Gyroscopic Stabilization of Unstable Vehicles

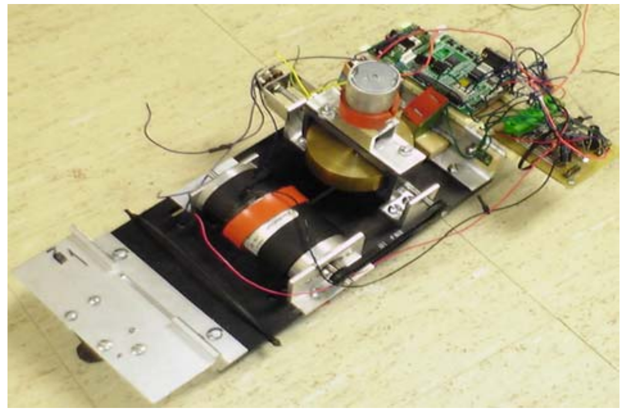

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TABLE OF CONTENTS

ABSTRACT

The purpose of this project is to design and build a cart that demonstrates active gyroscopic stabilization. The gyroscope balances the vehicle by countering external disturbances by use of the precession effect. When an external torque is applied to the gyroscope, it will react by turning on an axis that is perpendicular to both the spin axis and torque axis. The active control of this external torque forms the basis of the actuating mechanics. The goal of this project is to manufacture a functioning model that stabilizes itself if its equilibrium is disturbed.

INTRODUCTION

Monorails and other vehicles with only two wheels do not have enough points of contact to stabilize themselves, so they require a stabilizing mechanism. One such mechanism is an internal gyroscope, which counteracts external torques. The purpose of this project is to test the concept of gyroscopic stabilization in vehicles, by building a model that stabilizes itself using a gyroscope. This will be done by using electronic sensors that detects instabilities in the system and feed that information into a microcontroller, which controls an actuator that moves the gyroscope axis back and forth to counteract the instability. Our goal in this project is to construct a two-wheeled cart that stabilizes itself while idle. If this goal is met, we will create a track for the cart to follow that will have varying curves and inclines.

INFORMATION SEARCH

A gyroscope is a device that uses the principles of angular momentum to maintain a prescribed orientation. A gyroscope consists of a wheel mounted on an axis that can turn freely in any direction; when in rotation, it resists any change in the orientation of its spin axis. If a torque is applied to the spin axis, the axis will not turn in the direction of the torque, but will instead move in a direction perpendicular to it. This motion is called precession. The gyroscope's ability to stabilize itself was the foundation of many early 20th century inventions, the most notable of which, was Brennan's gyroscopic monorail.

Louis Brennan, an Irish-Australian inventor, was one of the first to patent a gyroscopic stabilizing vehicle [1]. In 1903, Brennan patented a gyroscopically balanced monorail system that he designed for military use; he successfully demonstrated the apparatus in 1909. By mounting one or more gyrostats (a modified gyroscope) along the body, the monorail balanced itself when its equilibrium was disturbed. Brennan feared that the gyrostats would fail in use, causing total system failure; thus, he prevented the monorail from being mass-produced.

More recently, a group from Columbia University manufactured a modernized version of Brennan's monorail [2]. Unfortunately, the group was unable to create a working model. The electronic component of the model continuously overheated during operation, causing the motor to burn out. The electronic segment was improperly modeled, which led to the mechanism's inability to perform.

We intend to improve upon this design and create a functioning model that stabilizes itself in a stationary position and maintains its equilibrium while in motion. In order for us to achieve this, we need to first derive the applicable equations of motion. This would allow us to design a mathematical model of the system. Second, we need to determine the inconsistencies in the Columbian design, which would permit us to avoid similar mistakes in our design. Lastly, we need to determine the power consumed by the design, which would allow us to supply the model with sufficient power.

CUSTOMER REQUIREMENTS

Since our project is a proof of concept, as opposed to a commercial design, we determined that our audience comprise of two groups of people. The first group consisted of the ME 450 professors and our project sponsor. The second group was the general audience at the Design Expo. With these two segments of the audience in mind, we developed al list of customer requirements.

From a technical standpoint, the most important aspect of our project is gyroscopic stabilization. To that end, we decided that stability is most important design requirement. Additionally, the manufacturing of the model must be cost-efficient.

From the standpoint of a Design Expo presentation, we added additional components to the customer requirements to ensure that our project was appealing to our audience. First, since our model was on display during the Design Expo, we incorporated safety and interactivity into our design. Hence, onlookers could operate the model without fear of being injured. Second, we wanted the device to be as quiet as possible; a noisy device could potentially disturb observers and passersby. Third, we wanted the model to be as light as possible; a lighter model would put less stress on the motors, consequently, decreasing the amount of time needed to stabilize the cart once it is disturbed.

ENGINEERING SPECIFICATIONS

Our problem definition stipulates that certain characteristics are present in our design. These constraints restrict our design and reduce the number of decisions that must be made. Additionally, they allow us to select specifications based specifically on this approach to stabilization, in addition to more general specifications that would apply to any similar problem.

In order to generate target goals, we estimated the necessary physical parameters by doing a rough, non-dynamical model of the system. The requirements reflecting the dynamics of the system are estimates of what would be desirable in a real-world application. Additionally, aesthetic requirements are selected for ease of maintenance and transport. The engineering specifications and their target values are presented in Appendix A.

QUALITY FUNCTION DEPLOYMENT

A Quality Function Deployment (QFD) diagram is used to relate customer needs to engineering specifications. It also tracks interdependencies between engineering specifications, and records target values. Finally, it can be used to compare the product development to various other products by using the engineering specifications to set benchmarks.

The customer requirements are written on the left side of the chart and are ordered in rows. The relative importance is written next to each requirement. The technical specifications are written across the top and are ordered in columns. At the bottom of each column, the target goal for each specification is written. Additionally, the importance of meeting that particular engineering specification is calculated by comparing the center of the chart with the left.

The center of the chart records how strong a correlation there is between each customer requirement and engineering specification. Each cell is assigned a 0 for no relationship, a 1 for a weak relationship, a 3 for a moderate relationship and a 9 for a strong relationship. This value is then multiplied with the weight of that particular customer requirement. Finally, each of these products is added vertically for each engineering specification to determine their relative weight. This way, the importance of every customer need is translated into added importance for every engineering specification based on relevance.

We determine the relative importance of the various customer requirements, by pairing each requirement against every other, and used this to rank them in order of importance. In order to relate each requirement to each engineering specification, we went through each cell and examined the nature of the relationship.

Another important part of the QFD is the triangular section on top. This describes the relationship between each of the technical specifications. It is not used to record their relationships so that a change later in the design process does not bring unexpected changes in related technical areas.

Finally, the QFD can be used to compare the product or idea in question to other similar products. We compared the various concepts in this space.

From our QFD, we determined that the following as the most important engineering specifications: the torque of gyroscope, the tilt limit of the cart, and the size of the cart. Now that we were aware of the importance of these specifications, we ensured that we met each specification in any proposed designs. For our full QFD, see Appendix B.

CONCEPT GENERATION

A two-wheeled vehicle is naturally unstable and requires an external force to provide balance. A gyroscopic device can be used to provide a force that resists gravity and balance the unstable vehicle. This balancing mechanism must be able to do the following: sense a disturbance of equilibrium, feed this information to a controller, and actuate a device that provides the necessary force to balance the system. We generated four concepts that fulfilled the aforementioned requirements. The concepts are as follows: a single gyro and single wheel, a single gyro and two wheels, two wheels and two gyros, and a fluid-based gyro with two wheels.

Single Gyro Single Wheel

The Big Wheel is a gyroscopically stabilizing system enclosed inside of single wheel. The system is comprised of counterweights, a gyroscope, and motors for both the gyroscope and the external wheel. We later discovered that Carnegie Mellon created a model that paralleled our single-wheel design concept, which they called the Gyrover. The Gyrover is a single wheel gyroscopically stabilized mobile robot; an internal pendulum serves as a counterweight for a drive motor that causes fore/aft motion, while a tilt-mechanism on a large gyroscope provides a mechanism for lateral actuation. [3] (See preliminary sketch in Appendix C)

The complexity of the design and the lack of information were the fundamental reasons for not selecting the Big Wheel concept. Although the concept is feasible, the system modeling, CAD drawings and manufacturing are difficult to create. Unlike the two-wheel cart, we do not have access to background research, equations of motion, or an expert on the project. With a lack of information, the time required to complete the project exceeds our schedule.

Single Gyro Two Wheels

This concept consists of a base plate, two wheels, a gyroscope, motors that operate a flywheel and gimbal, and a system that senses equilibrium disturbances and balances the cart. The gyroscope balances the vehicle by countering external disturbances by use of the precession effect. When an external torque is applied to the gyroscope, it will react by turning on an axis that is perpendicular to both the spin axis and torque axis. The active control of this external torque forms the basis of the actuating mechanics. (See preliminary sketch in Appendix C)

There were various advantages for selecting this design. First, the overall cart is compact. All the components support the gyroscope in the most compact way possible, thus making it easier for the gyroscope to balance the vehicle. Furthermore, the protective cage that houses the flywheel gives this concept a good safety factor. In order to balance the cart, the flywheel spins at very high velocities, thus it is important that a protective components shield bystanders in case of system failure. This concept also displays symmetry about the cart tilt axis, which is a significant factor in gyroscopic stabilization.

On the other hand, this concept would not be very useful for larger vehicle applications. The size and weight of the gyroscope are directly correlated to the size and weight of the vehicle. The gyro must be large and spin at high speeds for larger vehicles. Therefore, a large gimbal motor would be needed to rotate flywheel and a large motor would be needed to spin the gyro, potentially producing a substantial amount of noise and requiring a considerable amount of power to operate. These factors would make the system quite expensive.

Twin Gyros Two Wheels

This concept, most notably used in the Brennan monorail, uses two counter rotating gyroscopes to provide stability to an unstable vehicle. This concept was also applied to monorails, two-wheel cars, and bikes.

Although there are notable advantages to the design, the disadvantages associated with the design outweigh the advantages. The Brennan model is very similar to the baseline standard design with the addition of an extra gyroscope. The advantages of having two gyroscopes are the following; the ability to provide more precession force than a single gyroscope, and the second gyroscope provides safety net in case the first gyro fails. (See preliminary sketch in Appendix C)

Conversely, there are various disadvantages to selecting this concept; the complexity of the system makes it difficult to create a control algorithm, and this concept would require additional parts. Since the concept requires two gyroscopes, the model would require four motors as compared to two motors. The extra motors increase overall cost significantly, add additional weight to the model, and increases the noise level. Consequently, the amount gained does not merit giving up all the tradeoffs in this model; therefore, we did not select this design concept.

Single Fluid Gyro Two Wheels

This concept uses a fluid that rotates inside a circular tube to generate the angular momentum necessary for precession force actuation. An onboard water pump pumps the fluid at high speeds. A separate motor that tilts the gyroscope actuates the precession force. The chief reason for this design is to remove the potentially dangerous spinning flywheel and replace it with something less likely to destabilize. (See preliminary sketch in Appendix C)

However, upon closer analysis, we found that the concept was lacking in several areas. This concept has the potential to be more compact than any flywheel designs because the ability to place components inside the ring. On a flywheel design, the space inside the main mass of the flywheel is off-limits because of the necessary support structures like wheel spokes or even a thin disk. A fluid gyro need only support the pipes, so the inside area is available for component placement. Upon closer examination, however, the power required to generate adequate angular momentum was found to be too high. This would result in an unacceptable current draw, large batteries, and large actuating motor. Finally, there is the potential of the operating fluid damaging the electronics if there is a leak.

All the issues associated with the concepts analysis presented here are shown in Appendix D in detail.

Pugh Chart Analysis

In order to accurately and systematically analyze all the different design concepts together, a Pugh Chart was created to organize and compare features of each design and weigh their pros and cons. This chart can be seen in Appendix E. The chart compares the four new design concepts to the standard baseline design. This baseline is the cart design created by the Columbia University team. The four design concepts were then compared in the categories determined by the customer requirements from the QFD diagram.

The results of this diagram show that the standard model design concept outperformed all of other concept. For example, the water gyro would be unsafe; the large wheel cart would not be stable under static conditions; the Brennan monorail concept would be too expensive and heavy. Although there were benefits related to each design, they did not merit a design change in light of all the negatives associated with them. From the Pugh chart, we decided to pursue the original design of our predecessors, and try to improve on their execution of the concept.

ENGINEERING ANALYSIS

In order to reach our goal of fabricating a functional gyroscopic stabilization mechanism, we modeled our system using Matlab, Simulink, and CAD. Modeling and simulating our system enabled us to optimize stabilization and predict possible problems. It also gave us an inexpensive method in which to test our system with a sufficient level of accuracy. These procedures are essential in the process of manufacturing our final product.

Modeling and Simulation

We designed the simulation system of our model by first deriving a system of equations, linearizing the system of equations, and creating both a linear and nonlinear model.

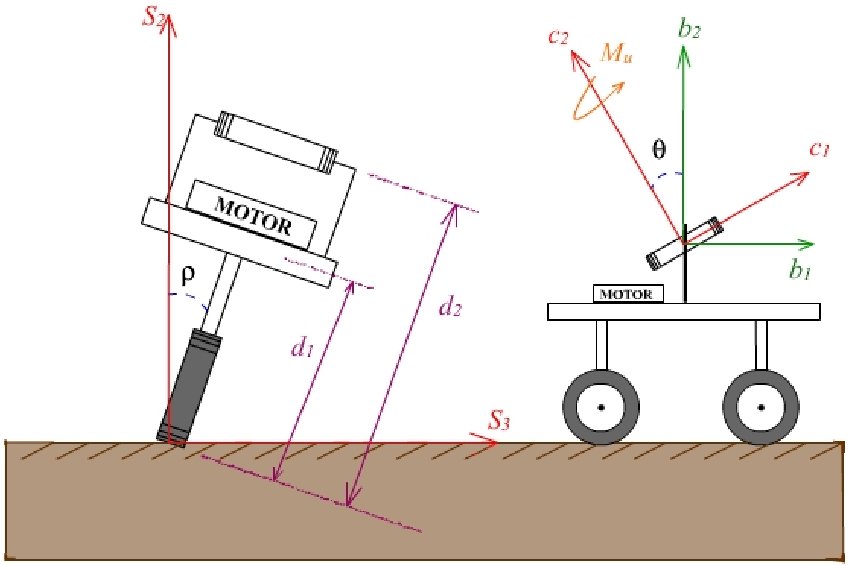
To determine the system of equations, we created Free Body Diagrams (FBDs) and defined reference frames for both the cart and the gyroscope to simplify calculations (See Figure 1). Since the cart and the gyroscope are considered as two separate rigid bodies, we created a separate FBD for each body. Next, we identified the input variable, Mu, and the state variables: θ = angle of gyroscope, $\dot{\theta}$ = angular velocity of the gyroscope, ρ = angle of the cart, and $\dot{\rho}$ = angular velocity of the cart. From the FBDs and the defined parameters, we derived the equations of motion (EOM) of the cart and gyroscope using Lagrange's method (see Eq. 1 and Eq. 2). A written derivation of EOMs is located in Appendix F. The systems of equations comprise of two coupled nonlinear deferential equations:

$$(m_B d_1^2 + m_G d_2^2 + I_{B11} + \cos^2\theta I_{G11} + \sin^2\theta I_{G22})\ddot{\rho} + 2\cos\theta\sin\theta(I_{G22} - I_{G11})\dot{\theta}\dot{\rho} \\ -\Omega\cos\theta I_{G22}\dot{\theta} - (m_B d_1 + m_G d_2) g \sin\rho\cos\psi = 0 \qquad \text{Eq. 1}$$

$$I_{G33}\ddot{\theta} - \dot{\rho}^2 \cos\theta \sin\theta (I_{G22} - I_{G11}) + \Omega \cos\theta I_{G22} \dot{\rho} = M_u \qquad \text{Eq. 2}$$

