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PREDICTING MIRROR ADJUSTMENT RANGE FOR DRIVER ACCOMMODATION

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16. Abstract

Although the question of how large a driver's outside rearview mirror must be in order to see a specified target has been addressed in other publications, the related problem of required adjustment range has not. In this paper, we present a series of equations that predict, for a given vehicle, the size and location of the mirror adjustment range needed in order to accommodate some percentage of the driver population (e.g., 96%). To complete the calculations for 96% accommodations, eye locations in the vehicle are represented by the 99% SAE J941 eyellipse. Because the transformation from eye location to target location in the mirror will not preserve the tangent properties of the eyellipse, we propose a method in which the side and plan views of the eyellipse are treated separately. Eye location in plan view affects only horizontal adjustment of the mirror, and eye location in side view affects only vertical adjustment of the mirror. In each view, there are two points that lie on lines that are tangent to the eyellipse and pass through the mirror center. These two points are used to represent two extremes of mirror adjustment. Thus, we exclude the 2% of driver eye locations that lie outside either of the tangent lines (no cases lie outside both, so each tangent excludes 1%). In plan view, eye locations must first be adjusted for head turn. We also present equations to calculate mirror adjustment, referenced to an arbitrary line, for each of the four tangent points, given a specified target. We discuss various choices of target location and type, including centered point targets, centered extended targets, and targets that are located at the edge of the field of view. For the latter target type, the calculation of head turn is somewhat different than for centered targets, but the rest of the calculations are the same. The end result of these equations is a rectangle in two-dimensional mirror-adjustment space such that 96% of drivers can find a suitable mirror position within those bounds. An example is carried out using dimensions from a specific vehicle and a target located at the inner edge of the field of view, in order to illustrate the procedure.

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Introduction

A number of technical papers (e.g., Seeser, 1976, Olson & Winkler, 1985) as well as federal regulations (e.g., FMVSS 111) concern the field of view provided by the driver's outside rearview mirror. Olson and Winkler (1985) measured the field of view chosen by a number of drivers in their own vehicles. Seeser (1976) developed a model of field of view to be used to determine how big a mirror needs to be for drivers to see a specified target.

Both Seeser's (1976) model and FMVSS 111, which defines minimum requirements for field of view of rearview mirrors, base calculations of field of view on the eyellipse, which is defined in SAE Recommended Practice J941. The eyellipse is a tangent ellipse that defines where drivers' eyes are located in a given vehicle. For a 95% eyellipse, any plane tangent to the eyellipse separates the space into two regions, one of which contains 95% of predicted eye locations and one which contains the other 5%. For a 99% eyellipse any tangent plane separates the space into regions containing 99% and 1% of eye locations. In theory, an eyellipse could be generated to separate the space into regions containing any percentage of eye locations. However, SAE J941 gives definitions for only 95% and 99% ellipses.

The present report focuses on a related problem, but one which is finessed in both Seeser's (1976) work and FMVSS 111. In each of those documents, the mirror is effectively assumed to be sufficiently adjustable to achieve required mirror positions. In other words, the models of field of view assume that the driver can adjust the mirror to any position he or she requires, leaving only the size of the field of view at issue. In FMVSS 111, a person of specified size or eye location must be able to see a specified target. In order to achieve this, the mirror must have sufficient adjustability.

In contrast, the model presented in this report identifies the necessary range of adjustability to accommodate a population of drivers in a given vehicle. Knowing the necessary range of adjustability is useful for two reasons. First, the field-of-view regulations use models to estimate field of view and these models assume sufficient adjustability. If the mirror does not adjust far enough one way or another, a mirror deemed large enough to be safe by the field-of-view model may, in fact, not provide the predicted field of view simply because it cannot be moved far enough. Thus, appropriate adjustability is necessary for safety. Second, appropriate adjustability is necessary for comfort. Drivers have different preferences for the way their mirror is adjusted. If the mirror cannot achieve the position a driver wants, he or she will not be accommodated. It is in the interest of manufacturers to provide enough adjustability to accommodate most drivers without having to provide more range than necessary.

Overview of the Problem

The basic outline of the problem of predicting necessary mirror adjustment range is illustrated in Figure 1. For each eye location in a given vehicle, there is a vector, $\bar{u}$, extending from the mirror center to the eye. There is a second vector, $\bar{v}$, that extends from the mirror (coinciding with the endpoint of $\bar{u}$) to a target that is seen in the center of the mirror. The angle between these two vectors, α, can be separated into vertical and horizontal components, relative to the vehicle. When the target is to be seen at the center of the mirror, the mirror will be aimed at the bisector of α in each dimension.

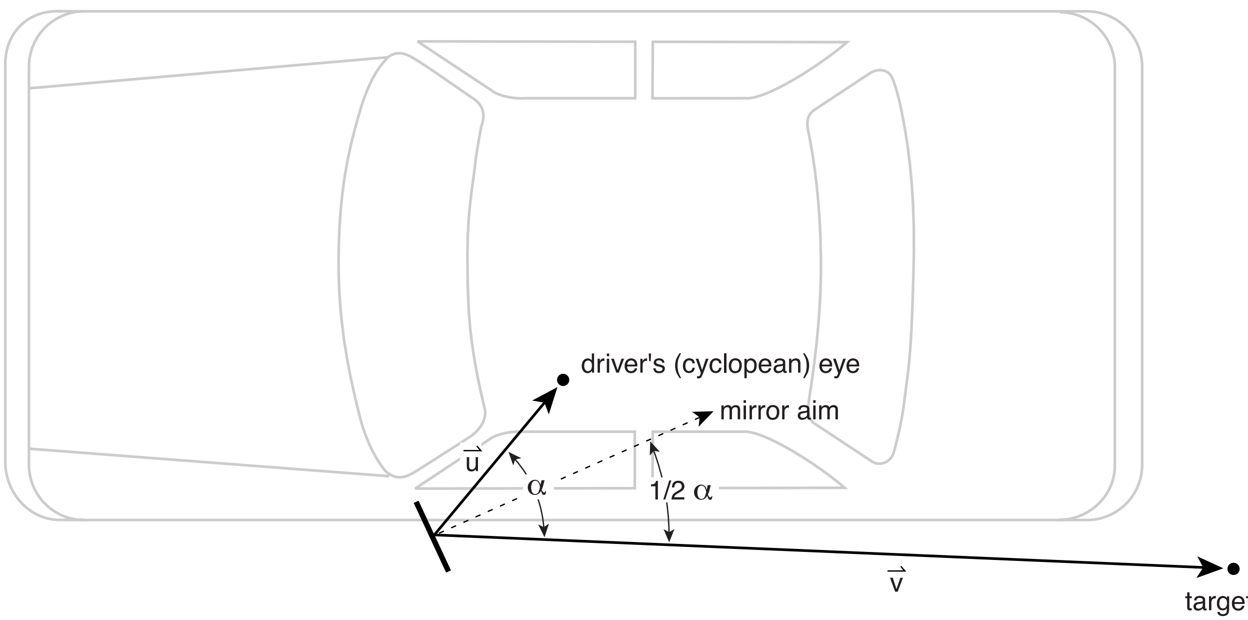

Figure 1. Components of the mirror adjustment problem.

