SYNTHESIS OF PRODUCTS, PROCESSES AND CONTROL FOR DIMENSIONAL QUALITY IN RECONFIGURABLE ASSEMBLY SYSTEMS

by

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ABSTRACT

SYNTHESIS OF PRODUCTS, PROCESSES AND CONTROL FOR DIMENSIONAL QUALITY IN RECONFIGURABLE ASSEMBLY SYSTEMS

by

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Co-Chairs: Shixin Jack Hu and Jianjun Shi

Reconfigurable systems and tools have given manufacturers the possibility to quickly adapt to changes in the market place. Such systems allow the production of different products with simple and quick reconfiguration. Another advantage of reconfigurable systems is that the accuracy of the tools provides a unique opportunity to compensate errors and deviations as they occur along the manufacturing system, hence improving product quality. This dissertation deals with the design of products, processes and controllers to enhance dimensional quality of products produced in reconfigurable assembly processes. The successful synthesis of these topics will lead to new levels of quality and responsiveness.

Fundamental research has been conducted in dimensional control of reconfigurable multistation assembly systems. This includes three topics related to the design of products, processes, and controls. These are:

- *Development of feedforward controllers:* Feedforward controllers allow deviation compensation on a part-by-part basis using reconfigurable tools. The control actions are obtained through the combination of multistation assembly models, in-line measurements (used to measure deviations along the process), and the characteristics and requirements of products/processes, in an optimization framework. Simulation results show that the proposed control approach is effective on reducing variation.
- *Optimal selection and distribution of actuators in multistation assembly processes:* The availability of reconfigurable tools in the process enables error correction; however, it is too expensive to install at every location. The selection and distribution of the actuators is focused on cost effectively reducing variation in multistation assembly processes. Simulations results prove that dimensional variation could be significantly reduced through an appropriate distribution of actuators.
- *Robust fixture design for a product family assembled in a reconfigurable multistation line:* The assembly of a product family in a reconfigurable line demands fixtures sharing across products. The sharing impacts the products robustness to fixture variation due to frequent systems reconfiguration and tradeoffs made in the design of fixtures to accommodate the family in the single system. A robust fixture layout for a product family is achieved by reducing the combined sensitivity of the whole family to fixture variation and considering product and process constraints. Simulations results show the existence of tradeoff between production flexibility and robustness.

CHAPTER 1

INTRODUCTION

1.1 Motivation

Today's manufacturing industry is faced with continuous and rapid changes in its environment. These changes, driven by more demanding customers, the emergence of new technologies, more strict regulations and globalization, have led manufacturers to new levels of competition (Koren *et al.,* 1999). Manufacturers must be able to respond to those changes rapidly and cost-effectively through the fast development of products, and the design and launch of manufacturing systems capable of delivering high quality products. Therefore, manufacturers are in need of production systems that can be easily reconfigured according to the environmental changes. The reconfigurability characteristics of the new production systems not only allow production flexibility, i.e. product family production, but also create the opportunity to improve quality by means of error compensation.

Reconfigurable production systems, consisting of reconfigurable tools and controls, have led to improvement in manufacturing responsiveness to customer preferences in terms of volume and variety. An example of such reconfigurable tool is the FANUC C-Flex robot that is used as a fixture to hold parts in automobile assembly lines (Figure 1.1). This type of reconfigurable tools is also known as Programmable Tooling (PT). As the product changes from one model to another, the PTs change their positions to locate the new parts in the appropriate location, allowing the assembly of different products in the same production line (e.g., a product family). The sharing of fixtures among different products affects the robustness to fixture variation cause by the frequent reconfiguration exacerbated by the fact that an optimal layout for one product may be suboptimal for another product. Therefore, it is necessary to determine an appropriate layout of fixtures used to assemble a product family.

Figure 1.1 View of a C-Flex unit used in auto assembly (Fanuc, 2007)

Counterbalancing the increase on fixture variation, PTs provide the capability to implement active dimensional control through the automatic compensation of product and process deviations. By doing so, they can help to enhance the final product dimensional quality. Traditionally, dimensional quality control has been done using robust design methodologies and Statistical Process Control (SPC). By using robust design methodologies, designers/manufacturers can reduce the effect that intermediate products and process variations have on the final product variation. However, robust design does not guarantee complete elimination of variation. On the other hand, SPC methodologies have been successfully used to detect out of control conditions in processes (e.g., mean shifts or variation changes) and for root cause identification (e.g., identify predetermined variation patterns). However, the SPC alone does not provide systematic means to automatically correct, or compensate, dimensional variation. Hence, one of the major limitations of SPC methodologies is that they cannot be used to compensate errors on a part-by-part basis.

Automatic deviation control presents an opportunity to increase production quality through part-by-part adjustment of tools. In multistation assembly processes, the tooling adjustment or compensation has been approached at the single station level (Svensson, 1985; Sekine *et al.,* 1991; Wu *et al.,* 1994; Pasek and Ulsoy, 1994; Khorzard *et al.,* 1995). Following this approach, adjustments are determined to improve the output of a particular station. However, this strategy may not necessarily lead to an effective improvement of final product quality because the single station scope does not consider the effect that deviations and control actions have on down-stream processes and on the final product. Therefore, there is an opportunity to further improve quality using active dimensional control in multistation processes by considering the multistation variation propagation.

1.2 Research objectives

The objective of this research is to improve dimensional quality in reconfigurable assembly systems by synthesizing product, process, and in-process control compensation. The effective synthesis of these subjects will significantly improve dimensional quality and responsiveness, while also reducing costs. The cost reduction will be due to the time-to-market shrinkage, the investment cost reduction (many products share the same production system), and improvement in yield.

The specific tasks for achieving the proposed objective are:

1. To develop a model of multistation manufacturing processes that includes control action capabilities. Such amodel will help to efficiently evaluate the performance that different controllers and actuators distributions have on variation reduction.
2. To develop a feedforward control strategy that considers process and product characteristics and requirements when determining part-by-part control actions.
3. To determine the optimal selection and distribution of reconfigurable fixtures for control of deviations in multistation assembly processes (cost effective reduction of variation).
4. To develop tools for evaluating the impact that a reconfiguration of the assembly line has on dimensional variation of final products.
5. To propose a method for a robust fixture layout design of a product family assembled in a reconfigurable multistation line.

The successful accomplishment of the objective will result in design procedures for products and processes, considering reconfigurable systems and correction capabilities, with the main goal of efficiently producing high quality products.

1.3 Outline of this dissertation

This dissertation is presented in a multiple-manuscript format. Each of Chapters 2, 3 and 4 is written as an individual research paper, including abstract, introduction, main body sections, conclusions and a reference section.

Chapter 2 describes the design of a feedforward controller used to reduce variation in multistation assembly processes. The proposed design is based on using in-line measurements obtained before assembly to determine the control actions. The derivation of the control actions considers specification and constraints of product and processes. The results of a case study indicate that this approach can efficiently reduce dimensional variation.

Chapter 3 addresses the problem of selecting and distributing PTs in a multistation assembly system with the goal of cost effectively reduce variation. The problem is formulated as a multiobjective optimization, including the derivation of the objective function indices (variation and total equipment cost). In addition, a controlled multistation manufacturing process model is developed. The usage of this model helps to efficiently evaluate the impact that different selection/distribution of PTs and use of different controller designs have on variation. A case study is performed to illustrate the impact that PT placement has on variation reduction.

Chapter 4 is devoted to the robust design of the fixture layout for a product family assembled in a single reconfigurable assembly line. The fixture layout is formulated as an optimization problem, where the objective function is to reduce the combined sensitivity of the product family to fixture variation. Constraints are incorporated into the formulation to account for restrictions that products and processes impose on the fixture layout. In a case study, the solution of a single line is benchmarked against the use of several lines (use of a dedicated line for each product) to quantify the effect that production flexibility has on dimensional quality.

Finally, Chapter 5 summarizes the conclusions and contributions of the dissertation. Several topics for future research are also proposed.

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CHAPTER 2

FEEDFORWARD CONTROL OF MULTISTATION ASSEMBLY PROCESSES USING PROGRAMMABLE TOOLING

Abstract

The combination of feedforward control and programmable tooling has emerged as a promising method to reduce product variation in multistation manufacturing systems. Feedforward control allows compensation of deviations on a part-by-part basis using programmable tooling. This paper[1] addresses the problem of designing an optimal feedforward control law that improves quality. The controller design involves estimation of deviations (from in-line measurements), variation propagation modeling and analysis, and process/parts constraints. Therefore, a control law is obtained using constrained optimization. A case study is conducted on a multistation assembly of a vehicle side frame to illustrate the developed methodology.

2.1 Introduction

Variation reduction is an important but challenging task in multistation manufacturing processes. As an example, dimensional variation in automobile body may lead to wind noise and water leakage, thus variation should be minimized whenever possible. The autobody assembly process involves up to 150 parts assembled in up to 100 stations, where variation may come from any part or assembly operation. Therefore, in such a complex process, determining the deviations and the appropriate correction for variation reduction are always difficult and time consuming tasks.

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[1] Izquierdo, L. E., Shi, J., Hu, S. J. and Wampler, C. W., 2007, Feedforward control of multistation assembly processes using programmable tooling. *Trans. of the NAMRI/SME*, vol. 35, pp 295-302.

There are three approaches commonly used in variation reduction in manufacturing: robust design, Statistical Process Control (SPC) and automatic deviation control. Robust design methodologies help to develop products and processes that are less sensitive to part/process variation and disturbances. However, robustness does not guarantee complete elimination of variation; therefore, parts and process errors may still impact final product quality. SPC methodologies have been successfully used to detect out of control conditions (e.g., detect mean shifts and variation changes in product/process), and for root cause identification (e.g., identify predetermined variation patterns). However, SPC alone does not provide systematic means to automatically correct, or compensate, dimensional variation. Hence, one of the major limitations of SPC methodologies is that they cannot be used to compensate errors on a part-by-part basis. This type of compensation can only be achieved using automatic deviation control, which allows the control of product/process deviations through corrections for each assembly.

The enablers of automatic deviation control are:

- Programmable Tooling (PTs): PTs allows the automatic adjustment of fixtures (locators and clamps) used to hold parts. Because of the high precision of the PTs and their capability to perform part-to-part adjustments, they provide the capability to compensate part/tooling deviations. One example of a PT is the Fanuc robot F-200iB (Fanuc, 2007), which was one of the first robots introduced in assembly to serve as a fixture carrier to allow the assembly of mixed models in the same line.
- In-line dimensional measurement sensors: The development of accurate non-contact sensors that can endure real process conditions has brought the possibility to obtain reliable in-line quality information on the assembly stations (Perceptron, 2006).
- Stream-of-Variation (SoV) modeling tools: SoV tools allow modeling the variation propagation process in multistation assembly processes (Hu, 1997; Jin and Shi, 1999; Shi, 2006). These models can be used to determine the impact that deviations and control actions have on the final product quality.

Automatic deviation control in multistation assembly processes can be approached in two ways: feedback control and feedforward control. Feedback control implies that the control actions (corrections) are determined using downstream measurements usually obtained at the end of the process or in certain intermediate stations. On the other hand, feedforward control uses distributed sensors to determine deviations of parts/process, and then apply control actions before the joining takes place. In this way, feedforward control proactively compensates current deviations instead of reacting to past deviations as feedback control does.

Product and process deviations in assembly can be understood as mean shifts and variance changes. Due to the usual absence or low autocorrelation of the variation sources in multistation assembly systems (Hu, 1990; Hu and Wu, 1990), feedback control can only be used to compensate mean shifts, but not to reduce variability. Thus, feedforward control scheme is preferable in assembly processes to perform corrections prior to the joining by adjusting the position of the PTs. Following this approach, deviations are compensated, and quality is improved. Other benefits of using a feedforward control in assembly processes include reducing process ramp-up time (time to market) and improving the disturbance response time. These advantages not only improve quality, but also enhance process responsiveness and reduce cost.

One of the first attempts to use feedforward control on assembly was done by (Svensson, 1985). With the help of a vision system, he modified the trajectory of a robot to achieve better fit of doors and windshields in car assembly. Similar applications were reported by several authors (Sekine *et al.,* 1991; Wu *et al.*, 1994; Khorzard *et al.,* 1995), where different techniques were used to determine the appropriate fitting of parts.

The aforementioned feedforward control strategies were related to the variation in one particular station, without considering downstream processes. This single station approach is most effective in reducing variation in multistation assembly processes if the station involved is the last one, or the Key Product Characteristics (KPC) of the product controlled in the station is minimally affected by later processes. However, if neither of these conditions hold, the single station control is not appropriate because it does not consider the impact of deviations and control actions on the downstream part dimension.

Therefore, the control actions obtained for the single station scheme may not optimally improve final product quality.

Feedforward control in a multistation process needs a model to determine the impact that control actions at one intermediate station have on the final product. Mantripragada (Mantripragada and Whitney, 1999) proposed a multistation model and the use of optimal control theory to determine control actions during the assembly. Using measurements of the parts before assembly, they were able to calculate the control actions that minimize the final product variation. They assumed that parts are the only source of variation in the process. More recently, Djurdjanovic (Djurdjanovic and Zhu, 2005) proposed the use of feedback and feedforward control using a state space model to control deviations in multistation machining applications by modifying the position of the fixtures and tool path. However, those papers do not address the feedforward control of the multistation assembly process including parts/process requirements and specific engineering constraints on the control actions.

This paper presents a methodology to design an optimal feedforward control that improves product quality by considering process/parts characteristics, multistation variation propagation, and constraints in process and control actions due to actuator characteristics, interference with other components, and other factors. Thus, the determination of the control actions can be formulated as a constrained optimization problem, where the requirements to determine the optimal actions are:

- Obtain an expression of the final product deviations (objective function) as a function of the control actions and the estimated parts deviations obtained from distributed measurements;
- Define the search space for the control actions considering the PT's constraints and parts/processes characteristics; and
- Determine the control actions that minimize the effects that the estimated deviations have on final product quality without violating the constraints, by using a suitable optimization method.

The remainder of the paper is organized as follows: Section 2.2 presents the multistation process model and the control estimation problem. Section 2.3 addresses the part/process deviations estimation problem and the development of the optimal feedforward control law in details. A case study is presented in Section 2.4, and the conclusions are given in Section 2.5.

2.2 Feedforward control of multistation assembly

This section formulates the optimal feedforward control problem for multistation assembly processes including part and process constraints. First, the SoV model is presented, which is used to determine the impact that the control actions have on the final product quality. Second, is addressed the determination of the control action as a constrained optimization problem using estimated deviations is addressed.

2.2.1 SoV model

The model used here to describe the variation propagation in multistation assembly of rigid parts is the state space model developed by Jin (Jin and Shi, 1999).

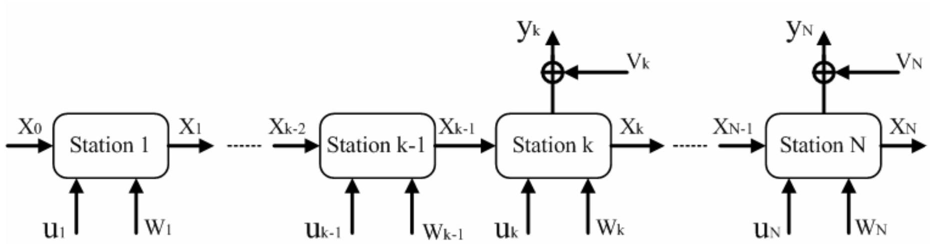

Figure 2.1 Multistation manufacturing process

A schematic of a multistation assembly process is presented in Figure 2.1. As the subassemblies are moved from one station to the next station, they sequentially accumulate errors (Shiu *et al.*, 1996). This process can be modeled as,

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \quad (1)$$

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k; \quad k = 1 \cdots N, \quad (2)$$

where Eq. (1) is the state equation, variable $\mathbf{x}_k \in \Re^n$ represents the state of the system in station k (part deviations from nominal). Variables $\mathbf{u}_k \in \Re^p$ and $\mathbf{w}_k \in \Re^n$ represent the fixture deviations and the disturbances respectively. Matrix $\mathbf{A}_{k-1} \in \Re^{n\times n}$ stands for the reorientation matrix, which relates the fixture layout of two adjacent stations (k-1 and k). The effects of fixture deviations into the state of the system are determined by matrix $\mathbf{B}_k \in \Re^{n\times p}$. The observation equation, Eq. (2), is used to determine the deviations of the measurement points $\mathbf{y}_k \in \Re^m$, which usually corresponds to the KPCs of the product. Their deviations are obtained from the state using the observation matrix $\mathbf{C}_k \in \Re^{m\times n}$ and adding the measurement noise $\mathbf{v}_k \in \Re^m$. For details on how to derive each matrix please refer to Jin and Shi (1999), Ding *et al.* (2000) or Shi (2006).

The state transition matrix $\mathbf{\Phi}_{k,i}$ describes the deviation transmission between stations *i* and *k*, and it is calculated as $\mathbf{\Phi}_{k,i} \equiv \mathbf{A}_{k-1}\mathbf{A}_{k-2}\cdots\mathbf{A}_{i+1}\mathbf{A}_i$, $k > i \geq 0$, otherwise $\mathbf{\Phi}_{i,i} \equiv \mathbf{I}$ (**I** is the identity matrix). Then, Eq. (2) can be written as,

$$\mathbf{y}_N = \mathbf{\Psi}_0\mathbf{x}_0 + \sum_{k=1}^{N}\mathbf{\Gamma}_k\mathbf{u}_k + \sum_{k=1}^{N}\mathbf{\Psi}_k\mathbf{w}_k + \mathbf{v}_N, \tag{3}$$

where $\mathbf{x}_0$ represents the deviation of the incoming parts, $\mathbf{\Gamma}_k = \mathbf{C}_k\mathbf{\Phi}_{N,k}\mathbf{B}_k$ and $\mathbf{\Psi}_k = \mathbf{C}_k\mathbf{\Phi}_{N,k}$.

The deviations of the incoming parts, fixtures deviations, disturbances and noise are considered as random variables with mean of zero and covariances of Σ_{x_0}, Σ_u, Σ_w and Σ_v respectively. They are considered to be independent within stations (e.g., $Cov(\mathbf{u}_k, \mathbf{v}_k) = 0$) and to be independent between different stations (e.g., $Cov(\mathbf{w}_i, \mathbf{w}_j) = 0,\ \forall i \neq j$).

In assembly, in-line measurements are usually obtained using OCMM sensors (Optical Coordinate Measurement Machine), which provide information on the displacement of the measurement points in 1D or 2D. The measurement points are usually selected to coincide with the KPCs. Therefore, they correspond to features that are important for the functionality, cost and safety of the product.

2.2.2 Feedforward control problem formulation

The feedforward control formulation is based on compensating deviations of parts/ subassemblies and process before the joining process takes place as presented in Figure 2.2.

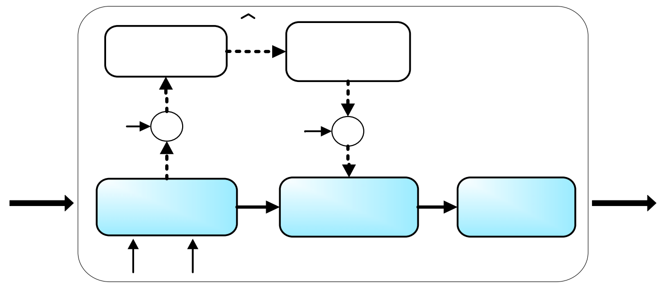

Figure 2.2 Procedure to correct deviations at station *k*

After the parts/subassemblies are mounted on the fixtures, measurements are performed to determine the deviations of the parts. Since the measurements are corrupted with noise $\mathbf{v}_k^B$, the true state of the system can only be estimated. Using the estimation, it is possible to determine the control action vector $\mathbf{s}_k \in \Re^p$, which will be applied by the PTs. When applying the control actions, due to imperfections, the PTs introduce an error $\mathbf{e}_k \in \Re^p$. Finally the state of the system at station k can obtained as

$$\mathbf{x}_k = \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{w}_k + \mathbf{B}_k{}^C (\mathbf{s}_k + \mathbf{e}_k), \tag{4}$$

where matrix $\mathbf{B}_k^C \in \Re^{n \times p}$ relates the control actions and errors with the state at station k through the control of the fixture (in particular the pins). If a part is mounted on a PT, depending on the PT characteristics, some or all the degrees of freedom (dof) of the part can be controlled. Therefore, each row of matrix $\mathbf{B}_k^C$, corresponding to a specific dof that can be controlled in a part, is equal to the same row in $\mathbf{B}_k$; otherwise, it is a rows of zeros.

Using the state space model, it is possible to determine the effect that the estimated deviations and control actions have on the estimated final product deviations $\tilde{\mathbf{y}}_{N/k}$, given the information available in station k (the derivation of $\tilde{\mathbf{y}}_{N/k}$ is presented in detail in section 2.3.2). By doing so, the control action determination can be formulated as a constrained optimization problem, where the objective function is the weighted sum of the squares of $\tilde{\mathbf{y}}_{N/k}$ as presented in Eq. (5).

$$\begin{aligned} J = \min_{\mathbf{s}_k} \quad & \tilde{\mathbf{y}}_{N/k}^T \mathbf{Q}_k \tilde{\mathbf{y}}_{N/k} \\ \text{s.t. } & \mathbf{g}(\tilde{\mathbf{y}}_{N/k}^T, \mathbf{s}_k) \leq \mathbf{0} \end{aligned}, \tag{5}$$

where matrix $\mathbf{Q}_k \in \Re^{m \times m}$ is the weight matrix, $\mathbf{Q}_k$ is a diagonal and positive definite matrix. The values of the weighting coefficients account for the relative importance of the KPCs. The set of constraints $\mathbf{g}(\cdot,\cdot)$ include the design and manufacturing requirements for the location of the KPCs and station/PT characteristics.