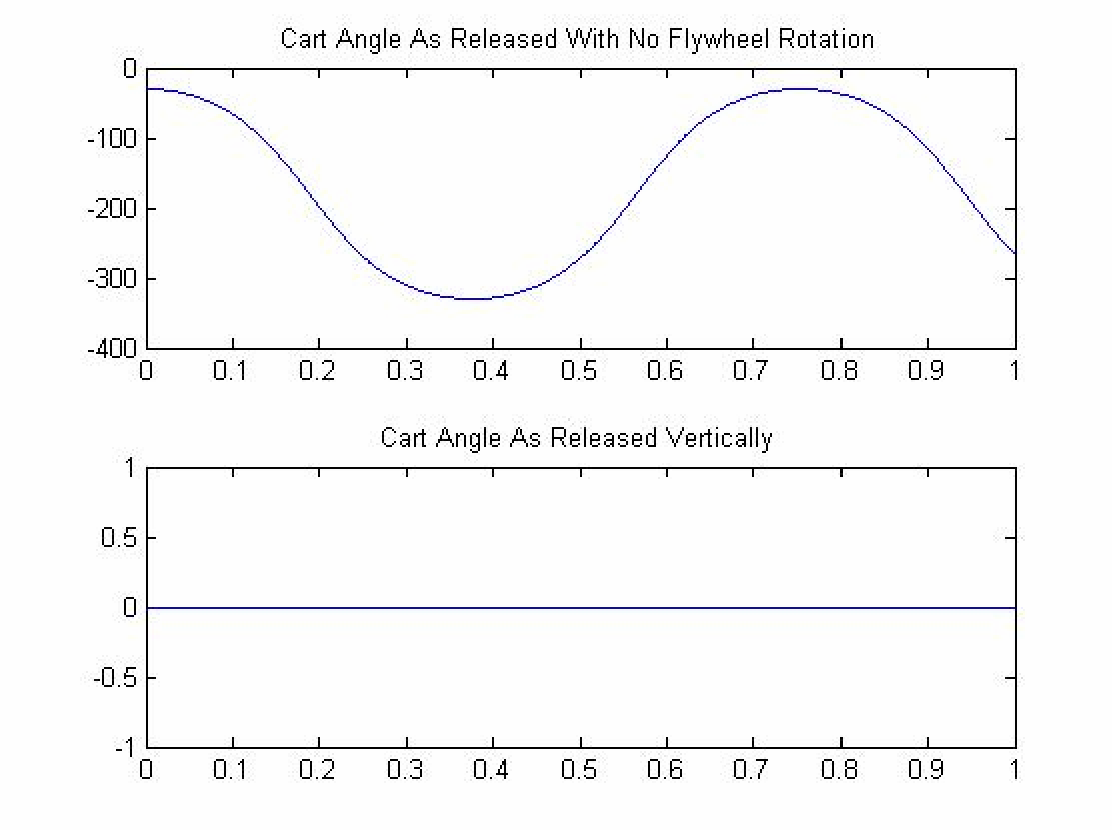
We converted the EOMs into state-space form and linearized around the vertical position. We defined the desired operating position of the system as, ($\dot{\rho}_{eq} = \dot{\theta}_{eq} = \rho_{eq} = 0$) (See Figure 1). Then we linearized the system around this equilibrium point and created a linear controller in MatLab.

Figure 1: The Defined Reference Frames for both the Gyro and the Cart



Since real world devices are typically non-linear as shown by Eq. (1) and Eq. (2), we constructed a nonlinear model in Simulink. We created the nonlinear model from the previously described EOMs (See Appendix G). To ensure the accuracy of the nonlinear system, we performed preliminary tests to observe if the system would respond as expected. The tests that we conducted have predetermined outcomes, which enabled us to verify if our simulation was working correctly. The first test comprised of setting the gyroscope tilt angle and the flywheel angular velocity to zero. In conjunction with those settings, an initial tilt angle was also imposed on the cart. Using these settings and simple physics knowledge, we recognized that the system response would be that of an oscillating pendulum (See figure 2). For the second test, we set the cart tilt angle to zero (which was upright) and the flywheel angular velocity to zero. Again, we knew that if our nonlinear model performed accurately, then the response of the system with these conditions would produce no movement – the cart would stay upright until disturbed (See figure 2). The nonlinear simulation passed both tests.

Figure 2: The Test Result of the Gyroscopic System



Subsequently, we obtained control gains from our Matlab linear approximation model and input those values into the nonlinear simulation (See Appendix H). The linear approximation model calculates the control vector, "K", given the desired pole positions using Ackerman's formula (in Matlab). These gains enable the motor to output a torque that is accurate enough to provide cart stabilization, which enables the nonlinear Simulink model to function. They are related as follows:

$$Mu = \begin{bmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{bmatrix} \bullet \begin{bmatrix} \dot{\theta} \\ \theta \\ \dot{\rho} \\ \rho \end{bmatrix} \qquad \text{Eq.3}$$

We used the nonlinear Simulink model to test the stability of the closed loop system. We discovered how the mechanism would respond with the change of various physical parameters. The desired response criteria for our system are fast settling time for both the gyro and cart, a peak overshoot less than 45° for the gyro, a peak overshoot less than the max tilt angle of the cart, and minimal control effort. In order to determine which parameters have a significant effect on the stabilization characteristics of our cart, we held every other parameter constant and varied each parameter by 10%, up and down. We then recorded the response time and peak overshoot, which are both good ways of measuring how effectively the cart stabilizes. The response time and max overshoot should both be small. In order to organize this information, we produced the chart shown in Appendix I.

As can be seen in the table of parameters—see Appendix I, certain parameters have no significant effect on the stabilization of our mechanism, such as the cart width, cart thickness, gyro radius and gyro thickness. Notably, changing the gyro mass has a sever effect on stabilization, but changing just the thickness, without changing mass has little effect. On the other hand, changes in the height of the gyro center of mass (COM) and the cart COM produced considerable responses to the stabilization of our mechanism. This was due to the change in the moment created by the COM of the gyro and cart. Decreasing these heights, lowered the peak overshoot values of the gyro and cart tilt and reduced settling time. It also increased the ability of the system to stabilize.

Modifying the mass of the gyro and cart produced significant effects on the stabilization of the system as well. Reducing the mass of the system decreased the peak values for the gyro and cart, settling time for the system, and the amount of power needed to stabilize the system. Increasing the angular velocity of the flywheel decreased peak values for both the gyro and cart but requires more control effort to operate.

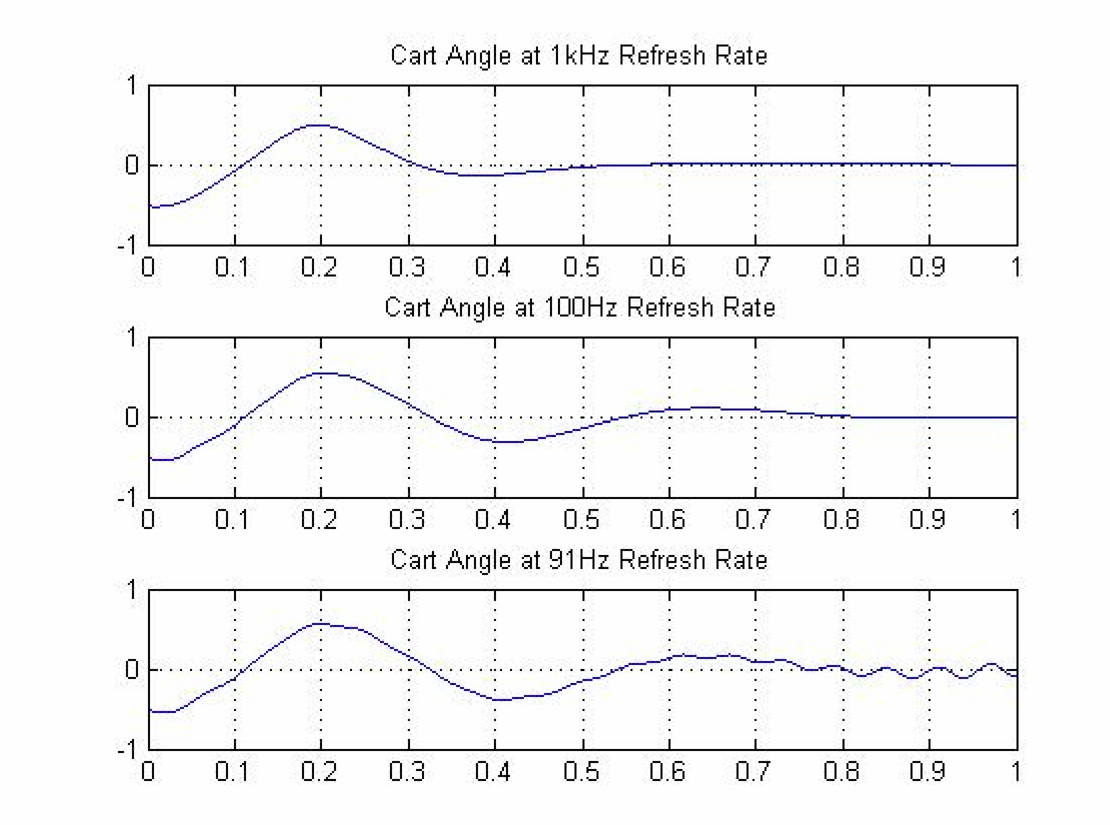
Using the knowledge gained from testing, we concluded that, in order to optimize the response of our system, it would be best to:

- Make the COM of the cart as low as possible
- Make the COM of the gyro as low as possible
- Increase the rotational speed of the gyro if we have the battery power to do so
- Decrease the mass of the gyro (to a point)

With these new design criteria, we devised a basic schematic for the model, which would produce optimal stability for our system. We carried out more tests in the simulation to discern the bounds of our system. Appendix J displays the maximum cart tilt angle; the gimbals tilt angle, and the corresponding control effort for this system.
The gyroscope can not stabilize the system for any cart angle larger than 30^{o}, and after the gyro tilts 45^{o}, it becomes ineffective. Changing physical parameters in the system produced various values of the control vector K. The control vector obtained for these parameters was
K = [-.5088 .0527 1.2489 -.0702].

In order to simulate the effect of using a discrete digital controller in our system, we modeled our controller as a discrete subsystem, with a fixed refresh rate. This allowed us to explore the stability of the system as a function of controller refresh rate. We determined that the system became unstable at about 100Hz, with slight variations depending on specific system parameters. We decided that our controller must update considerably faster than this in order to avoid this source of instability. This is evident in Figure 3, where the cart angle is plotted against time. Notably in the unstable example with a refresh rate of 91 Hz, the seemingly small vibrations in the cart angle are accompanied by a much large control effort of continuously increasing magnitude.

Figure 3: Stability Decreases as Controller Refresh Rate Drops



Flywheel and Gimbal supports

The most important component of our device is the gyroscope. The major variables of this component are its diameter, thickness, and mass. When we were creating the final design of our cart, we let the dimensions of the gyroscope determine everything around it. To avoid cost in ordering a flywheel or trying to buy stock and machine it ourselves, we were fortunate enough to find a brass flywheel in the scrap bin. It was very close to the size we needed at 75mm diameter and 30mm thick with a half inch whole through the center. We ended up keeping the dimensions as we found it, after determining we needed all the force from the gyroscope that we could get, which meant keeping as much mass as possible. We wanted to have a brass flywheel because of the higher tolerance of press fitting brass to steel—the material of flywheel axle—as compared to press fitting a steel shaft into a steel flywheel.

Two aluminum flanges support the rotating gimbal axis. These pieces will be 50 mm tall and have a 30 mm base. We chose this height because we wanted the gyroscope to be able to rotate at least 50º. We found this height by using the radius of the gyroscope and the angle at which we want it to be able to tilt, and seeing how far vertically the edge goes down. The shape of this part was chosen because the stock of this shape was in high supply in the machine shop, and would be easily modified to our purpose. Two dowels go through these supports and into a PVC cage; this cage supports the gyro axis from the top and bottom with bearings. (See Appendix K)

Base Supports

The base supports of the cart are comprised of two types of sections. The lower base, which supports the gyroscope, has dimensions of 275 x 150 mm. The material choice for this component was PVC plastic, because we wanted the lightest material possible, while still being stiff enough for our application.

The second type of section is two adjoining pieces of 2” angle aluminum that is 3/16” thick. These pieces are located at both ends of the cart and are used for mounting both the wheels and the circuits. This stock was chosen because this piece will experience significant stresses and will need to have a certain level of strength and stiffness, yet we still wanted the cart itself to be as light as possible. One piece has 3 mm wide slots that are 20 mm long so that the height of the cart can be adjustable. (See Appendix K)

Wheels and Wheel Supports

The rubber wheels and axles were purchased at Ryders’ Hobby. It was much easier to by rounded rubber wheels than to try to manufacture them. The size of the wheels was selected so that they would be big enough to keep the cart off the ground, and the thickness was chosen so that the wheels would be strong enough to not buckle under the load of the cart, while at the same time not being too thick that the cart balances itself on them. The wheels attach to the angle aluminum by two L bars. Their length was made in the shop to ensure that at the wheels would be able to touch the ground at the upper most slot of the vertical base. (See Appendix K)

Motors and Support

The major variables for the gyroscope motor are its torque and mass. We wanted the smallest possible motor; because it will be located high on our cart, a heavy motor could potentially make the cart top-heavy. We decided that we wanted the gyroscope to spin up to a speed 1000 RPM in one minute, which would give us the required angular acceleration necessary to operate the model. Working with the angular momentum of the flywheel, we determined that the motor must provide a torque of 80 g·cm. We acquired a 12VDC stepper motor from our sponsor Professor Girard. This motor is small and able to apply the necessary torque for the system.

The second motor in our system tilts the gyroscope gimbal. As with the flywheel motor, the major variables for this motor are its torque and mass. To find the necessary peak torques that this motor would have to apply, we found the parameters of the cart and gyroscope, and plugged them into our Simulink model. After running a simulation and verifying that the cart did stabilize, we pulled out what the maximum torque applied to the system. From the simulation, we calculated the torque to be 0.35 N·m. Since this motor was very important to the system, we chose a motor that could exceed this torque by a factor of 5—1.7 N·m. We chose

G240 series motor from Electrocraft, because it was a relatively small motor compared to others of its capabilities; also it provides well over the torque that was required.

Another component is the motor tensioner, which mounts the gimbal motor to the board, and moves the motor backwards when the belt creeps. The tensioner is two aluminum brackets that are attached by screws tightened into tapped holes on the motor faces. (See Appendix K)

Transmission
The tilt motor transfers the power via a Kevlar reinforced belt. The belt, which was purchased at Ryder's hobby store, is lined with small teeth, and came with two pulleys. This was ideal because the teeth make the belt more precise by preventing the belt from slipping on the gears and Kevlar reinforcement will help reduce the belt creep.

Control Electronics
The control electronics are the components that implement the control algorithm, from the microcontroller, to the high-powered drivers for the motors. The microcontroller selection took a long time because the decision included many different requirements. The driving components and sensors were chosen based on the motors that were selected, primarily based on the voltage and current required.

Microcontroller
The microcontroller was selected based on speed, number of input-output ports and cost. Additional peculiarities specific to each microcontroller were also considered. Our modeling determined that updates must occur significantly above 100Hz. Therefore, our benchmark for processor speed would be to fully update the controller and send an output in less than 5 ms. We determined that we would need between 7 and 11 input-output ports to drive our motors and receive input from our sensors.