Figure 1 contains all of the basic components of the problem, which will be addressed in detail. First, Figure 1 illustrates the problem as seen by a single driver with a single eye location. However, the complete problem must be based on a distribution of eye positions, generally represented by the eyellipse. Furthermore, there are really two eyellipses, one for each eye. For some targets, the right or left eyellipse will be used alone (representing one side of the ambinocular field of view). For others, the cyclopean eyellipse will be sufficient. As a final complication, a driver will generally need to turn his or her head to look in the mirror, so the corresponding change in eye location must be accounted for.

At the other end of the basic problem lies the target. Defining an appropriate target for mirror aim is important but difficult. Targets may be single points, or they may be extended lines, areas, or objects. Some targets are more naturally placed in the center of the field of view (e.g., the center of an adjacent lane) while others are more likely to be placed at the edge of the field of view (e.g., the rear corner of one's own vehicle). Some targets simplify the mathematics of the problem, but these may not be the most realistic targets in terms of how drivers actually choose to aim their mirrors. Other targets are prescribed for purposes of vehicle design (e.g., FMVSS 111), but they may not represent how drivers choose to aim their mirrors. In this paper, we will only begin to address the question of appropriate targets for mirror aim. We will use some relatively simple targets to solve the equations for identifying mirror adjustment range, and we will raise questions for future research.

The third piece of the problem is how to define how many drivers are being accommodated by the mathematical solution. The issue would be easily solved if the eyellipse defined a cloud of eye positions such that those inside the eyellipse would be accommodated by the solution and those outside would not. However, the eyellipse is designed to be used with tangent lines that represent lines of sight. Thus, some unknown number of eyes that lie outside the eyellipse may still lie inside a set of sight lines. We present a solution to this problem that sacrifices a small amount of precision for a large amount of simplicity. As an added complication, there is variability in how drivers prefer to aim their mirrors. In other words, using a single target does not realistically reflect driver behavior. Variability in aiming preference not only complicates the solution to the target problem, but it affects the number of drivers accommodated. The effects of variability in aim will be discussed, but again, only rudimentary solutions will be offered. We will start with a discussion of percent accommodation, which is the end of the problem, and then move to the eyellipse, which is at the beginning.

Percent Accommodation and the Eyellipse

Ideally, the end result of a model of mirror adjustability would be a shape in two dimensions that contains all of the positions the driver's-side rearview mirror needs to achieve in order to accommodate some designated percentage (e.g., 95%) of the population of drivers. The axes of the space in which the shape resides would represent angular deviation of the mirror in the vertical direction against angular deviation of the mirror in the horizontal direction (see Figure 2). The reference lines for deviations are shown in the figure. For both dimensions, the reference line is centered on the mirror, parallel to the main (fore-aft) axis of the vehicle, and perpendicular to the left-right and up-down planes of the vehicle. The aim of the mirror would be defined by a line extending from the center of the mirror, perpendicular to the mirror plane.

Unfortunately, generating such an accommodation region turns out not to be a simple problem. At the beginning of the problem lies the eyellipse. The eyellipse embodied in SAE Recommended Practice J941 represents the location of drivers' eyes in a given vehicle. By design, the eyellipse is a *tangent* ellipse, not a *density* ellipse. A 95% density ellipse defines a boundary within which 95% of eyes should lie. With a tangent ellipse, 95% of eye locations lie to one side of any plane that is tangent to the ellipse.

If the eyellipse were a density ellipse, its boundaries could be entered into a function that translates them into a new shape in two dimensions. However, the tangent ellipse is the appropriate form to use when lines of sight are to be drawn and analyzed, as in this case. There is no simple way to draw sightlines from a three-dimensional ellipse that represent boundary conditions to be transformed into the two-dimensional mirror-adjustment space.

It is possible to solve the problem empirically using Monte Carlo methods. Although we may pursue that solution in the future, it is very complex, probably even worth a report in its own right. For present purposes, we have identified a compromise that retains simplicity but sacrifices some precision in the percent of drivers accommodated. Instead of describing a complex curved shape in mirror-adjustment space, the model will produce a square that separately predicts necessary adjustability in the horizontal and vertical directions.

The simplified method works on the basis of two observations. First, adjustment of the mirror in each dimension is necessitated by differences in eye position in only two of three dimensions. That is, the horizontal adjustment of the mirror is only affected by the fore-aft and left-right location of the eyes. Differences in vertical eye location do not require horizontal adjustment of the mirror to see a particular target. Similarly, the vertical adjustment of the mirror is only affected by the vertical and fore-aft position of the eyes. Moving the eye location to the left or right does not affect the necessary vertical adjustment of the mirror. Thus, for each dimension, a two-dimensional projection of the eyellipse may be used.

