kappa = 1/2$, and have additionally rescaled the supersymmetry parameter ϵ.

approaches, after an appropriate rescaling of the shrinking Hopf fibres, the $(\text{Minkowski})_4 \times S^2$ solution found long ago in [1].

The construction of our family of $\mathrm{AdS}_3 \times S^3$ solutions proceeds straightforwardly. We make the following ansatz for the metric and other bosonic fields:

$$\begin{aligned} \mathrm{d}s_6^2 &= \mathrm{d}s_3^2 + a^2\big(\sigma_1^2 + \sigma_2^2\big) + b^2\sigma_3^2, \\ F_{(2)} &= k\sigma_1 \wedge \sigma_2, \\ H_{(3)} &= P\sigma_1 \wedge \sigma_2 \wedge \sigma_3 + \frac{P}{a^2 b}\varepsilon_{(3)}. \end{aligned} \tag{4}$$

Here, a, b, k and P are constants, and the σ_i are left-invariant 1-forms on the 3-sphere, satisfying the exterior algebra

$$\mathrm{d}\sigma_i = -\tfrac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k. \tag{5}$$

They can be represented, in terms of Euler angles (θ, φ, ψ), by

$$\sigma_1 + \mathrm{i}\sigma_2 = \mathrm{e}^{-\mathrm{i}\psi}(\mathrm{d}\theta + \mathrm{i}\sin\theta\, \mathrm{d}\varphi), \qquad \sigma_3 = \mathrm{d}\psi + \cos\theta\, \mathrm{d}\varphi. \tag{6}$$

The 3-form $\varepsilon_{(3)}$ appearing in the ansatz for $H_{(3)}$ in (4) denotes the volume form in the metric $\mathrm{d}s_3^2$. Note that $H_{(3)}$ in (4) is explicitly constructed to be self-dual. Locally, we can choose the potential for $F_{(2)}$ to be given by $A_{(1)} = -k\sigma_3$.

An elementary calculation shows that in the natural vielbein basis $e^1 = a\sigma_1, e^2 = a\sigma_2, e^3 = b\sigma_3$, the torsion-free spin connection for the S^3 factor in the metric $\mathrm{d}s_6^2$ in (4) is given by

$$\omega_{23} = -\frac{b}{2a^2}e^1, \qquad \omega_{31} = -\frac{b}{2a^2}e^2, \qquad \omega_{12} = \left(\frac{b}{2a^2} - \frac{1}{b}\right)e^3, \tag{7}$$

and the non-vanishing vielbein components of the Ricci tensor are given by

$$R_{11} = R_{22} = \frac{1}{a^2} - \frac{b^2}{2a^4}, \qquad R_{33} = \frac{b^2}{2a^4}. \tag{8}$$

The equation of motion for $H_{(3)}$ in (2) implies, in view of the ansatz (4), that ϕ is a constant, which without loss of generality we can take to be zero. The equations of motion then lead to the following relations:

$$k^2 = 16g^2a^4, \qquad b^2 = P, \qquad a^2 = b^2 + \tfrac{1}{2}k^2, \tag{9}$$

together with

$$R_{\mu\nu} = -\frac{b^2}{2a^4}g_{\mu\nu}. \tag{10}$$

Thus we can view the 3-form charge P as a free parameter, with the remaining parameters in the ansatz (4) determined by

$$b^2 = P, \qquad a^2 = \frac{k}{4g}, \qquad k(1 - 2gk) = 4gP. \tag{11}$$

We have, without loss of generality, made a specific sign choice when taking the square root of the first equation in (9). When P is non-vanishing, the solution is of the form $\mathrm{AdS}_3 \times S^3$, and since the ratio b/a is not equal to 1, the metric on the S^3 is squashed along the Hopf fibres. The equation determining a^2 has two branches, with

$$a^2 = \frac{1}{16g^2}\big(1 \pm \sqrt{1 - 32Pg^2}\big). \tag{12}$$

If we choose the + sign in (12), then a is non-vanishing in the limit $P \longrightarrow 0$, while b tends to zero. If the Euler angle ψ is rescaled to $\psi = b^{-1}z$ before taking the limit, then the metric takes the form

$$\mathrm{d}s_6^2 = \mathrm{d}s_3^2 + a^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2) + (\mathrm{d}z + b\cos\theta\,\mathrm{d}\varphi)^2. \tag{13}$$

If b goes to zero, we see from (10) that $\mathrm{d}s_3^2$ becomes Ricci flat. This leads to an 'untwisting' of the Hopf fibre coordinate z, leaving in the limit a metric that is the direct sum of the metric on S^2 and a Ricci-flat 4-metric, which could be taken to be $(\text{Minkowski})_4$. Thus one recovers the $(\text{Minkowski})_4 \times S^2$ solution of [1] as a special limiting case of our family of AdS_3 times squashed S^3 solutions, as the 3-form charge P is sent to zero[6]. The $(\text{Minkowski})_4 \times S^2$ solution can therefore be viewed as a fine tuning in which the 3-form charge P is set to zero, implying that the strength of the 2-form flux takes the specific value