Our initial search narrowed down microcontroller selection to three different concepts. The first was the BASIC STAMP 2e, which uses an old PIC controller with additional memory. It was programmable in a version of BASIC, but its particular hardware implementation drastically slows down the chip, thus, making it unsuitable for our uses. The second concept was a full Linux implementation on a miniaturized computer from Gumstix. We determined this to be expensive, far too powerful and generally in excess of what we needed. Lastly, we examined the PIC16/18 series of microcontrollers. While it is a chip of similar capabilities to the STAMP mentioned above, the lack of an information bottleneck in hardware allows these to operate much faster. We selected this chip to implement our control algorithm. In addition to being relatively fast, the PIC16/18 series of chips has an adequate number of input-output ports and has hardware analog-digital conversion and pulse-width modulation. This means that we can implement these procedures without interrupting or slowing program flow. The chip was programmable with a hardware programmer and code can be written in C.

Driving Components and Sensors

The controller we used has four inputs. These were the cart angle with respect to the vertical, the gimbals angle with respect to its "zero" position, and the respective angular velocities.

We needed an accelerometer in order to provide the cart angle. We ruled out liquid and temperature-based models and decided to go with a micro machined MEMs inertial type, because the accelerometer had to have a bandwidth considerably greater than 100Hz. The response time on this type of accelerometer was considerably smaller than 5ms.

We used a rate gyro to sense the rate of the cart angle. It was possible to derive this rate from the accelerometer signal. However, the high likelihood of slight vibration from the rotating flywheel, as well as other noise, was unacceptable due to the way that differentiation amplifies noise.

We decided to use a potentiometer to determine the gimbal angle. Other devices are available to determine rotation, such as optical encoder and Hall Effect sensors. However, these devices are complicated and produce a digital signal, which we must then read in software. We determined that analog was better for our purposes, because reading analog signals can be done in hardware using an analog-digital converter. Finally, having the potentiometer, instead of other pulse-based methods of sensing, allowed us to manually measure the position using a multi-meter, for use in troubleshooting.

Because a potentiometer was far less sensitive that an accelerometer, and because the gimbal shaft was unlikely to experience much angular vibration from the gyroscope, we decided that it was acceptable to differentiate the potentiometer signal in order to determine the angular velocity of the gimbal.

By noting the very high torque requirements during our modeling, we decided that it was unlikely our microcontroller could provide enough current to run our motors. Therefore, we decided that we would need motor drivers that are controlled by the MCU. Selecting the motor drivers was a straightforward procedure. We selected motors based on torque requirements. We determined our torque requirements from our model, as noted above, selected the motors that we would use and purchased the drivers based on our motors' current and voltage needs.

In order to drive the gimbal motor, we determined that we would need an H-Bridge. The key element in this decision was the fact that the motor must be able to rotate in both directions in order to meaningfully control the gimbal angle. The flywheel motor does not need to rotate in both directions since the flywheel never reverses direction. Therefore, the controllers can be a simple transistor to amplify the pulsed signal from the microcontroller.

MANUFACTURING

We divided the manufacturing into two components: stationary and dynamic. First, we used the engineering sketches (CAD model drawing) to manufacture all the stationary parts. These parts consist of the cart base and the wheel and motor supports. Once we completed the stationary parts, we manufactured all the moving parts accordingly. The sequential manufacturing of stationary then dynamic components ensured that all parts fit and operated as desired.

Most of the materials utilized during manufacturing were scraps found in the machine shop, which conserved time and money. Thus, much of our design was contingent upon the material available in the machine shop. The flywheel, which determined the size of the gimbal bracket and the flywheel shaft, was found in the scrap pile. The largest pieces of angled aluminum that we could find were 2 inches in width and so our design was adjusted accordingly. Most of the dimensions on our design were not restrictive; hence, we able to alter our design in accordance with the available materials.

The manufacturing process for the prototype took much longer than anticipated. Originally, we scheduled the manufacturing to take no longer than a week, with only two team members working on the construction of the prototype. Due to many setbacks, all five members were needed to accurately create an acceptable physical prototype. This resulted in an additional two weeks of manufacturing time spent on the prototype. A major unforeseen problem was performing very precise manufacturing with a limited amount of time and knowledge. Several parts were remade because of imprecise manufacturing. For example, the two pulleys on the gimbal motor and the gimbal itself had to be perfectly parallel to ensure even power transmission to the gimbal. The mounts for both the motor and gimbal were remade with flat parallel surfaces to enable references for mounting. In addition, the dowels on which the gimbal rotated were to be well aligned (axially) to ensure proper gimbal rotation and system stability. The shafts and the gimbal cage were both remade because the shafts were not axially aligned.

Other problems that were resolved included fitting the smaller bearings into holes on our prototype. The bearings we used were small and delicate so problems arose when attempting to press fit them into holes. Drilling and reaming the holes did not suffice; the holes would be either too large or too small. We solved this problem by making a slightly bigger hole and setting the bearing in it with epoxy. This solution ended up working well for the prototype, but was not the ideal solution.

Some moving parts required padding to guarantee nothing was damaged during operation. Since the gimbal could hypothetically rotate 360º, we installed cushioning where the gimbal assembly interfered with other stationary parts. Specifically, the motor atop the flywheel had a motion that was obstructed by the circuit boards on one side and the gimbal motor on the other

(See Appendix L). We placed silicone rubber around the flywheel motor as well on a post protecting the circuits and around the gimbal motor to prevent impact damage.

Since our part was a one-time prototype, we were able allow for unplanned, necessary adjustments. High scale production of system parts was not a requirement for this project. Additionally, the safety of the operator was not a major concern since there was not a high level of physical interaction. Although, we did consider the safety concerning the brass flywheel, which if dislodged while spinning, could cause serious injury. We decided to mount the flywheel using a ½" diameter steel shaft that was attached to a ¼" thick PVC cage by ¾"diamater bearings. Thus, the shaft, bearings and PVC cage would all need to break in order for the flywheel to detach. We felt this was an adequate safety factor built into our design.

Manufacturing of the electronics was problematic, mainly due to unforeseen problems that were exacerbated by the short amount of time we had to address them. First, we arranged the electronics into modules, where each sensing or control element was a module. Each module had wires soldered to it to connect to the main microcontroller. The components were then tested for functionality, disassembled, mounted on the cart, and then reassembled.

The modular design of our electronics system allowed us to add or remove components without much difficulty. However, this meant that we needed to prepare some components to be interconnected to our microcontroller, without necessarily connecting them to a PC board. The accelerometer and the initial H-Bridge were particularly tricky because both are in very small surface-mount packages. We soldered small wires to the surface mount components, verified their functionality, and applied epoxy to cover the pin-wire interface. The epoxy acted both as an insulator and as a strain relief. During this soldering, we damaged both the primary and backup H-Bridges and had to switch to a L298 H-Bridge, provided by Prof. Brent Gillespie. We also soldered wires to every other sensor, but had little difficulty with the other components. For detailed information and layout, see the section entitled Final Design.

After the components were prepared for interconnectivity, they were attached to the microcontroller and individually tested for basic functionality. After we established that they worked, we attached them to the cart using cardboard over epoxy adhesive as the base and a combination of screws and wire to tighten the cardboard to the boards. A pair of power strips from a breadboard was also installed to provide a common ground and +5V.

During this assembly process, several things went wrong. We burned both our H-Bridges, a replacement L298 H-Bridge, and both our PIC18F microcontrollers. The loss of H-Bridges was offset by replacements provided by Prof. Gillespie. We borrowed an ooPic board, also from Prof. Gillespie to replace the microcontroller. This board houses a PIC16F877 MCU and is much slower than our original microcontroller, but we assumed having some control capability was better than having none. We could have ordered replacements instead if we had more time. (A detailed Bill of Material (BOM) can be found in Appendix M)

The control system, components, interface pins and expected signals are all detailed in the Final Design section.

FINAL DESIGN

The final design of the cart changed slightly from its original plan once we started manufacturing. Several parts that were made from the original design did not work as planned. The flywheel gimbal was too weak in its original design, so we had to redesign it to be a fully enclosed support around the flywheel. Also, the gimbal motor mount was too weak and did not provide an accurate way to mount the motor. We had to remake these parts with thicker pieces with flat, parallel faces to be able to mount the motor parallel to the gimbal mounts to ensure even power transmission.

The final design was created with flexibility in mind. We knew that some part dimensions would be determined by the scrap pieces available in the machine shop. This caused some issues with manufacturing, since many dimensions in the CAD model were left as an estimate, which could change based on part availability. The essential parts of our cart still remained intact, but there were several changes made as we built the cart in the machine shop. Still, our final product maintained the same goals we had in mind when we designed it originally. All the functionality of the cart was still the same; a few minor changes and improvements were made along the way that did not alter our overall system design.

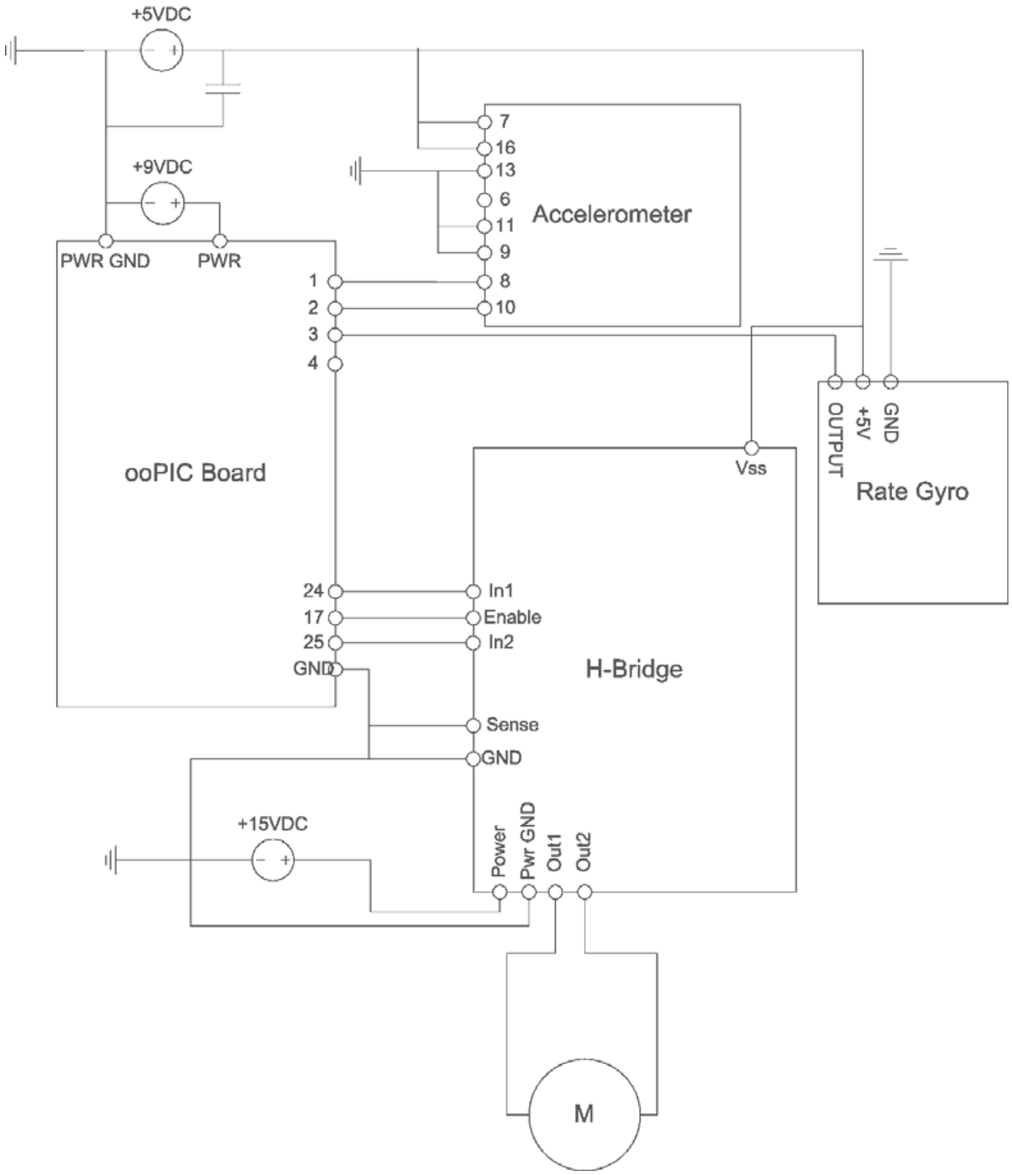
Because of the aforementioned difficulties with our first pick of MCUs, we stuck with the OOPic board for the Design Expo. The schematic in Figure XXX describes the connections that we made between our various components. Pin numbers are given for the OOPic board and the accelerometer. Specific pin usage can be found on their datasheets. The rate gyro was mounted on a PCB with callouts to the soldering holes. Thus, the schematic reflects the callouts. The H-Bridge was also mounted on a PCB, with flyback diodes and removable connectors. Because the pins on the board are not called out, the schematic labels the pin names on the H-Bridge.

The OOPic receives a ratiometric analog signal from the accelerometer and the rate gyro on ports 1 through 4. Since one port is unused, the potentiometer could also be connected, although we have not done so because of time constraints.

The OOPic sends directional information to the H-Bridge through pins 24 and 25, which are both digital I/O pins. The PWM of the motor is implemented over the 17th pin, attached to the Enable on the H-Bridge. The 17th port is one of only two on the OOPic that are capable of utilizing the chip's onboard hardware PWM.

The motor depicted in the schematic is the gimbal control motor, as the gyroscope motor was left to spin without feedback control. (See Figure 4)

Figure 4: The Schematic Diagram of The Feedback Control System



TESTING

Since our project goal was a proof of concept, testing involved simply running the system and watch if it stabilized. It would either stabilize or crash. Due to the lack of time we had for testing, we were not able to complete a test where the cart stayed upright for longer then a moment, and thus all our tests were considered failures. Our entire cart system with electronics was completed a few hours before the design expo. This left us with relatively no time to experiment with our electronics to find a setting that would stabilize the cart.

When the cart and electronics were completely built, we performed tests to see if it would stabilize. These tests were also used to refine the control effort so that it would work better. We accomplished these tests by first updating the controller to the desired level, and then turned on the power to the electronics board and the two motors. We would then set the cart so that it was completely vertical, and hold the gyroscope so that its axis was also completely vertical. After the gyroscope had reached its maximum velocity, we would gently let it go, so as not to physically add a disturbance to the system, and then observe if it stayed upright.

Aside from the cart testing as a whole system, we also tested each component individually to ensure they would perform as expected in the system. The motors in our system were tested by applying their operational voltage and observe if they worked. Both motors performed at or above our requirements, so we were able to use them. We were unable to measure the rotational velocities and torques because we did not have the necessary equipment — an optical encoder. We also tested to see that our microprocessor worked by first programming it to do a simple task, which was to turn on lights, and observed that it performed as anticipated. The sensors and H-bridge that we used were tested by connecting them, applying the correct voltage, and adding a disturbance to the system. We saw what these sensors read out and verified that they were working properly.

FUTURE IMPROVEMENT

If given the chance to do this project again, our team would do several things differently. We list them here in case any other students wish to replicate our project, or make one similar to it, so that they may learn from our mistakes. The fundamental problem our project had was lack of planning for manufacturing the device, which led to very long delays during the production process. Because of these delays, our group was unable to spend enough time on electronics assembly, programming, or controller tuning. We suggest the following be taken into account to address these issues.

A future group should manage its time better than we did. We separated our team into two mostly independent wings, one for manufacturing the physical components, and one to manufacture the electronic components. This allowed the electronic section of the team to be ignorant of the difficulty the manufacturing team was having, and exacerbated the delay greatly. A future team may still wish to separate responsibilities, but should meet more often to discuss progress in detail.