2.3 Determination of the control actions

This section presents the procedures to determine the optimal control actions using the estimated parts deviations.

2.3.1 Deviation estimation

Being in station *k*, the system equations before the control actions are applied can be described as:

$$\mathbf{x}_k^B = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \tag{6}$$

$$\mathbf{y}_k^B = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k^B; \quad k = 1 \cdots N, \tag{7}$$

where the super index ^{B} stands for the condition before applying the control. By using Eq. (7), it is possible to estimate the state of the system $\hat{\mathbf{x}}_k$ using the Weighted Least Squares (WLS) estimation method as,

$$\hat{\mathbf{x}}_k = \mathbf{C}_k^{\dagger}\mathbf{y}_k^B, \tag{8}$$

where, $\mathbf{C}_k^{\dagger}$ is the weighted pseudoinverse of matrix $\mathbf{C}_k$, and it is calculated as $\mathbf{C}_k^{\dagger} = (\mathbf{C}_k^T\mathbf{R}_k\mathbf{C}_k)^{-1}\mathbf{C}_k^T\mathbf{R}_k$. Here, matrix $\mathbf{R}_k \in \Re^{m \times m}$ is a weighting coefficient matrix, which accounts for differences in the importance and characteristics of the measured points, and it is a positive definite diagonal matrix. If matrix $\mathbf{R}_k$ contains on its diagonal the inverses of the sensors noise variances, then $\hat{\mathbf{x}}_k$ is the best linear unbiased estimator of $\mathbf{x}_k^B$ (Franklin *et al.,* 1998).

2.3.2 Control action determination

At station k, it is possible to write down the effects that the different variation sources and the control actions have on the final product deviations $\tilde{\mathbf{y}}_N$ as presented in Eq. (9).

$$\tilde{\mathbf{y}}_N = \mathbf{\Psi}_k \mathbf{x}_k^B + \mathbf{\Gamma}_k^C (\mathbf{s}_k + \mathbf{e}_k) + \mathbf{v}_N, \tag{9}$$

where matrix $\mathbf{\Gamma}_k^{\ C}$ is the with-control version of $\mathbf{\Gamma}_k$ obtained by using $\mathbf{B}_k^{\ C}$ on its derivation, i.e., $\mathbf{\Gamma}_k = \mathbf{C}_k \mathbf{\Phi}_{N,k} \mathbf{B}_k^C$.

The PTs error vector $\mathbf{e}_k$ is assumed to be a random variable with mean of zero and covariance $\mathbf{\Sigma}_e$, where the value of the covariance depends on the precision (repeatability) of the PTs utilized.

The expected deviations of the final product measurements, given the information available up to station k, can be obtained by calculating the expectation of Eq. (9) as,

$$\tilde{\mathbf{y}}_{N/k} = \mathbf{\Psi}_k \hat{\mathbf{x}}_k + \mathbf{\Gamma}_k^C \mathbf{s}_k. \tag{10}$$

As presented in section 2.2 the control actions are obtained based on the constrained optimization of Eq. (5). Writing down the constraints, the control problem can be formulated as follows,

$$\begin{aligned} J = \min_{\mathbf{s}_k} \quad & \tilde{\mathbf{y}}_{N/k}^T \mathbf{Q}_k \tilde{\mathbf{y}}_{N/k} \\ \text{s.t. } & \tilde{\mathbf{y}}_{N/k} \in \left[LSL_{\tilde{\mathbf{y}}_{N/k}}, USL_{\tilde{\mathbf{y}}_{N/k}} \right] \\ & \mathbf{s}_{\min} \le \mathbf{s}_k \le \mathbf{s}_{\max} \\ & \mathbf{s}_k = \begin{cases} \mathbf{s}_k & \text{if } |\mathbf{s}_k| \ge \mathbf{\Delta}_s \\ \mathbf{0} & \text{otherwise.} \end{cases} \end{aligned} \tag{11}$$

This general formulation includes the existence of constraints on the position of the KPCs and the control actions. The first constraint ensures that the final product KPCs are within the Upper and Lower Specification Limits (USL and LSL). The second constraint restricts the control actions to be within the upper and lower PT actuation limits ($\mathbf{s}_{\min}$ and $\mathbf{s}_{\max}$) that can be applied on each part/subassembly. The control action limits consider PTs workspace limitations and interferences with other station components. Finally, the third constraint is an or-type one, where there are two possibilities for $\mathbf{\Delta}_s$: it is either bigger than or equal to a threshold $\mathbf{\Delta}_s$ ($\mathbf{\Delta}_s > \mathbf{0}$), or it is zero.

This type of threshold is used to avoid obtaining control actions that cannot be performed by the PTs. The value of the threshold can be obtained according to the accuracy of the PTs.

Varying $\mathbf{\Delta}_s$ in Eq. (11) can be understood as using different types of PTs. Therefore, such study can lead to identify the appropriate PTs to be used based on an effectiveness analysis.

Figure 2.3 presents the procedure proposed to determine the control actions, which is based on determining first the unconstrained optimal solution of Eq. (11) (the unconstrained solution is presented next). If this solution does not violate any constraints, then control action can be directly applied. If one or more constraints are violated, then, the constrained optimization problem has to be solved.

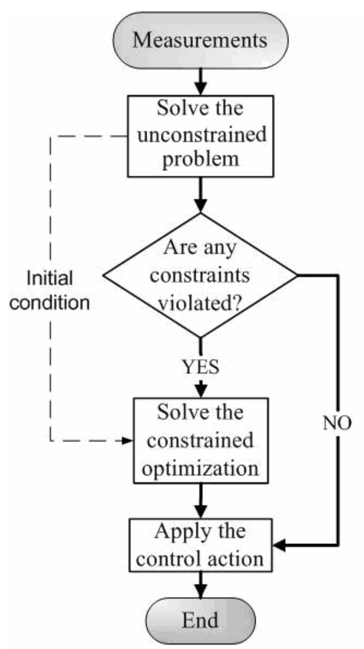

Figure 2.3 Procedure to determine the control action

The unconstrained solution ($\mathbf{s}_k^{*Unc}$) of problem (11) can be obtained by replacing Eq. (10) into Eq. (11) and solving it as a WLS problem (similar to the one in Section 2.3.1). Following this approach, the control action can be written down in terms of the measurements before control as,

$$\mathbf{s}_k^{*Unc} = -\mathbf{K}_k \mathbf{y}_k^B, \tag{12}$$

where the control gain matrix $\mathbf{K}_k$ is obtained as,

$$\mathbf{K}_k = \left[(\mathbf{\Gamma}_k^C)^T \mathbf{Q}_k \mathbf{\Gamma}_k^C \right]^{-1} (\mathbf{\Gamma}_k^C)^T \mathbf{Q}_k \mathbf{\Psi}_k \mathbf{C}_k^{\dagger} . \qquad (13)$$

The first constraint in Eq. (11) may cause the nonexistence of an optimal solution. This happens when the incoming parts and subassemblies at station k are so severely deviated from their nominal that it is impossible to satisfy the first constraint. Therefore, there is no control action capable to adjust the KPCs to make them be within their specification limits. If that is the case, the unconstrained control action should be apply and a notification to the maintenance department should be done.

2.4 Case study

The case study used to test the proposed methodology simulates the assembly of a Sport Utility Vehicle (SUV) side frame (Figure 2.4), and was proposed by Ding *et al.* (2002). The side frame is formed by four parts, which are assumed to be rigid and free to move in the x-z plane only (3 dof per part).

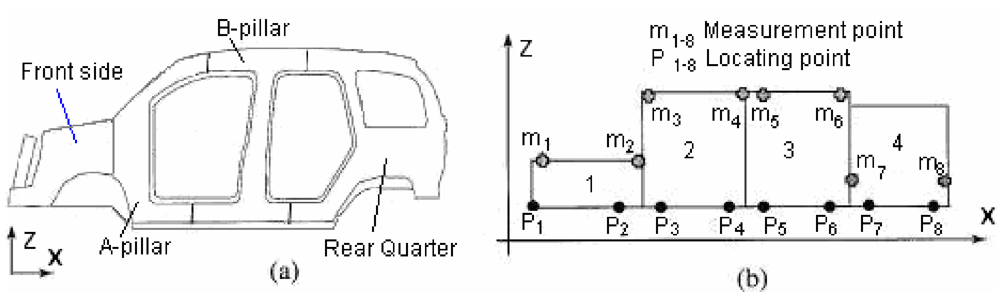

Figure 2.4 Schematic of a SUV side frame and its simplification (Ding *et al.,* 2002)

The assembly is performed in three stations with final measurement taken in a final inspection station. The assembly sequence is summarized as follows: in the first station the fender is attached to the A-pillar, then the B-pillar is added in the second station, and in the third station the rear quarter is attached. Afterwards, the complete assembly is moved to station four for final inspection. The locators used are: {(P1, P2), (P3, P4)}$\text{Station}_{\text{I}}$, {(P1, P4), (P5, P6)}$\text{Station}_{\text{II}}$, {(P1, P6), (P7, P8)}$\text{Station}_{\text{III}}$ and {(P1, P8)}$\text{Station}_{\text{IV}}$. It is assumed that (i) all the required measurement points (marked in Figure 5b) are available at each station, (ii) PTs are used to hold all the parts in stations I and III, and (iii) there are not fixture errors and disturbances in the measurement station due to tighter tolerances and a better maintenance policy. The parts are assumed to be rigid and the variation happens only in the x-z plane

The performance using the control algorithm derived in Section 3.2 is analyzed through (a) calculating the Quality Index (*QI*) defined in Eq. (14) as the reduction of the 2-norm of the final measurement standard deviation (σ) with and without using control actions, and (b) checking if some KPC's deviation exceeds the 2 mm/6σ threshold, which is the standard in the automobile industry.

$$QI = \frac{\left\|\sigma_{y_{N/k}}^{w/o\ Control}\right\|_2 - \left\|\sigma_{y_{N/k}}^{Control}\right\|_2}{\left\|\sigma_{y_{N/k}}^{w/o\ Control}\right\|_2} \times 100\,. \tag{14}$$

The parameters used in the simulations are $\mathbf{\Sigma}_{x_0} = 0.04 \cdot \mathbf{I}$, $\mathbf{\Sigma_u} = 0.0017 \cdot \mathbf{I}$, $\mathbf{\Sigma_w} = 0.0001 \cdot \mathbf{I}$, $\mathbf{\Sigma_e} = 0.0017 \cdot \mathbf{I}$, and $\mathbf{\Sigma}_{\mathbf{v}^B} = \mathbf{\Sigma_v} = 0.0009 \cdot \mathbf{I}$, where the units are mm^2, and **I** stands for the identity matrix with appropriate dimensions. The USL and the LSL were set to 0.8 mm and -0.8 mm respectively for all the KPCs. The values of $\mathbf{s}_{\min}$ and $\mathbf{s}_{\max}$ were set to -5 mm and 5 mm respectively to all the PTs, and the value of the threshold $\mathbf{\Delta}_s$ was set to 0.1 mm for all the PTs. The weighting coefficients matrices $\mathbf{Q}_k$ and $\mathbf{R}_k$ (*k*=1, 2 and 3) were set equal to the identity matrix. The results reported are based on the simulation of 1500 assemblies.

The standard deviations of the KPCs in the x and z directions for the cases with and without control are presented in

Table 2.1 and Figure 2.5. In the with-control case, both with and without constraints scenarios are included. The solution obtained with the unconstrained control were later filtered with the actuators constraints (second and third constraints in Eq. (11)) to analyze the effect that not considering these constraints will have on the performance of the controller.

Table 2.1 Effect of the control and the constraints on the measurement points quality (units: mm)

Measurement point	Without control		With control constrained		With control unconstrained	
	Stdev in x	Stdev in z	Stdev in x	Stdev in z	Stdev in x	Stdev in z
M_1	0.17	0.06	0.05	0.03	0.05	0.03
M_2	0.16	0.33	0.05	0.08	0.05	0.09
M_3	0.35	0.24	0.07	0.08	0.08	0.09
M_4	0.35	0.28	0.08	0.16	0.08	0.17
M_5	0.27	0.24	0.27	0.15	0.27	0.15
M_6	0.27	0.25	0.27	0.14	0.27	0.14
M_7	0.22	0.25	0.05	0.05	0.1	0.09
M_8	0.22	0.07	0.05	0.04	0.1	0.04

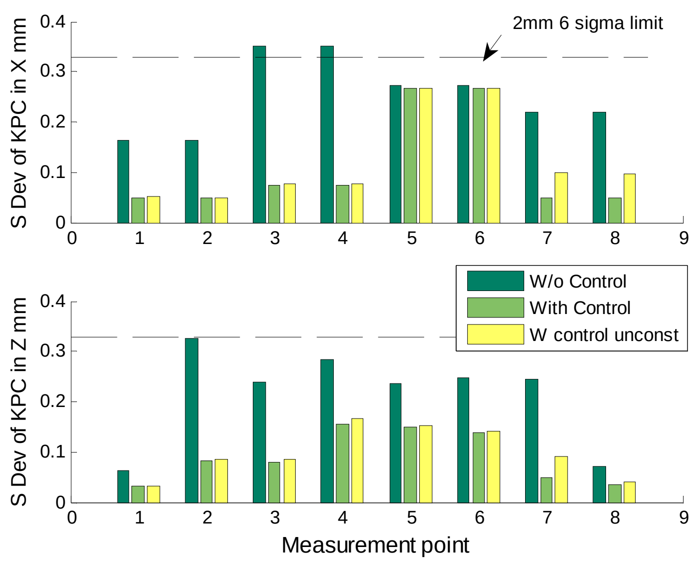

Figure 2.5 Standard deviations of the measurement points (KPCs)

The effect of using control significantly improves quality. The values of *QI* are 49.7% and 46.8% for the constrained and unconstrained control respectively. As can be expected, the constrained control improvement is bigger than the unconstrained one because it incorporates more information when determining the control actions. By analyzing the figure, it is possible to observe that only the uncontrolled case exceeded the

2mm 6σ (σ =0.33 mm) limit. Due to the lack of actuators in station II, the effects of both controls do not significantly improve the position of KPCs M_5 and M_6 in the x direction. However, both controllers help in the z direction. The reason is that if part three is joined in station II in the wrong position, the deviations that this part has in the x direction cannot be corrected by relocating the subassembly formed by parts 1, 2 and 3 in station III (the subassembly cannot be stretched or compressed to correct the errors). However, a significant portion of the deviations in the z direction (~30%) can be corrected through a proper relocation of the subassembly in station III.

Next, different scenarios are analyzed to study the impact of the threshold $\mathbf{\Delta}_s$ has on the quality improvement of constrained control. This analysis may help to select the appropriate PT for a given process. Table 2.2 and Figure 2.6 present the results using different values of the threshold. The effect on the improvement has a sigmoid shape, where for small thresholds, equivalent to using accurate PTs, there is a small drop in the *QI*. However as the threshold increases (greater than 0.1 mm) the *QI* tends to decay asymptotically to zero.

Table 2.2 Effect of the threshold $\mathbf{\Delta}_s$ on the quality improvement

Δ_s mm	*QI* %	Exceed 2 mm (6σ) ?
0	44.5	No
0.05	44.0	No
0.075	42.6	No
0.7	40.2	No
0.15	33.7	No
0.2	23.8	No
0.25	14.8	No
0.3	9.3	No
0.35	4.8	No
0.4	1.2	Yes

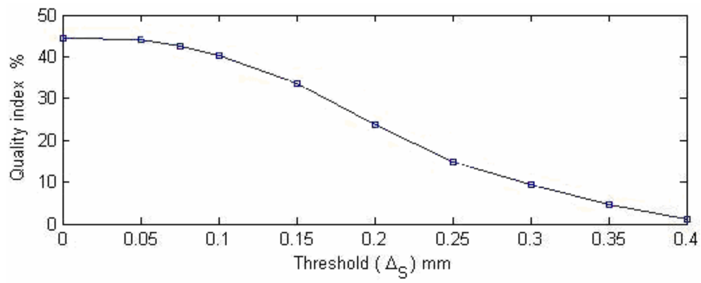

Figure 2.6 Effect if the threshold Δ_s on the quality improvement

2.5 Conclusions

This paper proposes a new approach to improving product dimensional quality in multistation assembly processes by deviation compensation using feedforward control. The proposed method uses distributed sensing and programmable fixturing technologies in determining and correcting deviations on a part-by-part basis. The problem of determining the optimal corrections or control actions is formulated as a constrained optimization by considering design specifications and actuator/process characteristics. A method is proposed to obtain the optimal control actions by solving first the unconstrained problem, and then, searching inside the constrained space to find a global optimal solution. A case study that considers the assembly of a SUV side frame in three stations is presented considering the existence of PTs in only two stations. The results proved that feedforward control including product and process constraints reduces the variation of the final product KPCs by more than 49 %, which is a better than the improvement achieved without considering the constraints. The effect of PTs accuracy on the resulting quality improvement is also analyzed. From this analysis, it can be concluded that for high PT accuracy the effect is almost constant. However, as the PT accuracy diminishes, there is a significant decrease in the amount of variation that can be reduced.

Acknowledgements

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CHAPTER 3

OPTIMAL ACTUATOR PLACEMENT FOR DIMENSIONAL CONTROL OF MULTISTATION ASSEMBLY PROCESSES

Abstract

The use of active control has emerged as a promising technique to reduce dimensional variation in multistation manufacturing processes. Through the correction of errors and deviations as they happen in the process, smaller variation can be achieved in the final products. However, the effectiveness of correction is limited by the availability and characteristics of actuators used in the process. This chapter proposes a methodology for the cost effective selection and distribution of actuators in multistation assembly processes for variation reduction. To this end, the problem of selecting/distributing actuators is formulated as a multiobjective combinatorial optimization one, where the objectives are to minimize variation and total actuator cost. Several constraints are added to the formulation according to the engineering problem to reduce the search space. The constraints are obtained based on the controllability analysis of the multistation assembly process. A new concept of introduced, which permits the identification of conditions where adding more actuators does not contribute to reduce variation. A case study is conducted on a multistation assembly of an automobile side frame to illustrate the proposed methodology. An optimal distribution of actuators leads to enhance quality by more than 86 % compared without control, and ratify that using more than necessary actuators (imperfect actuators) leads to increase variation instead of reducing it.

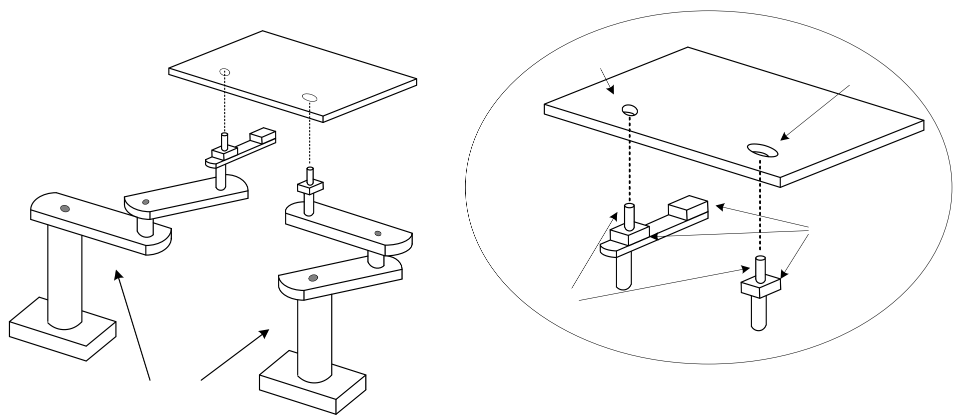
3.1 Introduction

Complex products such as airplanes, automobiles, and home and medical appliances are assembled in Multistation Assembly Processes (MAP) through the sequential aggregation of parts. As an example, an automobile body structure may have around 150 parts assembled in about 70 stations. In such a complex process, errors or deviations in parts and processes propagate downstream affecting the functional, aesthetics, and safety characteristics of the final product. Although, variation reduction is important, it is also a challenging due to the multiple source of variation, process complexity, and the difficulty in determining the origin of deviations and the proper correction.

Traditional methods for variation reduction are based on robust design and Statistical Process Control (SPC). By using robust design, designers/manufacturers can reduce the effect that intermediate products and process variations have on the final product variation. However, robust design does not completely eliminate variation since components and processes still introduce variation. SPC methodologies have been successfully used to detect out of control conditions in manufacturing processes (e.g., mean shifts or variation changes), or occasionally, to find root causes of variation. However, SPC alone does not provide a systematic means to automatically correct, or compensate, dimensional variation. One of the major limitations of SPC methodologies is that they cannot be used to compensate errors on a part-by-part basis.