$$k = \frac{1}{2g}. \tag{14}$$

It is straightforward to verify that our solution is supersymmetric for all values of P. First of all, we see from (3) that we shall have $\delta\chi = 0$, since $\phi = 0$ and $H_{(3)}$ is self-dual. In fact, the tensor multiplet is completely decoupled from this solution. Next, we see that $\delta\lambda = 0$ implies

$$\frac{k}{a^2}\Gamma_{12}\epsilon = 4\mathrm{i}g\epsilon, \tag{15}$$

which requires, since we chose $k = +4ga^2$, that

$$\Gamma_{12}\epsilon = +\mathrm{i}\epsilon. \tag{16}$$

Note that the Dirac matrices Γ_1, Γ_2 and Γ_3 are given with frame indices corresponding to the S^3. Condition (16) implies a halving of the original six-dimensional supersymmetry. Turning finally to the gravitino variation, from $\delta\psi_M = 0$ with M in the S^3 directions, we find for $\delta\psi_1 = 0$ and $\delta\psi_2 = 0$ that $b^2 = P$, which is consistent with (9). From $\delta\psi_3 = 0$ we find $b^2 = a^2(1 - 2gk)$, which, since $k = 4ga^2$, is also consistent with (9). Note that we have $\partial_i\epsilon = 0$, i.e. the Killing spinors are independent of the coordinates of S^3. Finally from $\delta\psi_\mu = 0$ in the AdS_3 directions we obtain

$$\nabla_\mu\epsilon = -\frac{\mathrm{i}P}{4a^2b}\Gamma_3\Gamma_\mu\epsilon = -\frac{\mathrm{i}}{2}\sqrt{-\frac{\Lambda}{2}}\Gamma_3\Gamma_\mu\epsilon, \tag{17}$$

where $\Lambda \equiv -P^2/(2a^4b^2) = -b^2/(2a^4)$ is, from (10), the cosmological constant of the AdS_3 spacetime. Equation (17) is nothing but a statement that ϵ must be a Killing spinor in AdS_3. This completes the demonstration that our $\mathrm{AdS}_3 \times S^3$ solutions preserve one half of the original six-dimensional supersymmetry.

[6] Since the $U(1)$ fibre coordinate ψ on S^3 has period 4π, it follows that the period of the rescaled coordinate z will tend to zero as $b = \sqrt{P}$ goes to zero. Once the limit $b = 0$ is reached, one can 'unwrap' the collapsed circle and allow z to range over the entire real line, thus giving a $(\text{Minkowski})_4 \times S^2$ topology. An alternative viewpoint would be to start from the 'fine-tuned' $(\text{Minkowski})_4 \times S^2$ solution at $P = 0$, using the coordinate z ranging over the entire real line. When P is then allowed to become non-zero, one would encounter conical singularities in the solution, since the coordinate z would be covering the $U(1)$ fibre in S^3 infinitely many times (and the 3-form charge $\int H_{(3)} = 4\pi\sqrt{P}(\Delta z)$ would be infinite owing to the infinite period (Δz) for z). These solutions can then be made regular, giving $\mathrm{AdS}_3 \times S^3$, by restricting z to have period $(\Delta z) = 4\pi\sqrt{P}$.

4. Dyonic string solutions in $N = (1, 0)$ gauged supergravity

Having shown that the $N = (1, 0)$ gauged supergravity admits a family of AdS_3 times squashed S^3 solutions, it is natural to ask whether such vacua may be related to string-like objects. In this section we show that this is indeed the case. In particular, we construct dyonic string solutions preserving $1/4$ of the original six-dimensional supersymmetry. In the near-horizon limit, these strings approach the AdS_3 times squashed S^3 solutions of the previous section, whereupon supersymmetry is partially restored from $1/4$ to $1/2$ of the original supersymmetry.

4.1. The equations of motion, and Killing spinor conditions

Our starting point is the following ansatz for string solutions:

$$\begin{aligned}
\mathrm{d}s_6^2 &= c^2\,\mathrm{d}x^\mu\,\mathrm{d}x_\mu + a^2\big(\sigma_1^2+\sigma_2^2\big) + b^2\sigma_3^2 + h^2\,\mathrm{d}r^2,\\
H_{(3)} &= P\sigma_1\wedge\sigma_2\wedge\sigma_3 + u\,\mathrm{d}^2x\wedge\mathrm{d}r,\\
F_2 &= k\sigma_1\wedge\sigma_2,
\end{aligned} \tag{18}$$

where a, b, c, h, u and ϕ are now all functions of r. The magnetic charge P and the coefficient k are constants, by virtue of the Bianchi identities for $H_{(3)}$ and $F_{(2)}$. While h may be removed by a coordinate transformation, $\mathrm{d}r' = h(r)\,\mathrm{d}r$, the string solution simplifies for a suitable choice of h (as will be apparent below). For this reason we will retain h in the following. When it is necessary to use explicit numerical vielbein component labels, we use the following basis:

$$\begin{aligned}
&e^{\tilde 0} = c\,\mathrm{d}t, &&e^{\tilde 1} = c\,\mathrm{d}x, &&e^1 = a\sigma_1,\\
&e^2 = a\sigma_2, &&e^3 = b\sigma_3, &&e^4 = h\,\mathrm{d}r.
\end{aligned} \tag{19}$$