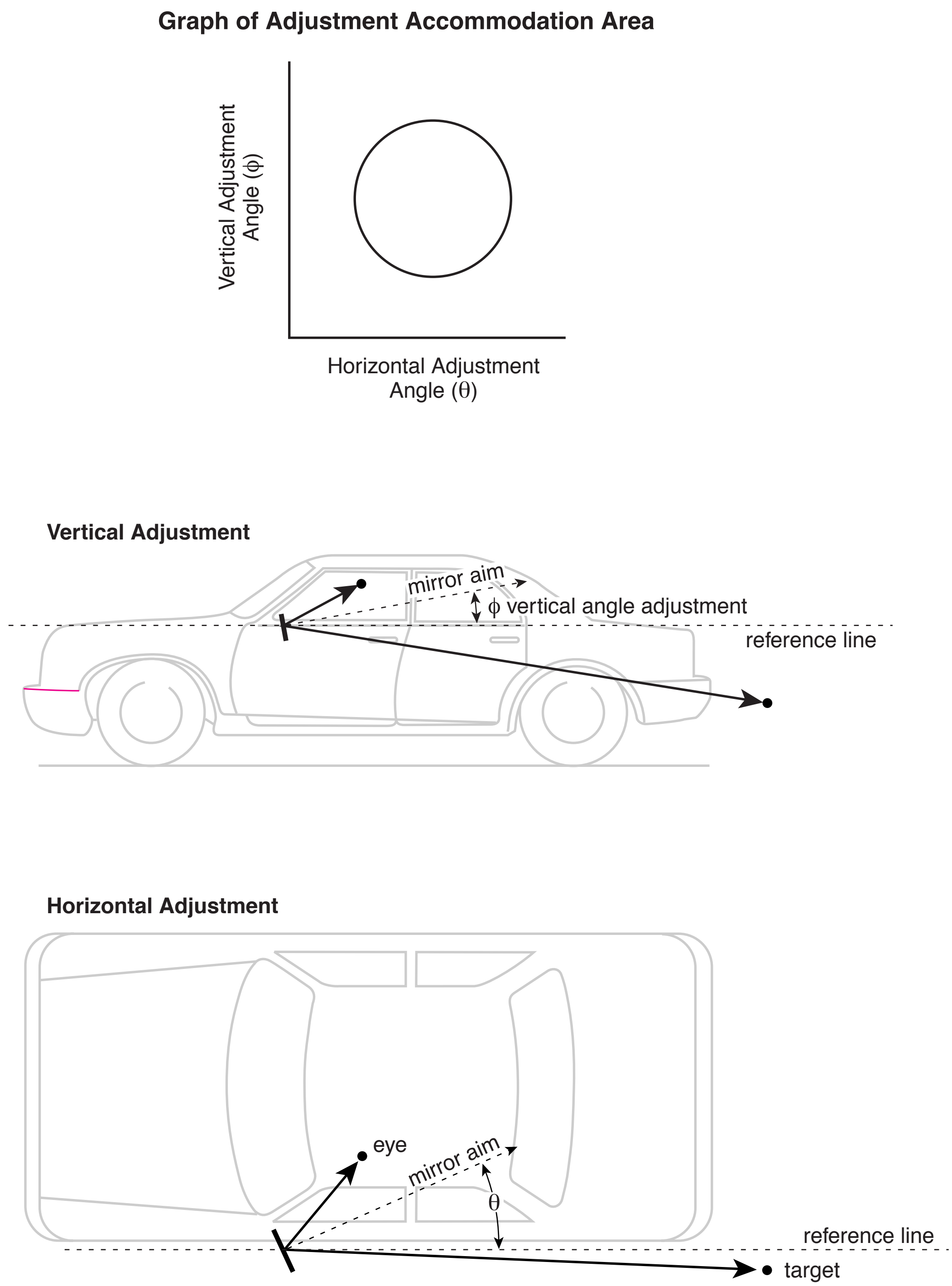

Figure 2. Illustration of reference coordinate system and the relationship between the calculations performed and the end product, which is a region in two-dimensional mirror-adjustment space (illustrated here by a circle, but actually of undetermined shape).

The second observation is illustrated in Figure 3. For the horizontal mirror adjustment, the plan-view eyellipse can be used. First, the two points on the eyellipse that represent extremes of horizontal mirror adjustment must be identified. These can be found graphically by drawing two lines from the center of the mirror, tangent to the eyellipse on each side. As described in the rest of this report, each of these lines can be translated into mirror adjustment, given in terms of angular offset from the reference line. If the 99% eyellipse is used, each tangent line excludes 1% of drivers' eyes. The two tangent lines meet at the mirror, but there are no eyes located beyond the meeting point, so the mirror adjustment range identified by the two tangent lines will accommodate 98% of drivers in the horizontal dimension.

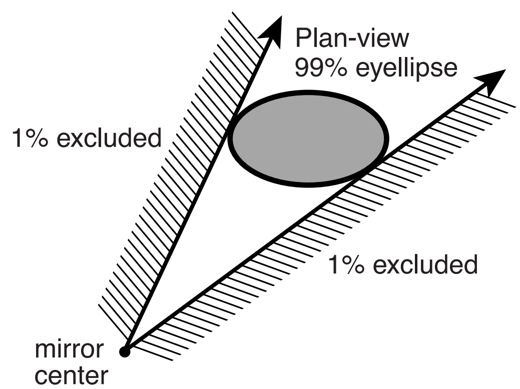

Figure 3. Illustration of technique for determining 98% accommodation in one dimension of mirror adjustment (e.g., horizontal).

The same process is repeated for mirror adjustment in the vertical dimension, using the side-view 99% eyellipse. The two adjustment ranges combined form a square in mirror-adjustment space. If the likelihood of a driver needing adjustability outside the horizontal range is unrelated to the likelihood of a driver needing adjustability outside the vertical range, then the two ranges are independent. If so, the square would accommodate $(.98)^2$, or about 96% of drivers. A limiting case, and an implausible alternative, is one in which being outside the range in the horizontal dimension guarantees that the driver will be outside the range on the other dimension. The same 2% would be excluded on both dimensions, leaving 98% of drivers accommodated. Calculating the true percent of drivers accommodated by this scheme requires complex methods beyond the scope of this work. However, the percent of accommodated drivers ranges from 96% to 98%, with the true value probably falling close to 96%. Other percentages of drivers can be accommodated by using a different eyellipse.

Possible Changes to the Eyellipse

The eyellipse embodied in J941 was developed over 30 years ago to predict the three-dimensional location of drivers' eyes in different vehicles. Manary, Flannagan, Reed, and Schneider (1998) presented more recent eye-position data that suggest that the eyellipse mispredicts current eye positions in certain consistent ways. First, current eyes tend to be located rearward and above the eye positions predicted by J941. In addition, Manary et al. suggest that the prediction equations for the eyellipse should be changed, though they do not provide specific coefficients. Development of a new eyellipse model is still underway.

Because a new eyellipse model is not yet available, we will use the J941 eyellipse in this paper as the starting point for a model of mirror adjustability. However, at relevant points in the model description, we will note the consequences of this new information about eye position.

Predicting Mirror Adjustment

Using the Eyellipse

We will start with the simplest version of the mirror-adjustment problem, in which the target is a single point to be centered in the field of view. To determine necessary horizontal adjustment range, the plan-view eyellipse will be used because it contains the two dimensions that affect horizontal mirror adjustment (see Figure 4). With a centered target, the cyclopean eyellipse can be used because its center is the same as the center of the ambinocular field of view. The latter more realistically represents the total field of view, and it will be used later in the report for targets that are not centered.