Active dimensional control has emerged as a promising technique to reduce dimensional variation in multistation manufacturing processes (Mantripragada and Whitney, 1999; Fenner *et al.,* 2005; Djurdjanovic and Zhu, 2005; Izquierdo *et al.,* 2007). The enablers of active dimensional control in a MAP are the advancements in multistation assembly modeling, control, and actuators and in-line sensing technologies. Multistation models, as called Stream-of-Variation (SoV) models, can be used to predict the impact that part and process variations have on final product (Jin and Shi, 1999; Camelio *et al.,* 2003; Zhou *et al.,* 2003; Shi 2006). The development of control algorithms has permitted determining optimal control actions in a MAP considering the process as a whole and not considering each station isolated from the rest of the process (Mantripragada and Whitney, 1999; Djurdjanovic and Zhu, 2005; Izquierdo *et al.,* 2007). Advances in sensing technology have opened the possibility to perform measurements directly in assembly stations. A new generation of embedded sensors can endure the harsh conditions of the assembly process and provide accurate in-line information on the process condition (e.g., Optical Coordinate Measurement Machines (OCMM) sensors

(Perceptron, 2007)). This in-line information is further used to determine control actions to be applied by the actuators. Advances in robotics and actuators technology, which were first introduced for reconfiguration purposes, have opened the possibility to actively compensate process deviations in a MAP due to the high precision of the PTs. Figure 3.1a presents a schematic of this type of actuator, also known as reconfigurable fixtures or Programmable Tooling (PT). In the figure, two PTs are used to carry the fixture elements (fixels) used to locate a sheet metal part in an assembly station. Figure 3.1b presents a close view of the 3-2-1 fixture-type commonly used in sheet metal assembly. This fixture is formed by three NC blocks, two of which having pins that fit into the hole and slot pierced on the parts, and a set of clamps (not shown in the figure) to ensure part-blocks contact.



a) Schematic of the PTs b) Details of a 3-2-1fixture

Figure 3.1 Schematic of the PTs used to hold and control deviations in assembly

The availability and characteristics of sensors and actuators limit the capability to detect and correct errors. Their availability and characteristics (number, location, and type of sensors and actuators used) are constrained by budget and product/process, e.g., space limitations in assembly stations. Therefore, it is necessary to determine the optimal selection and distribution of these devices that helps reduce dimensional variation in MAPs at minimum cost while satisfying the constraints. In this paper, we address the optimal selection and distribution of actuators in a MAP. We assume that all the necessary sensors are available along the process. The reasons for not including sensors in the resource allocation are: (i) sensors are required for dimensional quality monitoring

Part

purposes, so they are installed in the process nevertheless, and (ii) the relative low cost of sensors with respect to actuators makes them easily available. Hence, the questions addressed in this paper are the number, type and distribution of the actuators to cost-effectively reduce variation in a MAP.

Since the actuators are imperfect tools, they may introduce some errors when performing control actions, which depend on the actuator's quality (repeatability). Therefore, actuators characteristics will be considered when determining the appropriate tool for a given process.

The actuator selection/distribution problem is formulated as a multiobjective optimization problem as presented in Equation 1. In this formulation, the objective function includes dimensional variation and cost. The design variables are the location (i.e. which pins are controlled at each station) and type of PT used, and finally, the set of constraints $\mathbf{g}(\cdot)$, which account for space, budget and other constraints.

$$J = \min_{\substack{\text{location \&} \\ \text{actuator type}}} \left(Cost,\ Variation\right) \qquad \text{s.t. } \mathbf{g}(\text{budget, space}) \leq 0 \tag{1}$$

To solve the actuator selection /distribution problem, it is necessary to:

1. Obtain a model to represent the final product variation of a controlled MAP as a function of the type and distribution of actuators.
2. Define a cost function of the process considering the type and number of actuators used.
3. Specify the search space for the type and location of the actuators that includes process/products constraints.
4. Determine the selection/distribution of actuators that minimizes Eq. (1) by using a suitable optimization method.

The two objectives, minimize variation and minimize cost, are clearly antagonistic. Variation reduction will usually demand more and better actuators, while cost reduction looks for trimming down the number of actuators and their quality. Since the solution implies tradeoff between variation and cost reductions, a good way to resolve the conflict is to solve the problem by means of Pareto set. The Pareto sets can be used to determine sets of solutions where one objective (either cost or variation in this case) cannot be further improved without sacrificing the other. By using these sets, designers can identify tradeoffs, evaluate/visualize multiple solutions, and finally make decisions (Fellini *et al*., 2005).

As aforementioned, the design variables are the location and type of actuators used. The location of the actuators will be modeled as a binary variable (i.e. PT present or absent on each possible location), and the type of actuators will be modeled as an integer, where each value represents a different actuator type selected from a finite set of available PTs in the market. Consequently, the type of problem addressed in this work may be a large combinatorial optimization problem depending on the number of parts, and the type and number of actuators that can be used. On top of that, the final product variation, as it will be shown later, is an implicit nonlinear function of the distribution and characteristics of the actuators. The combinatorial and nonlinear characteristics of the problem make it hard to solve. To improve the solvability of the problem, by means of speeding the variation evaluation and reducing of the search space, some characteristics of the model used to track variation in a MAP, as well as the controllability of the system, will be investigated.

The remainder of the paper is organized as follows; Section 3.2 presents the state of the art in multistation variation modeling, control and actuator placement in other disciplines. Section 3.3 presents the formulation of the multistation process model including control. Section 3.4 addresses the selection of the objective function. Section 3.5 presents the proposed methodology to efficiently solve the problem of selecting/distributing the actuators. A case study is presented in Section 3.6; and the conclusions are given in Section 3.7.

3.2 Relevant work

The review of relevant work covers the following three areas related to the proposed research: multistation assembly models, controller design for MAPs, and optimal actuator placement.

a) Multistation assembly models

This subsection reviews models used to determine dimensional variation propagation in MAPs.

Several models have been proposed in the literature to represent dimensional variation and its propagation in MAPs considering rigid parts. Among these are statistical models such as the Autoregressive model AR(1) model proposed by Lawless *et al.* (1999), and the physics based ones proposed by Jin and Shi (1999), Mantripragada and Whitney (1999) and Ding *et al.* (2000). The physics based models use kinematics relationships to relate the system state or part's deviations with the different variation inputs (incoming parts variation, process disturbances, and fixture variation) and the deviation propagation in a multistation process (Mantripragada's model does not consider the effect of fixture deviations in the process). The models also include the effect that parts deviations have on the important features measured on the final product, also known as the Key Product Characteristics (KPC), and the effect that sensor noise has when measuring the KPCs.

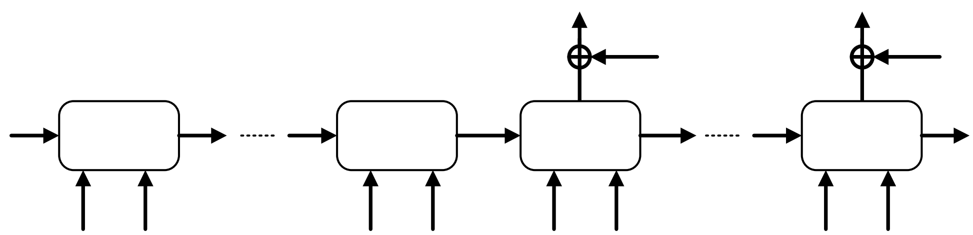


Figure 3.2 Schematic of a multistation process

A schematic of a multistation system is presented in Figure 3.2, where the incoming parts and subassemblies are sequentially transferred from station to station, accumulating deviations along the process. Using the state space approach, this sequential process can be modeled as

$$\mathbf{x}_k = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k \tag{2}$$

$$\mathbf{y}_k = \mathbf{C}_k\mathbf{x}_k + \mathbf{v}_k; \quad k = 1 \cdots N. \tag{3}$$

Equation (2) is known as the state equation, where vector $\mathbf{x}_k \in \Re^n$ represents the state or dimensional deviations of the parts from their nominal position after station *k*.

The input vector $\mathbf{u}_k \in \Re^p$ stands for fixture deviations (caused by wear, loose, bent or even missing pins) at station *k*; and the disturbance vector $\mathbf{w}_k \in \Re^n$ accounts for other external disturbances and unmodeled high-order terms. The reorientation matrix $\mathbf{A}_{k-1} \in \Re^{n\times n}$ relates the fixture layout of two adjacent stations (i.e. stations *k-1* and *k*) and its effect on the state at station *k*. The effects of fixture deviations in the system state are determined by matrix $\mathbf{B}_k \in \Re^{n\times p}$. The rows of matrix $\mathbf{B}_k$ determine the contribution that each fixture deviation has on the state. Equation (3) is known as the observation equation and is used to determine the deviations of the measurement points $\mathbf{y}_k \in \Re^m$, which usually corresponds to product's KPCs. The measurement deviations are obtained by multiplying the system state with the observation matrix $\mathbf{C}_k \in \Re^{m\times n}$ and adding the measurement noise $\mathbf{v}_k \in \Re^m$. Matrix $\mathbf{C}_k$ depends on the relative position of the measurement points and the part reference point. Details of the derivation of each matrix can be found in Ding *et al.* (2000) and Shi (2006).

The state transition matrix $\mathbf{\Phi}_{k,i}$ describes the deviation transmission between stations *i* and *k,* and it is calculated as $\mathbf{\Phi}_{k,i} \equiv \mathbf{A}_{k-1}\mathbf{A}_{k-2}\cdots\mathbf{A}_{i+1}\mathbf{A}_i$, $k>i\geq 0$; $\mathbf{\Phi}_{i,i} \equiv \mathbf{I}$ (**I** is the identity matrix). Applying recursively Eq. (2) from station 1 to *N*, the final state of the system can be written as,

$$\mathbf{x}_N = \mathbf{\Phi}_{N,0}\mathbf{x}_0 + \sum_{k=1}^{N}\mathbf{\Phi}_{N,k}\mathbf{B}_k\mathbf{u}_k + \sum_{k=1}^{N}\mathbf{\Phi}_{N,k}\mathbf{w}_k\,, \tag{4}$$

where $\mathbf{x}_0$ represents the deviation of the incoming parts. Combining Eqs. (3) and (4), it is possible to determine the final product KPCs' deviations as a function of all the process and products deviations.

$$\mathbf{y}_N = \mathbf{\Psi}_0\mathbf{x}_0 + \sum_{k=1}^{N}\mathbf{\Gamma}_k\mathbf{u}_k + \sum_{k=1}^{N}\mathbf{\Psi}_k\mathbf{w}_k + \mathbf{v}_N\,, \tag{5}$$

where, $\mathbf{\Gamma}_k = \mathbf{C}_k\mathbf{\Phi}_{N,k}\mathbf{B}_k$ and $\mathbf{\Psi}_k = \mathbf{C}_k\mathbf{\Phi}_{N,k}$.

An important property of the state transition matrix $\mathbf{\Phi}_{k,i}$ (i>1) is that it is a singular matrix. Its singularity is due to the singularity of matrices $\mathbf{A}_k$ (k>1), which is caused by the reuse of a subset of locators when holding subassemblies along the process (Ding *et al.,* 2004; Kim and Ding, 2004).

b) Controller design for MAPs

There are mainly two approaches to obtain control action in a MAP, feedback control and feedforward control. In a feedback approach, the control actions are determined based on downstream measurements, e.g., using end-of-process measurements or intermediate process measurements. Feedforward control uses information about parts/subassemblies deviations prior to assembly (from in-line measurements available on the assembly stations) to determine the control actions.

Variation reduction using feedback control in MAPs requires the existence of autocorrelation in the variation sources. Since the control actions for a current product are based on previous product measurements, the absence of autocorrelation will lead to “over control” actions resulting in variation growth, similarly to the funnel experiment reported by MacGregor (1990). Hu (Hu, 1990; Hu and Wu, 1990) reported that in the automobile body assembly, autocorrelation does not exist or is weak if present. Therefore, feedback control should not be used for variation reduction in such a process. In this kind of processes, feedback should only be used to compensate for mean shifts, which reveals some level of autocorrelation of inputs.

Since feedforward control is based on pre-assembly measurements, it can be used to reduce variation through part-by-part correction. Following this approach, deviations can be compensated as they happen in the process to reduce variation. Next, it is presented a procedure for determining the control actions for feedforward control in a multistation framework (see Figure 3.3).

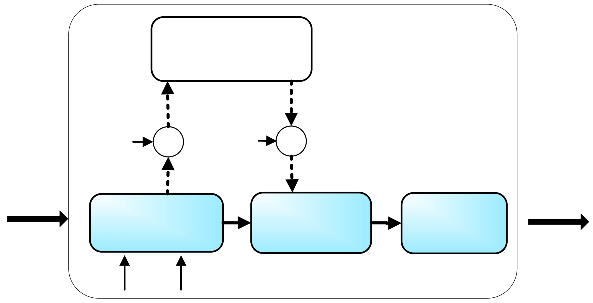


Figure 3.3 Procedure to control deviations using feedforward control

Equation (6) represents the deviations of the parts in station *k* when they are mounted on the fixtures *before* the control actions are applied, hence the superscript B. Similarly, $\mathbf{y}_k^B$ represents the measurements performed on the parts before control including the effect of sensors noise.

$$\mathbf{x}_k^B = \mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k\,, \tag{6}$$

$$\mathbf{y}_k^B = \mathbf{C}_k\mathbf{x}_k{}^B + \mathbf{v}_k{}^B;\quad k = 1,\cdots,N \quad . \tag{7}$$

As presented in Figure 3.3, the controller uses the measurements $\mathbf{y}_k^B$ to determine the control actions $\mathbf{s}_k \in \Re^p$ to be performed by the PTs (later is presented on how $\mathbf{s}_k$ is obtained). Because of PTs imperfections, there is an error $\mathbf{e}_k \in \Re^p$ when applying the control actions. Consequently, the state of the system after control is a function of the state before control, and the influence of the control actions combined with the PT errors as presented in Eq. (8).

$$\mathbf{x}_k = \mathbf{x}_k{}^B + \mathbf{B}_k{}^C(\mathbf{s}_k + \mathbf{e}_k)\,, \tag{8}$$

where matrix $\mathbf{B}_k^C \in \Re^{n\times p}$ relates the control actions and errors with the state at station *k* through the control of the appropriate fixels (in particular the pins). If a pin is mounted on a PT, depending on the PT characteristics, some or all the degrees of freedom (dof) of the pin can be controlled (for simplicity controllable dof is simplified as c-dof). Therefore, each row of matrix $\mathbf{B}_k^C$, corresponding to a specific dof that can be controlled in a part, is equal to the same row in $\mathbf{B}_k$; otherwise, it is a rows of zeros.

Following the feedforward concept, the control actions are obtained as a linear function of the measurements before control with the control gain matrix $\mathbf{K}_k$ as presented in Eq. (9).

$$\mathbf{s}_k = -\mathbf{K}_k \cdot \mathbf{y}_k^B \tag{9}$$

In the literature, mainly two methods have been proposed for determining the control gain matrix for feedforward control in multistation manufacturing processes. First, Mantripragada and Whitney (1999) and Fenner *et al.* (2005) used optimal control theory to derive the control gain matrices. Second, Djurdjanovic and Zhu (2005) and Izquierdo *et al.,* (2007) determined the control gain by minimizing the weighted expected effect that deviations and control actions have on the final product KPCs given the information available at station *k*. Following Izquierdo *et al.* (2007), the control gain matrix $\mathbf{K}_k$ can be determined as

$$\mathbf{K}_k = \left[\mathbf{Q}_k^{1/2} \cdot \ \mathbf{\Gamma}_k{}^C\right]^{\dagger} \cdot \mathbf{Q}_k^{1/2} \cdot \mathbf{\Psi}_k \cdot \mathbf{C}_k^{\dagger} \ , \tag{10}$$

where matrix $\mathbf{\Gamma}_k{}^C$ is the with-control version of $\mathbf{\Gamma}_k$ obtained by using $\mathbf{B}_k{}^C$ on its derivation, and matrix $\mathbf{Q}_k \in \Re^{m \times m}$ is a weighting matrix that accounts for differences in the importance of the final product KPCs. Matrix $\mathbf{Q}_k$ is positive definite and usually a diagonal matrix. Symbol † stands for the pseudoinverse of a matrix.

c) Optimal actuator placement

The problem of efficient actuator selection and distribution has attracted the attention of researchers in a wide range of engineering disciplines. This problem has been faced in many fields; such as structural design (buildings, satellites, airplanes, membranes, cantilevers, etc.), plant design (chemical plants and reactors), communications (network design), civil infrastructure (water supply networks) and mechanical devices (robotics and automobiles) (see Kubrusly and Malebranche (1985) for an extensive review). The common goal in all such applications is to improve system's performance at a minimum cost. Sometimes the actuator placement problem is simultaneously addressed with sensor placement, with the consequent increase in the problem complexity (Padula and Kincaid, 1999).

The actuator and sensor placement problem is formulated and solved in the following way: First, an index is defined to evaluate the performance of the system for a given configuration of actuators/sensors. Second, using the index, the hardware distribution problem is formulated as an optimization problem, which may or may not have constraints. Third, the problem is solved using a suitable optimization method. The most frequently used index is hardware cost, while ensuring a certain system performance (usually defined as a constraint); other indices include disturbance rejection, control energy minimization, maximization of the system damping, etc.. Some of the actuator placement problems also address the determination of the control parameters or gains when formulating the problem (e.g., the set of weighting matrices **Q** and **R** in a LQR and a LQG control). Following this approach, researchers determined not only the optimal actuator placement, but also the appropriate controller. The inclusion of the controller design in the formulation significantly increases its complexity, and makes the solution search more difficult (Chiemielewsky and Peng (2006), and reference therein).

The optimization methods used to solve the sensor and actuator problem vary depending on the problem dimension. For small problems, exhaustive or complete enumeration can be a viable approach. However, this approach is not viable in medium and large problems. For medium-large problems, usually heuristic methods are used; among these, genetic algorithms, simulated annealing and tabu search are the most popular ones (Padula and Kincaid, 1999). The drawback of these types of methods, is that they cannot guarantee optimality of the solution (suboptimal solution).

Regarding the characteristics of the actuators, most researchers have assumed that they are perfect. Therefore, the control actions commanded are perfectly executed (there may exist disturbances in the system; however, they are not considered to be dependent on the actuators characteristics). One of the pioneer works to include actuator imperfections when determining optimal actuator placement is the one reported by Skelton and DeLorenzo (1985). They studied the effect that the actuator's characteristics (actuator noise or error) have on the performance of the system. Due to actuator noise, Skelton and DeLorenzo concluded that it is not necessarily true that more actuators always improve the response of the system. In the same context of noisy actuators, Chiemielewsky and Peng (2006) used robust optimization tools to solve the actuators selection/distribution and controller design problem including bounded levels of uncertainty of the actuators noise. To this end, they solved the problem through the combination of a branch and bound algorithm to determine the optimal actuator distribution/selection, and a linear matrix inequality solver to account for the uncertainty during the derivation of the control gains.

Even though there is a vast literature on the actuator placement problem in several fields, to the best of our knowledge, no work has been reported on the optimal actuator placement for multistation manufacturing processes. The special characteristics of this type of process (e.g., discrete process with finite number of stations) demands the development of specific tools to solve the actuator placement.

3.3 Variation propagation model including feedforward control

This section presents the model derivation of the final product KPCs variation (covariance matrix) as a closed-form function of the process/product characteristics, the use of feedforward control, and the distribution and type of PTs. This model will be useful to efficiently determine the impact that different PT configurations have on the final product dimensional quality, without having to run Monte Carlo simulations to estimate the final product variation. In the derivation, there is no assumption on how the control gain matrices are determined. The only consideration is that the control actions are a linear function of the measurement before control, as proposed in Eq. (9).

Equation (11) presents the final deviations after control in station *k*, which can be obtained through the replacement of the control action (Eq. (9)), and the state/measurements of the system before control (Eqs. (6) and (7)) into the after control state equation (Eq.(8)).

$$\begin{aligned}\mathbf{x}_k &= \left(\mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k\right) \\ &\quad + \mathbf{B}_k{}^C \left[\mathbf{K}_k\left[\mathbf{C}_k\left(\mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{w}_k\right) + \mathbf{v}_k^B\right] + \mathbf{e}_k\right]\end{aligned} \tag{11}$$

By grouping terms, Eq. (11) can be rewritten as

$$\begin{aligned}\mathbf{x}_k &= \left(\mathbf{I} - \mathbf{B}_k{}^C\,\mathbf{K}_k\mathbf{C}_k\right)\mathbf{A}_{k-1}\mathbf{x}_{k-1} + \left(\mathbf{I} - \mathbf{B}_k{}^C\,\mathbf{K}_k\mathbf{C}_k\right)\mathbf{B}_k\mathbf{u}_k \\ &\quad + \left(\mathbf{I} - \mathbf{B}_k{}^C\,\mathbf{K}_k\mathbf{C}_k\right)\mathbf{w}_k - \mathbf{B}_k{}^C\,\mathbf{K}_k\mathbf{v}_k^B + \mathbf{B}_k{}^C\,\mathbf{e}_k\end{aligned} \tag{12}$$

Now, introducing the control-effect matrix, $\mathbf{F}_{k-1} = \left(\mathbf{I} - \mathbf{B}_k{}^C\, \mathbf{K}_k \mathbf{C}_k\right)$, Eq. (12) can be simplified as,

$$\mathbf{x}_k = \mathbf{F}_{k-1}\mathbf{A}_{k-1}\mathbf{x}_{k-1} + \mathbf{F}_{k-1}\mathbf{B}_k\mathbf{u}_k + \mathbf{F}_{k-1}\mathbf{w}_k - \mathbf{B}_k{}^C\mathbf{K}_k\mathbf{v}_k^B + \mathbf{B}_k{}^C\mathbf{e}_k\,. \tag{13}$$

The control-effect matrix determines the impact that the use of feedforward control has on the state of the system. This matrix combines information of the existence or absence of PTs holding the pins in station k through matrix $\mathbf{B}_k^C$, the control gain matrix $\mathbf{K}_k$, and the observation matrix $\mathbf{C}_k$.