A future group should also plan the construction and layout of the hardware with greater detail than our group did. We did not include anything beyond large, conceptual blocks on our first CAD model, which hurt our group. In particular, we suggest making sure the CAD is detailed enough to generate technical drawings of all the relevant parts and joints. Particular attention should be paid to any rotary axes, their alignment, and how they are fixed to the model, to the position and alignment of brackets, and to the mounting and fastening of any flexible couplings.

In addition, a future group should make an effort to manufacture things correctly the first time. We had difficulty early on with construction quality and parts fitting together. Many aspects of this would have been solved by having the correct technical drawings with all the necessary dimensions needed to make the part, the relevant tolerances, as well as the likely drill and reamer bits necessary to make holes. These drawings should be printed and taken to the machine shop during manufacturing. However, be sure to not waste time by making parts unnecessarily precise since it adds on valuable time. Find out what dimensions are critical to the design and only focus on their accuracy.

Finally, we believe that any future group must realize that at some point, they must spend time on electronics. We spent only about five days total on electronics, and our project suffered for it. Future groups should take time away from manufacturing to work on electronics, even if it does not look like the manufacturing will be finished.

Continuing Work

This section is intended for any groups that wish to continue using the prototype we have built. It will discuss what our prototype can be used for, as well as ways to improve on the prototype without rebuilding everything. Detailed electronic schematics are shown in Appendix XX.

Our prototype is intended to demonstrate the nature of nonlinear control systems, the difficulty in nonlinear controller design, and the effectiveness of linearized controllers in controlling nonlinear systems. These things are possible simply by reprogramming the control logic, in our case the ooPic board. A future group will need to use the proprietary ooPic programming language.

Should this control board prove inadequate; the logic can be replaced with another control chip, including the PIC18F2520 we originally planned to use, the ubiquitous PIC16F877, or any of a variety of cheap microcontroller solutions available on the market. The control and feedback signals are so simple that they could actually be run through a personal computer running LabView, or similar simulation software.

If there is a desire to improve the mechanical nature of the prototype, we feel the following areas are most in need of it, in order of importance:

1. Replace the flywheel motor with something smaller.
2. Replace the flywheel-motor coupling.
3. Reface or replace the flywheel so that it is a perfect cylinder and does not vibrate when spun. The flywheel is currently press fit, and should come out under a one ton Arbor press.
4. Reduce the cart weight by removing the material from unnecessary areas in the board and brackets.

CONCLUSIONS

This group was commissioned to build a setup that will test gyroscopic stabilization of a two-wheeled cart. We have researched what other groups have done in the past to use as a benchmark in our design process. We found the troubles those groups encountered so that we could try to avoid the same difficulties to accomplish our goal. A Quality Function Deployment diagram was created to relate the customer's need to the engineering specifications, which allows us to determine the most important attributes for the final design. Finally, a Gantt chart was made to map how we hope this project progresses, and so that we will be able to stay on track.

Our solution consisted of spinning a brass flywheel, via a DC motor, inside a bracket that was made out of PVC. This entire bracket was then supported by two pieces of aluminum that kept the bracket high enough so that it could rotate. We determined the best way to tilt the gyro was by putting our motor axis parallel to the gimbal axis and then transmit its power with a reinforced toothed belt and pulley system. This would help prevent creep and slipping in the belt. When we designed this prototype, we wanted the center of gravity to be as low as possible. We accomplished this by creating mounting boards that were higher then the base for the wheels to attach to. This allowed us to get the bottom of the cart as low to the ground as we wanted.

A variety of sensors were needed to control the motors. This was accomplished by using a rate gyro, accelerometer, and potentiometer to feed information into a microprocessor that would then control the motors. These different sensors were used so that our microprocessor could have as much information as possible and thus give more accurate instructions.

Our prototype never worked as well as our group wanted, and in this way was not an ideal solution. Yet, there are many aspects of the design that make it a good solution. For instance, most of the mechanical structures are simple designs. Also, the weight distribution is fairly symmetrical which makes it easier for the cart to stabilize. Electronically, the design of the different sensors allowed us to get more information on the state of the stability of the cart. With more time, the controller could be tweaked so the cart works to the expectations of this group.

ACKNOWLEDGEMENTS

We would like to thank the following individuals, whose assistance was vital to the success of our project. First, we would like to thank our sponsor Professor Anouck Girard, who funded our project, and worked extensively with us to derive the equations of motions. Professor Girard also assisted our group in creating a MatLab simulation of our model. Second, we would like to thank our discussion leader Professor Shorya Awtar for aiding us through the entire design process. Professor Awtar helped us create a Simulink model of our prototype and provided sound manufacturing advice. Third, we would like to thank Bob Coury, Marv

Cressey, Steve Emanuel, and all the other machine shop attendants who helped us manufacture our prototype. Finally, we would like to thank Professor Gillespie who provided us with replacement electronics when our electronic components crashed the night before the senior design expo.

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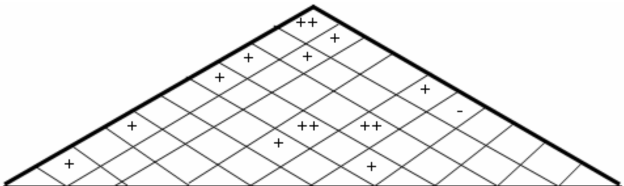
APPENDICES

APPENDIX A: ENGINEERING SPECIFICATIONS

Engineering Spec	Target
Typical Torque Provided by Gyro	2.5 Nm
Response Time of Actuator	0.1 sec
Time to Stabilize	0.5 sec
Sensor Resolution	0.5 degrees
Power Consumption	T.B.D.
Gyro Tilt Limit	Min 30 degrees
Cart Tilt Limit	Min 10 degrees
Size of Cart	35x20x20 cm max
Size of Battery	4x D cells max
Microprocessor Sampling rate	0.050 msec

Appendix B: QFD



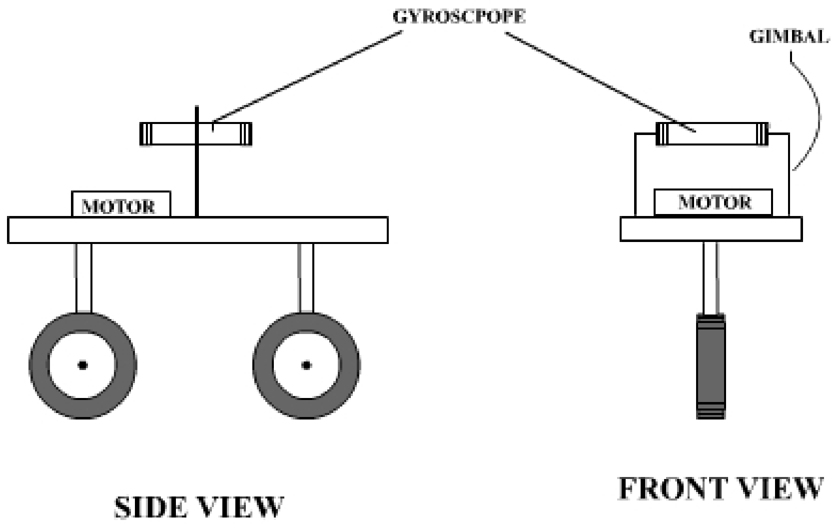
	Weight*	Torque gyro can provide	response time of actuator	Sensor Resolution	Power Consumption	Gyro tilt limit	Size of cart	Size of Battery	Cart tilt limit	Microprocessor sampling rate	Time to stabilize
Stability	6	9	3	3	0	3	1	1	3	0	9
Safety	5	3	0	0	0	1	3	0	3	0	1
Noise	2	3	3	1	1	1	0	0	0	3	1
Weight	1	1	0	0	3	3	9	9	1	0	0
Cost	4	3	1	3	3	1	9	9	3	3	0
Interactivity	3	1	0	0	0	1	3	0	9	0	3
Measurement Unit		Nm	sec	degrees	tbd	degrees	cm	-	degrees	msec	sec
Target Value		2.5	0.1	0.5	W	min 30	35	D	min 10	50	0.5
Importance Rating		**1**	**8**	**7**	**10**	**6**	**2**	**5**	**3**	**9**	**4**
Total		91	28	32	17	35	75	51	73	18	70
Normalized		0.19	0.06	0.07	0.03	0.07	0.15	0.10	0.15	0.04	0.14

Benchmarks

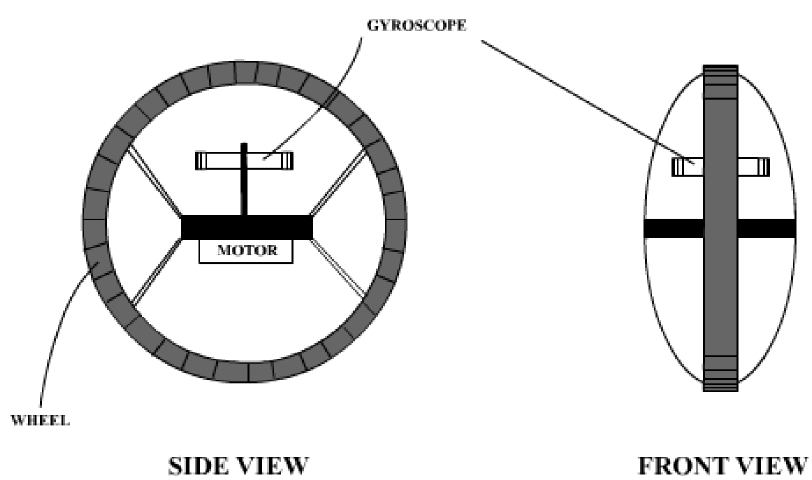
	Standard (Columbia)	Fluid Gyroscope	Large Wheel	Brennen Model	Bicycle
Stability	0	-2	-3	1	0
Safety	0	0	-2	2	0
Noise	0	1	0	-2	0
Weight	0	1	1	-2	-3
Cost	0	-2	-1	-3	-2
Interactivity	0	0	1	0	3

Appendix C: Preliminary Sketches of Each Concept

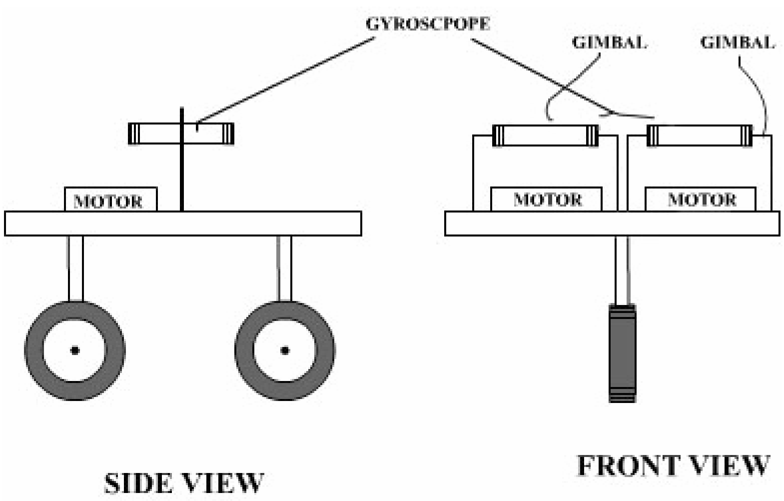
Single Gyro Single Wheel



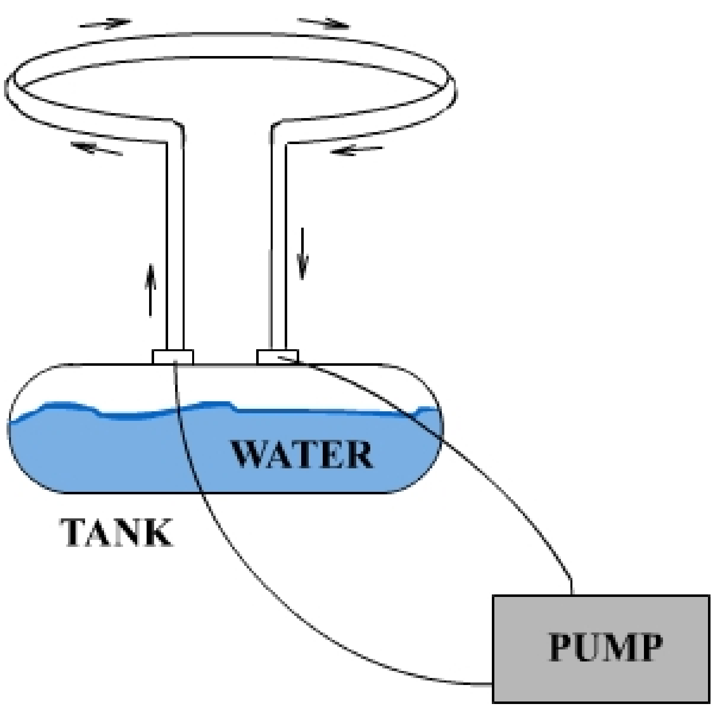
Single Gyro Two Wheels



Twin Gyro Two Wheel



Single Fluid Gyro Two Wheels



APPENDIX D: CONCEPT ANALYSIS

Single Gyro Single Wheel

Functional requirement	How well design meets FR	Prior references	Analysis/Calculation	Risk/Drawback	Mitigation
Stability	Poor, a dynamically stable but statically unstable	Brown, H. B.	Rough estimation proves that the system was unstable statically and stable dynamically	Vulnerability to wheel damage when model stops	Place protection on both sides of the model
Safety	Poor, has highest chance of rolling into someone or something	Brown, H. B.	Lack of support against rotation about vertical means it's unlikely to stay moving in one direction		
Noise	Moderate. One gyro means comparable noise levels to other concepts	Brown, H. B.	Noise was generated primarily by motors	Extra noise from drive mechanics	Use good (and expensive) drive mechanics
Weight	Good, some extra weight in drive mechanics, but otherwise good.	Brown, H. B.	Only one gyro.		
Cost	Poor, material require for assembly was expensive	Brown, H. B.	Lack of single shaft for wheel means a circular basis for rotation was necessary	Components were too expensive to mass produce	
Interactivity	OK, similar to other designs, but since it was enclosed, less issue with tampering			If unable to stand still, ability to interact severely hampered	Design with intent of standing still

Single Gyro Two Wheels

Functional requirement	How well design meets FR	Prior references	Analysis/Calculation	Risk/Drawback	Mitigation
Stability	Good. Design was compact.	Gallaspy, J.	Tilt sensor must have high response rate.	If correct components were not chosen, electronics could overload.	Make sure power source was able to supply sufficient energy foroperation of motor.
Safety	Good. Protective cage houses spinning flywheel.	Gallaspy, J.	Cage should be symmetric about gyro spin axis so vehicle balance was easily met.	Larger flywheel and fast rotations were required to balance larger vehicles.	Unavoidable. Size of flywheel was directly dependent on vehicle weight.
Noise	Moderate. Motor size was relatively small.	Gallaspy, J.	Motor must generate sufficient torque.	Some motors were very loud.	Choose motor carefully.
Weight	Good. Cart was compact and symmetric.	Gallaspy, J.	Size and mass of gyroscope dependent on size and mass of vehicle.	Larger vehicles require more power to stabilize.	Unavoidable. Gyroscope size and mass were dependent on vehicle mass.
Cost	Good. Materials and components used were relatively inexpensive.	Gallaspy, J.	Cost of system was greatly dependent on size.	If vehicle was large, flywheel must be large and spin very fast. This would make system expensive.	Unavoidable. Flywheel size and mass were dependent on vehicle mass.
Interactivity	Moderate, if control system works, should function similar to other concepts	Gallaspy, J.	Actuation force would be similar to other concepts.	Average functionality for average complexity	None

Twin Gyros Two Wheels

Functional requirement	How well design meets FR	Prior references	Analysis/Calculation	Risk/Drawback	Mitigation
Stability	Stable, less worry about non-intentional actuation due to presence of second gyro	Brennen Paper	More	people won't buy it if it doesn't seem safe	set up augers
Safety	Good, multiple gyros allow redundancy		Laws of physics, one gyro can still actuate, although not as well	More parts means more failures	None
Noise	Poor, multiple gyros and control mechanisms mean more noise		More parts tend to equal more noise, all things being equal		
Weight	Poor, multiple gyros and control mechanisms mean more weight		In general, more parts tends to equal more complexity, and thus more weight, even if the parts were smaller		
Cost	Poor, double the parts should increase cost significantly	Brennen Paper	double the cost	too expensive	cut cost in other areas
Interactivity	Moderate, if control system works, should function similar to pther concepts		Actuation force would be similar to other concepts, because there was no sense in creating oversized gyros	More cost and complexity compared to similar functionality	None

Single Fluid Based Gyro Two Wheels

Functional requirement	How well design meets FR	Prior references	Analysis/Calculation	Risk/Drawback	Mitigation
Stability	Poor, Moment of Inertia was very low compared to cart mass		Rough back of napkin calc shows too much power drain		
Safety	Moderate, no large moving parts		Fluid was not as dangerous as a moving flywheel	Possibility of fluid leakage. Not dangerous, but annoying	Seal all electronic components. Keep spare fluid and a rag
Noise	Moderate, pump has potential to be quieter than a flywheel motor		Still two motors, but potential to isolate pump enough to dampen noise	Pump hard to reach for maintenance	
Weight	Moderate, fluid would have to be heavier due to it moving slower than flywheel.		Fluid cannot move as quickly as flywheel due to viscous forces		
Cost	Moderate, lack of precision machining cuts down on cost, but need to waterproof increases it.		Flywheel was not solid and does not have to be machined	Potential damage to electronics if fluid leaks	Waterproofing on electronics
Interactivity	OK, Similar to other designs.		If similar moment of inertia attained, interactivity should be comparable	Lots of tubes were unsightly	Unavoidable, tube access necessary for maintenance.