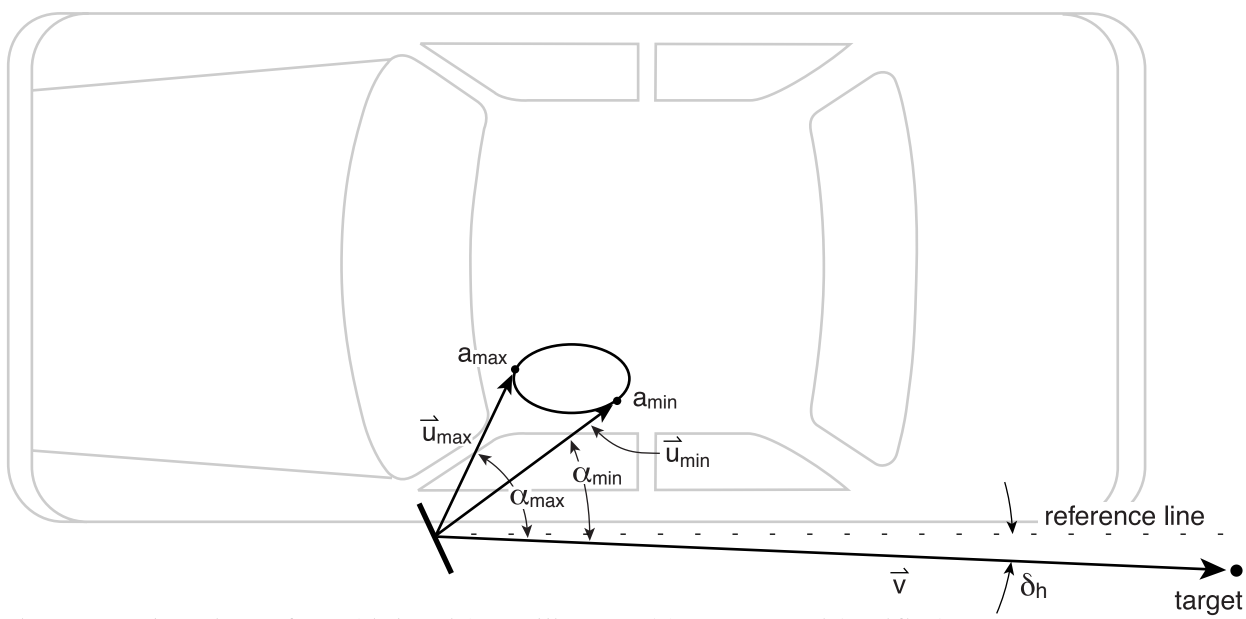

Figure 4. Plan view of a vehicle with eyellipse and key vectors identified.

Figure 1 shows the vectors and angles that must be identified or calculated in order to predict mirror adjustment for a single eye location. In Figure 4, the eyellipse has been added, so that the problem now addresses a distribution of eye positions instead of a single eye location. As discussed earlier, we will use the 99% eyellipse and from it will identify boundary conditions for accommodation. Thus, the calculations will be performed twice to determine the two endpoints of the necessary range of horizontal adjustment.

The vector $\bar{v}$ in Figure 4 is the same for both calculations because it extends from the mirror center to the target, which will be seen in the mirror center. The vector $\bar{u}$ differs for the two calculations because there are two lines that intersect the mirror center and are tangent to the eyellipse. The minimum adjustment is calculated using $\bar{u}_{\min}$, which extends from the center of the mirror to $a_{\min}$, the tangent point closest to the left side of the vehicle. The maximum adjustment is calculated using $\vec{u}_{\max}$, which extends from the center of the mirror to $a_{\max}$, the tangent point closest to the center of the vehicle.

All of the vectors, $\vec{u}_{\min}$, $\vec{u}_{\max}$, and $\vec{v}$, are located in three dimensions, but only their projections on the x-y plane need to be considered for this portion of the calculation. Thus, the z-component of the vectors can be set to zero. The angle between $\vec{v}$ and $\vec{u}_{\min}$ is labeled $\alpha_{\min}$ and the angle between $\vec{v}$ and $\vec{u}_{\max}$ is labeled $\alpha_{\max}$. The equations to calculate $\alpha_{\min}$ and $\alpha_{\max}$ are shown below as Equations 1 and 2.

$$\alpha_{\min} = \cos^{-1}\left(\frac{\vec{u}_{\min} \bullet \vec{v}}{\|\vec{u}_{\min}\|\|\vec{v}\|}\right) \tag{1}$$

$$\alpha_{\max} = \cos^{-1}\left(\frac{\vec{u}_{\max} \bullet \vec{v}}{\|\vec{u}_{\max}\|\|\vec{v}\|}\right) \tag{2}$$

In addition, there is a third angle, δ_h, which lies between the reference line and $\vec{v}$ in the x-y plane. This angle can be seen in Figure 4. If $\vec{v}$ lies between $\vec{u}$ and the reference line, then δ_h is positive. Otherwise, δ_h is negative. The mirror must be adjusted to the bisector of $\alpha_{\min}$ or $\alpha_{\max}$, and then must be added to δ_h in order to be located in the reference coordinate system. Equations 3 and 4 describe these transformations, resulting in angles $\theta_{\min}$ and $\theta_{\max}$ which define the minimum and maximum required horizontal adjustment to accommodate 98% of the population in the horizontal dimension.

$$\theta_{\min} = 0.5\alpha_{\min} + \delta_h \tag{3}$$

$$\theta_{\max} = 0.5\alpha_{\max} + \delta_h \tag{4}$$

Calculation of necessary adjustment range in the vertical plane works the same way, except that the side-view eyellipse is used instead of the plan-view eyellipse. Equations 5 through 8 are analogous to Equations 1 through 4, with vectors $\vec{i}$ replacing vectors $\vec{u}$, vector $\vec{j}$ replacing vector $\vec{v}$, angles ß replacing angles α and angle δ_v replacing δ_h. The vector $\vec{i}_{\min}$ extends from the mirror center to $b_{\min}$, the lower of the tangent points on the eyellipse. The vector $\vec{i}_{\max}$ extends from the mirror center to $b_{\max}$, the higher of the tangent points on the eyellipse. The angles ϕ_{min} and ϕ_{max} define the corresponding minimum and maximum vertical adjustment to accommodate 98% of the population in the vertical dimension.