Introducing the with-control version of the state transition matrix as,

$$\begin{aligned} &\mathbf{\Omega}_{k,i} = \mathbf{F}_{k-1}\, \mathbf{A}_{k-1}\, \mathbf{F}_{k-2}\, \mathbf{A}_{k-2} \cdots \mathbf{F}_{i+1}\, \mathbf{A}_{i+1}\, \mathbf{F}_i\, \mathbf{A}_i \qquad \forall k > i \geq 0 \\ &\mathbf{\Omega}_{i,i} = \mathbf{I} \end{aligned} , \tag{14}$$

the effect that all the variation sources have on the state of the system at the end of the process or station N can be written as (see Appendix I for its derivation)

$$\begin{aligned} \mathbf{x}_N = {} & \mathbf{\Omega}_{N,0}\, \mathbf{x}_0 + \sum_{i=1}^{N} \mathbf{\Omega}_{N,i}\, \mathbf{F}_{i-1}\, \mathbf{B}_i\, \mathbf{u}_i + \sum_{i=1}^{N} \mathbf{\Omega}_{N,i}\, \mathbf{F}_{i-1}\, \mathbf{w}_i \\ & - \sum_{i=1}^{N} \mathbf{\Omega}_{N,i}\, \mathbf{B}_i{}^C\, \mathbf{K}_i\, \mathbf{v}_i^B + \sum_{i=1}^{N} \mathbf{\Omega}_{N,i}\, \mathbf{F}_{i-1}\, \mathbf{B}_i{}^C\, \mathbf{e}_i \end{aligned} . \tag{15}$$

Next are explained the different elements on the right side of Eq. (15):

- The first term ($\mathbf{\Omega}_{N,0}\, \mathbf{x}_0$) accounts for the impact that incoming parts deviations have on the final product.
- The second term ($\mathbf{\Omega}_{N,i}\, \mathbf{F}_{i-1}\, \mathbf{B}_i\, \mathbf{u}_i$) considers the effects that fixture deviations at station *i*-th have on the final state.
- The third term ($\mathbf{\Omega}_{N,i}\, \mathbf{F}_{i-1}\, \mathbf{w}_i$) determines how the process disturbances at the *i*-th station affect the final product.
- The fourth term ($\mathbf{\Omega}_{N,i}\, \mathbf{B}_i{}^C\, \mathbf{K}_i\, \mathbf{v}_i^B$) accounts for the impact that the in-line measurement noises at station *i*-th, have on the final state.
- Finally, the fifth term ($\mathbf{\Omega}_{N,i}\, \mathbf{F}_{i-1}\, \mathbf{B}_i{}^C\, \mathbf{e}_i$) determines the impact that control action errors at the *i*-th station have on the final state.

By using Eq. (15), it is possible to write down the deviations of the final product KPCs, measured at station N, including the final measurement noise as

$$\begin{aligned}\mathbf{y}_N &= \mathbf{C}_N\mathbf{\Omega}_{N,0}\mathbf{x}_0 + \sum_{i=1}^{N}\mathbf{C}_N\mathbf{\Omega}_{N,i}\mathbf{F}_{i-1}\mathbf{B}_i\mathbf{u}_i + \sum_{i=1}^{N}\mathbf{C}_N\mathbf{\Omega}_{N,i}\mathbf{F}_{i-1}\mathbf{w}_i \\ &\quad - \sum_{i=1}^{N}\mathbf{C}_N\mathbf{\Omega}_{N,i}\mathbf{B}_i^{\,C}\mathbf{K}_i\mathbf{v}_i^B + \sum_{i=1}^{N}\mathbf{C}_N\mathbf{\Omega}_{N,i}\mathbf{F}_{i-1}\mathbf{B}_i^{\,C}\mathbf{e}_i + \mathbf{v}_N\end{aligned}. \qquad (16)$$

In order to simplify Eq. (16) we introduce the following matrices and vectors (formed by the aggregation or stack up of the corresponding matrices and vectors)

$$\mathbf{E}_N \equiv \mathbf{C}_N\mathbf{\Omega}_{N,0}$$

$$\mathbf{G}_N \equiv \begin{bmatrix}\mathbf{C}_N\,\mathbf{\Omega}_{N,1}\,\mathbf{F}_0\,\mathbf{B}_1 & \mathbf{C}_N\,\mathbf{\Omega}_{N,2}\,\mathbf{F}_1\,\mathbf{B}_2 & \cdots & \mathbf{C}_N\,\mathbf{\Omega}_{N,N}\,\mathbf{F}_{N-1}\,\mathbf{B}_N\end{bmatrix}$$

$$\tilde{\mathbf{u}}_N \equiv \begin{bmatrix}\mathbf{u}_1^{\,T} & \mathbf{u}_2^{\,T} & \cdots & \mathbf{u}_N^{\,T}\end{bmatrix}^T;$$

$$\mathbf{H}_N \equiv \begin{bmatrix}\mathbf{C}_N\,\mathbf{\Omega}_{N,1}\,\mathbf{F}_0 & \mathbf{C}_N\,\mathbf{\Omega}_{N,2}\,\mathbf{F}_1 & \cdots & \mathbf{C}_N\,\mathbf{\Omega}_{N,N}\,\mathbf{F}_{N-1}\end{bmatrix};$$

$$\tilde{\mathbf{w}}_N \equiv \begin{bmatrix}\mathbf{w}_1^{\,T} & \mathbf{w}_2^{\,T} & \cdots & \mathbf{w}_N^{\,T}\end{bmatrix}^T;$$

$$\mathbf{R}_N \equiv \begin{bmatrix}\mathbf{C}_N\,\mathbf{\Omega}_{N,1}\,\mathbf{K}_1\mathbf{B}_1^{\,C} & \mathbf{C}_N\,\mathbf{\Omega}_{N,2}\,\mathbf{K}_2\,\mathbf{B}_2^{\,C} & \cdots & \mathbf{C}_N\,\mathbf{\Omega}_{N,N}\,\mathbf{K}_N\,\mathbf{B}_N^{\,C}\end{bmatrix};$$

$$\tilde{\mathbf{v}}_N^{\,B} \equiv \begin{bmatrix}\mathbf{v}_1^{\,T} & \mathbf{v}_2^{\,T} & \cdots & \mathbf{v}_N^{\,T}\end{bmatrix}^T;$$

$$\mathbf{T}_N \equiv \begin{bmatrix}\mathbf{C}_N\,\mathbf{\Omega}_{N,1}\,\mathbf{B}_1^{\,C} & \mathbf{C}_N\,\mathbf{\Omega}_{N,2}\,\mathbf{B}_2^{\,C} & \cdots & \mathbf{C}_N\,\mathbf{\Omega}_{N,N}\,\mathbf{B}_N^{\,C}\end{bmatrix};$$

$$\tilde{\mathbf{e}}_N^{\,B} \equiv \begin{bmatrix}\mathbf{e}_1^{\,T} & \mathbf{e}_2^{\,T} & \cdots & \mathbf{e}_N^{\,T}\end{bmatrix}^T.$$

Then, the final product measurements or KPCs are obtained as,

$$\mathbf{y}_N = \mathbf{E}_N\,\mathbf{x}_0 + \mathbf{G}_N\,\tilde{\mathbf{u}}_N + \mathbf{H}_N\,\tilde{\mathbf{w}}_N - \mathbf{R}_N\,\tilde{\mathbf{v}}_N^B + \mathbf{T}_N\,\tilde{\mathbf{e}}_N + \mathbf{v}_N. \qquad (17)$$

The dimensional quality of the final product is usually characterized by the KPCs variation (process output variation), which are contained in the final measurement covariance matrix. Before calculating the output covariance matrix, it is necessary to characterize the system inputs. With respect to the inputs, there are two major assumptions: first, the independence of the variables, and second, their characteristics. Regarding to their independency, it is assumed that all the input variables are independent of each other (e.g., $Cov(\mathbf{u}_k, \mathbf{v}_k) = 0\,;\forall\ k = 1\cdots N$), and independent of the same variable and of other variables at different stations (e.g., $Cov(\mathbf{w}_i, \mathbf{w}_j) = 0,\ \forall i \neq j$). It is reasonable to assume the independences in and in-between stations because the different variation inputs have different origins (e.g., worn out pins, loosing pins, measurement noise), which in general are not related to each other. On top of these assumptions, it is also assumed that the inputs have zero mean and covariance matrices as:

$$\mathbf{x}_0 \rightarrow \mathbf{\Sigma}_{\mathbf{x}_0}; \ \ \tilde{\mathbf{u}}_N \rightarrow \mathbf{\Sigma}_{\tilde{\mathbf{u}}_N}; \ \ \tilde{\mathbf{w}}_N \rightarrow \mathbf{\Sigma}_{\tilde{\mathbf{w}}_N};$$
$$\tilde{\mathbf{v}}_N{}^B \rightarrow \mathbf{\Sigma}_{\tilde{\mathbf{v}}_N{}^B}; \ \ \tilde{\mathbf{e}}_N \rightarrow \mathbf{\Sigma}_{\tilde{\mathbf{e}}_N}; \ \ \tilde{\mathbf{v}}_N \rightarrow \mathbf{\Sigma}_{\tilde{\mathbf{v}}_N}.$$

Using the aforementioned assumptions and the input descriptions, the covariance matrix of the final product KPCs can be determined as

$$\begin{aligned}\mathbf{\Sigma}_{\mathbf{y}_N} &= \mathbf{E}_N\,\mathbf{\Sigma}_{\mathbf{x}_0}\mathbf{E}_N{}^T + \mathbf{G}_N\,\mathbf{\Sigma}_{\tilde{\mathbf{u}}_N}\mathbf{G}_N{}^T + \mathbf{H}_N\,\mathbf{\Sigma}_{\tilde{\mathbf{w}}_N}\mathbf{H}_N{}^T \\ &\quad + \mathbf{R}_N\mathbf{\Sigma}_{\tilde{\mathbf{v}}_N{}^B}\mathbf{R}_N{}^T + \mathbf{T}_N\mathbf{\Sigma}_{\tilde{\mathbf{e}}_N}\mathbf{T}_N{}^T + \mathbf{\Sigma}_{\tilde{\mathbf{v}}_N}\end{aligned} \tag{18}$$

3.4 Design criteria

This section presents the formulation of the objective function or design criteria for the optimal selection and distribution of actuators that reduce variation at minimum cost.

As mentioned in Section 3.1, the objectives of minimizing both variation and cost are antagonistic. Therefore, it was proposed to solve the problem through the generation of the Pareto sets. To this end, we selected the objective function to minimize the final product variation, while ensuring that the cost is less than or equal to a certain limit. Then, the Pareto set is obtained by solving the problem for different cost limits. The reason for maintaining the variation in the objective function, instead of a constraint, is to be consistent with the principle of continuous improvement or variation reduction in quality engineering. Following this approach, the problem described in Eq. (1) is reformulated as

$$\begin{aligned} J = \min_{\mathbf{p}_t,\mathbf{\Sigma}_{\tilde{\mathbf{e}}_N}} \ & \mathbf{\Sigma}_{\mathbf{y}_N} \\ \text{s.t. } & Cost \le Cost_{Limit}, \\ & \tilde{\mathbf{g}}(\text{space}) \le 0 \end{aligned} \tag{19}$$

where the design variables $\mathbf{p}_t$ and $\mathbf{\Sigma}_{\tilde{\mathbf{e}}_N}$ contain information of how many, where, and which PT-type are used. *Cost* stands for the total cost of the actuators used (the cost calculation is presented in Sec 4.2), and $Cost_{Limit}$ is the maximum allowed cost. Now,

$\tilde{\mathbf{g}}(\cdot)$ represents the subset of constraints only related to space limitation. From this point forward, these constraints are dropped from the formulation to simplify the analysis.

The first design variable, $\mathbf{p}_t \in \{0,1\}^n$, is a binary vector containing the information of the presence (represented as 1) or absence (represented as 0) of a PT on each possible location. Then, the number of PTs used is equal to the summation of all the components of $\mathbf{p}_t$. Vector $\mathbf{p}_t$ is formed through the aggregation of the vectors containing the information of presence/absence of PTs on each station as $\mathbf{p}_t = [\mathbf{p}_1{}^T \quad \mathbf{p}_2{}^T \quad \cdots \quad \mathbf{p}_N{}^T]^T$, where $\mathbf{p}_k$ corresponds to the binary vector containing the information of the presence/absence of PTs on each possible location at station *k*. The second design variable, $\mathbf{\Sigma}_{\tilde{\mathbf{e}}_N} \in \mathbb{R}^{p\times p}$, corresponds to a covariance matrix of the actuators error. This matrix depends on the type of PTs selected for the process. Recalling that $\tilde{\mathbf{e}}_N$ is the vector containing error introduced by all the PTs used in the process, each element in the diagonal of $\mathbf{\Sigma}_{\tilde{\mathbf{e}}_N}$ represents the error variance introduced by each PT, which can be determined from the PTs' repeatability. In general, the diagonal elements of $\mathbf{\Sigma}_{\tilde{\mathbf{e}}_N}$ can be all different reflecting the use of different PT-types along the process. However, to facilitate maintenance and reduce backup equipment inventory, manufacturers prefer to use the same equipment-type everywhere. This approach leads to equipment homogeneity simplifying the error covariance matrix to $\mathbf{\Sigma}_{\tilde{e}} \cong \sigma_e{}^2\mathbf{I}$, where $\sigma_e{}^2$ is the error variance or the square of the selected PT-type repeatability.

The objective function in Eq. (19) is a matrix function ($\mathbf{\Sigma}_{\mathbf{y}_N}$), which is not appropriate from the optimization point of view. Therefore, it is necessary to transform it into a scalar function. A matrix function can be transformed into a scalar one for optimization purposes in many ways. Next are described some matrix-to-scalar transformations borrowed from optimal design of experiments (Atkinson and Donev, 1992).

- A-optimality, which is to minimize the trace of $\mathbf{\Sigma}_{\mathbf{y}_N}$ divided by the dimension of $\mathbf{\Sigma}_{\mathbf{y}_N}$
- D-optimality, which is to minimize the determinant of $\mathbf{\Sigma}_{\mathbf{y}_N}$

- E-optimality, which is to minimize the extreme (maximum or minimum) eigenvalue of $\boldsymbol{\Sigma}_{\mathbf{y}_N}$
- M_s-optimality, which is to minimize the square root of the maximum element in the diagonal of $\boldsymbol{\Sigma}_{\mathbf{y}_N}$

The A-optimality criterion focuses on minimizing the trace or the sum of the diagonal elements of $\boldsymbol{\Sigma}_{\mathbf{y}_N}$ divided by the dimension of the $\boldsymbol{\Sigma}_{\mathbf{y}_N}$ (total number of KPCs). Thus, in the case analyzed in this paper, this criterion is equivalent to minimize the average of the all the KPCs variation (the variation of the KPCs are contained in the diagonal of the covariance matrix $\boldsymbol{\Sigma}_{\mathbf{y}_N}$). The D-optimality criterion corresponds to minimizing the multiplication of all the eigenvalues of $\boldsymbol{\Sigma}_{\mathbf{y}_N}$. This criterion corresponds to minimize the multiplication of all the KPCs variations. The E-optimality criterion will minimize the maximum eigenvalue of $\boldsymbol{\Sigma}_{\mathbf{y}_N}$, which can be understood as minimizing the amplifying factor (eigenvalue) of the worst-case combination between variations and covariances of the KPCs represented by the associated eigenvector. The last proposed criterion, M_s-optimality, corresponds to the squared root of the M-optimality criterion proposed by (Elfving, 1959). The M-optimality criterion is focused on minimizing the maximum element in the diagonal of $\boldsymbol{\Sigma}_{\mathbf{y}_N}$. Therefore, the M_s-optimality reduces the maximum standard deviation among the KPCs.

The D-optimality is a very popular criterion among experimental designers because i) it maintains invariant under scaling; and ii) it has a clear meaning in estimation, which is to minimize of the variance of parameters estimated using least-squares (Pukelsheim, 1993). However, as Kim and Ding (2004) pointed out, those properties may not have an important role in engineering design for three reasons. First, the scaling property does not work well when the variables have constraints; second, the D-optimal meaning is not obvious in engineering systems design; and third, the complexity of engineering design problems sometimes leads to have singular matrices (with determinant equal to zero), which rules out the possibility to use the D-optimality criterion. In the case addressed in this research, the scaling is not relevant, and the physical interpretation that the multiplication of the KPCs variances has on the distribution/selection of the PTs is not intuitive. Therefore, this criterion is discarded for actuator placement.

In this research context, the E-optimality criterion, similarly to the D-optimality one, does not have an obvious meaning, unless the atypical situation where $\mathbf{\Sigma}_{\mathbf{y}_N}$ is a diagonal matrix (no covariance elements in $\mathbf{\Sigma}_{\mathbf{y}_N}$). Only in this case, the maximum eigenvalue correspond to the KPC with maximum variation, which is equivalent to use the M-optimality criterion.

The A-optimality criterion considers all the KPCs variances when determining their mean. However, the use of the mean of variations may not be an appropriate index. The KPCs' variation mean does not incorporate information of the dispersion among the variances. Therefore, two solutions with the same mean and cost may have different dispersion in the KPCs variances, with one of them having a large dispersion and the other with low dispersion. The large dispersion solution may not be a good one because high dispersion solutions are more likely to have some KPCs variances exceeding some industry limits (e.i., the two millimeters six sigma limit used in automobile assembly). Therefore, such solution should not be selected. In the case addressed in this research, the limitation of A-optimality is that it does not capture the dispersion or variability among the KPCs' variances. One possibility to implement the A-optimality, without violating an industry limit, is to include the limit as an extra constraint for the KPCs variances in the problem formulation, at the expense of adding more complexity to the formulation and the search for the solution.

The M_s-optimality criterion focuses on minimizing the largest KPC variance. Therefore, the search for solutions with small largest variation leads to improve quality. On top of this, if the largest KPC variation does not violate the industry limit, neither do the remaining KPCs. This will make the M_s-optimality to have clear meaning and to make analysis of the Pareto sets easy to understand. For these reasons, and according to our experience in the automotive industry, the M_s-optimality is more likely to represent the needs of dimensional control in assembly processes.

Using the M_s-optimality criterion, the optimal distribution and selection of PTs in a process can be obtained by solving the following problem

$$J = \min_{\mathbf{p_t}, \sigma_e} \left\| \mathbf{diag}\left(\mathbf{\Sigma}_{\mathbf{y}_N}\right)^{1/2} \right\|_\infty , \qquad \text{s.t. } Cost \le Cost_{Limit} \tag{20}$$

where $\mathbf{diag}(\cdot)$ is a vector containing the diagonal elements of a matrix, and $\| \cdot \|_\infty$ stands for the infinity norm of a vector, which extracts the element of a vector with highest absolute value (Moler, 2004).

The cost of the actuators is assumed to be inversely proportional to their repeatability, which is the amount of error that they introduce when performing a control action. Following this approach, the cost of each unit is $\alpha \cdot \sigma_e^{-1}$, where α is a proportionality constant. Then, the total cost of the equipment used in the process is equal to the summation of the number of actuators times their cost, i.e.

$$Cost = \frac{\alpha}{\sigma_e} \sum_{i=1}^{p} \mathbf{p}_t(i) = \frac{\alpha}{\sigma_e} \left\| \mathbf{p}_t \right\|_2^2 . \tag{21}$$

3.5 Optimal selection and distribution of actuators

The actuator placement problem, as proposed in Eq. (19), belongs to the family of combinatorial nonlinear problems, which are usually hard to solve. As described in Section 3.2 c), there are several optimization methods to solve this type of problem. All these methods involve searching for solutions in a search space defined by the different number and actuators type, which can be very large for medium to large problems (e.g., a problem involving 32 possible locations of PTs has a search space of 2^{32} = 4.29e9 possible distributions for each PT-type). Therefore, the search speed and the optimality of the result can be significantly improved if the searching space can be reduced. In this section, we propose a methodology to efficiently solve the actuator placement problem based on reducing the search space by analyzing the controllability of the process. This analysis helps to determine upper bounds, at the system and station level, for the number of c-dof needed to efficiently correct deviations. Finally, we propose an algorithm to find the optimal distribution and selection that is independent of the optimization method used.

3.5.1 Controllability of MAPs

This subsection proposes the concept of controllability and further discusses how this concept can be used to determine the appropriate number of c-dof to be controlled. Before explaining the controllability, some the terminologies used to characterize it will be introduced first.

In this chapter, we distinguish between actuators and c-dof because the number of actuators in a given system may not be necessarily the same as the number of c-dof. The number of c-dof depends on the actuators characteristics (number of dofs that each actuator has), the type of locator controlled (hole or slot), and the existence or absence of constraints to perform control actions (e.g., interference with other equipment in the station that may blocks some dofs). For the case of in-plane motion of the parts, if a PT holds a pin that fits into a hole, then it may control as much as two dofs (see Figure 3.1). If it holds a pin that fits into a slot, then it may control only one dof.

A concept that will be used later is the number of necessary dofs (n-dofs) in a process, which corresponds to the minimum number of c-dofs that are necessary to control all the deviations in a given process (later it is presented how to determine the n-dofs limit). An additional concept is the number of effective dofs (e-dofs) of a given PT distribution, which corresponds to the minimum number of c-dofs that are necessary to control the same type of errors as controlled with the original number of c-dofs. In case that the number of c-dofs is greater than the e-dof number, there is one or more unnecessary or redundant c-dofs not contributing to correcting more error types. This mismatch reflects that the resources may not be well utilized. On top of this, ensuring that the number of e-dof in a system is equal to the number of c-dofs is important from the quality point of view. Since the actuators are not perfect, controlling deviations using more than the necessary number of c-dofs will introduce an additional variation source in the process. Therefore, it is recommended that the number of e-dofs should always be equal to the number of c-dofs. The upper limit for e-dofs in a given system is n-dofs; once the n-dofs level is reached, no more c-dofs (PTs) are required to control deviations.