In this orthonormal frame, the non-vanishing components of the spin connection are given by

$$\begin{aligned}
&\omega_{23} = -\frac{b}{2a^2}e^1, &&\omega_{31} = -\frac{b}{2a^2}e^2, &&\omega_{12} = \left(\frac{b}{2a^2}-\frac{1}{b}\right)e^3,\\
&\omega_{14} = \frac{a'}{ah}e^1, &&\omega_{24} = \frac{a'}{ah}e^2, &&\omega_{34} = \frac{b'}{bh}e^3, \qquad \omega^\mu{}_4 = \frac{c'}{ch}e^\mu,
\end{aligned} \tag{20}$$

and the non-vanishing components of the Ricci tensor are given by

$$\begin{aligned}
R_{\mu\nu} &= -\left[\frac{c'^2}{h^2c^2} + \frac{2a'c'}{ach^2} + \frac{b'c'}{bch^2} + \frac{1}{ch}\left(\frac{c'}{h}\right)'\right]\eta_{\mu\nu},\\
R_{11} = R_{22} &= -\frac{2a'c'}{ach^2} - \frac{a'b'}{abh^2} - \frac{a'^2}{a^2h^2} - \frac{1}{ah}\left(\frac{a'}{h}\right)' - \frac{b^2}{2a^4} + \frac{1}{a^2},\\
R_{33} &= -\frac{2b'c'}{ach^2} - \frac{2a'b'}{abh^2} - \frac{1}{bh}\left(\frac{b'}{h}\right)' + \frac{b^2}{2a^4},\\
R_{44} &= -\frac{2}{ah}\left(\frac{a'}{h}\right)' - \frac{1}{bh}\left(\frac{b'}{h}\right)' - \frac{2}{ch}\left(\frac{c'}{h}\right)'.
\end{aligned} \tag{21}$$

The field equations for $F_{(2)}$ and $H_{(3)}$ given in (2) imply

$$b^2 = P\,\mathrm{e}^{\frac{1}{2}\phi}, \qquad u = \frac{hc^2Q}{a^2b}\,\mathrm{e}^{-\phi}, \tag{22}$$

respectively, where Q is a constant characterizing the electric charge carried by $H_{(3)}$. The Einstein and dilaton equations of motion give

$$
\begin{aligned}
&\frac{c'^2}{h^2c^2} + \frac{2a'c'}{ach^2} + \frac{b'c'}{bch^2} + \frac{1}{ch}\left(\frac{c'}{h}\right)' = \frac{k^2}{8a^4}\,\mathrm{e}^{\frac{1}{2}\phi} + \frac{1}{4}\,\mathrm{e}^{\phi}\left(\frac{u^2}{h^2c^4} + \frac{P^2}{a^4b^2}\right) - 2g^2\,\mathrm{e}^{-\frac{1}{2}\phi},\\
&\frac{2a'c'}{ach^2} + \frac{a'b'}{abh^2} + \frac{a'^2}{a^2h^2} + \frac{1}{ah}\left(\frac{a'}{h}\right)' + \frac{b^2}{2a^4} - \frac{1}{a^2}\\
&\qquad = -\frac{3k^2}{8a^4}\,\mathrm{e}^{\frac{1}{2}\phi} - \frac{1}{4}\,\mathrm{e}^{\phi}\left(\frac{u^2}{h^2c^4} + \frac{P^2}{a^4b^2}\right) - 2g^2\,\mathrm{e}^{-\frac{1}{2}\phi},\\
&\frac{2b'c'}{ach^2} + \frac{2a'b'}{abh^2} + \frac{1}{bh}\left(\frac{b'}{h}\right)' - \frac{b^2}{2a^4} = \frac{k^2}{8a^4}\,\mathrm{e}^{\frac{1}{2}\phi} - \frac{1}{4}\,\mathrm{e}^{\phi}\left(\frac{u^2}{h^2c^4} + \frac{P^2}{a^4b^2}\right) - 2g^2\,\mathrm{e}^{-\frac{1}{2}\phi},\\
&\frac{2}{ah}\left(\frac{a'}{h}\right)' + \frac{1}{bh}\left(\frac{b'}{h}\right)' + \frac{2}{ch}\left(\frac{c'}{h}\right)' + \frac{\phi'^2}{4h^2} = \frac{k^2}{8a^4}\,\mathrm{e}^{\frac{1}{2}\phi} + \frac{1}{4}\,\mathrm{e}^{\phi}\left(\frac{u^2}{h^2c^4} + \frac{P^2}{a^4b^2}\right) - 2g^2\,\mathrm{e}^{-\frac{1}{2}\phi},\\
&\frac{1}{a^2bc^2h}\left(\frac{a^2bc^2\phi'}{h}\right)' = \frac{k^2}{2a^4}\,\mathrm{e}^{\frac{1}{2}\phi} + \mathrm{e}^{\phi}\left(\frac{P^2}{a^4b^2} - \frac{u^2}{h^2c^4}\right) - 8g^2\,\mathrm{e}^{-\frac{1}{2}\phi}.
\end{aligned}
\tag{23}
$$