$$\beta_{min} = \cos^{-1}\left(\frac{\vec{i}_{min} \bullet \vec{j}}{\left\|\vec{i}_{min}\right\|\left\|\vec{j}\right\|}\right) \tag{5}$$

$$\beta_{max} = \cos^{-1}\left(\frac{\vec{i}_{max} \bullet \vec{j}}{\left\|\vec{i}_{max}\right\|\left\|\vec{j}\right\|}\right) \tag{6}$$

$$\phi_{min} = 0.5\beta_{min} + \delta_v \tag{7}$$

$$\phi_{max} = 0.5\beta_{max} + \delta_v \tag{8}$$

Incorporating Head Turn

The eyellipse describes eye locations of drivers who are looking straight ahead in a normal driver posture. When using the outside mirror, however, drivers use a combination of head turn and eye turn to see the mirror. Devlin and Roe (1968) performed a laboratory study from which they concluded that drivers turn their eyes 30 degrees and use head turn to complete the motion to look directly at an offset target. In addition, the center of rotation is located at the atlanto-occipital joint, which is estimated to lie 3.88 in (98 mm) behind the cyclopean eye. This model has been incorporated into SAE J1050a, and it will be used in the present model.

Interestingly, the head-turn model just described will produce a different amount of rotation for every eye location. When the eyellipse is used to generate sight lines with exterior mirrors, the most appropriate way to define the sight lines is from a new "eyellipse" to which the head-turn transformation has been applied. However, because head turn is different for every point, the resulting shape will not be elliptical and will be difficult to work with mathematically. This presents a complication to the eyellipse that SAE J1050a gets around by using v-points. Without going into detail, v-points essentially replace the eyellipse with a single point (for each eye) that can be used to approximate the appropriate tangent to the eyellipse under limited circumstances. Thus, the head-turn model can be applied to that one point without raising the question of whether head-turn should be applied before or after the tangent is calculated.

In the present application, we cannot make use of v-points or other single-point approximations to the eyellipse because some of the tangent lines of interest are not in the range that is legitimate for approximation by v-points. Our solution, instead, is to apply the head-turn model *after* selecting the tangent points from the original eyellipse. Although it is possible that the specific point selected in this way will be different from one that would be selected if the head-turn model were applied first, we argue that the differences will be so small as to be inconsequential.

Head turn is most relevant for horizontal mirror adjustment. To incorporate head turn, the vectors $\vec{u}_{\min}$ and $\vec{u}_{\max}$ in Equations 1 and 2 must first be adjusted according to the head-turn model. As shown in Figure 5, using $\vec{u}_{\max}$ as an example, first create a new vector, $\vec{h}$, which extends from the center of the mirror to the head-turn pivot point, *p*, exactly 98 mm rearward of $a_{\max}$ (the tangent point closest to the center of the vehicle). Next, calculate the angle, γ, between $\vec{h}$ and a line extending from *p* forward along the fore-aft axis of the vehicle, according to Equation 9, where h_1 and h_2 are the fore-aft and left-right components of $\vec{h}$, respectively. If γ is less than 30°, the eyes will account for all of the necessary movement and there will be no need to adjust for head turn. Otherwise, subtract 30° from γ to get γ', which represents the required head-turn angle (Equation 10). Finally, translate $a_{\max}$ to $a'_{\max}$ along an arc, centered at *p*, with a radius of 98 mm, until the angular change is equal to γ'. Equations 11 and 12 give the formulas for calculating the *changes* to the coordinates of $a_{\max}$. Fore-aft and left-right changes are labeled d_1 and d_2, respectively. The new vector, $\vec{u}'_{\max}$, extends from the center of the mirror to $a'_{\max}$. The same process should be repeated for $\vec{u}_{\min}$. Once $\vec{u}'_{\min}$ and $\vec{u}'_{\max}$ have been calculated, they replace their original counterparts in equations 1 through 4.

$$\gamma = \tan^{-1}\left({h_2}/{h_1} \right) \tag{9}$$

$$\gamma' = \gamma - 30^\circ \tag{10}$$

$$d_1 = 98 - 98\cos\gamma' \tag{11}$$

$$d_2 = 98\sin\gamma' \tag{12}$$

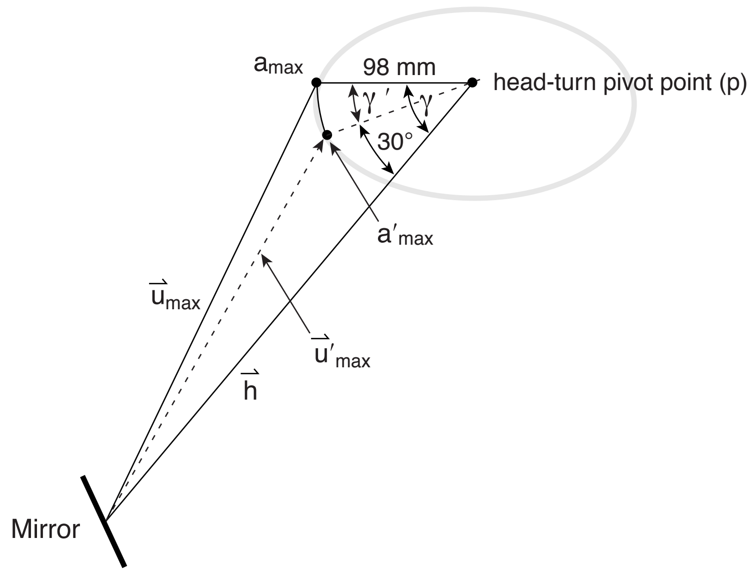

Figure 5. Illustration of calculation of eye location adjustment to account for head turn. (The fact that in this example the eye location after head turn, a'_{max}, lies on the eyellipse boundary is coincidental.)

The Target

So far, we have presented a solution to predicting necessary mirror adjustment that makes use of a single, centered target. Using such a target simplifies the problem by enabling us to use the cyclopean eyellipse. However, from a human factors standpoint, it is unlikely that drivers adjust their mirrors based on a single-point, centered target. In fact, there often may not be anything located at the center of a typical driver's mirror field of view.

Another type of target that might be used is one with some width and height. Such a target is used in FMVSS 111 for determining required mirror width. The present method could be used to determine necessary adjustability to see the FMVSS 111 target, or a similar extended target (e.g., the next lane at some distance) might be chosen as plausible. Interestingly, if an extended target is assumed to be centered in the field of view, the same equations are used as for the single-point target. The vertical and horizontal center of the extended target becomes the single point, to be used with the procedure described above.