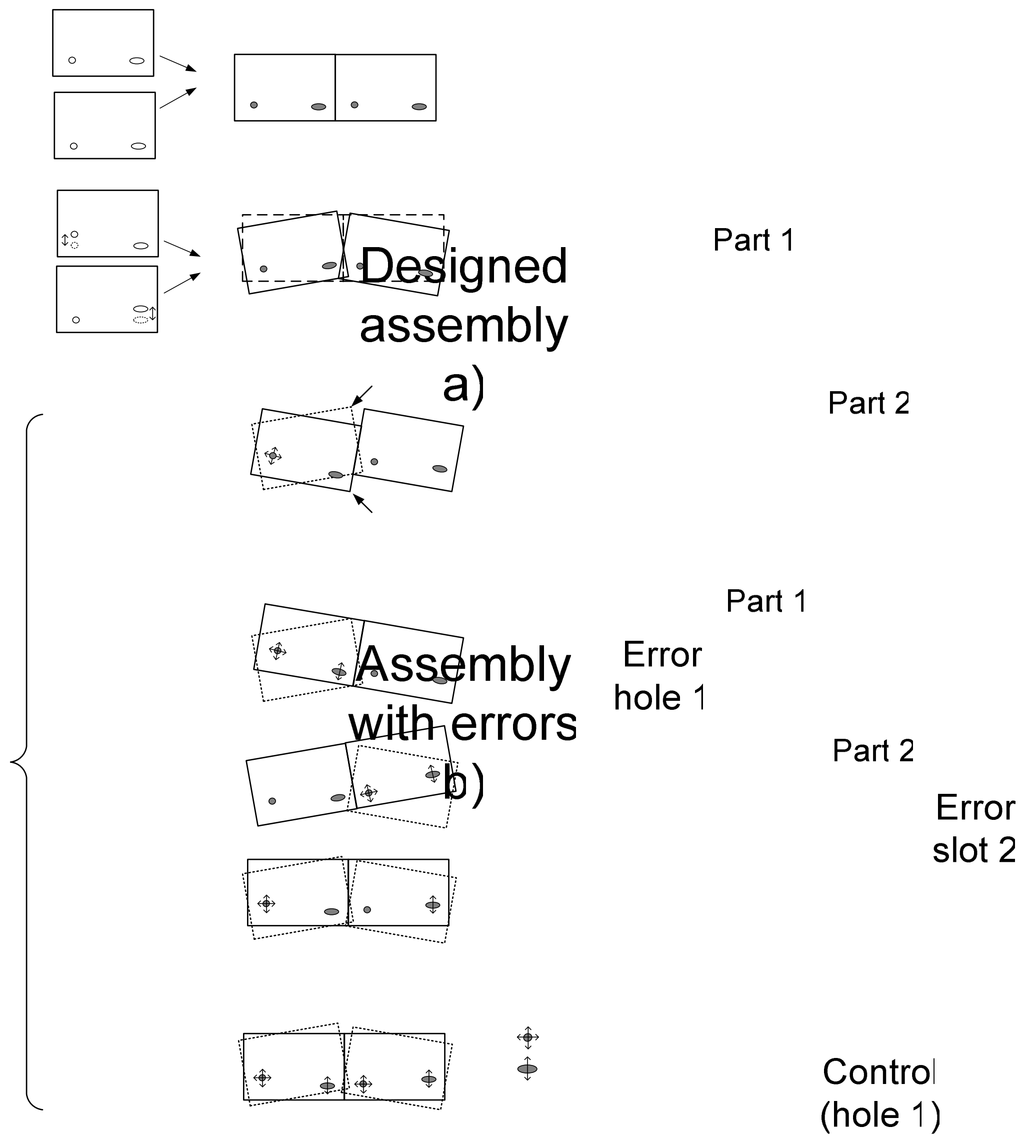


Figure 3.4 Assembly of two parts and possible ways to control deviations

Figure 3.4a and b present the original design for the assembly of two parts, where the parts may have initial deviations (errors in the location of the hole and the slot due to piercing operation errors) causing in-plane deviations. Figure 3.4c presents some possible ways in which the deviations can be controlled depending on the type and number of locator controlled. The analysis of the possible combinations or distributions of PTs in this example provides a better understanding on the effects that the combinations have on controlling deviations. In the assembly depicted in Figure 3.4, the number of n-dofs is three because controlling three c-dof will allow one to modify the position of the parts to achieve a perfect assembly (aligned parts) for all types of incoming errors (see center figures in Figure 3.4c, where the control of three dof allows correcting the deviations). Therefore, to control all types of incoming deviations it is necessary to control at least three dof in the process (n-dof). The analysis of possible distribution of PTs in this process considers four cases:

- The first case is to consider layouts with only one PT. In these cases, the single PT used can control either a hole or a slot. Thus, the number of c-dof and e-dof is two or one respectively. Controlling one or two dofs do not allow correcting all possible deviations of the incoming parts to achieve a perfect alignment of the parts (case where e-dofs is less than n-dofs).
- The second case considers the use of two PTs in the process. This case leads to four subcases to be analyzed:
    - The first subcase considers that both PTs control one part (one PT controlling the hole and one controlling the slot of the same part); then, the error of both parts can be corrected by moving the controlled part until it is aligned with other one achieving a perfect assembly. In this layout, the number of c-dof corresponds to three as well as the e-dof. Therefore, the two PTs are efficiently distributed.
    - In the remaining three subcases, one locator (either a hole or a slot) is controlled on each part. The possible numbers of c-dof for this configuration are two, three, or four, depending on the type of locators controlled. When both PTs are used to control the slots (one on each part), the resulting number of c-dof is two as well as the number of e-dofs. In this case, there is a lack of capability to control deviations. The subcase where three c-dof is achieved corresponds to having one PT controlling a slot in one part and the other PT controlling a hole in the other part. With this configuration all deviations can be controlled because the number of e-dof is three (e-dofs is equal to n-dofs). The last subcase

considers that each PT controls a hole (one on each part); then, there are four c-dof, and the number of e-dof is three ensuring that all deviations can be controlled. However, there is an extra c-dof, which is due to the type of locators controlled.

- The third case corresponds to the use of three PTs in the process; two of them used to hold one part and one for the other part. In this case two subcases can be identified depending on the locator type used to control the second part
    - The first subcase considers that in the second part the PT controls the hole resulting in five c-dof (three dof for the first part and one for the second part). In this case the number of e-dof is three and there is an excess in the number of PTs.
    - In the second subcase, the slot is controlled in the second part ensuring four c-dofs, again the number of e-dofs is three meaning that there are extra c-dofs.
- The fourth case corresponds to the one where two PTs are used to control each part. Consequently, there are six c-dof (three c-dof per part), three e-dofs, and an unnecessary use of PTs.

In automatic control fields, the controllability is defined as the capability to drive a system from any arbitrary initial state to the origin (zero state) in a finite time using finite control inputs (Bay, 1999). The controllability concept in the framework of MAP can be understood as the capability to drive the assembly from any starting state (any initial deviation of the parts) to the zero final state or perfect assembly in the finite number of stations of the process using a finite-magnitude control ignoring actuation errors. In other words, the controllability tries to answer the following question: does the process has sufficient and properly distributed PTs to perform the necessary control actions to drive the system to zero final deviation?.

The aforementioned controllability question will be answered borrowing some results from the control theory related to discrete time-varying systems (the differences between stations in a MAP resembles a discrete time-varying system). However, it is important to point out that a discrete MAP is not exactly the same as a discrete time-varying systems. First, MAPs have a finite number of stations. Second, due to the permanent joining of the parts, the occurrence of errors during the assembly process in a given station cannot be completely corrected in later stations without deforming the parts. This means that errors occurring in a particular station can only be completely corrected there. It may be possible to find a good fit for an imperfect subassembly with other parts in later stations; however, errors will stay in the process once they occur. In contrast, if errors occur in a discrete time-varying system at a given time, then it maybe possible to be completely compensated for them later and still reach the target.

The controllability analysis of a MAP is analyzed at two levels: system level and station level. The system level analysis not only permits determining if the process is or is not controllable, but also provides information on the required number of c-dof needed to ensure controllability of the whole system or n-dofs. On the other hand, the station level analysis helps to determine the number of c-dof needed to ensure deviations controllability at the station level or station level n-dofs. Once the station-level bound is reached, then, no more actuators should be assigned to that station. On the other hand, when the system-level bound is reached, no more actuators should be assigned to the whole system. Therefore, the use of those bounds will be very useful to significantly reduce the search space of the combinatorial problem addressed in this paper.

a) System level controllability

Considering that the only variation source in the system described in Figure 3.2 are part errors ($\mathbf{x}_0$). Then, by recursively applying Eq. (8) from station one through station N, it is possible to obtain the final state as a linear combination of the initial deviations and control actions as presented in Eq. (22).

$$\mathbf{x}_N = \mathbf{\Phi}_{N,0} \cdot \mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{\Phi}_{N,k} \cdot \mathbf{B}_k{}^C \cdot \mathbf{s}_k \, . \tag{22}$$

Introducing the reachability matrix $\mathbf{L}_N^C = \left[\mathbf{\Phi}_{N,1} \cdot \mathbf{B}_1{}^C \;\; \mathbf{\Phi}_{N,2} \cdot \mathbf{B}_2{}^C \;\; \cdots \mathbf{\Phi}_{N,N} \cdot \mathbf{B}_N{}^C \right]$ (Bay, 1999) and the control actions stack-up vector $\tilde{\mathbf{s}}_N = \left[\mathbf{s}_1{}^T \quad \mathbf{s}_2{}^T \quad \cdots \quad \mathbf{s}_N{}^T \right]^T$; then, Eq. (24) can be rewritten as

$$\mathbf{x}_N = \mathbf{\Phi}_{N,0} \cdot \mathbf{x}_0 + \mathbf{L}_N{}^C \cdot \tilde{\mathbf{s}}_N \, . \tag{23}$$

Now considering the case where the initial deviations are known and the desired final state $\mathbf{x}_N$ is $\mathbf{0}$ (e.g., perfect final product); then, the control actions can be obtained by solving Eq. (24).

$$-\mathbf{\Phi}_{N,0} \cdot \mathbf{x}_0 = \mathbf{L}_N^C \cdot \tilde{\mathbf{s}}_N . \tag{24}$$

Since $\mathbf{x}_0$ is an arbitrary initial deviation, the general condition to ensure that there is a unique solution to Eq. (24) is that matrix $\mathbf{L}_N^C$ is full rank or nonsingular. This is equivalent to have a completely controllable or reachable system as known in the automatic control field (Weiss, 1972; Kwakernaak and Sivan, 1972; Bay, 1999).

One important property of discrete time-varying systems is that even though $\mathbf{L}_N^C$ may not be full rank, the system defined in Eq. (22) may still be controllable; even though not reachable or completely controllable (Bay, 1999). This means that the singularity of $\mathbf{L}_N^C$ does not completely rule out the possibility to find a set of control actions that leads to $\mathbf{0}$ as a final condition. The singularity rules outs the possibility to ensure uniqueness of the control actions that leads the systems to $\mathbf{0}$ independent of the initial condition or to any arbitrary final state. The condition to ensure that a discrete time-varying system is controllable is the following (Bay, 1999)

$$rank(\mathbf{\Phi}_{N,0}) = rank(\mathbf{L}_N{}^C) . \tag{25}$$

In Section 3.2, we explained that the state transition matrix $\mathbf{\Phi}_{N,0}$ in a MAP is singular, and so is the reachability matrix. Consequently, a MAP is not reachable; however, it may be controllable if Eq. (25) holds. In the assembly context, Eq. (25) can be interpreted as having sufficient and properly distributed actuators along the system such that the deviations of the incoming parts can be controlled to reach a perfect final assembly.

Matrix $\mathbf{\Phi}_{N,0}$ represents the mapping between the initial and final deviations in the system; its columns determine the effect that each initial deviation component has on the final deviation. The number of linearly independent columns of $\mathbf{\Phi}_{N,0}$ determines the dimension of the final deviations space or range of $\mathbf{\Phi}_{N,0}$ (Noble and Daniel, 1988). The singularity of this matrix means that not all its column vectors are linearly independent:

initial deviations, belonging to the null space of $\mathbf{\Phi}_{N,0}$, will not be reflected in the final deviation, and there is no need to control them. Consequently, the dimension of the deviations space that should be controlled is equal to the *rank*($\mathbf{\Phi}_{N,0}$), and it determines the required number of dof that is necessary in the system, which is equivalent to n-dof.

The term *rank*($\mathbf{L}_N^C$) represents the dimension of the control action space spanned by the columns of $\mathbf{L}_N^C$. This dimension depends on the number, distribution, and type of the locators controlled. If the number of c-dof in a system is equal to the *rank*($\mathbf{L}_N^C$), then this means that all the c-dofs contribute to controlling the deviations in the system. On the other hand, if the number of c-dof is larger than the *rank*($\mathbf{L}_N^C$), then there are redundant or unnecessary c-dofs in the system. The extra c-dofs do not contribute to expand the control capabilities of the system, measured in terms of the dimension of the control actions space (they only add linearly dependent columns to $\mathbf{L}_N^C$). Since the *rank*($\mathbf{L}_N^C$) determines the number of c-dof that can effectively control deviations, it is equivalent to the number of system's e-dof.

The use of extra c-dofs in the system (i.e., case when c-dofs greater than n-dofs) makes the solution of Eq. (24) not unique. In fact, for such cases, there are infinite number of solutions. This situation is depicted in Figure 3.5 using the same assembly example as before, where there is one PT controlling each locator. Thus, the number of c-dof is six, the number of e-dof is three, and the number of n-dof is three. The number of control actions to compensate the deviation is infinite; any control action that aligns the parts is a possible solution.

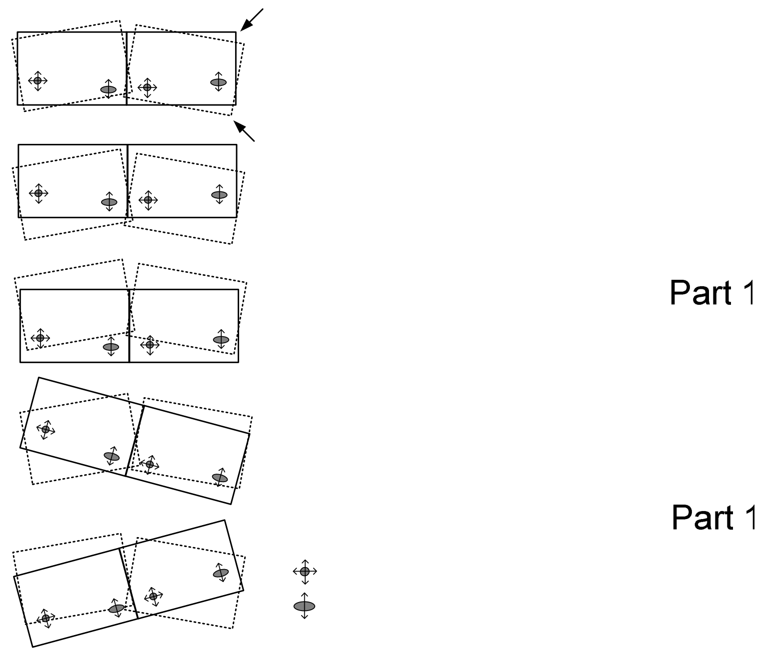


Figure 3.5 Possible ways to correct deviations by controlling all the locators

Even though the left hand side of Eq. (25) provides an upper bound for the total number of c-dof required in the system (n-dofs), it is uninformative about how the c-dof should be distributed along the system (where the actuators should be placed). The study of the station-level controllability will help to obtain more detailed information about appropriate PTs distributions.

b) Station level controllability

The study of station-level controllability is aimed to provide an upper bound in the number of c-dof required at the station level to ensure station controllability. The station-level controllability analysis follows the same approach as the system-level analysis does.

Assuming that the only deviation source at station k are the incoming parts (the incoming subassembly from station k-1 is assumed perfect), the output at station k can be determined as,

$$\mathbf{x}_k = \mathbf{\Phi}_{k,k-1} \cdot \mathbf{x}_{k-1} + \mathbf{\Phi}_{k,k} \cdot \mathbf{B}_k^{\,C} \cdot \mathbf{s}_k \,. \qquad (26)$$

Defining the station-level reachability matrix as ${}^{s}\mathbf{L}_{k}^{C} = \left[\mathbf{\Phi}_{k,k} \cdot \mathbf{B}_{k}^{C}\right]$ and following the same steps as in the system-level controllability analysis, the controllability at the station-level can be verified through the rank comparison of the matrices in Eq. (26) as

$$rank(\mathbf{\Phi}_{k,k-1}) = rank({}^{s}\mathbf{L}_{k}^{C}) . \qquad (27)$$

A station is said to be controllable if the equality of Eq. (27) holds, which means that the number of e-dof at station *k*, determined by $rank({}^{s}\mathbf{L}_{k}^{C})$, is equal to the number of the number of n-dof at the station level, determined by $rank(\mathbf{\Phi}_{k,k-1})$.

3.5.2 Proposed optimization methodology

In this subsection, a general methodology is presented to solve the actuator placement problem in a MAP. It takes advantages of the previously presented controllability bounds to reduce the search space. The proposed methodology allows one to construct (or approximate) the Pareto set for each PT-type, by following the steps described next:

1. Generate a state space model of the system (obtain matrices **A**'s, **B**'s, **C**'s and **Φ**'s)
2. Determine the upper bound for the number of c-dof required in the system (using the left hand side of Eq. (25))
3. Determine the upper bound for the number of c-dof required on each station (using the left hand side of Eq. (27))
4. For each PT-type solve the combinatorial problem Eq. (19) including the constraints obtained in steps 2 and 3 as presented in Eq. (28)

$$\begin{aligned} J = \min_{\mathbf{p_t},\sigma_e} & \left\| \mathbf{diag}\left(\mathbf{\Sigma}_{\mathbf{y}_N}\right)^{1/2} \right\|_{\infty} \\ \text{s.t. } & Cost \le Cost_{Limit} \\ & c\text{-}dof(\mathbf{p}_k) \le rank(\mathbf{\Phi}_{k,k-1}); \quad \forall\ k = 1 \cdots N-1 \\ & c\text{-}dof(\mathbf{p}_t) \le rank(\mathbf{\Phi}_{N,0}) \end{aligned} \qquad (28)$$

where, *c-dof(.)* determines the total number of c-dof of the corresponding vector. Here, the function *c-dof(.)* is used instead of the ${}^{s}\mathbf{L}_{k}^{C}$ or $\mathbf{L}_{k}^{C}$ ranks because they do not capture the use of an excessive number of c-dof in the system. These two ranks functions saturate at corresponding number of n-dof for the station and system level.

The total number of combinations of possible PTs distribution as a function of the number of parts is presented in Figure 3.6. This figure also includes the number of combinations that are required for each number of part after removing the combinations that violates the controllability constraints in Eq. (30). The use of the constraints results in a significant reduction of the combinations. However, if the number of parts is larger than 14, even though the constraints are included, the number of combinations is intractable. Therefore, for assemblies with 14 or more parts, the only hope is to obtain a good approximation of the Pareto set using heuristic methods and exploring only distributions that satisfy the station and system level constraints.

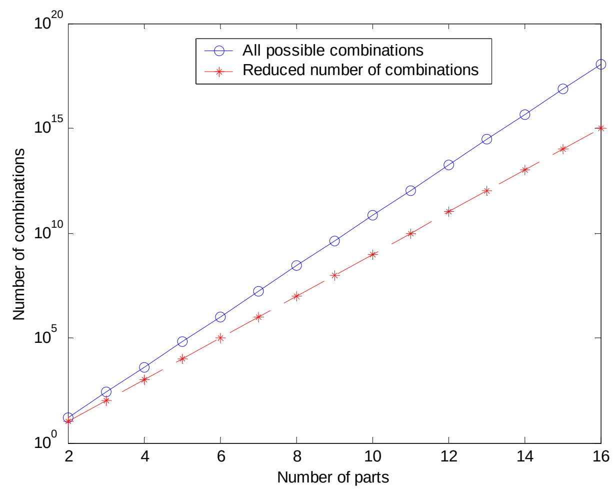


Figure 3.6 Intractability of the actuator placement problem for large number of parts

3.6 Case study

The case study used to prove the proposed methodology was taken from Ding *et al.* (2002b). It simulates the assembly of a Sport Utility Vehicle (SUV) side frame as depicted in Figure 3.7. The side frame is formed by four parts (simplified as four rectangles), which are assumed rigid and free to move in the x-z plane only (3 dof per part).

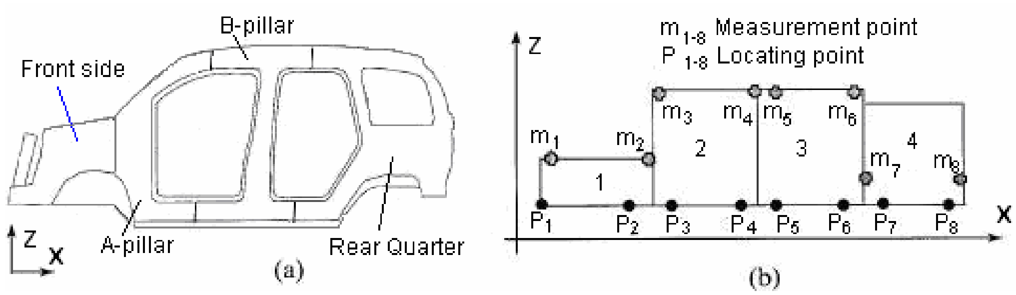


Figure 3.7 Schematic of an SUV side frame and its simplification (Ding *et al.*, 2002b)

The assembly process is performed in three assembly stations and a final inspection station where the sensors check the eight measurement points or KPCs defined for this assembly. The assembly sequence is the following: in the first station, the A-pillar is attached to the fender; then, in the second station, the B-pillar is added to the subassembly generated in station one; and in the third station, the rear quarter is attached to the main subassembly. Afterwards, the complete assembly is moved to the measurement station for final inspection. The locators sequence used in the process is: $\{(P_1, P_2), (P_3, P_4)\}_{\text{StationI}}$, $\{(P_1, P_4), (P_5, P_6)\}_{\text{StationII}}$, $\{(P_1, P_6), (P_7, P_8)\}_{\text{StationIII}}$ and $\{(P_1, P_8)\}_{\text{StationIV}}$. In this process it is assumed that:

- all the required measurement points (marked in Figure 3.7b) are available at each assembly station for the parts existing in that station;
- each PT is used to hold a single pin and they have the necessary dof to control the pin in the plane
- the measurement station is free of fixture errors (due to tight tolerance and good maintenance program)

Because each PT holds a single pin, the maximum number of PTs per assembly station is four, and the maximum number of PT used in the assembly is 12. Therefore, the total number of combinations or possible distributions of the 12 PTs used in the process is 2^{12} or 4096.