Rather than trying to solve the rather complicated second-order Einstein and dilaton equations of motion, we shall look directly for supersymmetric solutions by studying the conditions for the existence of Killing spinors. Having found such configurations, we then verify that they do indeed satisfy the second-order field equations. First, the condition $\delta\lambda = 0$ implies

$$\frac{k}{a^2}\Gamma_{12}\epsilon = 4\mathrm{i}g\,\mathrm{e}^{-\frac{1}{2}\phi}\epsilon. \tag{24}$$

Without loss of generality, we can require that ϵ satisfies the same projection condition (16) as in section 3, and hence we have

$$a^2 = \frac{k}{4g}\,\mathrm{e}^{\frac{1}{2}\phi}. \tag{25}$$

Next, from $\delta\chi = 0$, we find using (3) that

$$\phi'\Gamma_4\epsilon = \frac{h}{a^2b}\,\mathrm{e}^{\frac{1}{2}\phi}(P - Q\,\mathrm{e}^{-\phi})\Gamma_{123}\epsilon, \tag{26}$$

where Q is given in (22). This implies a further halving of supersymmetry by a projection condition which, without loss of generality, we take to be

$$\Gamma_{1234}\epsilon = +\epsilon, \tag{27}$$

and hence we have the first-order equation

$$\phi' = -\frac{h\,\mathrm{e}^{\frac{1}{2}\phi}}{a^2b}(P - Q\,\mathrm{e}^{-\phi}). \tag{28}$$

Turning to the supersymmetry variations of the gravitino, we find that $\delta\psi_\mu = 0$ in the string worldsheet directions implies

$$\frac{c'}{hc} = -\frac{\mathrm{e}^{\frac{1}{2}\phi}}{4a^2b}(P + Q\,\mathrm{e}^{-\phi}), \tag{29}$$

where ϵ is independent of the string worldsheet coordinates x^μ. From the $\delta\psi_i = 0$ conditions in the S^3 directions, we find from $i = 1, 2$ and $i = 3$

$$\frac{a'}{ha} = \frac{\mathrm{e}^{\frac{1}{2}\phi}}{4a^2b}(P + Q\,\mathrm{e}^{-\phi}) - \frac{b}{2a^2}, \tag{30}$$

$$\frac{b'}{hb} = \frac{\mathrm{e}^{\frac{1}{2}\phi}}{4a^2b}(P + Q\,\mathrm{e}^{-\phi}) + \frac{b}{2a^2} - \frac{(1-2kg)}{b}, \tag{31}$$

respectively, where ϵ is independent of the Euler-angle coordinates on S^3. (We have made use of the projection conditions (16) and (27) here.) Finally, the variation $\delta\psi_4 = 0$ allows us to solve for the r-dependence of the Killing spinor ϵ. We have

$$\delta\psi_4 = \frac{1}{h}\frac{\partial\epsilon}{\partial r} + \frac{1}{8a^2b}\,\mathrm{e}^{\frac{1}{2}\phi}(P + Q\,\mathrm{e}^{-\phi})\Gamma_{1234}\epsilon, \tag{32}$$

whence, using (27) and comparing with (29), we find that the r-dependence of ϵ is given by

$$\epsilon(r) = c^{1/2}\epsilon_0, \tag{33}$$

where ϵ_0 is a constant spinor satisfying the same projection conditions (16) and (27).

From (25), (22), (28) and (31) we also find that the constant parameters P, g, k must be related as

$$4gP = k(1-2gk). \tag{34}$$

So far we have shown that the F and H field equations and Bianchi identities, and the Killing spinor conditions are satisfied. The remaining field equations are the Einstein and dilaton field equations. We have verified by explicit computation that they are also satisfied. In fact, this is not surprising as we will show in the next section where we study the integrability of the Killing spinor conditions.

In summary, we have a dyonic string solution given by (4), (22), (25) and (34), with h arbitrary and c, ϕ determined by (29) and (28). In section 4.3, we shall make a convenient choice for h and study various properties of our solution.

4.2. Integrability of the Killing spinor equations

In this section we shall show that once the F and H field equations and Bianchi identities, and the Killing spinor conditions are satisfied by our ansatz, the remaining Einstein and dilaton field equations are automatically satisfied as well as a consequence of the Killing spinor integrability conditions. As a by-product we will determine the full Killing spinor integrability conditions and observe that the first-order Killing spinor equations by themselves are in general insufficient to guarantee that all the equations of motion are satisfied.

We now determine the Killing spinor integrability conditions. For the gravitino variation, we may take the usual commutator of (generalized) covariant derivatives. After considerable algebra, we obtain