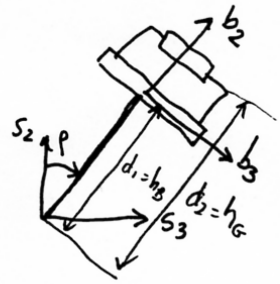
APPENDIX E: PUGH CHART

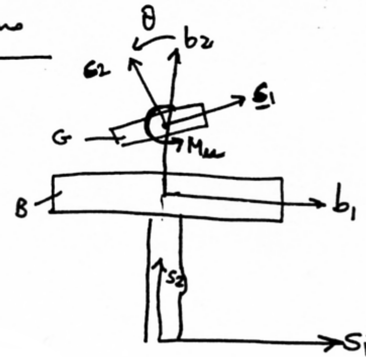
Customer Requirements	Weight	Standard (Columbia)	Fluid Gyroscope	Large Wheel	Brennen Model
Stability	0.3	0	-2	-3	1
Safety	0.2	0	0	-2	2
Noise	0.1	0	1	0	-2
Weight	0.05	0	1	1	-2
Cost	0.2	0	-2	-1	-3
Interactivity	0.15	0	0	1	0
	1				
Sum (compared to standard)		--	-0.85	-1.3	-0.2

Appendix F: Hand written Derivation of Equation of Motion (EOM) for both Cart and Gyroscope

Gyro Stabilized Platforms Equations (1)

Back View





→ ignore ϕ

→ for level track, ignore s

Frames : A : earth - fixed

S : moves with platform, $a_3 // s_3$, s_1 at angle ψ below horizontal

B : fixed on platform

C : attached to gyro cage

G : gyro wheel

$\underline{b}_1 = \underline{c}_1 \cos\theta - \underline{c}_2 \sin\theta$ notation : s for sin, c for cos

$= \underline{c}_1 c\theta - c_2 s\theta$

Write expressions for:

${}^A\underline{V}^B =$

${}^A\underline{V}^G =$ T =

${}^A\underline{\omega}^B =$ V =

${}^A\underline{\omega}^G =$ L = T − V

Kinetic NRJ T_B = Lagrange condition

T_G = ⇒ EOM

Tell me how you would change this

Potential NRT V_B = to move down a slope ...

(2)

$$ {}^A\underline{V}^B = \dot{\rho}\, d_1 \underline{b}_3 + \dot{s}\, \underline{s}_1 $$

$$ {}^A\underline{V}^G = \dot{\rho}\, d_2 \underline{b}_3 + \dot{s}\, \underline{s}_1 $$

$$ {}^A\underline{\omega}^B = \dot{\rho}\, b_1 $$

$$ {}^A\underline{\omega}^G = {}^A\underline{\omega}^B + {}^B\underline{\omega}^C + {}^C\underline{\omega}^G $$

$$ = \dot{\rho}\, \underline{b}_1 + \dot{\theta}\, \underline{c}_3 + \Omega\, \underline{c}_2 $$

$$ = \dot{\rho}\, c\theta\, \underline{c}_1 + (\Omega - \dot{\rho}\, s\theta)\, \underline{c}_2 + \dot{\theta}\, \underline{c}_3 \qquad \text{using } \underline{b}_1 = c\theta\, \underline{c}_1 - s\theta\, \underline{c}_2 $$

$$ T_B = \frac{1}{2} m_B \left(\dot{\rho}^2 d_1^2 + \dot{s}^2 \right) + \frac{1}{2} \dot{\rho}^2 I_{B11} $$

$$ T_G = \frac{1}{2} m_G \left(\dot{\rho}^2 d_2^2 + \dot{s}^2 \right) + \frac{1}{2} \left[\dot{\rho}^2 c^2\theta\, I_{G11} + (\Omega - \dot{\rho}\, s\theta)^2 I_{G22} + \dot{\theta}^2 I_{G33} \right] $$

Applied force M_u $\qquad \delta W = M_u\, \delta\theta$

$$ V_B = m_B\, g\, d_1\, c\rho\, c\psi $$

$$ V_G = m_G\, g\, d_2\, c\rho\, c\psi $$

$$ T = \frac{1}{2} \left(m_B d_1^2 + m_G d_2^2 \right) \dot{\rho}^2 + \frac{1}{2} (m_B + m_G)\, \dot{s}^2 + \frac{1}{2} \left[\dot{\rho}^2 \left(I_{B11} + c^2\theta\, I_{G11} \right) + \underbrace{(\Omega - \dot{\rho}\, s\theta)^2}_{\Omega^2 - 2\Omega\dot{\rho}\, s\theta + \dot{\rho}^2 s^2\theta} I_{G22} + \dot{\theta}^2 I_{G33} \right] $$

$$ V = (m_B d_1 + m_G d_2)\, g\, c\rho\, c\psi $$

$$ L = T - V \qquad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i $$

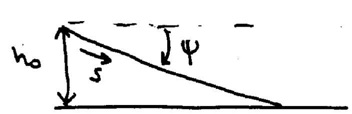
use $q = \begin{bmatrix} \rho \\ \theta \end{bmatrix}$ (with $s \equiv 0$)

$$\left\{\begin{aligned} &\frac{d}{dt}\left[\left(m_B d_1^2 + m_G d_2^2 + I_{B_{11}} + c^2\theta I_{G_{11}} + s^2\theta I_{G_{22}}\right)\dot{\rho} - \Omega s\theta I_{G_{22}}\right] \\ &\qquad - (m_B d_1 + m_G d_2) g\, s\rho\, c\Psi = 0 \\ &\frac{d}{dt}\left[I_{G_{33}}\dot{\theta}\right] - \left[\dot{\rho}^2 c\theta s\theta (I_{G_{22}} - I_{G_{11}}) - \Omega c\theta \dot{\rho} I_{G_{22}}\right] = M_u \end{aligned}\right.$$

$$\begin{aligned} \rho \quad &\left\{\begin{aligned} &\left(m_B d_1^2 + m_G d_2^2 + I_{B_{11}} + c^2\theta I_{G_{11}} + s^2\theta I_{G_{22}}\right)\ddot{\rho} + 2c\theta s\theta (I_{G_{22}} - I_{G_{11}})\dot{\theta}\dot{\rho} \\ &\qquad - \Omega c\theta I_{G_{22}}\dot{\theta} - (m_B d_1 + m_G d_2) g\, s\rho\, c\Psi = 0 \\ \theta \quad & I_{G_{33}}\ddot{\theta} - \dot{\rho}^2 c\theta s\theta (I_{G_{22}} - I_{G_{11}}) + \Omega c\theta I_{G_{22}}\dot{\rho} = M_u \end{aligned}\right. \end{aligned}$$

Now, allow the system to move down a sloped track

$$q = \begin{bmatrix} \rho \\ \theta \\ s \end{bmatrix} \qquad V_{new} = V + (m_B g + m_G g)(h_0 - h' s)$$

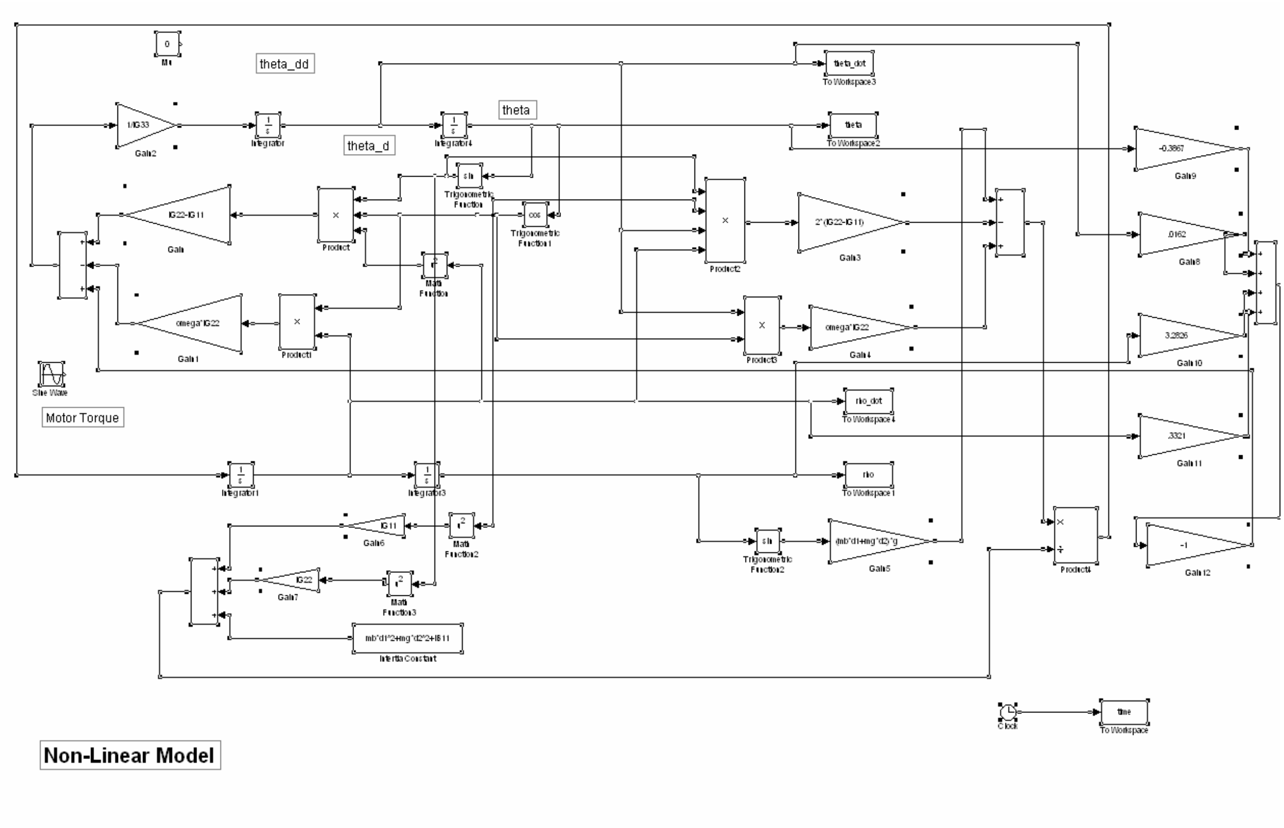


$$h' = \frac{\partial h}{\partial s} = s\Psi$$

$$(m_B + m_G)\ddot{s} - (m_B + m_G) g h' = 0$$

$$s \Rightarrow \boxed{\ddot{s} = g\, s\Psi}$$

APPENDIX G: Nonlinear Simulink Model of the gyroscopic stabilizing device



Appendix H: Matlab Program of the System

Gyro.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       Gyro-stabilized platform code
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clear

global u G first_time

%Simulation Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%



deg2rad = 0.0174532925;
rad2deg = 57.2957795;

IC = [-20*deg2rad 0*deg2rad -10*deg2rad 0*deg2rad];
Tf = 1.5;
del =.01;
N = Tf/del;

DATA = [0 IC];
UDAT = [0];

first_time = 0;

for k = 1:N

   %Get end values from last xx msec simulation
   len = length(DATA(:,1));
   t1 = DATA(len,1);
   x(1) = DATA(len,2);
   x(2) = DATA(len,3);
   x(3) = DATA(len,4);
   x(4) = DATA(len,5);



   %Reset the Initial Conditions
   IC = [x(1) x(2) x(3) x(4)];

   %Do the simulation
   k*del
   [T,Y] = ode45('gyroderiv',[(k-1)*del k*del],IC);
```

```
    %Record the data for the last time period
    DATA = [DATA; T(length(T)) Y(length(Y),:)];
    UDAT = [UDAT; u];

end;

t=DATA(:,1);
theta=DATA(:,2)*rad2deg;
theta_dot=DATA(:,3)*rad2deg;
rho=DATA(:,4)*rad2deg;
rho_dot=DATA(:,5)*rad2deg;

uplot=UDAT(:,1);

figure;
subplot(311)
plot(t, theta)
title('theta vs time')
grid
subplot(312)
plot(t,rho)
title('rho vs time')
subplot(313)
plot(t,uplot)
title('control effort vs time')
```

Gyroderiv.m

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%        Gyroscopically stabilized cart
%         Derivatives file
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
function dx = gyroderiv(t,x)

global u G first_time



%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%      System Constants
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%




mb = 1; % kg unit mass     a block of aluminum of 10cm x 25cm x 1.48cm
      % density of aluminum is roughly 2700 kg/m^3
wb = .1; % meters, body is rectangle, 10 cm by 25 cm
hb = .0148; %m

mg = .314; % in kg, 10 cm diameter alumninum cylinder, 1.48cm thick
rg = 0.05; % radius of gyro, in meters
hg = 0.0148; % in meters, height of gyro, gyro is assumed to be cylinder.

d1 = 0.1; % m
d2 = d1+0.05; %m

IG11 = (mg*(rg^2)/4) + (mg*(hg^2)/12);
IG22 = mg*(rg^2)/2;
IG33 = IG11;

IB11 = (mb/12)*wb^2*hb^2;




g=9.81;

% "extra" dynamics
omega = 1000*.10472; % (rpm * conversion factor = rad/sec)
phi = 0;

% desired dynamics
```

```
theta_des = 0;
rho_des = 0;

% constants
g = 9.81;   % acceleration of gravity

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%CONTROLLER PARAMETERS:
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%% linearized coefficients
a1 = -omega*IG22/IG33;
a2 = omega*IG22/(mb*d1*d1+mg*d2*d2+IB11+IG11);
a3=g*(mb*d1+mg*d2)*cos(phi)/(mb*d1*d1+mg*d2*d2+IB11+IG11);

b1 = 1/IG33;



if 0 %(first_time == 0)
   % run once to get K
   A=[0 1 0 0; 0 0 0 a1; 0 0 0 1; 0 a2 a3 0];
   B=[0; b1; 0; 0];
   C=[1 0 1 0];
   D = 0;

   SYS = ss(A,B,C,D);

   G = tf(SYS);

   P=[-20 -20 -20 -20];
   K=acker(A,B,P);

   first_time =1;
end



K=[-0.3867    0.0162    3.2826    0.3321];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       State Definitions
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% x1 = theta
% x2 = theta_dot
% x3 = rho
% x4 = rho_dot
```