The ideal target for our purposes would be one that matches the way drivers choose to adjust their mirrors. However, there is little research that addresses the question of what criteria drivers use for mirror adjustment. The best data we know of are from Olson and Winkler (1985), who measured the locations and sizes of the fields of view of many drivers' own exterior rearview mirrors. However, this information speaks only to the variability and mean of where mirrors are adjusted. They did not analyze the relationship between, for example, mirror size and field-of-view location, which would more directly indicate the basis on which drivers adjust their mirrors. The large variability found by Olson and Winkler does suggest that there may be no single answer to how drivers choose to adjust their mirrors, but a study in which parameters of the mirror and vehicle are manipulated and mirror adjustment is measured might shed light on the question.

Another approach to the problem is taken by Seeser (1976) and by FMVSS 111. In these documents, the author(s) determined a priori what drivers *should* see from their exterior rearview mirrors and based further calculations on that target. Although this approach is useful and clearly appropriate in some contexts (e.g., federal regulations), the present work is on accommodation, and as such, involves accommodating driver preferences rather than prescribing appropriate adjustments.

It seems plausible that one guide that drivers use for mirror adjustment is their own vehicle. For illustration, we might designate a single target location on the side of the vehicle that we hypothesize drivers want to see at the inner edge of their field of view. Equations 1 through 4, for calculating the horizontal adjustment, can be altered in the following way.

Because the target is at the inner edge of the field of view, we must consider the ambinocular field of view rather than the cyclopean field of view. In this case, the left eye will see the target at the inner edge of its field of view. Thus, the mirror endpoint of $\underline{u}_{\min}$ in Equation 1 will be the inboard edge of the mirror, and the other endpoint, $a_{\min}$, will be the most outboard tangent point on the left eyellipse rather than the cyclopean eyellipse. In addition, $a_{\min}$ must be adjusted for head turn.

Because the point of rotation is located behind the cyclopean eye, Equations 11 and 12 cannot be used. Equations 9 and 10, which calculated the change in head angle, can be used with a slight adjustment. In equation 9, the vector $\vec{h}$ extends from the mirror center to p, the head-turn pivot point, located 98 mm behind the cyclopean eye point. In the present case, $a_{\min}$ lies on the left eyellipse, so its left-right coordinate, h_2, must be translated to its position on the cyclopean eyellipse. In addition, the driver's head will turn to center the eyes on the mirror center, not the left edge. Thus, 32.5 mm is added in Equation 13 to account for half the interpupillary distance, which translates the left-right component of the left eye to the left-right component of the cyclopean eye, h_2'. The value d_m, which is equal to half the mirror width, is also added to translate the origin of $\vec{h}$ to the mirror center. The fore-aft coordinate, h_1, is equal to the fore-aft coordinate of $a_{\min}$ plus 98 mm, as before. Equations 14 and 15 repeat equations 9 and 10 for the adjusted vector, $\vec{h}$.

$$\vec{h} = h_2 + 32.5 + d_m \tag{13}$$

$$\gamma = \tan^{-1}\left(h_2' / h_1 \right) \tag{14}$$

$$\gamma' = \gamma - 30^\circ \tag{15}$$

The head-turn angle for the left eye is the same as for the cyclopean eye. However, the simplicity of equations 11 and 12 depends on the angular change from turning the head being referenced to the fore-aft axis of the vehicle. Because the center of rotation is not directly aft of the left eye, the rotation from head turn is relative to an angle not aligned with any axis. Thus, equations 16 through 18 must be used for determining the new left-eye location after head turn. In these equations, the center of rotation of the head is designated as the origin, and $\vec{t}$ is a vector extending from the point of rotation to the left-eye point, $a_{\min}$. Because the center of rotation is now the origin, the vector $\vec{t}$ has fixed coordinates, t_1 and t_2, as in Equation 16. The vector, $\vec{s}$, extends from the center of rotation to the new left-eye point, and has components s_1 and s_2. Equations 17 and 18 must be solved simultaneously to determine values of s_1 and s_2.

$$\vec{t} = (t_1, t_2) = (-98, -32.5) \tag{16}$$

$$\gamma' = \cos^{-1}\left(\frac{\vec{t} \bullet \vec{s}}{\|\vec{t}\|\|\vec{s}\|}\right) \tag{17}$$

$$\|\vec{s}\| = \|\vec{t}\| \tag{18}$$

The components of the vector $\vec{s}$ measure the distance from the turned-head left-eye point to the center of rotation of the head. These components must be translated into the coordinate system centered at the inboard edge of the mirror in order to determine the mirror aim. Once translated, equations 1 through 4 can be used as before, but the origin of the vectors will be the inboard edge of the mirror instead of the center. The mirror is then treated as though it rotates around its inboard edge, but this will have a negligible effect on the outcome. The reference line can be translated to extend from the inboard edge of the mirror without any change to the equations.

With this target, the vertical adjustment of the mirror will be computed as before. In other words, the target is treated as being centered vertically for simplicity, and Equations 5 through 8 remain the same. In side view, the cyclopean, right, and left eyellipses are all the same.

Considering the options of centered point targets, centered extended targets, and edge targets, most plausible target choices should be covered. Other targets might be defined such that the solutions would depend on mirror width. However, this type of target seems unusual enough that it could reasonably be replaced by either an edge or centered target. More must be known about how drivers actually position their mirrors before the target issue can be settled definitively.

An Example

It is often helpful to see equations applied to an example. In this section, we will use the equations we have described to predict necessary mirror adjustment range for a specific vehicle, a 1994 Pontiac Trans Am[1]. The target will be a point on the side of the vehicle, 500 mm from the rear bumper, and 1075.5 mm above the ground. The choice of target is partly educated guess and partly based on previous research. Olson and Winkler (1985) showed that some proportion of drivers could see part of their vehicle in the inner edge of their field of view. Although the proportion they found was less than might be expected (about 40%), it is reasonable to guess that many drivers use the edge of their vehicle as a basis for aiming their mirror. Choosing one-half meter forward of the rear bumper is just a guess on our part as to how much of the vehicle drivers might want to see. The vertical location of the target is based on the Olson and Winkler data for vertical mirror adjustment. They found that at the 50th percentile, drivers see at the vertical center of their mirror a point 10° (1016 mm) above the ground, 19 ft (5791 mm) behind the mirror. In the Trans Am, this translates to 1075.5 mm above the ground at the target point (2833 mm behind the mirror).