$$\begin{aligned}\Gamma^N[\tilde{D}_M, \tilde{D}_N] = {} & -\tfrac{1}{2}\big[R_{MN} - \tfrac{1}{4}\partial_M\phi\partial_N\phi - \tfrac{1}{2}\,\mathrm{e}^{\frac{1}{2}\phi}\left(F^2_{MN} - \tfrac{1}{8}g_{MN}F^2\right) \\ & - \tfrac{1}{4}\,\mathrm{e}^{\phi}\big(H^2_{MN} - \tfrac{1}{6}g_{MN}H^2\big) - 2g^2\,\mathrm{e}^{-\frac{1}{2}\phi}g_{MN}\big]\Gamma^N \\ & - \tfrac{1}{48}\,\mathrm{e}^{\frac{1}{2}\phi}\big(\partial_{[N}H_{PQR]} - \tfrac{3}{4}F_{[NP}F_{QR]}\big)\Gamma^{NPQR}\Gamma_M \\ & - \tfrac{1}{16}\,\mathrm{e}^{-\frac{1}{2}\phi}\nabla^N(\mathrm{e}^{\phi}H_{NPQ})\Gamma^{PQ}\Gamma_M \\ & - \tfrac{1}{8}\big(\partial_M\phi + \tfrac{1}{12}\,\mathrm{e}^{\frac{1}{2}\phi}H_{NPQ}\Gamma^{NPQ}\Gamma_M\big)\Delta_\phi + \tfrac{1}{8}\,\mathrm{e}^{\frac{1}{4}\phi}F_{MN}\Gamma^N\Delta_F \\ & - \tfrac{1}{64}\Gamma_M\big(\mathrm{e}^{\frac{1}{4}\phi}F_{NP}\Gamma^{NP} + 8\mathrm{i}g\,\mathrm{e}^{-\frac{1}{4}\phi}\big)\Delta_F,\end{aligned} \tag{35}$$

where the last two lines vanish when acting on Killing spinors. (The quantities Δ_ϕ and Δ_F are defined in (3) and are supersymmetry transformations on χ and λ, up to unimportant numerical factors.) We see that once the H field equation, Bianchi identity and the Killing spinor conditions are satisfied, and given that the Ricci tensor is diagonal, the Einstein equation is then satisfied as well.

Additional integrability conditions may be derived from the $\delta\chi$ and $\delta\lambda$ variations. For the tensor multiplet, we find

$$\begin{aligned}\Gamma^M[\tilde{D}_M, \Delta_\phi] = {} & \left[\Box\phi - \tfrac{1}{4}\,\mathrm{e}^{\frac{1}{2}\phi}F^2 - \tfrac{1}{6}\,\mathrm{e}^{\phi}H^2 + 8g^2\,\mathrm{e}^{-\frac{1}{2}\phi}\right] \\ & - \tfrac{1}{6}\,\mathrm{e}^{\frac{1}{2}\phi}\left(\partial_{[M}H_{NPQ]} - \tfrac{3}{4}F_{[MN}F_{PQ]}\right)\Gamma^{MNPQ} - \tfrac{1}{2}\,\mathrm{e}^{-\frac{1}{2}\phi}\nabla^M(\mathrm{e}^{\phi}H_{MNP})\Gamma^{NP} \\ & + \tfrac{1}{24}\,\mathrm{e}^{\frac{1}{2}\phi}H_{MNP}\Gamma^{MNP}\Delta_\phi - \tfrac{1}{8}\left(\mathrm{e}^{\frac{1}{4}\phi}F_{MN}\Gamma^{MN} + 8\mathrm{i}g\,\mathrm{e}^{-\frac{1}{4}\phi}\right)\Delta_F.\end{aligned} \tag{36}$$

This shows once the H field equation, Bianchi identity and the Killing spinor conditions are satisfied, then the dilaton field equation is satisfied as well.

Finally, from the Killing spinor condition coming from the Maxwell multiplet we find

$$\begin{aligned}\Gamma^M[\tilde{D}_M, \Delta_F] = {} & \mathrm{e}^{\frac{1}{4}\phi}\partial_{[M}F_{NP]}\Gamma^{MNP} + 2\,\mathrm{e}^{-\frac{1}{4}\phi}\left[\nabla^M\left(\mathrm{e}^{\frac{1}{2}\phi}F_{MP}\right) - \tfrac{1}{2}\,\mathrm{e}^{\phi}H_{MNP}F^{MN}\right]\Gamma^P \\ & - \tfrac{1}{4}\Gamma^M\partial_M\phi\Delta_F + \tfrac{1}{2}\,\mathrm{e}^{-\frac{1}{4}\phi}F_{MN}\Gamma^{MN}\Delta_\phi + \tfrac{1}{4}[\Delta_\phi, \Delta_F],\end{aligned} \tag{37}$$

which is automatically satisfied as a result of the F field equation and the Killing spinor conditions.

The Killing spinor integrability conditions presented above can also be used to analyse in more general situations the extent to which they imply the field equations. We leave this to a future work, and we next analyse the properties of our dyonic string solution.

4.3. The properties of the supersymmetric string solution

In this section, we shall make a convenient choice for the 'coordinate gauge function' h and show explicit form of the dyonic string solution, and study its salient properties such as its behaviour in various limits. In particular, we choose h so that the solutions for ϕ and c will be identical to those in [16] for the gauge dyonic string. This is achieved by making the gauge choice

$$h = -\frac{2a^2bc^2}{r^3}, \tag{38}$$

and defining $\phi_\pm \equiv \phi \pm 4\log c$, whereupon the equations become diagonalized, with

$$\phi_+' = \frac{4P}{r^3}\,\mathrm{e}^{\frac{1}{2}\phi_+}, \qquad \phi_-' = -\frac{4Q}{r^3}\,\mathrm{e}^{-\frac{1}{2}\phi_-}. \tag{39}$$