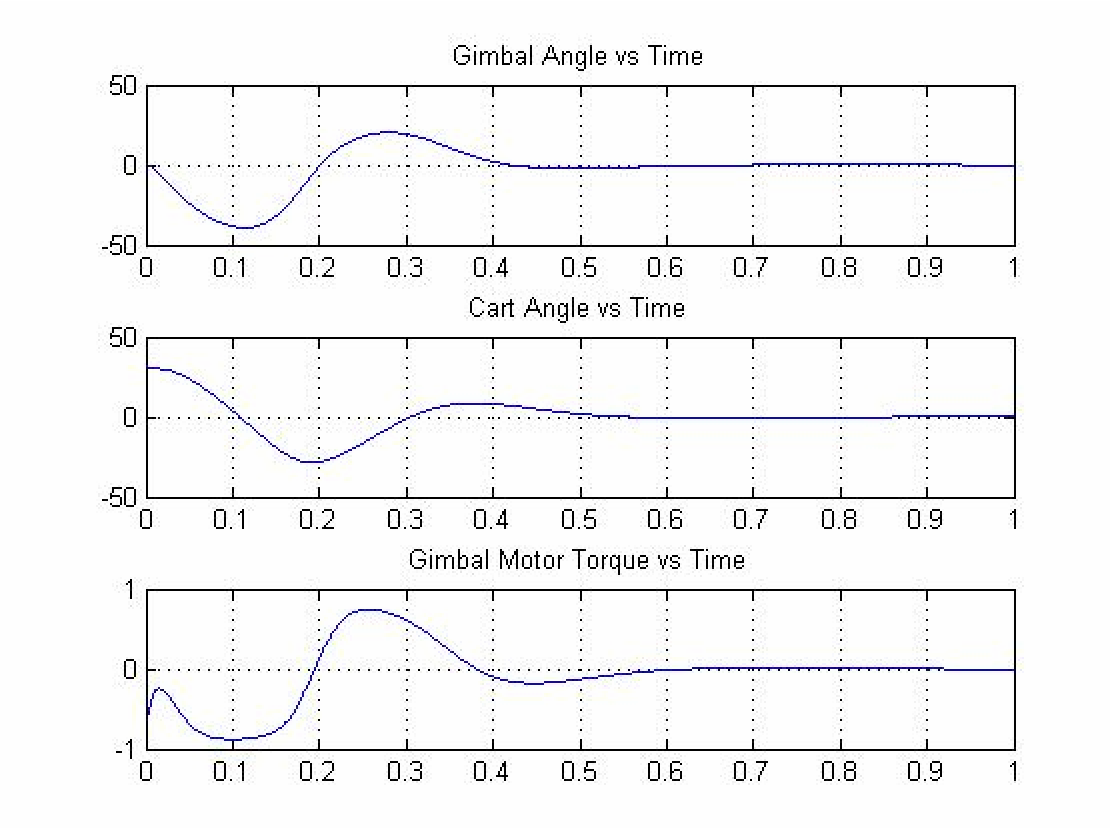
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Controller Calculations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%  simplified above equations for numerical reasons

M_u = -K*x;

u=M_u;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%       State Equations
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% intermediate term
int_term = mb*d1^2+mg*d2^2+IB11+(cos(x(1)))^2*IG11+(sin(x(1)))^2*IG22;

%Plant Equations
dx(1) = x(2);
dx(2) = ((x(4))^2*cos(x(1))*sin(x(1))*(IG22-IG11)-omega*IG22*cos(x(1))*x(4)+M_u)/IG33;  %% changed one
sign
dx(3) = x(4);
dx(4) = -(2*cos(x(1))*sin(x(1))*(IG22-IG11)*x(2)*x(4)-omega*cos(x(1))*IG22*x(2)-
(mb*d1+mg*d2)*g*sin(x(3))*cos(phi))/int_term;