The parameters used in the simulations are $\boldsymbol{\Sigma}_{x_0} = 0.2 \cdot \mathbf{I}$, $\Sigma_{\tilde{u}_N} = 0.04 \cdot \mathbf{I}$, $\Sigma_{\tilde{w}_N} = 0 \cdot \mathbf{I}$ (no disturbances), $\Sigma_{\tilde{v}_N^B} = 0.03 \cdot \mathbf{I}$, and $\Sigma_{\tilde{v}_N} = 0.03 \cdot \mathbf{I}$, where the units are mm^2. The weighting coefficients matrix $\mathbf{Q}_k$ (k = 1, 2 and 3), used to determine the control gain matrix in Eq. (9), was set equal to the identity matrix. Four different PT-types were assumed available, and their characteristics are reported in Table 3.1.

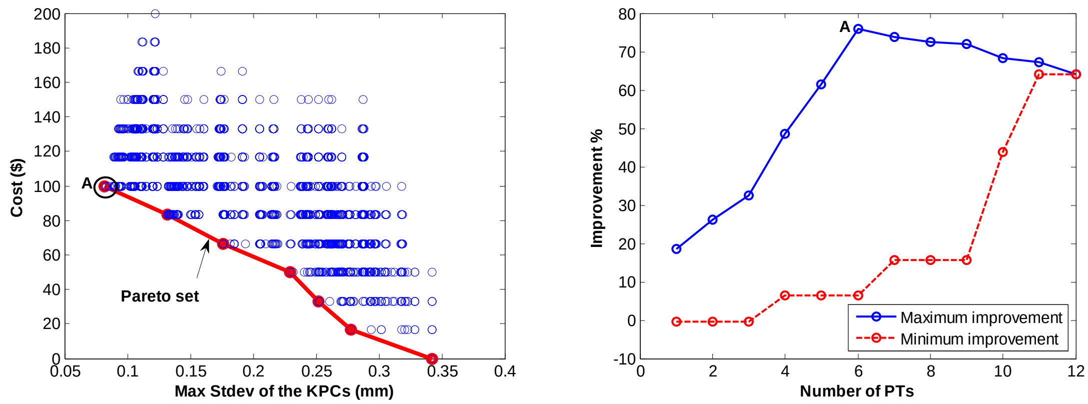
Table 3.1 Characteristics of the available PTs

PT-type	σ_e (mm)	Cost per unit ($\boldsymbol{\alpha}_i = 1;\ \forall i=1\cdots 4$)
PT_1	0.12	8.33
PT_2	0.06	16.67
PT_3	0.03	33.33
PT_4	0.015	66.67

The matrices used in this case study were constructed using information reported by Ding *et al.* (2002b). These matrices permitted the evaluation of the controllability at the system and station level. The evaluation resulted in the number of n-dofs for the system and station levels being nine and three respectively.

Results

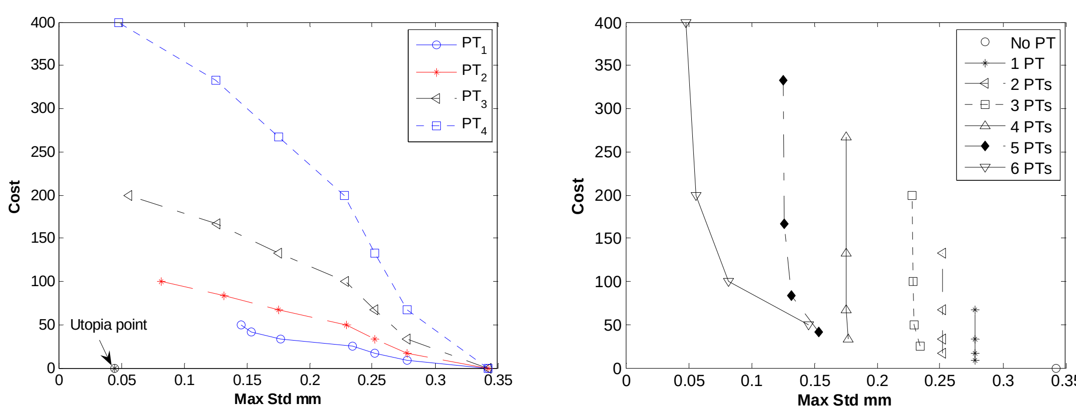
All the 4096 possible combinations of PT placements and the Pareto set for the case of PT_2 are presented in Figure 3.8a. The stratified levels in cost are due to the discrete number of PTs used, where each level of solution corresponds to the use of a different number of PTs (ranging from 0 to 12). It is evident from the figure that the distribution of the PTs plays an important role on reducing variation (e.g., analyze the variation range for a given cost or number of PTs). The importance of an appropriate distribution is more evident in Figure 3.8b. This figure presents the percentage of maximum and minimum possible variation reduction for each number of PTs that can be achieved by using feedforward control, measured with respect to the case of no control (no PTs in the process). For example, if six PTs are used, the minimum improvement that can be obtained is only 6.7 % compared with not using control. On the other hand, the best distribution of PTs achieves a 78 % improvement for the same amount of resources invested in the process. Then, it is critical that the distribution of the PT follows the maximum improvement curve (equivalent to follow the Pareto set in Figure 3.8a).



a) Pareto set b) Ranges of the improvement

Figure 3.8 Effect of the different PTs distributions on the variation reduction

The left limit of the Pareto set is attained for six PTs (point A in the Figure 3.8 a and b). If more than six PTs are used, then the maximum variation reduction tends to decay. Six PTs properly installed achieve the maximum improvement; more PTs not only does not help to increase the capability to perform corrections, but also they introduce more error into the system with the consequent variation increment. The distribution of six PTs resulting in point A in Figure 3.8 assigns two PTs on each station, with one PT controlling a hole and the other a slot. This layout results in three c-dofs and e-dofs per station, which means that the assembly is controllable on each station. The system level analysis of this assembly results in nine c-dofs and e-dofs, with the consequent controllability at the system level (recalling that the system n-dof is nine).



a) Pareto sets for different PT-type b) Pareto sets for different number of PTs

Figure 3.9 Pareto sets for different PT types and number of PTs

The Pareto sets obtained for each PT-type are presented in Figure 3.9a. In this case, simple and less expensive PTs (e.g., PT_1 and PT_2) can reduce the maximum standard deviation as much as 58 % and 77 % respectively. Further improvement on the dimensional quality requires using more repeatable and expensive PTs (85 % and 86 % improvement obtained for PT_3 or PT_4 respectively). This figure also includes the utopia point, which corresponds to the bounds of the Pareto sets (Fellini *et al.,* 2005). The variation level at the utopia point (Max Stdev = 0.0451 mm) corresponds to the minimum achievable level of the maximum KPCs standard deviation, which can only be achieved when perfect PTs are used.

The maximum improvement achievable for a given number of PTs is presented in Figure 3.9 b). Where each line is constructed by joining the points of the Pareto set of Figure 8a for a fixed number of PTs. Observing this figure, it is possible to notice that for a small to medium number of PTs (case of one to four PTs used), there is a small effect of the PT quality on the variation reduction. However, for a large number of PTs (five to six PTs), their quality starts playing an important role on variation reduction. Figure 3.9b can be understood as a guide to select the appropriate PT type when there are is a restriction in the number of PTs that can be used (e.g., due to space limitations on the stations). If the maximum number of PTs that can be used is less than or equal to four, then low repeatability PTs should be used. In case that five or six PTs can be used, the PT-type selection depends on the budget and desired variation reduction.

Figure 3.10 presents for each PT-type the contribution (measured as a percentage) that each PT unit has on the maximum possible variation reduction (obtained with six PT properly distributed) following the Pareto set solutions. This analysis permits visualizing the contribution that each extra unit has on the variation reduction. For instance, in the case of PT_1, the third PT added only contributes 8 % of the total improvement. However, the fourth PT contributes more than 31 %, meaning that it may be beneficial to add the fourth unit to the process to further reduce variation.

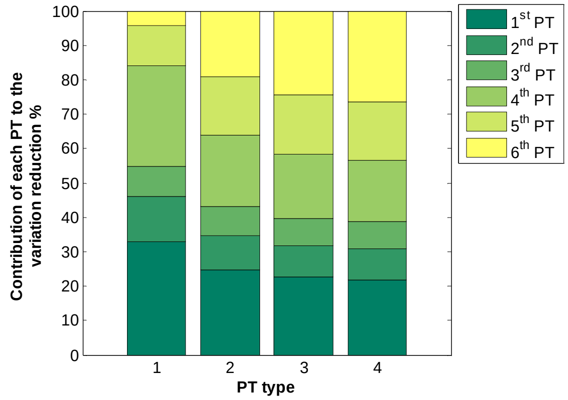


Figure 3.10 Contribution of each PT-type to the total variation reduction

It is interesting to note two characteristics of the results presented in Figure 3.10 The first is the large contribution that the first PT unit has on the total contribution to reduce variation. For all PT-types, at least 20 % of the variation reduction is due to the first unit. This reveals the potential for variation reduction that active control and appropriate distribution of actuators have. The second characteristics is that as the quality of the actuators increases (e.g., PT_3 and PT_4), so does the contribution of the sixth PT in variation reduction. The simultaneous achievement of system controllability and the use of high quality PTs lead to the largest quality enhancement.

3.7 Conclusions

This paper investigates a strategy for selecting and distributing PTs in multistation assembly processes to improve dimensional quality at minimum cost. To this end, the selection/distribution problem is formulated as a multiobjective combinatorial optimization problem that considers variation and cost as objectives and PTs selection/distribution as design variables. The problem was reformulated to construct Pareto sets, which help decision makers to select appropriate type and distribution of PTs by trading between cost and variation reduction.

The controllability of MAPs was analytically studied. This study revealed conditions of controllability at the system and station levels. These conditions can be understood as limits for the number of controllable dof (associated with the PTs) required in the system and the stations. When the number of PTs ensures controllability at the station level, no more actuators should be included in that station. Similarly, when the system level controllability is reached, no more PTs should be added at all. The addition of extra PTs will result in more process variation due to the PTs imperfections. Therefore, the obtained limits were used to significantly reduce the search space of solutions. A case study on the multistation assembly of a SUV side frame was used to test the proposed approach. The results showed the importance of controlling the appropriate dofs to avoid the introduction of unnecessary actuators variation in the process. Appropriate distributions of the PTs and the use of feedforward control can reduce the variation as much as 86 %.

Even though the proposed methodology is focused on multistation assembly processes, we believe that it is general enough to be applied in other type of multistation manufacturing processes. Obviously, such application will require some level of adaptation for the particularity of the process.

Acknowledgements

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3.8 Appendix I Derivation of Eq. (15)

From Eq. (12), the state of the system in station one can be represented as

$$\mathbf{x}_1 = \mathbf{F}_0\mathbf{A}_0\mathbf{x}_0 + \mathbf{F}_0\mathbf{B}_1\mathbf{u}_1 + \mathbf{F}_0\mathbf{w}_1 - \mathbf{B}_1{}^C\mathbf{K}_1\mathbf{v}_1^B + \mathbf{B}_1{}^C\mathbf{e}_1 . \tag{A.1}$$

Following a similar approach, the state of the system in station two can be obtained as

$$\mathbf{x}_2 = \mathbf{F}_1\mathbf{A}_1\mathbf{x}_1 + \mathbf{F}_1\mathbf{B}_2\mathbf{u}_2 + \mathbf{F}_1\mathbf{w}_2 - \mathbf{B}_2{}^C\mathbf{K}_2\mathbf{v}_2^B + \mathbf{B}_2{}^C\mathbf{e}_2 . \tag{A.2}$$

Replacing the value of A.1 into A.2, and using the appropriate state transition matrices is possible to write Eq A.2 as

$$\begin{aligned}\mathbf{x}_2 = \mathbf{\Omega}_{2,1}\left[\mathbf{\Omega}_{1,0}\,\mathbf{x}_0 + \mathbf{\Omega}_{1,1}\,\mathbf{F}_0\,\mathbf{B}_1\,\mathbf{u}_1 + \mathbf{\Omega}_{1,1}\,\mathbf{F}_0\,\mathbf{w}_1 - \mathbf{\Omega}_{1,1}\,\mathbf{B}_1{}^C\,\mathbf{K}_1\,\mathbf{v}_1^B + \mathbf{\Omega}_{1,1}\,\mathbf{F}_0\,\mathbf{B}_1{}^C\,\mathbf{e}_1\right] \\ + \mathbf{\Omega}_{2,2}\,\mathbf{F}_1\mathbf{B}_2\mathbf{u}_2 + \mathbf{\Omega}_{2,2}\,\mathbf{F}_1\mathbf{w}_2 - \mathbf{\Omega}_{2,2}\,\mathbf{B}_2{}^C\mathbf{K}_2\mathbf{v}_2^B + \mathbf{\Omega}_{2,2}\,\mathbf{B}_2{}^C\mathbf{e}_2 .\end{aligned} \tag{A.3}$$

Grouping similar terms in A.3 we obtain

$$\begin{aligned}\mathbf{x}_2 = &\, \mathbf{\Omega}_{2,1}\, \mathbf{\Omega}_{1,0}\, \mathbf{x}_0 \\ & + \mathbf{\Omega}_{2,1}\, \mathbf{\Omega}_{1,1}\, \mathbf{F}_0\, \mathbf{B}_1\, \mathbf{u}_1 + \mathbf{\Omega}_{2,2}\, \mathbf{F}_1 \mathbf{B}_2 \mathbf{u}_2 \\ & + \mathbf{\Omega}_{2,1}\, \mathbf{\Omega}_{1,1}\, \mathbf{F}_0\, \mathbf{w}_1 + \mathbf{\Omega}_{2,2}\, \mathbf{F}_1 \mathbf{w}_2 \\ & - \mathbf{\Omega}_{2,1}\, \mathbf{\Omega}_{1,1}\, \mathbf{B}_1^{\,C}\, \mathbf{K}_1\, \mathbf{v}_1^B - \mathbf{\Omega}_{2,2}\, \mathbf{B}_2^{\,C} \mathbf{K}_2 \mathbf{v}_2^B \\ & + \mathbf{\Omega}_{2,1}\, \mathbf{\Omega}_{1,1}\, \mathbf{F}_0\, \mathbf{B}_1^{\,C}\, \mathbf{e}_1 + \mathbf{\Omega}_{2,2}\, \mathbf{B}_2^{\,C} \mathbf{e}_2 .\end{aligned} \tag{A.4}$$

Then, A.4 can be rewritten as

$$\begin{aligned}\mathbf{x}_2 = &\, \mathbf{\Omega}_{2,0}\, \mathbf{x}_0 + \sum_{i=1}^{2} \mathbf{\Omega}_{2,i}\, \mathbf{F}_{i-1}\, \mathbf{B}_i\, \mathbf{u}_i + \sum_{i=1}^{2} \mathbf{\Omega}_{2,i}\, \mathbf{F}_{i-1}\, \mathbf{w}_i \\ & - \sum_{i=1}^{2} \mathbf{\Omega}_{2,i}\, \mathbf{B}_i^{\,C}\, \mathbf{K}_i\, \mathbf{v}_i^B + \sum_{i=1}^{2} \mathbf{\Omega}_{2,i}\, \mathbf{F}_{i-1}\, \mathbf{B}_i^{\,C}\, \mathbf{e}_i .\end{aligned} \tag{A.5}$$

From A.5 is possible to see the structure of Eq. (15).

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CHAPTER 4

ROBUST FIXTURE LAYOUT DESIGN FOR A PRODUCT FAMILY ASSEMBLED IN A MULTISTATION RECONFIGURABLE LINE

Abstract

Reconfigurable assembly systems enable a family of products to be assembled in a single system by adjusting and reconfiguring fixtures according to each product. The sharing of fixtures among different products impacts their robustness to fixture variation due to trade offs in fixture design (to allow the accommodation of the family in the single system) and to frequent reconfiguration. This paper[2] proposes a methodology to achieve robustness of the fixture layout design through an optimal distribution of the locators in a multistation assembly system for a product family. This objective is accomplished by: (1) the use of a multistation assembly process model for the product family, and (2) minimizing the combined sensitivity of the products to fixture variation. The optimization considers the feasibility of the locator layout by taking into account the constraints imposed by the different products and the processes (assembly sequence, datum scheme and reconfigurable tools workspace). A case study where three products are assembled in four stations is presented. The sensitivity of the optimal layout was benchmarked against the ones obtained using dedicated assembly lines for each product. This comparison demonstrates that the proposed approach does not significantly sacrifice robustness while allowing the assembly of three products in a single reconfigurable line.

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[2] Izquierdo, L. E., Du, H., Jin, R., Gee, H., Hu, S. J. and Shi, J., 2007, Robust fixture layout design for a product family assembled in a multistage reconfigurable line, Proceedings of the 2006 *ASME International Manufacturing Science & Engineering Conference,* Ypsilanti, MI. Under review ASME Journal of Manufacturing Sciences and Engineering.

4.1 Introduction

Traditionally, mass production of complex products has been done using dedicated manufacturing systems. Such systems are characterized by high productivity and low flexibility, which work well for a relatively static market. However, today's market features rapid changes in demand and short product lifecycle. Those changes have obliged manufacturers to increase product variety and reduce lot size. Therefore, manufacturers are continuously developing new products and production systems. The development of product families has helped manufacturers to meet customer requirements in terms of variety. An example of a product family is presented in Figure 4.1, where three car models of different sizes form the family. The use of reconfigurable manufacturing systems and controls has given manufacturers the possibility to cost effectively produce the family of products through systematic reconfigurations.

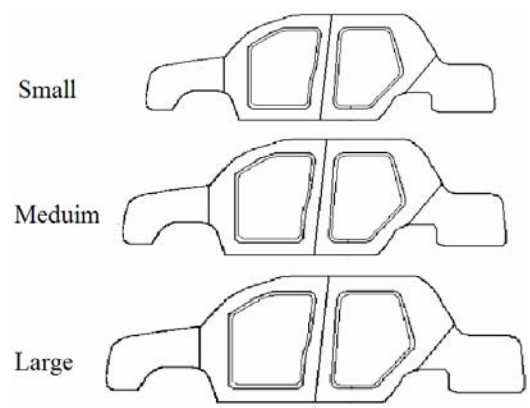


Figure 4.1 A product family consisting of sedans of small, medium and large sizes

In the automotive industry, the body assembly process is the less flexible than general assembly. Therefore, it has been receiving a lot of attention nowadays in pursuing flexibility. The auto body is usually assembled in a multistation sequential process (up to 70 stations), where at each station, fixtures are used to locate and clamp the parts for welding and joining. These fixtures play a critical role in controlling the position of the parts and subassemblies on each station, and on the final product quality. Traditionally, fixtures are dedicated to one product type, thus limiting the possibility to reuse them for other products. Since fabricating assembly systems for each product type in the family can be very expensive, there is a necessity for fixture flexibility to allow the assembly of a product family in a single line.

Reconfigurable assembly systems using flexible fixtures allow the assembly of different products in a single assembly line by sharing process tools. An example of such flexible fixture are the FANUC robot F-200iB and C-Flex (Fanuc, 2007), which can hold different part-types in automobile body assembly lines. Such robots are often called Programmable Tools (PT). As the product changes from one type to another, the robots change their positions as needed by the new part geometry, thus allowing the assembly of different product types in the same production line. The disadvantages of such systems are that assembling multiple products in a single reconfigurable line imposes additional constraints on product design, and the frequent change-over between products is an additional source of process variation, which impacts the final product quality.

Product quality is usually characterized by the fulfillment of customer's specifications and product functionality. In the auto industry, the parameters that determine product quality are known as the Key Product Characteristics (KPC). The KPCs are, in general, quantitative features of the product such as relative position of parts, flushes and gaps. Fixtures have a key role in determining the position of the parts, and doing so, on the achievement of the KPC specifications. For this reason, the fixtures form part of the Key Control Characteristics (KCC) of the process (Ding *et al.*, 2002a). Figure 4.2 represents a part (a rectangular sheet) mounted on a 3-2-1 fixture formed by three NC blocks. Two of the blocks have pins that restrict the in-plane motion of the part. The pins locate the part by fitting into a hole and a slot previously pierced on the part. The three blocks also position and restrain the part in the direction normal to the plane. The 3-2-1 locating points (hole and slot) are known as Principal Locating Points (PLP). The positions of the PLPs and their interaction with the fixture play an important role on the quality of the product (e.g., the position of the KPC points M_1 and M_2 in Figure 4.2).

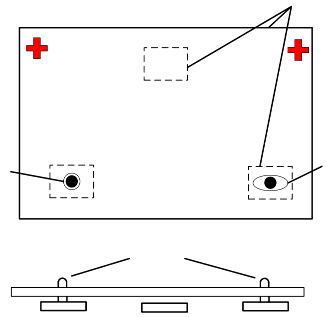


Figure 4.2 Top and side views of the 3-2-1 fixture layout

When dedicated fixtures are used for each product at each station, it is possible to optimize the location of the PLPs in terms of robustness to fixture variation. However, when multiple products of the same family are assembled in a single line, the products have to share fixtures. Sharing fixtures may result in a distribution of the PLPs that is not optimal for each individual product. Therefore, it is important to determine a robust distribution of the PLPs for the product family considering fixture sharing.

This paper presents a methodology to design robust fixture layouts for a product family assembled in a single line using reconfigurable fixtures, involving rigid parts. The requirements to solve such a problem are:

- To obtain an expression that relates the PLP layout (design variables) to the final product variation, applicable to all products in the family.
- To define the search space for the location of the PLPs and the constraints for their location mathematically. In the case of the product family, the constraints for the solution not only include product-parts geometry, but also consider the sharing of fixtures and the workspace of the reconfigurable fixtures.
- To minimize the effect that fixture variation has on product variation without violating the constraints, using an appropriate optimization method.