The solutions can be written as

$$\mathrm{e}^{-\frac{1}{2}\phi_+} = P_0 + \frac{P}{r^2}, \qquad \mathrm{e}^{\frac{1}{2}\phi_-} = Q_0 + \frac{Q}{r^2}, \tag{40}$$

and hence

$$\mathrm{e}^{\phi} = \left(Q_0 + \frac{Q}{r^2}\right)\left(P_0 + \frac{P}{r^2}\right)^{-1}, \qquad c^{-4} = \left(Q_0 + \frac{Q}{r^2}\right)\left(P_0 + \frac{P}{r^2}\right). \tag{41}$$

Were we indeed looking for the dyonic string solutions in the ungauged theory, we would then solve (30) and (31) for a and b, with $g = 0$. In our present case, however, we already have the algebraic equations (22) and (25), which came from solving the F_2 field equation and the $\delta\lambda = 0$ supersymmetry condition, respectively. (Both these conditions would have been vacuous in the dyonic string solutions in the ungauged theory.) Thus our solution is simply given by (41), together with the expressions for a and b in (25) and (22). Collecting the above results, we find that the dyonic string solution of the six-dimensional gauged $N = (1, 0)$ supergravity is given by

$$\begin{aligned} \mathrm{d}s^2 &= H_P^{-\frac{1}{2}} H_Q^{-\frac{1}{2}}\,\mathrm{d}x^\mu\,\mathrm{d}x_\mu + \frac{k^2 P}{4g^2 r^6} H_P^{-\frac{5}{2}} H_Q^{\frac{1}{2}}\,\mathrm{d}r^2 + \frac{k}{4g} H_P^{-\frac{1}{2}} H_Q^{\frac{1}{2}} \left(\sigma_1^2 + \sigma_2^2 + \frac{4gP}{k}\sigma_3^2\right), \\ H_{(3)} &= P\sigma_1 \wedge \sigma_2 \wedge \sigma_3 - \mathrm{d}^2 x \wedge \mathrm{d}H_Q^{-1}, \qquad F_{(2)} = k\sigma_1 \wedge \sigma_2, \qquad \mathrm{e}^\phi = H_Q/H_P, \end{aligned} \tag{42}$$

where

$$H_Q = Q_0 + \frac{Q}{r^2}, \qquad H_P = P_0 + \frac{P}{r^2}. \tag{43}$$

Note that the $H_{(3)}$ and $F_{(2)}$ charges must satisfy the algebraic constraint (34), namely $4gP = k(1 - 2gk)$.

Before turning to the properties of this solution, we may examine its relation to the dyonic string of the ungauged theory [16]. To highlight the similarities, we may reexpress the metric as

$$\mathrm{d}s^2 = (H_P H_Q)^{-\frac{1}{2}}\mathrm{d}x^\mu\,\mathrm{d}x_\mu + (H_P H_Q)^{\frac{1}{2}}\left[4\xi^2\,\Xi^{-3}\mathrm{d}r^2 + \Xi^{-1} r^2\left(\xi\left(\sigma_1^2 + \sigma_2^2\right) + \sigma_3^2\right)\right], \tag{44}$$

where

$$\xi = \frac{k}{4gP} = (1 - 2gk)^{-1}, \qquad \Xi = 1 + \left(\frac{P_0}{P}\right) r^2. \tag{45}$$

To obtain the ungauged theory, we may take the limit $g \to 0, k \to 0$ with the $H_{(3)}$ magnetic charge $P \to k/4g$ held fixed, so that $\xi \to 1$. The metric, (44), then approaches that of the dyonic string in the ungauged theory, provided $\Xi \to 1$. This latter condition is perhaps somewhat surprising, as this restriction is absent in the ungauged theory. Its origin is apparently related to the nature of turning on both $F_{(2)}$ and $H_{(3)}$ fluxes over the squashed S^3.

We now return to the gauged theory, and consider the properties of the dyonic string solution (42). For small r, the functions H_P and H_Q blow up as $1/r^2$ (we take both P and Q positive). Furthermore, in this limit, we see that $\Xi \to 1$. Hence the near-horizon limit of the string may be read off from the metric (44) by taking $\Xi = 1$ and retaining ξ as a squashing parameter. As a result, we see that this limit in fact precisely yields the AdS_3 times squashed 3-sphere family of solutions that we found in section 3.

Turning to the asymptotics away from the horizon, we note that some care must be involved in handling the constant P_0. For $P_0 > 0$, $H_P \to$ const as $r \to \infty$. However $\Xi \sim r^2$ in this limit, and this drastically modifies the asymptotics. In particular, $\mathrm{d}s^2 \sim \mathrm{d}r^2/r^6$ at large r, so that the r interval has a finite range. On the other hand, for $P_0 = 0$, the function Ξ is identically 1, and in this fashion we are able to recover large-distance asymptotics. For $P_0 < 0$, the function H_P goes through zero when $r^2 = |P/P_0|$, thus putting a natural limit on the coordinate, $r \in (0, |P/P_0|^{1/2})$.