dx=dx';
```

Appendix I: Peak Values and Settling Times for Varied Parameters

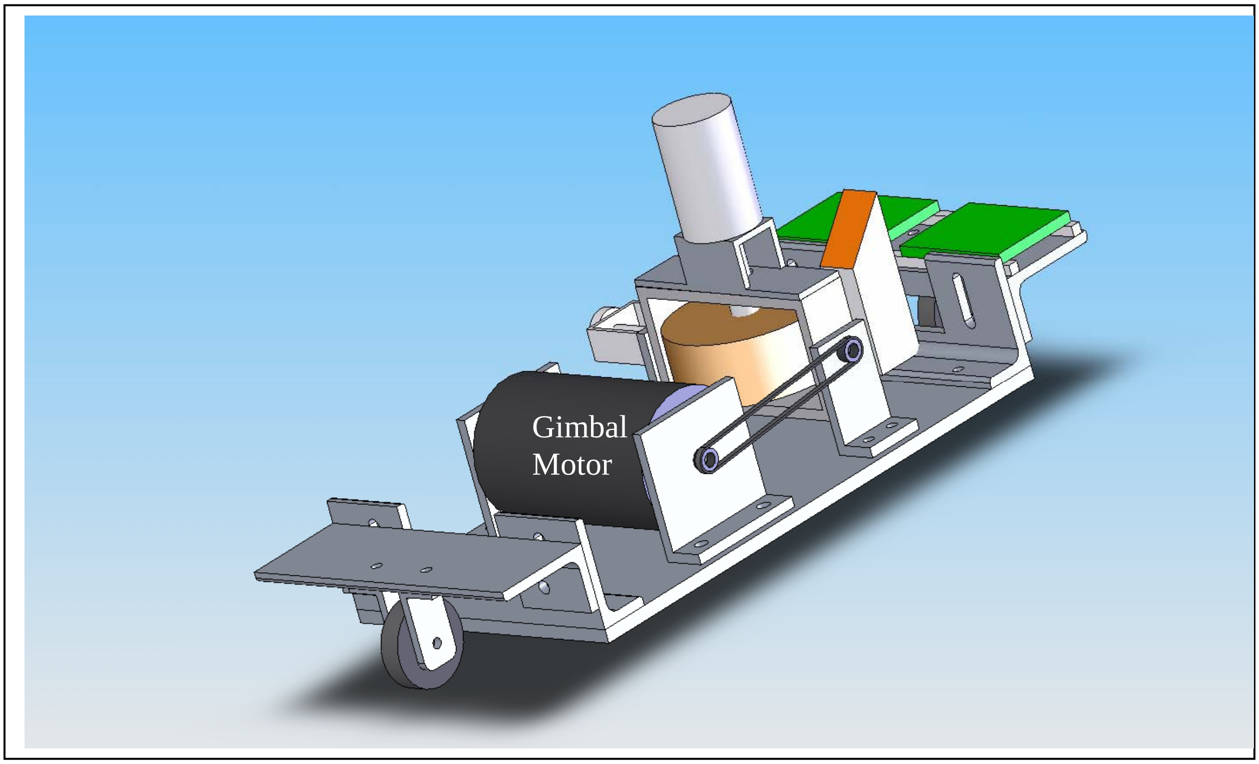
	Peak Value (deg)		Settling Time (sec)	
	Gyro Tilt Angle	*Cart Tilt Angle*	*Gyro*	*Cart*
Base Values	43	3	0.5	0.5
Mass of Cart + 10%	48	3	0.75	0.5
Width of Cart + 10%	negligible change	negligible change	negligible change	negligible change
Thickness of Cart + 10%	negligible change	negligible change	negligible change	negligible change
Height of Cart COM + 10%	58	7.5	1	0.75
Mass of Gyro + 10%	45	3.1	0.6	0.5
Radius of Flywheel + 10%	negligible change	negligible change	negligible change	negligible change
Thickness of Gyro + 10%	negligible change	negligible change	negligible change	negligible change
Height of Gyro COM + 10%	50	7.1	1.1	0.8
Gyro Angular Velocity + 10%	41	1	0.5	0.6
Mass of Cart - 10%	37	2	0.3	0.5
Width of Cart - 10%	negligible change	negligible change	negligible change	negligible change
Thickness of Cart - 10%	negligible change	negligible change	negligible change	negligible change
Height of Cart COM - 10%	33	1	0.4	0.4
Mass of Gyro - 10%	41	2.9	0.5	0.5
Radius of Flywheel - 10%	negligible change	negligible change	negligible change	negligible change
Thickness of Gyro - 10%	negligible change	negligible change	negligible change	negligible change
Height of Gyro COM - 10%	38	1.3	0.5	0.3
Gyro Angular Velocity - 10%	45	4.5	0.6	0.71

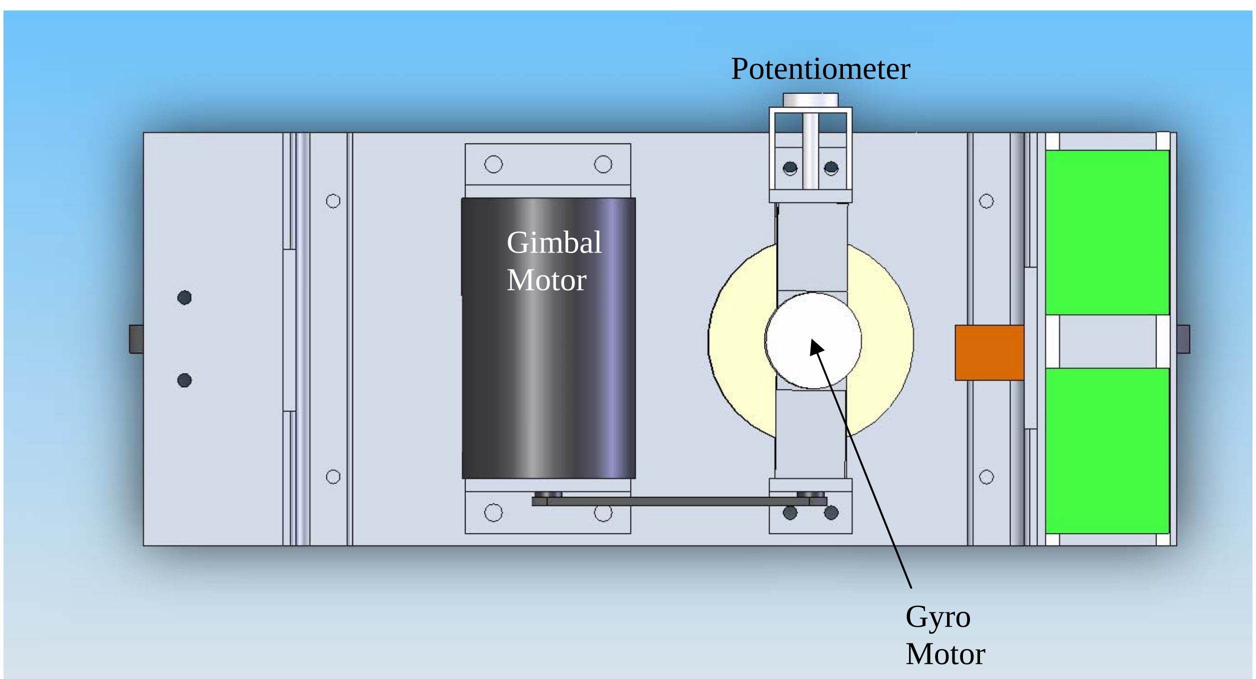
APPENDIX J: Modeling Results

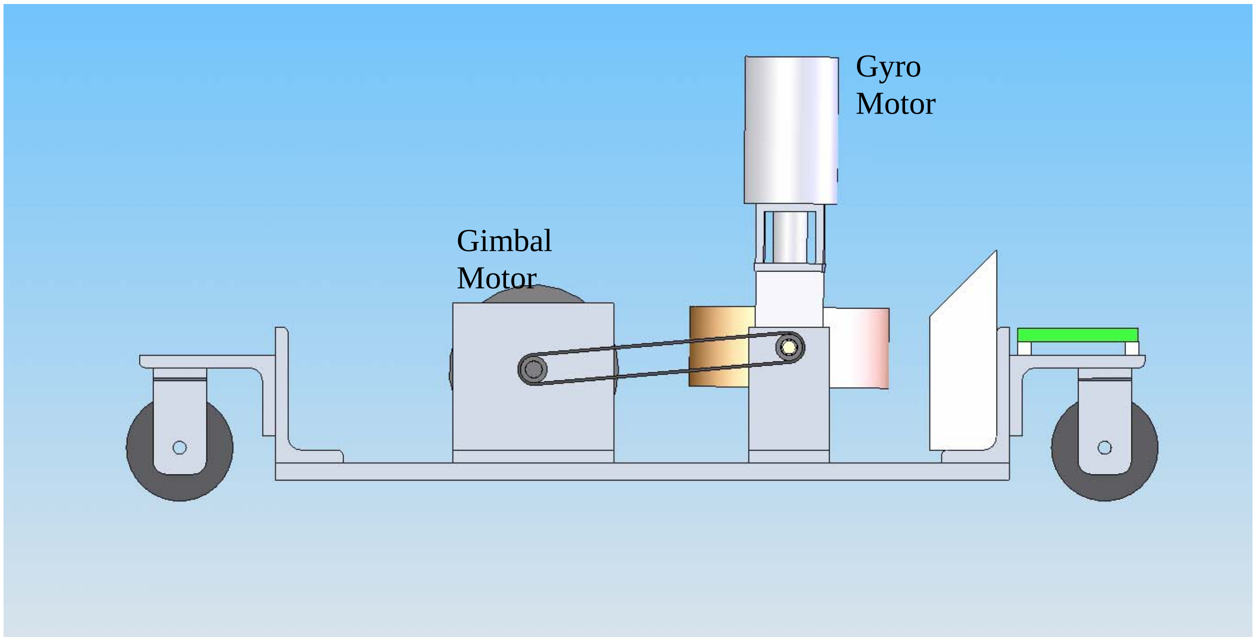
Figure 2: Maximum Cart Angle with Corresponding Gyro Angle and Control Effort

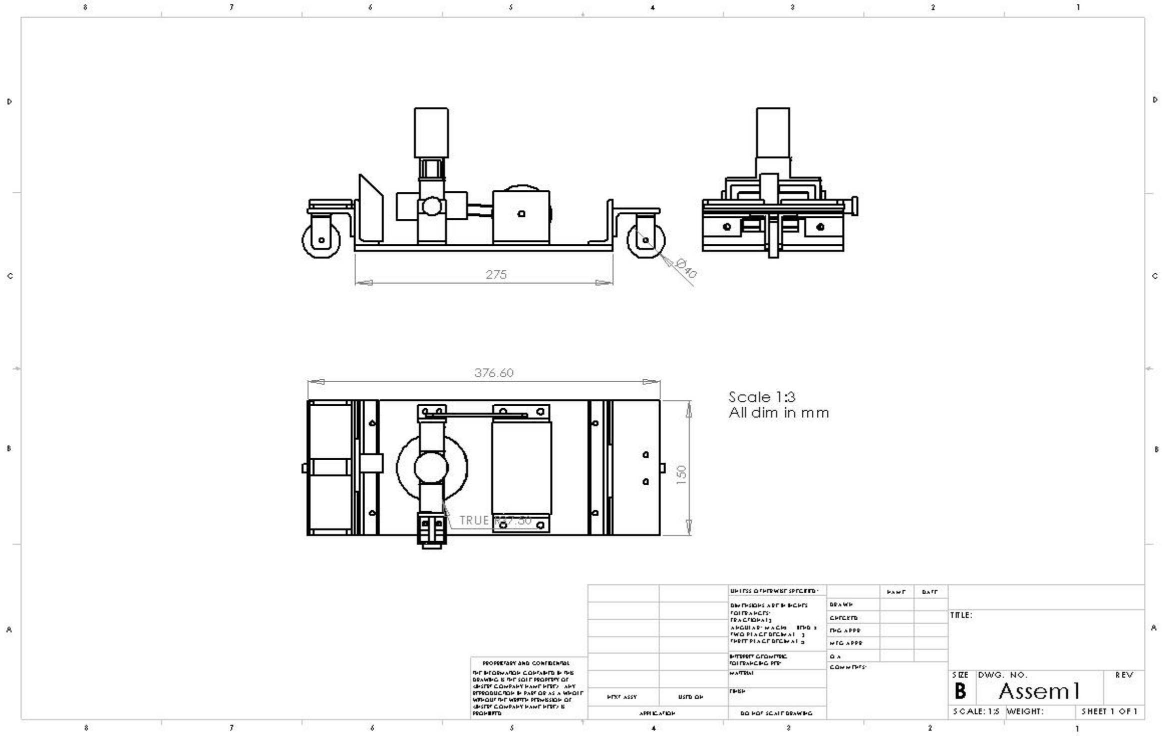


APPENDIX K: CAD files and Engineering Drawings

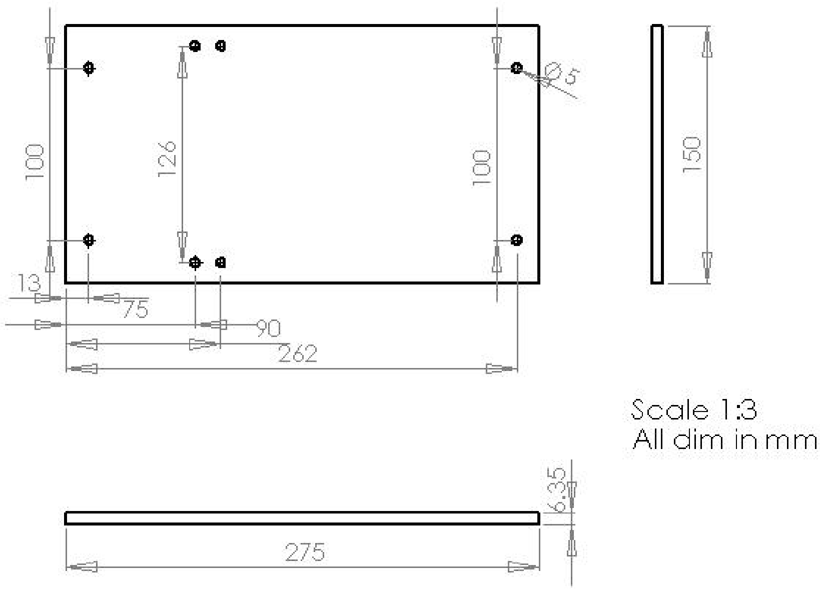




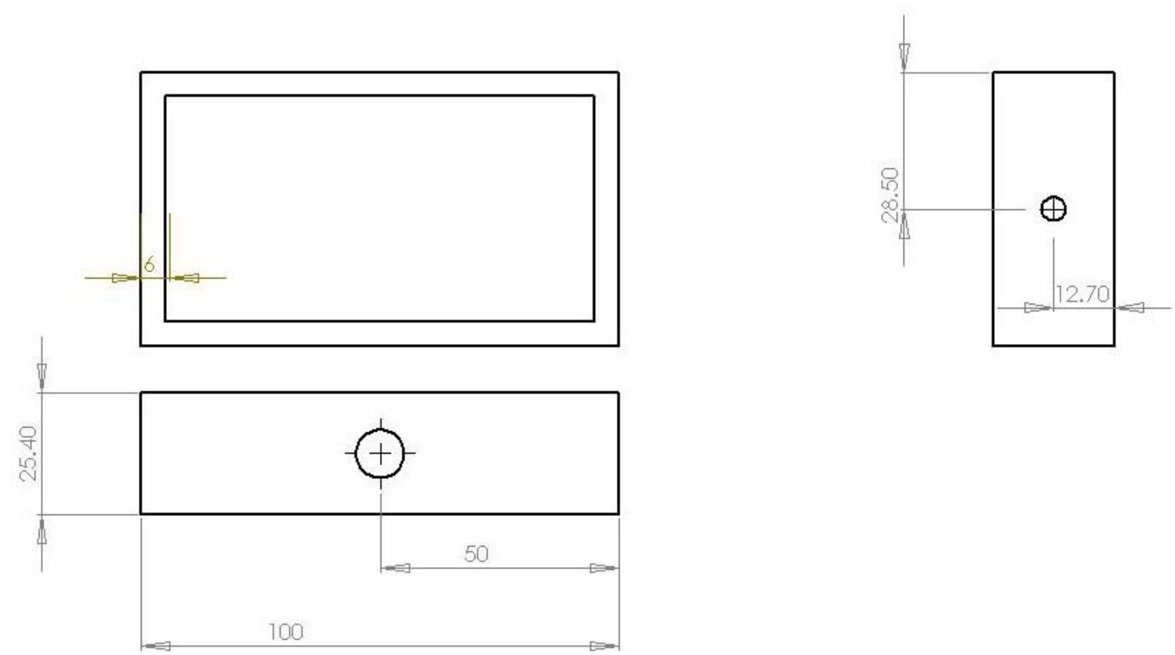

Gyro
Motor
Gimbal
Motor


275
Ø40
376.60
150
Scale 1:3
All dim in mm
TITLE:
SIZE DWG. NO. REV
B Assem1
SCALE: 1:5 WEIGHT: SHEET 1 OF 1

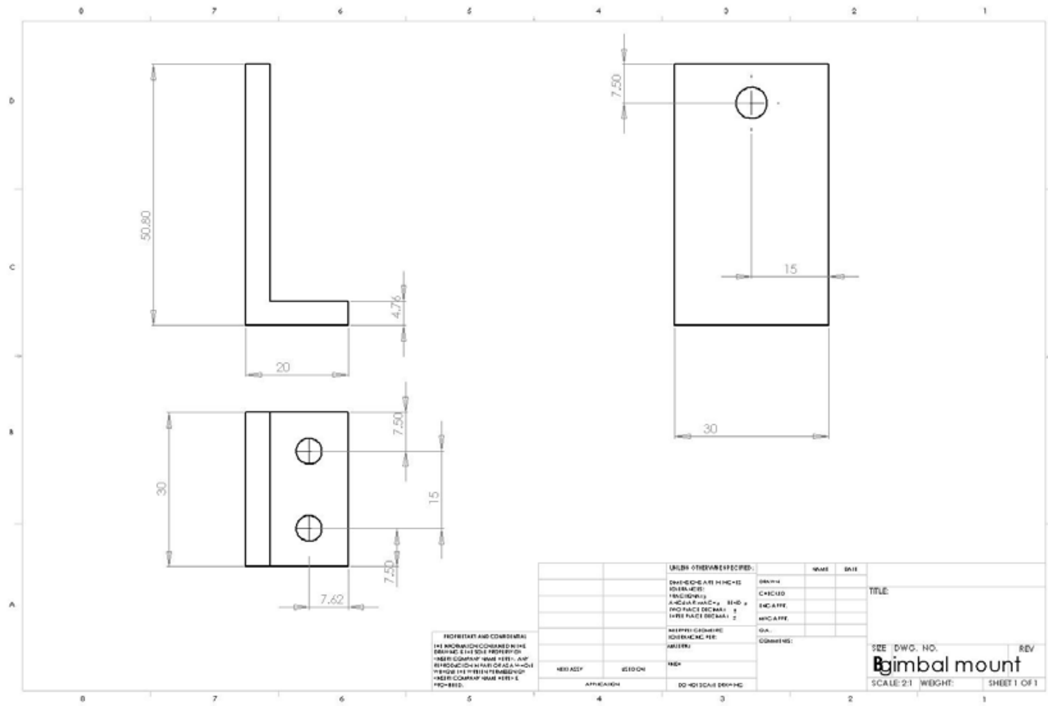
PVC Base



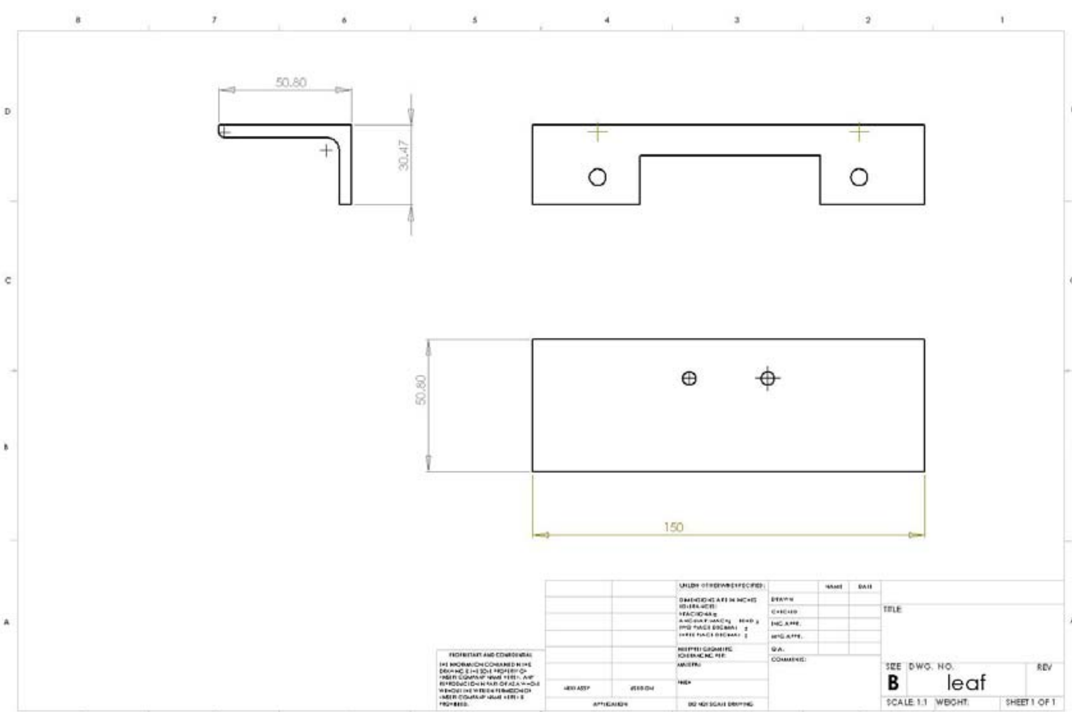
Gimbal Bracket



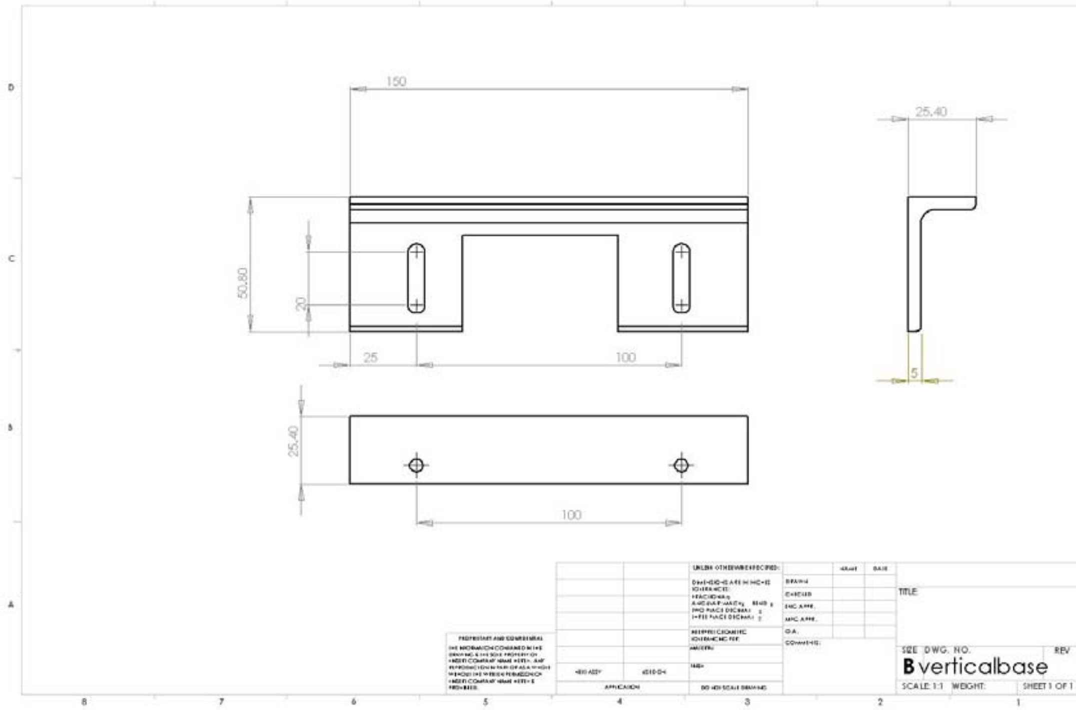
Gimbal Mount



Mounting Platforms

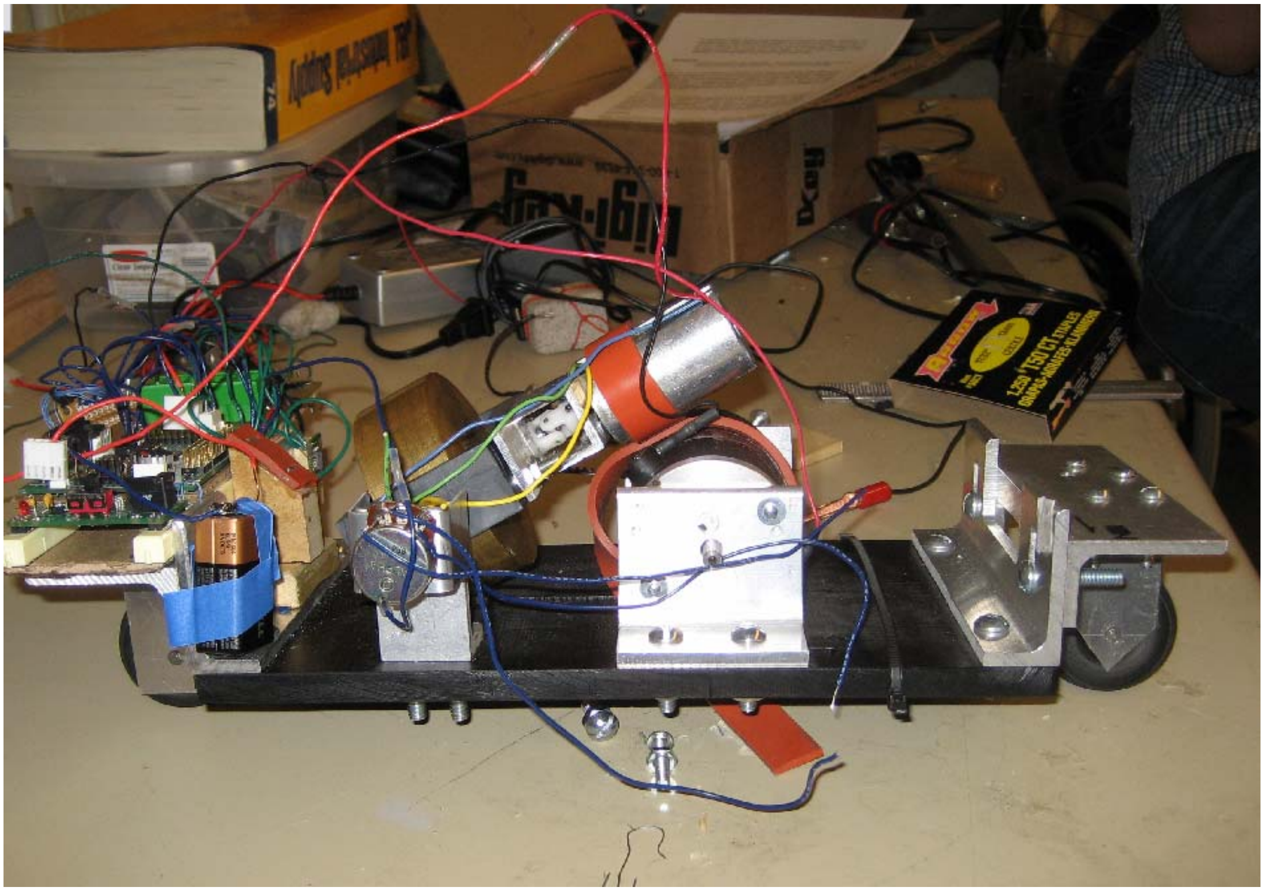


Vertical Supports



Appendix L: Photo of Model

Full Model



Appendix M: BOM

Bill of Materials

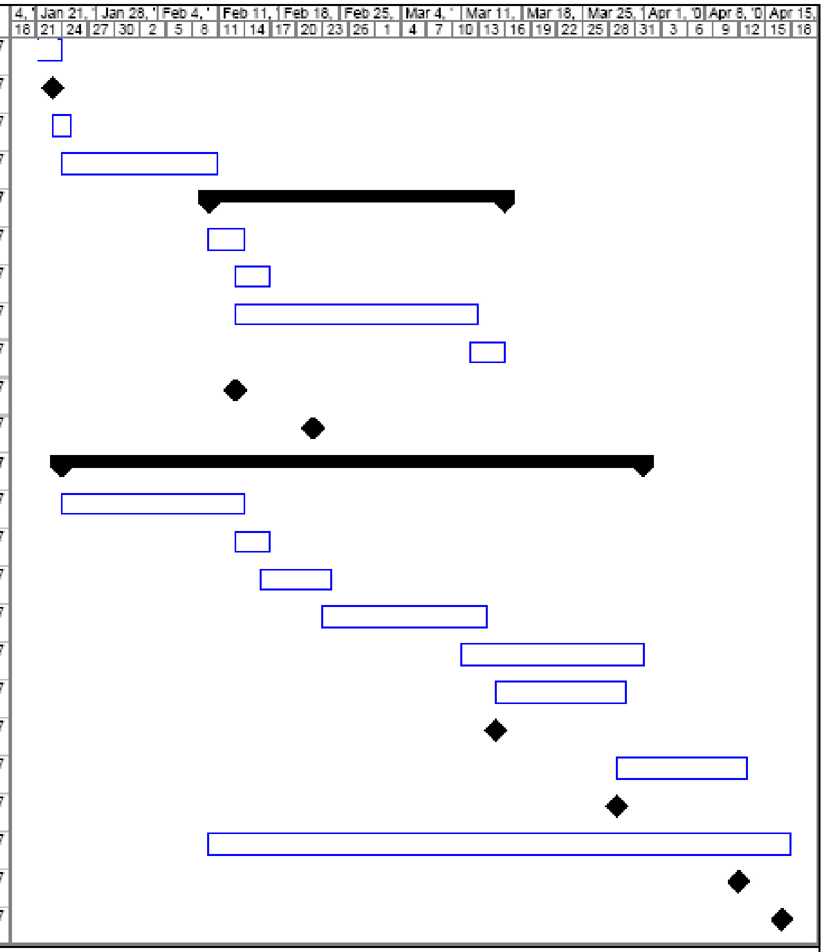
Quantity	Part Description	Purchased From	Part Number	Price (each)
1	DC Motor - Gimbal	ElectroCraft[1]	0240-03-017	$224.65
1	DC Motor - Flywheel	University of Michigan		$0.00
1	Li-Ion Rechargeable Battery Pack	All-Battery.com	L18650-2200-4	$32.99
1	Universal Smart Charger (incl. S&H)	All-Battery.com	TLP_2000	$30.00
1	Rubber Belt	Ryder's Hobby		$5.99
1	Pulley Set	Ryder's Hobby		$4.99
1	6mm x 10mm bearing set	Ryder's Hobby		$7.99
1	1/4" x 1/2" bearing set	Ryder's Hobby		$7.99
1	1/2" x 3/4" bearing set	Ryder's Hobby		$12.99
1	Wheel Set	Ryder's Hobby		$7.99
2	H-Bridges	Digikey.com	MC33186VW1-ND	$15.28
2	Accelerometers	Digikey.com	497-4127-ND	$21.86
2	Potentiometers	Digikey.com	3852A-162-103AL-ND	$18.32
5	Transitors	Radio Shack		$1.99
1	PVC Base	University of Michigan		$0.00
1	Brass Flywheel	University of Michigan		$0.00
	Electronic Boards	University of Michigan		$0.00
	Aluminum Angle Stock	University of Michigan		$0.00
	Silicon Rubber	University of Michigan		$0.00
	Fasteners	University of Michigan		$0.00
			Total =	$393.03

[1] electrocraft.com

APPENDIX N: Gantt Charts

Projected Gantt chart

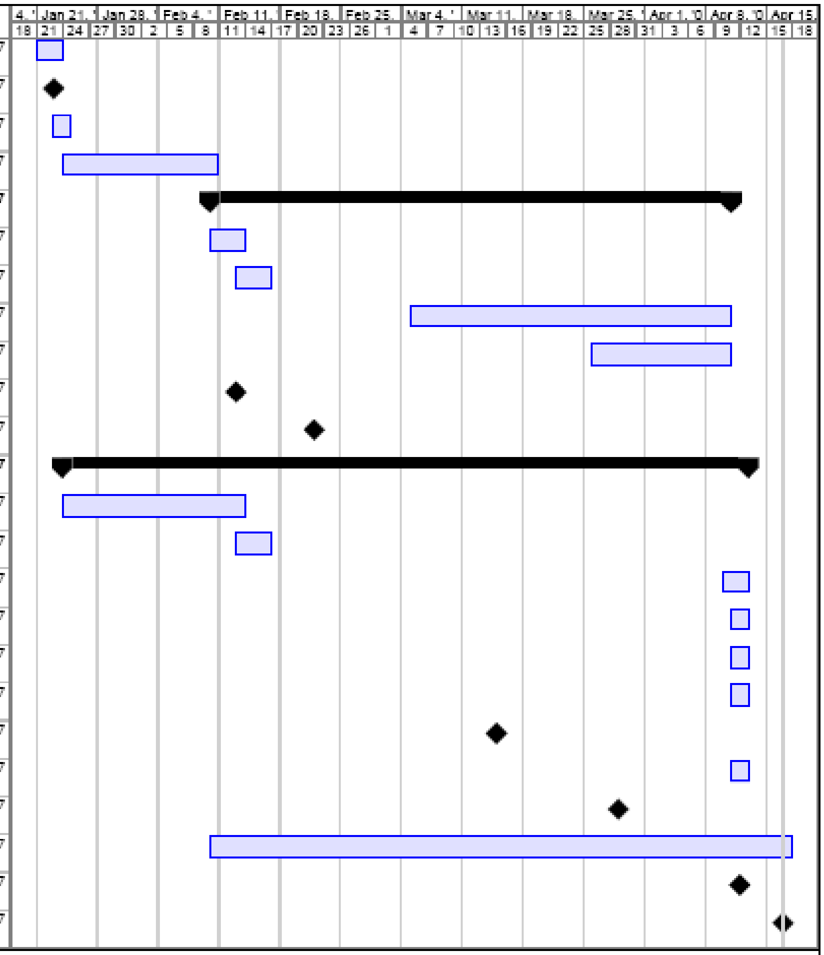
ID	Task Name	Start	Finish
1	Technical Specifications	Sun 1/21/07	Tue 1/23/07
2	DR 1: Info Search and Enginee	Tue 1/23/07	Tue 1/23/07
3	Brainstorming	Tue 1/23/07	Wed 1/24/07
4	Modeling	Wed 1/24/07	Sat 2/10/07
5	**Hardware**	**Sat 2/10/07**	**Thu 3/15/07**
6	CAD	Sat 2/10/07	Tue 2/13/07
7	Buy Parts	Tue 2/13/07	Fri 2/16/07
8	Manufacture Parts	Tue 2/13/07	Mon 3/12/07
9	Subassembly	Mon 3/12/07	Thu 3/15/07
10	DR2: Concept Work	Tue 2/13/07	Tue 2/13/07
11	Milestone: Order Parts	Thu 2/22/07	Thu 2/22/07
12	**Software**	**Wed 1/24/07**	**Sat 3/31/07**
13	Design	Wed 1/24/07	Tue 2/13/07
14	Buy Components	Tue 2/13/07	Fri 2/16/07
15	Subassembly	Fri 2/16/07	Fri 2/23/07
16	Test	Fri 2/23/07	Tue 3/13/07
17	Develop Control Software	Sun 3/11/07	Sat 3/31/07
18	Final Assembly	Thu 3/15/07	Thu 3/29/07
19	DR3: Final Design	Thu 3/15/07	Thu 3/15/07
20	Debugging	Thu 3/29/07	Thu 4/12/07
21	DR4: Alpha Prototype	Thu 3/29/07	Thu 3/29/07
22	Final Report	Sat 2/10/07	Tue 4/17/07
23	Milestone: Design Expo	Thu 4/12/07	Thu 4/12/07
24	Final Report Due	Tue 4/17/07	Tue 4/17/07



Page 1

Actual Gantt chart

ID	Task Name	Start	Finish
1	Technical Specifications	Sun 1/21/07	Tue 1/23/07
2	DR 1: Info Search and Enginee	Tue 1/23/07	Tue 1/23/07
3	Brainstorming	Tue 1/23/07	Wed 1/24/07
4	Modelling	Wed 1/24/07	Sat 2/10/07
5	**Hardware**	**Sat 2/10/07**	**Tue 4/10/07**
6	CAD	Sat 2/10/07	Tue 2/13/07
7	Buy Parts	Tue 2/13/07	Fri 2/16/07
8	Manufacture Parts	Mon 3/5/07	Tue 4/10/07
9	Subassembly	Mon 3/26/07	Tue 4/10/07
10	DR2: Concept Work	Tue 2/13/07	Tue 2/13/07
11	Milestone: Order Parts	Thu 2/22/07	Thu 2/22/07
12	**Software**	**Wed 1/24/07**	**Thu 4/12/07**
13	Design	Wed 1/24/07	Tue 2/13/07
14	Buy Components	Tue 2/13/07	Fri 2/16/07
15	Subassembly	Tue 4/10/07	Thu 4/12/07
16	Test	Wed 4/11/07	Thu 4/12/07
17	Develop Control Software	Wed 4/11/07	Thu 4/12/07
18	Final Assembly	Wed 4/11/07	Thu 4/12/07