The remainder of the paper is organized as follows: Section 4.2 reviews the state of the art in multistation assembly variation propagation models, fixture design and reconfigurable fixturing systems. Section 4.3 addresses the design problem of determining the optimal distribution of the PLPs for a product family. A case study is presented in Section 4.4, with the conclusions given in Section 4.5.

4.2 Literature review

The literature review covers the following three areas related to the proposed research: multistation assembly models, fixture design and reconfigurable fixturing systems.

4.2.1 Multistation manufacturing processes

To establish relations between part and process variation and the final product quality in a multistation assembly process, it is necessary to have a model of the process. Such a model was first developed for auto body assembly at the station level (Liu and Hu, 1995). The modeling of a multistation assembly process was first attempted by (Shiu *et al.,* 1996), where a kinematics-based model of the process was developed. One of their main contributions was the identification of the "relocation" effect that occurs in multistation assembly processes. This effect occurred when subassemblies are located again in downstream stations where the PLPs may not be the same as in prior stations. Figure 4.3 illustrates the effects of fixture deviation and the relocation, where Figure 4.3a presents the effect that a displacement of the 2-way pin (P_2) has on the part, and especially in the location of points M_1 and M_2. Figure 4.3b shows the relocation effect on a subassembly as it moves from station *k-1* to station *k*. Then, variation in station *k-1* is transmitted to station *k* due to relocation, which is the major difference between the single station and the multistation variation modeling.

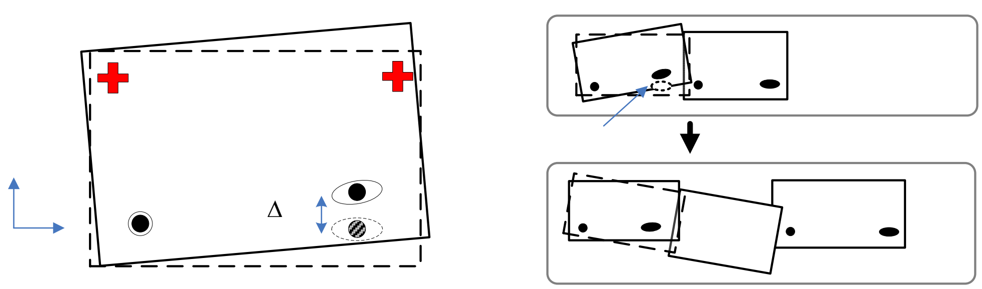


Figure 4.3 Effect of the fixture deviation and relocation
(adapted from Ding *et al.,* 2000)

A formal representation of the multistation assembly process was developed by Jin (Jin and Shi, 1999). They developed a state space representation of the assembly process to determine the final product variation given the variation of the incoming parts and fixtures for the case of rigid parts varying in the plane. Another multistation modeling method was proposed by Mantripragada (Mantripragada and Whitney, 1999). They used the state transition model to predict the variation propagation and to perform assembly corrections. Since the variation propagation model is fundamental to establishing the relation between KCCs and the KPCs deviation, the state space model is described next.

A schematic of a multistation assembly process is presented in Figure 4.4. Observing this figure, it is possible to understand how the subassemblies are transferred from one station to another, accumulating variation along the process. The variation accumulated up to station *k* (translations and rotations of the parts) is represented by the variable $\mathbf{x}_k \in \Re^n$ in Eq. (1). This variable depends on the deviation accumulated up to station *k-1* plus the deviation of the fixtures $\mathbf{u}_k \in \Re^p$, and other un-modeled deviation or disturbances sources $\mathbf{w}_k \in \Re^n$. The relocation effect of the subassembly coming from station *k-1* in station *k* is represented by matrix $\mathbf{A}_{k-1} \in \Re^{n\times n}$. This matrix relates the fixture layout of two adjacent stations and determines the re-positioning necessary for the subassembly entering station *k* (see Figure 4.3b). The impact of fixture deviations in station *k* is determined by matrix $\mathbf{B}_k \in \Re^{n\times p}$. On the other hand, the measurements or outputs $\mathbf{y}_k \in \Re^m$, if they exist at station *k*, depend on the position of the selected measurement points for the assembly (normally they correspond to the KPCs of the assembly). The relation between the variation of the part and the measurement points is given by matrix $\mathbf{C}_k \in \Re^{m\times n}$. Usually the measurements are not perfect and they are corrupted by noise represented by $\mathbf{v}_k \in \Re^m$. All the aforementioned matrices are obtained based on kinematic relationships, which are detailed in (Jin and Shi, 1999; and Ding *et al.,* 2000).

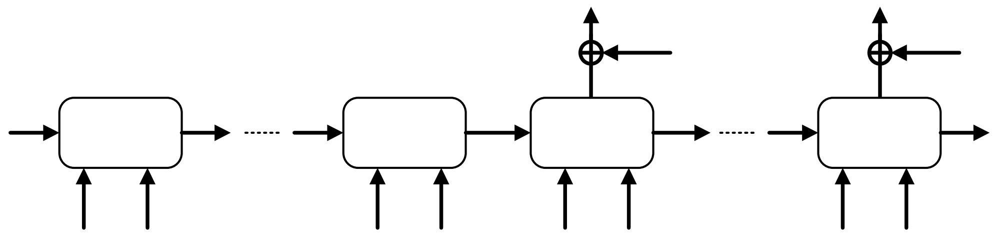


Figure 4.4 Diagram of the multistation assembly process with n stations (adapted from Ding *et al.*, 2002a)

The complete state space representation of the dimensional relationships is given below

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}_{k-1} \cdot \mathbf{x}_{k-1} + \mathbf{B}_k \cdot \mathbf{u}_k + \mathbf{w}_k \\ \mathbf{y}_k &= \mathbf{C}_k \cdot \mathbf{x}_k + \mathbf{v}_\mathbf{k} \end{aligned} . \tag{1}$$

Based on the linear properties of the model, it is possible to write the deviation of the measurement points in the last station N as,

$$\begin{aligned} \mathbf{y}_N = &\sum_{k=1}^{N} \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{B}_k \cdot \mathbf{u}_k + \mathbf{C}_N \cdot \mathbf{\Phi}_{N,0} \cdot \mathbf{x}_0 \\ &+ \sum_{k=1}^{N} \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{w}_k + \mathbf{v}_N \end{aligned} , \tag{2}$$

where $\mathbf{\Phi}$ is the state transition matrix, and it can be calculated as

$$\begin{aligned} \mathbf{\Phi}_{k,i} &\equiv \mathbf{A}_{k-1} \cdot \mathbf{A}_{k-2} \cdot \mathbf{A}_{k-3} \cdot \ldots\ldots \cdot \mathbf{A}_i \\ \mathbf{\Phi}_{i,i} &\equiv \mathbf{I} \end{aligned} . \tag{3}$$

Equation (2) can be simplified to

$$\mathbf{y}_N = \sum_{k=1}^{N} \mathbf{\Gamma}_k \cdot \mathbf{u}_k + \mathbf{\Psi}_0 \cdot \mathbf{x}_0 + \sum_{k=1}^{N} \mathbf{\Psi}_k \cdot \mathbf{w}_k + \mathbf{v}_N , \tag{4}$$

where

$$\mathbf{\Gamma}_k = \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{B}_k \text{ and } \mathbf{\Psi}_k = \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} . \tag{5}$$

Since the type of process analyzed in Figure 4.4 involves a serial assembly line with only one assembly station per stage, the words station and stage are used interchangeably in the remaining of the paper.

4.2.2 Fixture design

Early research in fixture design did not consider the existence of external variation sources (Ferreira *et al.*, 1985; Chou *et al.,* 1989). Later, researchers considered the existence of errors in fixtures and/or parts. In this area, the research is divided in two categories based on whether the workpiece is considered rigid or compliant. In both categories, the common approach is to determine the position of the locators and clamps that ensures a correct location of the workpiece and minimizes the effect of external variation sources.

In the case of rigid parts, the research has been focused on robust layout design of fixtures and clamps. Cai (Cai *et al*., 1997) proposed a variational method for robust fixture configuration design of 3-D rigid parts. Wang (Wang and Pelinescu, 2001) developed an algorithm for fixture synthesis for 3-D workpiece by selecting the positions of the clamps from a collection of discrete candidate locations called point set.

In the design of fixtures for complaint parts, Lee (Lee and Haynes, 1987) used finite element methods to model and analyze workpiece behavior including the effect of friction forces. Menassa (Menassa and Devries, 1991) used optimization to assist in the evaluation and selection of the 3-2-1 fixtures and clamps for prismatic parts aiming to minimize workpiece deflection. Cai (Cai *et al*., 1996) studied the use of more complex fixture scheme, the "N-2-1" fixture, to hold compliant parts by over-constrain the part, and used optimization to distribute the fixtures in order to reduce the part's deformation. Camelio (Camelio *et al*., 2004) determined the optimal fixture location to hold sheet metal parts considering variation of fixtures and welding guns position, and the springback effect of the subassembly after it is released from the station.

All the previous works are based on single station synthesis of locator layout. The problem of distributing the PLPs in a multistation process is more challenging due to relocation effect. This problem was first addressed by (Kim and Ding, 2004). They determined the distribution of PLPs for rigid parts that is robust to fixture variation for a single product assembled in a multistation process. To do so, they develop a sensitivity index that relates PLP layout to final product variation (KPC) and used several optimization methods to determine the distribution. Kim and Ding centered their effort on reducing the impact of fixture variation on the final product quality. Following this approach, Eq. (4) can be simplified as,

$$\mathbf{y}_N \equiv \sum_{k=1}^{N} \mathbf{C}_N \cdot \mathbf{\Phi}_{N,k} \cdot \mathbf{B}_k \cdot \mathbf{u}_k = \sum_{k=1}^{N} \mathbf{\Gamma}_k \cdot \mathbf{u}_k = \mathbf{D} \cdot \mathbf{u}, \tag{6}$$

where $\mathbf{u}$ is the stack up vector of all the fixture deviation, and matrix $\mathbf{D}$ is calculated as

$$\mathbf{D} = [\mathbf{\Gamma}_1 \quad \mathbf{\Gamma}_2 \quad \ldots\ldots \quad \mathbf{\Gamma}_N]. \tag{7}$$

In their model, Kim and Ding ignored the last term in matrix $\mathbf{D}$ ($\mathbf{\Gamma}_N$) because it is the final measurement station, which has fixtures with tighter tolerances and a better maintenance policy. Using that simplification, they proposed the calculation of a sensitivity index that relates the deviation sum squares of the output measurements $\mathbf{y}^T \cdot \mathbf{y}$ as presented in Eq. (7). The sub index N in Eq. (6) is now dropped for simplification, resulting in

$$\mathbf{y}^T \cdot \mathbf{y} = \mathbf{u}^T \cdot \mathbf{D}^T \cdot \mathbf{D} \cdot \mathbf{u}. \tag{8}$$

Then, the input/output sensitivity S can be calculated as the ratio of the sum of output variance of the KPCs to input variance as,

$$S \equiv \frac{\hat{\mathbf{y}}^T \cdot \hat{\mathbf{y}}}{\mathbf{u}^T \cdot \mathbf{u}} = \frac{\mathbf{u}^T \cdot \mathbf{D}^T \cdot \mathbf{D} \cdot \mathbf{u}}{\mathbf{u}^T \cdot \mathbf{u}}. \tag{9}$$

When analyzing the sensitivity index, it is possible to observe that if theproduct $\mathbf{D}^T \cdot \mathbf{D}$ is “small”, then the effect of the fixture variation is minimized. This is precisely the objective of a robust locator layout: minimizing the impact that fixture variation has on the KPC. To achieve this goal there are several criteria, most of which have an origin in optimal design of experiments:

- A-optimality, which is to minimize the trace of $\mathbf{D}^T \cdot \mathbf{D}$.
- D-optimality, which is to minimize the determinant of $\mathbf{D}^T \cdot \mathbf{D}$.
- E-optimality, which is to minimize the extreme (maximum or minimum) eigenvalue of $\mathbf{D}^T \cdot \mathbf{D}$.

The A-optimality criterion is equivalent to minimizing the sum of all the eigenvalues and can be understood as minimizing the sum of all the sensitivities of the process. The D-optimality criterion corresponds to minimizing the multiplication of the eigenvalues. This criterion has been widely use in design of experiments due to its clear interpretation, which is the minimization of the uncertainty on the parameters estimated using least squares. However, this criterion cannot be used in fixture design because matrix **D** is singular due to the singularity of the A's matrices used to form it (Ding *et al.*, 2004).

The E-optimality criterion is equivalent to minimizing the square root of the 2-norm of **D**. In practice, this is equivalent to minimizing the worst possible deviation in the process, which is associated to the maximum eigenvalue of **D**. Using the E-optimality criterion, the optimization problem can be stated as determining the location of the locators φ that minimizes the upper bound of the sensitivity and does not violate the constraints $\mathbf{g}(\varphi)$, that is,

$$\begin{aligned} \min_{\varphi} \quad & S_{\max} \equiv \lambda_{\max}(\mathbf{D}^T \cdot \mathbf{D}) \\ & \text{s.t.} \;\; \mathbf{g}(\varphi) \le 0 \end{aligned}, \qquad (10)$$

where $\lambda_{\max}(\cdot)$ stands for the maximum eigenvalue of a matrix, and the geometric constraints $\mathbf{g}(\varphi)$ consider that the locators have to be located in the feasible region inside the parts.

To solve the optimization problem in Eq. (10), Kim and Ding used several different methods, such as sequential quadratic programming, simplex, basic exchange, modified Fedorov and revised exchange. Since the problem in Eq. (10) is nonlinear and may have several local minima, global optimality of the solution cannot be guaranteed.

4.2.3 Reconfigurable fixturing systems

There have been many attempts to use reconfigurable fixturing systems in manufacturing aiming to reduce cycle time, fixture costs and process variation (Lee *et al.*, 1999). The first automatically reconfigurable assembly fixture was developed by Asada (Asada and By, 1985). They studied reconfigurable or adaptive fixture systems using a kinematical and mechanical approach. Since then, research has been done in the area such as assembly flexibility (Youcef-Toumi and Biutrago, 1988), and on quality by error compensation (Pasek and Ulsoy, 1994).

In machining, Walczyk (Walczyk and Longtin, 2000) studied the use of reconfigurable fixtures for compliant parts. They analyzed the performance of a reconfigurable system formed by a matrix of extendable pins, used to locate a workpiece, in terms of the forces applied and the system accuracy. More recently, Shen (Shen *et al.*, 2003) developed a reconfigurable fixturing system that can be relocated in the pallet as different parts enter the machining station.

The aforementioned efforts were mainly focused on the design of reconfigurable fixture devices. However, they do not consider the layout of the fixture (e.g., distribution and selection of reconfigurable devices). The single-station layout design for a family of products was first studied by (Lee *et al.*, 1999). They investigated the use of reconfigurable equipment to fixture a family of sheet metal parts using the N-2-1 scheme. The problem addressed was to determine the feasible position of the fixturing robots in the station to ensure that all the parts can be processed. They also determined the minimal size of the required working spaces in order to use small robots by using genetic algorithms.

Table 4.1summarizes the methodologies presented in this review section. Previous work in robust design and reconfigurable fixtures has been based on single machine (station) level for a single or multiple products. On the other hand, the multistation approach has only been considered for a single product. Therefore, there is a need to develop a methodology to design a robust reconfigurable fixture layout for a product family assembly in a single line.

Table 4.1 Comparison of modeling and fixture design methodologies

	Single product		**Multiple products**
Single station level	Modeling & Fixture Design	Ferreira *et al* 1985; Chou *et al.* 1989; Menassa and Devries 1991; Liu and Hu 1995; Cai *et al.* 1996; Cai *et al.* 1997	Asada and By 1985; Lee *et al.* 1996
Multistation level	Modeling	Shiu *et al* 1996; Jin and Shi 1999; Mantripragada and Whitney 1999; Camelio *et al.* 2004	Can be treated as a single product
	Fixture design	Kim and Ding 2004	***To be developed in this paper***

4.3 Optimal Locators layout for a product family

This section presents a methodology to solve the problem of distributing the locators for a product family in which the products share fixtures. This problem can be formulated as a constrained optimization problem, including the determination of the objective function, the definition of the constraints and the optimization method to search the solution.

4.3.1 Objective function

Minimizing the sum of squares of the final product deviations ($\mathbf{y}^T\mathbf{y}$) is equivalent to optimize dimensional quality. Therefore, we propose that the objective function *f(·)*, used to determine the optimal location of the PLPs, is a function of the upper sensitivity of all the products. Then, for a product family, consisting of *r* products or models sharing the same assembly line, the problem can be formulated as

$$\begin{aligned} &\min_{\varphi^1,\varphi^2\cdots\varphi^r} \; f(S_{\max-1}, S_{\max-2}, \cdots, S_{\max-r}) \\ &\qquad\qquad \text{s.t. } \mathbf{g}(\varphi^i) = 0; \quad \forall i = 1\cdots r, \end{aligned} \tag{11}$$

In particular, we consider the case where the function *f(·)* is the weighted sum of the sensitivities' upper bound for the whole family, as presented in Eq. (12). The reasons for selecting this function are the following: i) it directly incorporates all products into the objective function; ii) it allows the use of weights to accounts for difference in importance between the different products in the family; iii) in case that a cost-quality-sensitivity model were available, the use of the proposed objective function, using the sensitivities summation, will allow designers to quantify the maximum potential cost or the cost upper bound incurred (due to the increment of variation) by using a single reconfigurable line (the development of a cost model is a topic of future research); and finally, iv) for the reasons aforementioned and according to our experience, the proposed objective function can be easily accepted by practitioners.

$$f(S_{\max-1}, S_{\max-2}, \cdots, S_{\max-r}) = \sum_{i=1}^{r} w_i \cdot S_{\max-i} = \sum_{i=1}^{r} w_i \cdot \lambda_{\max}(\mathbf{D}_i^T \cdot \mathbf{D}_i). \tag{12}$$

After replacing Eq. (12) into Eq. (11) we have the formulation of the fixture layout problem as

$$\min_{\varphi^1,\varphi^2\cdots\varphi^r} \quad \sum_{i=1}^{r} w_i \cdot \lambda_{\max}(\mathbf{D}_i^T \cdot \mathbf{D}_i) \qquad (13)$$
$$\text{s.t.} \quad \mathbf{g}(\varphi^i) \le 0; \quad \forall i = 1\cdots r,$$

The use of weights w_i (w_i>0) allows designers to incorporate in the formulation the relative importance that each product has on the family. A possible criterion to select the weights is to consider the expected demand for each product (product with higher expected demand can have a higher weights, i.e. defining the weights as the product mix-ratio). Another possibility is to use the normalized expected profit of each product as weights. The normalized expected profit can be obtained by dividing the profit of each model by the total expected profit of the product family. Exploring possible selections of the weights is not the scope of this work; hence, we assume all the weights equal to one.

The constraints $\mathbf{g}(\varphi^i)$ correspond to the set of geometric constraints that limit the PLPs' location (φ^i) of the i-th product; they contain information about the feasible region where the locators can be placed, and the relations between the parts and the different products of the family (their derivation is presented in section 3.2). In this case, the design vector φ^i contains the location of the 2m PLPs required to hold the m parts or components of product i in the X-Z plane. The PLPs locations are directly related with the position of the two NC blocks containing pins (Figure 4.2). The location of the third NC block is not considered in this analysis because it does not impact the in-plane variation of the part. The PLPs locations for the i-th product are denoted by φ^i=$[\mathbf{p}_1^i \;\; \mathbf{p}_2^i \;\; \mathbf{p}_3^i \cdots \; \mathbf{p}_{2m}^i]$, where $\mathbf{p}_k^i$ ($\forall\; k = 1\cdots 2m$) has 2 coordinates (one in X and one in Z); consequently, the design variables can be rewritten for the i-th product as: $\varphi^i = [(x_1{}^i,\, z_1{}^i) \;\; (x_2{}^i,\, z_2{}^i) \;...\; (x_{2m}{}^i,\, z_{2m}{}^i)\,]$.

Since the objective is to minimize the maximum eigenvalue of $\mathbf{D}_i^T \cdot \mathbf{D}_i$, it is important to analyze the sensitivity of the eigenvalue calculation to modeling and computational errors. Model errors are caused by errors in the generation of the system matrices A's, B's and C's, and computational errors are inherent in calculations with floating point arithmetic (Moler, 2004). In this research, both errors can be seen as perturbations of the true matrix product $\mathbf{D}_i^T \cdot \mathbf{D}_i$. A special property of symmetric matrices, such as $\mathbf{D}_i^T \cdot \mathbf{D}_i$ (the multiplication of a non-symmetric matrix by its transpose results in a symmetric matrix), is that they have the lowest possible eigenvalue conditioning or sensitivity of the eigenvalue calculation to perturbations (Moler, 2004; Wilkinson, 1988). Therefore, the selected criterion, based on the minimizing the maximum eigenvalue (Eq. (10) and Eq. (13)) is robust to modeling and computation errors.

4.3.2 Constraints definition

The constraints define the feasible space where the PLPs can be located as well as the necessary conditions to ensure that the assembly is feasible. Thus, they define the viability of the assembly. Before describing the constraints for the product family design problem, it is necessary to present some process conditions or considerations that make the problem addressed in this research closer to the reality; those are:

- Each part has only one set of PLPs. This implies that in later stations each subassembly must be held using some of the previously used locating points available on the parts. The use of only one set of locators per part is a common practice in industry because it helps to minimize the parts cost.
- Each PT carries the set of fixture elements (blocks and pins) necessary to hold a part or subassembly. This condition avoids the use of multiple PTs to carry each part or subassembly, to save cost and space.
- To avoid increasing the mechanical complexity and cost of the PT, it is considered that the distance between the pins installed on the PT is constant. This distance is a design variable, which has to be the same for all the products, and cannot vary from product to product.