In fact, for either $P_0 = 0$ or $P_0 < 0$, the large-distance asymptotics originate when $H_P \to 0$. Both cases may be treated simultaneously by changing to a new radial coordinate ρ, related to r by

$$P_0 + \frac{P}{r^2} = \frac{P^2}{\rho^4}. \tag{46}$$

We also replace the constants Q_0 and Q by

$$\tilde{Q}_0 \equiv Q_0 - \frac{QP_0}{P}, \qquad \tilde{Q}^2 \equiv QP. \tag{47}$$

In terms of these redefined quantities, the dyonic string solution of (42) becomes

$$\begin{aligned} ds^2 &= \frac{\rho^2}{P\mathcal{H}^{\frac{1}{2}}}\,dx^\mu\,dx_\mu + 16\xi^2\mathcal{H}^{\frac{1}{2}}\,d\rho^2 + \mathcal{H}^{\frac{1}{2}}\rho^2\big(\xi\left(\sigma_1^2+\sigma_2^2\right)+\sigma_3^2\big),\\ H_{(3)} &= P\sigma_1\wedge\sigma_2\wedge\sigma_3 - d^2x\wedge d\mathcal{H}^{-1}, \qquad F_{(2)} = k\sigma_1\wedge\sigma_2,\\ e^\phi &= \frac{\mathcal{H}\rho^4}{P^2}, \qquad \mathcal{H}\equiv\tilde{Q}_0+\frac{\tilde{Q}^2}{\rho^4}. \end{aligned} \tag{48}$$

If $\tilde{Q}_0$ is positive, these solutions describe everywhere non-singular dyonic strings[7]. The previously noted horizon at $r = 0$ corresponds here to $\rho = 0$, and in the near-horizon limit where $\rho \longrightarrow 0$, the metric approaches

$$ds^2 \sim (PQ)^{\frac{1}{2}}\left(16\xi^2\frac{d\rho^2}{\rho^2}+\frac{\rho^4}{P^2Q}\,dx^\mu\,dx_\mu\right)+(PQ)^{\frac{1}{2}}\big(\xi\left(\sigma_1^2+\sigma_2^2\right)+\sigma_3^2\big), \tag{49}$$

while the dilaton approaches a constant; $e^\phi \longrightarrow Q/P$. In this limit the tensor multiplet is frozen, and the projection condition (27) is lost. As a result, the supersymmetry at the horizon is restored to $1/2$ of the original supersymmetry.

At large distance, $\rho\to\infty$, the metric approaches

$$ds^2 \sim 16\xi^2\tilde{Q}_0^{\frac{1}{2}}\left(d\rho^2+\frac{1}{16\xi^2}\rho^2\big(\xi\left(\sigma_1^2+\sigma_2^2\right)+\sigma_3^2\big)+\frac{Pg^2}{k^2\tilde{Q}_0}\rho^2\,dx^\mu\,dx_\mu\right), \tag{50}$$

which describes a cone over the product of a squashed S^3 and the $(\text{Minkowski})_2$ string worldsheet metric. Unlike the gauge dyonic string [16], or for that matter typical string solitons [17], the present dyonic string is not asymptotic to the usual vacuum attributed to this gauged $N = (1, 0)$ theory, namely $(\text{Minkowski})_4 \times S^2$. In fact, looking at the dilaton, one finds $e^\phi \sim \rho^4(\tilde{Q}_0/P^2)$, which blows up asymptotically. This is not necessarily surprising, since the potential is of a single-exponential form, which suggests the possibility of a domain-wall type solution with runaway dilaton. We note, however, that when the dilaton is active, the $\delta\chi = 0$ condition requires non-vanishing $H_{(3)}$ for the preservation of supersymmetry. This indicates that domain-wall solutions with $(\text{Minkowski})_5$ symmetry do not occur, and that the $(\text{Minkowski})_2$ times squashed S^3 geometry above is perhaps the most symmetric that may be obtained for a domain-wall configuration.

5. Discussion

The construction of a new family of solutions to the gauged $N = (1, 0)$ theory suggests that there exists a wider class of supersymmetric vacua than previously anticipated. In particular, the well-known $(\text{Minkowski})_4 \times S^2$ solution is but a special singular limiting case of a larger class of solutions of the form AdS_3 times squashed S^3. Thus it is apparent that some form of fine tuning is necessarily present to obtain a flat Minkowski spacetime. The half-supersymmetric vacua constructed in section 3 are parametrized by the $H_{(3)}$ flux P, with $P \longrightarrow 0$ corresponding to the Minkowski limit. For non-zero P, on the other hand, the 3-form singles out three of the six dimensions to form a 3-sphere, and one of the $(\text{Minkowski})_4$ dimensions is lost to become the Hopf fibre of S^3.

We now turn to the implications of this result on the recent braneworld models with football-shaped extra dimensions [6–8]. While we have little to say about the effect of non-supersymmetric 3-branes (i.e. deficit angles) on the cosmological constant, it should now

[7] If $\tilde{Q}_0$ is negative, there is a naked singularity at the value of ρ for which $\mathcal{H}$ vanishes. If $\tilde{Q}_0 = 0$, the metric (48) coincides precisely with the near-horizon metric (49), which is nothing but AdS_3 times the squashed 3-sphere.

be evident that a fine tuning of the bulk geometry is nevertheless required, even before the introduction of branes into the bulk. The vanishing of the cosmological constant in the $(\text{Minkowski})_4 \times S^2$ background does indeed arise after assuming an $M_4 \times S^2$ symmetry of the vacuum. However, as we have seen, supersymmetry itself is insufficient for selecting such a symmetry, and the theory itself is perfectly content to compactify spontaneously with an AdS_3 times squashed S^3 geometry. This feature is similar to what occurs in 11-dimensional supergravity, where $(\text{Minkowski})_{11}$ may be obtained as a limiting case of the $\text{AdS}_4 \times S^7$ Freund–Rubin compactification [18].