Because the chosen target is to be seen at the inner edge of the field of view, the calculations will center around the inboard edge of the mirror and the left eyellipse. In the Trans Am, the right (inboard) edge of the mirror is located at the point (2515.4, -845.6, 1124.3) in the vehicle coordinate system. In this coordinate system, values increase rearward, rightward (from the driver's viewpoint), and up, forming a right-hand coordinate system. The SAE J941 99% left eyellipse centroid is located at the point (3013, -398.4, 1238.3) with axis lengths of 268.2 mm fore-aft, 148.9 mm left-right, and 122 mm up-down. The angle in side view is 6.4° and the angle in plan view is -5.4°.

The eyellipse tangent points for each two-dimensional view are found by first translating the coordinate system to be centered at the eyellipse centroid, then rotating the axes to be aligned with the eyellipse axes, and finally scaling the coordinates so that the eyellipse becomes a unit circle. On a circle, tangent lines are perpendicular to a radius of the circle, so it is easy to find the two tangent lines that also pass through the mirror inboard edge. Once the two tangent points

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[1] The specifications for the Trans Am came from measurements taken in conjunction with a study of eye position (Manary et al., 1998). Some key dimensions, such as the SgRP-to-ground distance and the distance from the front of the vehicle to the mirror, were not obtained at the time and could not be obtained from published vehicle specifications. These dimensions were approximated with reasonable guesses. For the purposes of the example it is not important that these measurements be completely accurate, but the results of the example calculations should not be applied to any actual vehicles without taking precise measurements for those vehicles.

on the eyellipse are found, the coordinate system is rescaled, rerotated, and translated so that the origin is located at the right edge of the mirror. The coordinates of these points are given in Table 1, along with the coordinates of the target, referenced to the mirror edge.

Table 1
Coordinates of tangent and target points in the Trans Am.

	Coordinate (mm from mirror edge)		
Tangent Point	x (fore-aft)	y (left-right)	z (up-down)
a_{min} (outboard point)	665	342	
a_{max} (inboard point)	241	473	
b_{min} (lower point)	434		-11
b_{max} (upper point)	300		179
Target	2833	25	-49

The next step is to adjust a_{min} and a_{max} for head turn, according to Equations 13-18. Taking a_{min} first, the fore-aft component of the vector $\vec{h}$ is the x coordinate of a_{min} plus 98 mm (the distance to the pivot point, p). The left-right component of $\vec{h}$ is equal to the y coordinate of a_{min} plus 32.5 mm plus half of the mirror width (as in Equation 13). The mirror width is 6.5 in (165 mm). Equations 14 and 15 are applied to the coordinates of $\vec{h}$ to determine γ', the head-turn angle. Finally, Equations 17 and 18 must be solved simultaneously for s_1 and s_2. The calculations for a_{min} and a_{max} are shown in Table 2.

Table 2
Calculations to adjust for head turn.

a_{min}	a_{max}
$\vec{h} = (763, 457)$	$\vec{h} = (339, 588)$
$\gamma = \tan^{-1}\left(\frac{457}{763}\right) = 30.9^\circ$	$\gamma = \tan^{-1}\left(\frac{588}{339}\right) = 60.0^\circ$
$\gamma' = 30.9 - 30 = 0.9^\circ$	$\gamma' = 60.0 - 30 = 30.0^\circ$
$s_1 = -96.6$	$s_1 = -66.8$
$s_2 = -36.7$	$s_2 = -82.5$

Recall that the vector $\vec{s}$ has its origin at the head-turn pivot point and its endpoint at the new tangent point after head rotation. To return to the mirror-edge coordinate system, the component of $\vec{s}$ must be added to the location of the pivot point in mirror-edge coordinates. The resulting tangent points are given in Table 3. Obviously, head turn has a much greater effect on a_{max}, which is both more forward and more inboard, as compared to a_{min}.

Table 3
Coordinates of Trans Am tangent and target points, adjusted for head turn.

Tangent Point	Coordinate (mm from mirror edge) x (fore-aft)	y (left-right)	z (up-down)
$a_{\min}$ (outboard point)	666	338	
$a_{\max}$ (inboard point)	272	423	
$b_{\min}$ (lower point)	434		-11
$b_{\max}$ (upper point)	300		179
Target	2833	25	-49

At this point, the numbers are nearly in place to apply Equations 1 through 8 and determine mirror adjustment angles. The only missing components are δ_h and δ_v, the angles between the reference line and $\vec{v}$, the vector from the mirror to the target. These can be found easily by taking the inverse tangent of the ratio of the two components of $\vec{v}$. The values for the Trans Am are given below.

$$\delta_h = \tan^{-1}\left(\frac{25}{2833}\right) = 0.51^\circ$$
$$\delta_v = \tan^{-1}\left(\frac{-49}{2833}\right) = -0.99^\circ$$

Finally, Equations 1 through 8 can be applied to the four tangent points, as appropriate. Table 4 shows the arithmetic for each of the four points.

Table 4
Calculations for mirror adjustment angle for four tangent points.

Adjustment angle type	Angle between eye and target	Angle of mirror adjustment with respect to the reference line
Minimum Horizontal	$\alpha_{\min} = \cos^{-1}\left(\frac{666*2833+338*25}{\sqrt{666^2+338^2}\sqrt{2833^2+25^2}}\right) = 13.7^\circ$	$\theta_{\min} = 0.5*26.3+0.51 = 13.7^\circ$
Maximum Horizontal	$\alpha_{\max} = \cos^{-1}\left(\frac{272*2833+423*25}{\sqrt{272^2+423^2}\sqrt{2833^2+25^2}}\right) = 56.7^\circ$	$\theta_{\max} = 0.5*56.7+0.51 = 28.9^\circ$
Minimum Vertical	$\beta_{\min} = \cos^{-1}\left(\frac{434*2833+(-11)*(-49)}{\sqrt{434^2+(-11)^2}\sqrt{2833^2+(-49)^2}}\right) = 0.4^\circ$	$\phi_{\min} = 0.5*0.4-0.99 = -0.77^\circ$
Maximum Vertical	$\beta_{\min} = \cos^{-1}\left(\frac{300*2833+179*(-49)}{\sqrt{300^2+179^2}\sqrt{2833^2+(-49)^2}}\right) = 31.8^\circ$	$\phi_{\min} = 0.5*31.8-0.99 = 14.9^\circ$

For the Trans Am, the total adjustment angle needed is 15.2˚ horizontally and 15.7˚ vertically. The resulting rectangle in mirror adjustment space is shown in Figure 6. Approximately 96% of drivers should be able to adjust their mirror so that they can see half a meter of the rear of their vehicle in the mirror. Assuming this is a reasonable representation of what drivers would like to see, and that variation between drivers in mirror adjustment preference is negligible, then adjustability that matches the rectangle in Figure 6 will accommodate 96% of drivers.