19	DR3: Final Design	Thu 3/15/07	Thu 3/15/07
20	Debugging	Wed 4/11/07	Thu 4/12/07
21	DR4: Alpha Prototype	Thu 3/29/07	Thu 3/29/07
22	Final Report	Sat 2/10/07	Tue 4/17/07
23	Milestone: Design Expo	Thu 4/12/07	Thu 4/12/07
24	Final Report Due	Tue 4/17/07	Tue 4/17/07



Page 1

GROUP BIOS

Temitope Akinlua (ednut@umich.edu)
Temitope Akinlua (nicknamed Tem) was born at Hutzel Hospital on February 4, 1986 in Detroit, MI. It was a brisk and cloudy day in February, and the air was filled with uncertainty and uneasiness. Things didn't look too well for the soon-to-be third son of Debo and Nike Akinlua. The doctors informed them that Nike would be unable to go through a normal labor and that she must undergo a cesarean section. She was willing to do anything to ensure that her son made it into this world. After many tense hours, their baby boy was finally born. Due to the complications of his birth, he was given the name Temitope (*Tay-mee-tuh-peh*), which was a Nigerian name that means, "Thanks be to God".

Growing up in Detroit, Tem always had a fascination of how things work. He would always enjoy taking random objects apart (especially objects around the house) and then reconstruct them with a little "twist". He also liked to make chores and tasks easier by finding new and better ways of doing them. Often times he reprimanded for it, but that never stopped his passion. When he was intelligent enough to understand, he discovered that this was a form of engineering and he wanted to pursue it ever since. He chose mechanical engineering because he enjoys physics and design. He plans to run his own engineering firm someday and be the possessor of numerous patents.

Dmitriy Dryga (ddryga@umich.edu)
Dmitriy Dryga was originally from Russia, but has lived in many places of the U.S. for most of his life. He moved around a lot, but in particular likes the warm climate of North Carolina. Dmitriy has a fondness of learning because of his strong curiosity and a penchant for creative thought due to a very active imagination. He has liked mechanical machines from a young age. In particular, airplanes catch his interest.

He chose to go to the University of Michigan because of its strong mechanical engineering program. Mechanical engineering in particular fascinates Dmitriy, because it shares many aspects with aerospace engineering, but remains distinctly eclectic, so that it was possible to do many different things. Dmitriy's future plans involve finding work in the industry and getting his Master's degree after several years of work experience. He was undecided whether his final focus will be in industry or research at a university, as both seem very promising and exciting.

Adam Gunnett (agunnett@umich.edu)
Adam was a 4th year senior graduating in April of 2007. He was from Grand Rapids, MI where he has lived his whole life. Adam always wanted to go to U-M since he was little, since his Grandfather, Uncle, and Sister all graduated from the University. He declared Mechanical Engineering after his sophomore year when he became interested in cars. He then also joined the Society of Automotive Engineers (SAE) and became the student chapter Treasurer at U-M. He was intrigued by a gyroscope senior design project because of his newfound obsession with aircraft and flight. This was due to the fact that he just recently accepted a job offer to go work

for the U.S. Air Force at Edwards Air Force Base in California. He wanted to gain some basic knowledge of gyroscopes since they were frequently used in avionics.

Jonathan Montague (jontague@umich.edu)
Jon was from Grand Rapids, Michigan, and graduated from Forest Hills Central High School in the spring of 2003. He came to the University of Michigan without a clear idea of what he wanted his major to be, but eventually decided on Mechanical Engineering because of its diversity of subjects one can study in this area. Jon has been interviewing with companies for the last year for a job after he graduates in May 2007. His future plans include getting a masters degree while working in the next couple of years of getting his job.

Elobuike Oji (elooji@umich.edu)
The ability to plan ahead was not a philosophy that was reiterated to Elo time after time throughout his life; it was a trait that was given to him in a more discreet manner, his name. Elobuike, in Igbo, means "proper planning leads to success." At a young age, Elo understood the importance of planning for the future; he constantly wrote down goals and the date by which he expect to accomplish them. Although his goals have changed, which normally happens as one progress from an adolescent to adult, he remained fixated on one goal in particular, to become a CEO of a Fortune 500 company.

Elo's desire to start his own company was the sole reason he decided to pursue engineering. A family friend advised him that a strong technical background would be beneficiary in business world, and engineering would aid in the development of his deductive reasoning. He decided to major in Mechanical engineering at the University of Michigan, because of the extensive principles taught in the discipline. Ultimately, he plans to pursue a MBA, and later start a Fortune 500 company.