In fact, obtaining a naturally small cosmological constant in these braneworld models comes to the issue of balancing the $F_{(2)}$ and $H_{(3)}$ fluxes against the six-dimensional potential. Reduced onto the braneworld, this mechanism essentially replaces the effective cosmological constant by a dynamical variable. This is similar to earlier ideas where the cosmological constant is replaced by a 4-form field strength in four dimensions [19, 20]. In such models, the 4-form itself has no local dynamics, but may take on appropriate values so as to cancel the background vacuum energy. In the end, however, this effective 4-form may take on a range of values (quantized in the case of M-theory [21]), and one is again reduced to an anthropic argument to explain the smallness of the cosmological constant.

More generally, we would like to investigate whether any additional supersymmetric vacua of this gauged $N = (1, 0)$ theory may exist. In addition to the AdS_3 times squashed S^3 backgrounds preserving $1/2$ of the original supersymmetry, we have also identified a class of dyonic string solutions preserving $1/4$ of the original supersymmetry. Although we believe we have essentially exhausted the possibility of static vacua, a more systematic treatment would be necessary. Since the field content of the theory as well as its multiplet structure are relatively simple, it may be amenable to an analysis similar to that which was performed in [22, 23] for the case of minimal (gauged and ungauged) supergravity in five dimensions. Some preliminary analysis is currently under way.

Finally, whether the minimal gauged supergravity may be obtained from a higher-dimensional theory remains an unresolved issue. While ungauged $N = (1, 0)$ theories are easily obtained from, for example, heterotic strings reduced on $K3$, obtaining a chiral gauged supergravity from higher dimensions appears to be a more difficult task. One may naturally obtain gauged supergravities from compactifications with fluxes, and in fact gauged $N = 2$ supergravity in five dimensions may be obtained by a flux compactification on $K3 \times S^1$ [24]. However, it is not clear that this could be lifted up to six dimensions. In fact, it is argued in [24] that no background fluxes can be turned on for the heterotic on $K3$ reduction to six dimensions in itself.

Alternatively, the $(\text{Minkowski})_4 \times S^2$ background with monopole configuration is suggestive of a seven-dimensional interpretation with $(\text{Minkowski})_4 \times S^3$ vacuum, where the S^3 may be viewed as a $U(1)$ bundle over S^2 [1]. However, attempts at reducing the gauged $N = 2$ supergravity in seven dimensions to yield the gauged $N = (1, 0)$ theory in six dimensions have so far proved unsuccessful. Note that the ungauged $N = (1, 0)$ theory may be obtained in this fashion through a braneworld reduction for both pure supergravity [25] and supergravity coupled to a single tensor multiplet [26]. However, to reduce to the gauged theory, one needs to retain at least a vector multiplet in the braneworld reduction. So far this has only been accomplished in the bosonic sector [26].

In particular, the gauged theory involves a chiral six-dimensional gravitino charged under the Maxwell field. Ordinarily, this Maxwell field would be labelled as a graviphoton, namely a superpartner of the graviton and gravitino. However, in the $N = (1, 0)$ theory, it is actually part of an ordinary Maxwell multiplet and has a single spin-$1/2$ superpartner. For a braneworld reduction from seven dimensions, this Maxwell field can only arise as the $U(1)$ component of the $SU(2)$ graviphotons, as one needs to retain the gauged R-symmetry of the gravitino (and there is no $U(1)$ isometry in the reduction of the metric). However, this leads to an apparent incomplete gravitino multiplet in the reduction, and not the requisite Maxwell multiplet [26].

More importantly, it is not clear how one obtains a chiral charged gravitino from dimensional reduction of ten- or eleven-dimensional supergravities. Since the higher-dimensional theory necessarily involves uncharged gravitini, a gauged R-symmetry must somehow arise from the dimensional reduction. In this case, one would have to ensure chirality, either through singularities or non-perturbative effects, or perhaps by an overlooked mechanism in the braneworld reduction. An alternative approach would be to obtain chirality by consistent truncation of a non-chiral theory, such as the $N = (1, 1)$ gauged supergravity in six dimensions. However, a straightforward truncation of the bosonic sector of the $N = (1, 1)$ supergravity yields an incorrect scalar potential. Furthermore, it is not clear that the fermion sector (and especially the resulting $N = (1, 0)$ gravitino multiplet) may be consistently truncated.

Thus the dyonic string solutions that we have obtained are not obviously related to any of the well-known objects in M-theory, although they do share similarities with the dyonic strings of the ungauged theory [16]. Of course, the minimal theory considered here [1] is anomalous, and must be supplemented with additional matter for consistency [27–29]. It is presumably the anomaly-free models, if any, that may be lifted to higher dimensions.

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