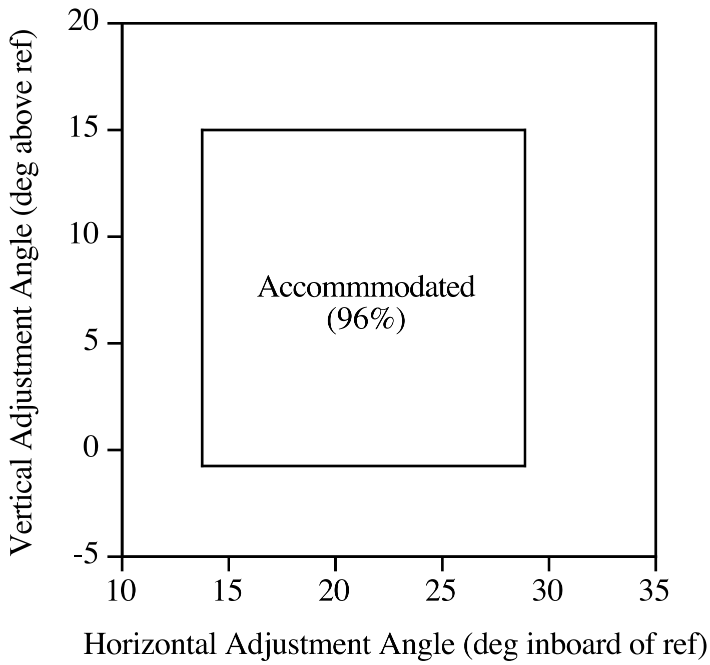

Figure 6. Mirror adjustment rectangle accommodating 96% of drivers in a Trans Am.

Summary and Conclusions

In this report, we present a series of equations that can be used to predict the range and location (relative to an arbitrary reference line) of mirror adjustability required to accommodate a given percentage of the drivers in any passenger vehicle (96% in our example). At various points in the solution, we met with challenges that required compromise solutions that might be considered practical but inelegant. As desirable as the elegant solution is to the mathematician, the practical solution is desirable to the engineer.

The solution begins with the SAE J941 eyellipse, which represents the distribution of eye locations in a given vehicle using a tangent ellipse. The J941 eyellipse may soon be replaced by an updated version. The new eyellipse, because of its more rearward centroid, will likely result in smaller adjustment angles for both maximum- and minimum-adjustment points. When it is available, the new eyellipse will work equally well with these equations.

The first challenge was to figure out how to determine the percent of drivers accommodated, given that the eyellipse could not simply be transformed "through the mirror" into a mirror-adjustment shape. The solution was to operate on the plan-view and side-view eyellipses independently to produce separate ranges for horizontal and vertical mirror adjustment. This results in a 96% accommodation rectangle in mirror-adjustment space when the 99% eyellipse is used.

Using side-view and plan-view eyellipses separately, we identified two points on each that lie on tangent lines passing through the mirror center (or mirror edge for some targets). These represent eye locations for which minimum and maximum mirror adjustments are required. The next challenge was to handle head turn for the plan-view tangent points. Ideally, head turn should be incorporated first, and minimum and maximum points chosen afterwards. This solution, however, is mathematically intractable, and so head turn was incorporated after the tangent points were selected. In exchange for simplicity, the potential error is extremely small.

Having handled head turn, the last challenge was to choose a target definition. This is a theoretical challenge rather than a mathematical challenge. Since little is known empirically about criteria for mirror aim, the user must make an educated guess. We proposed solutions for three types of potential targets: a single-point centered target, an extended centered target, and a target seen at one edge of the field of view. Once the target is defined, the appropriate equations will yield the minimum and maximum horizontal and vertical adjustment needed, relative to an arbitrary reference line. These four points form the corners of a rectangle that contains mirror adjustment positions that accommodate 96% of drivers.

The other critical assumption about the target in our example is that there is no variability in target choice across drivers. In reality, there will be at least some such variability, and Olson and Winkler's (1985) results show that, indeed, different drivers see different views in their mirrors. The primary effect of incorporating variability would be to expand the boundaries of the region that would accommodate 96% of drivers. In other words, the region that results from assuming a fixed target is too small. On the other hand, it is not clear how much bigger the region needs to be. As a place to start, Olson and Winkler report 5.5° degrees of difference between the 10^{th} and 90^{th} percentile in what drivers see at the inboard edge of their mirror. To achieve this change from a fixed eye point, the mirror would have to adjust horizontally by half that amount, or 2.75°. Thus, based on this provisional estimate of the effects of target preference, the range of horizontal mirror adjustment needed to accommodate differences in target preference may be considerably smaller than the range needed to accommodate differences in eye position (15.2° for the example discussed earlier in this paper). Also, to achieve a desired value for overall percent accommodation it is not necessary to add the entire target-preference range to the eye-position range. This is because, compared to modeling based on a single, unvarying target, target variability can be expected not only to put some drivers who were inside the eyellipse outside the overall accommodation region but also to put some who were outside the eyellipse boundary inside the overall accommodation region. On average, the people who are extreme in eye position are not likely to be equally extreme in their target preferences. Therefore the overall range of mirror adjustment needed to accommodate a given percentage of drivers may be dominated by the variable with the largest influence (apparently, eye position), and the other variable (apparently, target preference) may have only a minor influence. However, this issue cannot be fully resolved without further information about people's target preferences, and about how those preferences are related to people's eye positions.

It should be noted that the solutions given are based on the simplifying assumption that the mirror pivots around the point on the mirror at which the target is seen. This point is used as the origin for mirror-aim calculations. Although mirrors do not actually pivot around any point on the surface of the mirror, the adjustments needed to correct for the effect of the true pivot point are small. Moreover, different mirrors may use different pivot mechanisms, requiring unique adjustments for each mirror.

In spite of the challenges encountered in solving this problem, the solution has many good characteristics. First, it is relatively simple and can be calculated without CAD or other sophisticated computer programs. It could even be calculated by hand! Second, the solution provides a clear, simple recommendation for calibrating mirror adjustment in a particular vehicle. Finally, the solution can be adapted to many different target types and to future versions of the eyellipse, making it relatively flexible. Further work should be done on characterizing drivers' target preferences in order to achieve a comprehensive solution to the problem of determining mirror adjustment ranges.

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