Considering the aforementioned conditions, it is possible to define the constraints for the product family as follows (the mathematical description of the constraints can be found in Appendix I):

a) All the PLPs must be positioned within the feasible area of an individual part. This area includes all the part and excludes the internal holes on the part. A safety margin of 30 mm is defined along all the part contours (internal and external) to ensure that the locators are not too close to the edges. The verification of the belonging or not of a point to the feasible region of a part was done using an image-matrix of the geometric shape of every part. Then, a value of 0 was assigned to the “in” or feasible region and 1 to the outer or infeasible region (including cavities on the parts). Doing so, the verification of the in/out location of a point was done by checking if the coordinates of the point correspond to 0 or 1 in the appropriate image (part). The advantages of this method are that it is simple to check, and the image has to be calculated only once, then stored and used every time it is required. The generation of the image requires information of the position of both external and internal vertices that defines the part, and an algorithm to check if a point belongs or not to a certain region. There are many algorithms to perform this type of verification, one of those is the point inclusion test widely use in the CAD-CAM and the computational geometry field (Preparata and Shamos, 1998).

b) The distance between the locators on each part-type (*d*) and subassembly-type (*s*) should be the same for all models (see Figure 4.5). This means that the distance between the two locators used to hold the same type of part or subassembly is fixed. However, the position of the pins in the station can be adjusted using the PT to accommodate the different products. If the distance between the locators used to hold a given part-type or a subassembly-type are not the same for all the models, then one or more assemblies are not feasible because the parts or subassemblies do not fit into the fixtures. Figure 4.5a presents graphically the constraint for the part-type (products A, B and C), and Figure 4.5b presents the constraints for the subassembly-type (only products A and B are shown).

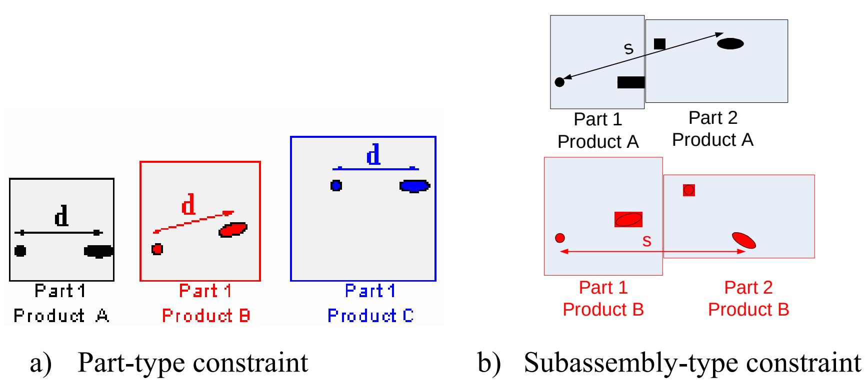


Figure 4.5 Distance constraint in parts and subassemblies for different products

c) The PT has to be able to locate the fixture elements in the appropriate position; therefore, at least one point in between both pins has to belong to the workspace of the PT (e.g., the middle point between the pins). Graphically this can be presented in Figure 4.6, where the locator's middle points, represented by triangles, are inside the PT workspace. For the case where the workspace is circular; the radius of the minimum circle that contains all the middle points must be smaller than the workspace radius. The problem of determining the circle with minimum radius that contains a set of points is known as the minimum circle enclosing problem. This problem has been extensively studied in the computational geometry field; a good review of the available methods used to solve it can be founded in (Preparata and Shamos, 1998).

d) Another constraint that can be included is that the PLPs on each part have to be aligned along one of the principal axes of the part. This prevents the coupling of the errors in the three axes. Therefore, having the PLPs aligned with the principal axis of the part is a recommended practice. Mathematically, the constraints can be represented as the product of the differences in location of the hole and the slots in the X and Z directions, which has to be equal to zero to ensure the correct alignment.

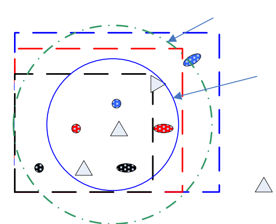


Figure 4.6 Workspace verification

4.3.3 Optimization and optimality

Due to the non-linear nature of the problem and the constraints, sequential quadratic programming was chosen to perform the optimization. This optimization method is frequently used for fixture design (Cai and Hu, 1996; Wang, 1999; Kim and Ding, 2004). One of the properties of the gradient-based method is that it tends to converge rapidly. A disadvantage of this method is that it can be easily entrapped in a local optimum. Therefore, different initial conditions can be used to perform the search for a good locator layout.

Due to the complexity of the objective function and the constraints, solving the problem as proposed in Eq. (11) is difficult. On top of this, obtaining a feasible initial condition that satisfies all the constraints is also challenging. Therefore, the problem was solved first using the relaxed formulation (Lagrange relaxation), which is, in general, easier to solve compared with the original one (Wolsey, 1998). Equation (14) presents the relaxed formulation, where the objective function directly includes the squares of the constraints multiplied by a constant factor or Lagrange multiplier β (β >0).

$$J = \min_{\varphi^1,\varphi^2\cdots\varphi^r} \sum_{i=1}^{r} w_i \cdot \lambda_{\max}(\mathbf{D}_i^T \cdot \mathbf{D}_i) + \beta \cdot \left[G(\varphi^i)^T \cdot G(\varphi^i) \right] \qquad (14)$$

The relaxed form of the problem has the advantage of allowing a slight violation of the constraints. Therefore, it can be used as a starting point for the solution of the constrained problem Eq. (13). The selection of the multiplier β is done to ensure a reasonable solution (low value of β) that tolerates some constraints violation, and then, it is increased to look for a solution that is closer to the one of the real problem. Finally the true problem Eq. (13) can be solved starting from the result of the one with the highest factor β.

4.4 Case study

The case study selected is the assembly of the side frame of the family of sedans presented in Figure 4.1. The side frames are composed of four parts each and are assembled in the process pictured in Figure 4.7. The process consists of three assembly stations and a final measurement station, where the location of the KPCs defined for this process are measured (points M).

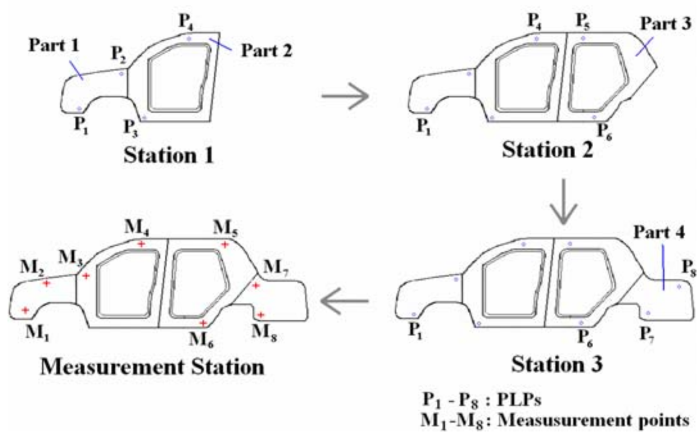
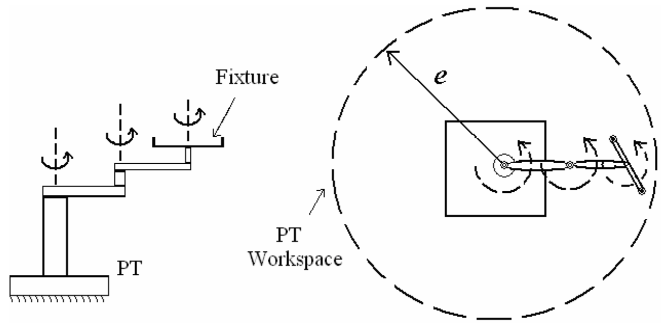


Figure 4.7 Assembly sequence of a sedan side frame

The datum scheme defined for this process is the following: in station one the locators used are $\{(P_1,P_2),(P_3,P_4)\}$, this means that the first part is held using locators P_1 and P_2, the second part using locators P_3 and P_4. In station two the locators used are $\{(P_1,P_4),(P_5,P_6)\}$, in station three $\{(P_1,P_6),(P_7,P_8)\}$ and in the measurements station $\{(P_1,P_8)\}$.

The relative sizes of the frames compared to the small one were selected as 1.06 and 1.12 for the medium and large frames respectively. The scale factors are used to define the geometry of the parts for each, which are defined based on the part vertices. The location of the vertices of each part and the location of the KPCs are presented in Appendixes II and III respectively. All the locations are reported for the small sedan. The locations for the medium and small can be obtained using the corresponding scaling factors.

The PTs used in the assembly were assumed to be robots with three degrees of freedom in the plane as presented in Figure 4.8 a, which corresponds to a revolute-revolute-revolute type robot. Due to the robot characteristics, they have a circular working space as shown in Figure 4.8 b. The radius *e* of the workspace was selected to be 500 mm.



(a) PT side view (b) PT top view and workspace

Figure 4.8 Views of a programmable tool and its workspace

The results of the PLP layout for a product family are presented next. The results are benchmarked with the optimal solutions obtained for each product as it were assembled in a dedicated assembly line (dedicated line for each product). This comparison provides information of the performance compromised, in terms of robustness to fixture variation, by using a single reconfigurable line.

Due to the existence of several local minima, in accordance with the results obtained by Kim and Ding, 100 random initial conditions were used to search for a good layout of the PLPs for both the product family and the single products (case of dedicated lines). In the product family case the multiplier β was first set to 5. Later, the layout with lower J was optimized after increase β to 50 and then to 750.

a) Fixture layout for a dedicated line

In the optimization for each single model, considering dedicated lines, two cases were analyzed. Case one has no constraint in the alignment of the locators and case two impose constraints on the alignments of the locators. In both cases the optimization was performed 100 times starting from random initial conditions of the locators for each model independently. Figure 4.9 and Figure 4.10 presents the location of the fixtures for each model for the cases with and without locator's alignment. Table 4.2 presents the values of λ_{max} for each configuration. The table also includes the sum of the λ_{max} for later comparison with the product family solution.

Table 4.2 Results of the optimization for each single model (λ_{max})

	Dedicated lines	Dedicated lines (aligned pins)	Reconfigurable line
Small car	18.04	20.28	23.93
Medium car	18.01	18.56	25.07
Large car	18.02	19.98	21.83
Sum λ	***54.07***	***58.70***	***70.93***

For the case of aligned pins, the solution obtained in height (z) is close to the "center of gravity" of the sensor points in the same direction. Therefore, for this case where the pins have to be aligned, their locations tend to be equally distant to the "upper" and "lower" set of measurement points. In that way, the effect of fixture variation will be minimized in average. The final locations of the locators are reported in Appendix IV.

b) Fixture layout for a reconfigurable line

Figure 4.11 presents the final location of the PLPs for the family, where the distances of the hole and the slot are the same across the three products. The values of the upper bound of the sensitivity (λ_{max}), obtained for the each model and for the product family (summations of the λ_{max}), are presented in Table 4.2.

The difference between the value of λ_{max} for the reconfigurable line and the dedicated lines (non-aligned and aligned cases) are 16.76 and 12.13 respectively, which corresponds to an increment of a 31 % and 21 % for each case. Those increases can be judged as reasonable considering the complexity of the geometries, the amount of constraints that the reconfigurable line imposes in the assembly and the differences in sizes of the cars. It is important to note that the value obtained with λ_{max} corresponds to the upper bound on the sensitivity. Therefore, it corresponds to the worst case scenario. The increase in the sensitivity of the product family can be compensated through an appropriate distribution of the tolerances in the fixtures and locators and a good maintenance strategy that keeps the variation low. The final location of the locators is reported in Appendix V.

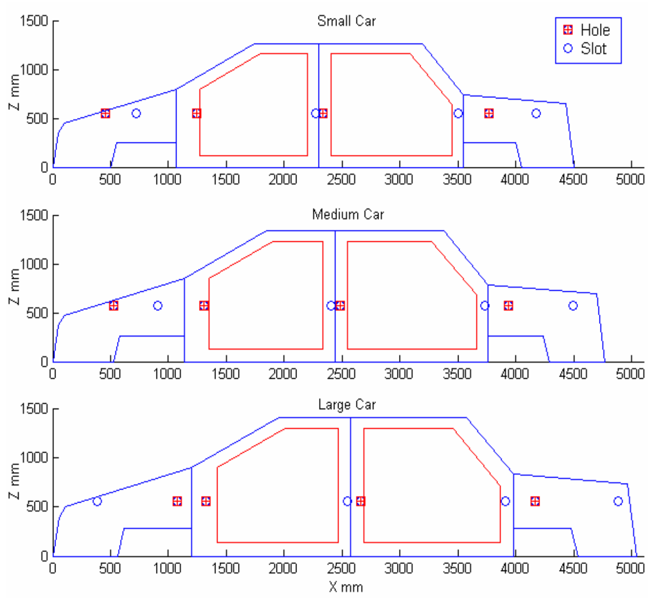


Figure 4.9 Location of the PLPs for dedicated lines with the alignment constraint

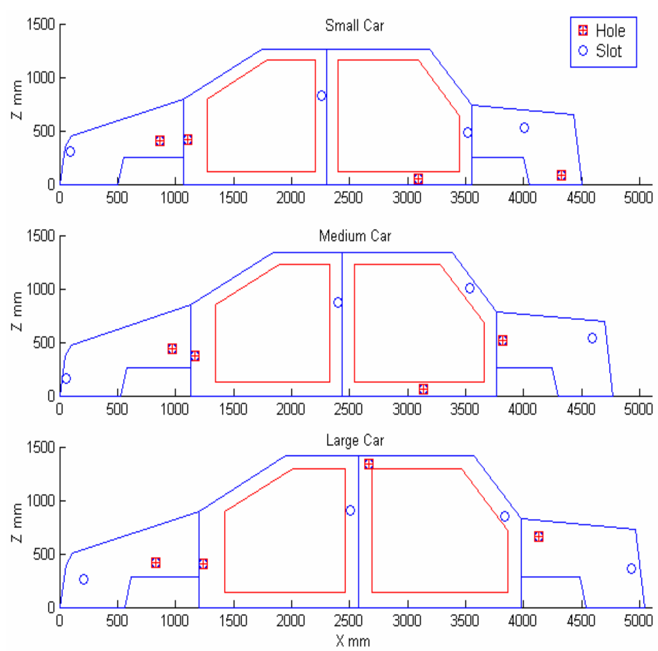


Figure 4.10 Location of the PLPs for dedicated lines without the alignment constraint

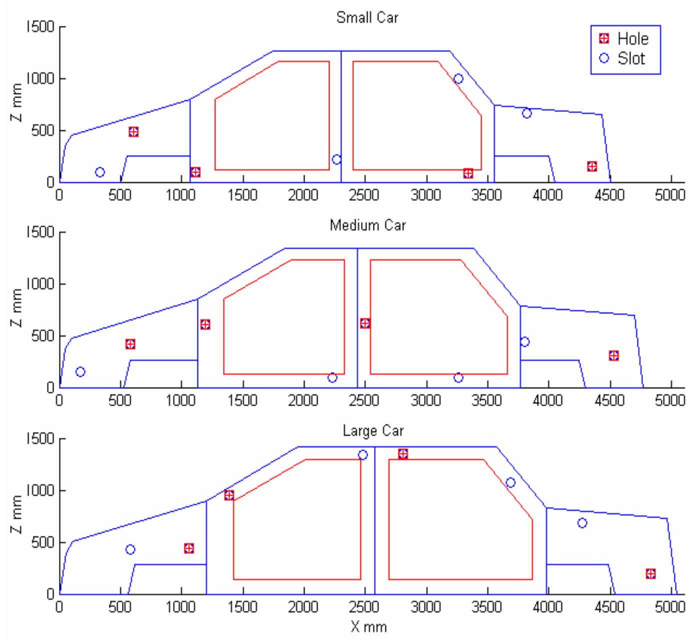


Figure 4.11 Location of the PLPs for reconfigurable line
(Note that the distance between the hole and the slot remains the same for each part-type across the three models)

No results are presented for the case of the product family with aligned pins since there is no feasible solution to that problem for the cases considered here.

4.5 Conclusions

This paper proposes a new approach for fixture configuration design for a family of products assembled in a single reconfigurable line. The problem is formulated as a constrained optimization by considering part geometry, fixture workspace and the alignment of the pins. Sequential quadratic programming was used to solve the optimization problem, and a relaxed formulation of the problem allowed searching for a robust layout. The resulting fixture layout using a reconfigurable line is compared with the case of single product dedicated lines in term of the quality of the solution. Two different scenarios were analyzed: no alignment restriction on the PLPs, and the PLPs has to be aligned (in X or Z directions). The result obtained for the product family is feasible; however, the sensitivity is 31 % (worst case) higher than the one for dedicated lines. This increment does not imply that the product family assembly is in general worse than the single lines. Obviously, there is a tradeoff between the achievement of production flexibility by using a reconfigurable line, and the robustness of the system to fixture variation for the product family. Using separated PTs for the each pin will significantly improve robustness; however, at a significant cost. An enterprise level evaluation of the pros and cons of both approaches (reconfigurable-dedicated) seems to be an appropriate method to decide which production scheme is better considering expected demands, product and process costs, flexibility and quality among other factors. It is the aim of this research to help that type of decision through the development of tools that help to perform such evaluation, and also help designers on the development of this type of assembly process.

Acknowledgments

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Appendix I: Constraints

First, it is presented the nomenclature used to formulate the constraints; the locators $\mathbf{p}_{p\text{-}j\text{-}hole}{}^{i}$ and $\mathbf{p}_{p\text{-}j\text{-}slot}{}^{i}$ are vectors containing the position (in the x-z directions) of the hole and slot respectively for the *j*-th part of model *i*-th. The terms $\mathbf{p}_{s\text{-}j\text{-}hole}{}^{i}$ and $\mathbf{p}_{s\text{-}j\text{-}slot}{}^{i}$ stand for the vectors containing the position (in the x-z directions) of the hole and slot respectively for the *j*-th subassembly of model *i*-th. The number of parts on each model is *m*, the number of subassemblies is *q*, and the number of models is *r*.

Constraint a: the locators should be inside the feasible region of the parts

$$\mathbf{p}_{p-i-hole}{}^{j} \ \ \& \ \ \mathbf{p}_{p-j-slot}{}^{i} \ \in \text{Feasible region of part } j \text{ of product } i, \ \forall j = 1...m, \ \forall i = 1...r$$

Constraint b: the distance d_j^i between the locators (hole and slot) used in the *i*-th part of model *j*-th has to be the same across models.

$$d_j^{\,i} = dist\left(\mathbf{p}_{p-j-hole}^{\;\;i}, \mathbf{p}_{p-j-slot}^{\;\;i}\right)$$

$$d_j^{\,1} = d_j^{\,2} = \cdots = d_j^{\,r};\ \forall\ j = 1\cdots m$$

where *dist*(**a**,**b**) stands for the Euclidian distance between vectors **a** and **b**.

Also for each subassembly, the distance s_j^i between the locators used in the *i*-th subassembly of model *j*-th has to be the same across models,

$$s_j^{\,i} = dist\left(\mathbf{p}_{s-j-hole}^{\;\;i}, \mathbf{p}_{s-j-slot}^{\;\;i}\right)$$

$$s_j^{\,1} = s_j^{\,2} = \cdots = s_j^{\,r};\ \forall\ j = 1\cdots q\ .$$

Constraint c: the position of the fixture containing the pins used to hold each part or subassembly must be inside the workspace of the PT that carries it. Then, the position of the point where the PT holds the fixture should, which lies between the two locators, must be inside the workspace of the PT. This point can be described, for the case of a part, as

$$\mathbf{f}_{p-j}^{\;\;i} = \mathbf{p}_{p-j-hole}^{\;\;i} + \alpha_j \cdot (\mathbf{p}_{p-j-hole}^{\;\;i} - \mathbf{p}_{p-j-slot}^{\;\;i})\ ,$$

and for a subassembly it can be described as

$$\mathbf{f}_{s-j}^{\;\;i} = \mathbf{p}_{s-j-hole}^{\;\;i} + \alpha_j \cdot (\mathbf{p}_{s-j-hole}^{\;\;i} - \mathbf{p}_{s-j-slot}^{\;\;i})\ ,$$

where α_j is a constant ($0 \le \alpha_j \le 1$), without lost of generality we can assume α_j=0.5, which corresponds to the midpoint between the locators. Then, the condition for the point to belong to the workspace of the corresponding PT can be written for a part and a subassembly as

$$\mathbf{f}_{p-j}^{\;\;i} \in \text{ Work space }_j ; \forall j \ = 1\cdots m\ ; \forall i \ = 1\cdots r\ ,$$

$$\mathbf{f}_{s-j}^{\;\;i} \in \text{ Work space}_j ; \forall j \ = 1\cdots q\ ; \forall i \ = 1\cdots r\ .$$

The workspace of each PT is defined by its own characteristics (e.g., dimensions, number of dof, type of joints, etc), and it represents all the points that a PT can reach holding the fixture in the appropriate direction.

Constraint d: the locators on each part and subassembly have to be aligned along one of the principle axes of the part or subassembly, then, the product of the differences between the location of the hole and the slot along each axis, for each part/subassembly, must satisfy the following condition,

$$\left(\mathbf{P}_{p-j-hole}{}^{i}(x)-\mathbf{P}_{p-j-slot}{}^{i}(x)\right)\cdot\left(\mathbf{P}_{p-j-hole}{}^{i}(z)-\mathbf{P}_{p-j-slot}{}^{i}(z)\right)=0;\ \forall j=1\cdots m;\ \forall i=1\cdots r\ ,$$

$$\left(\mathbf{P}_{s-j-hole}{}^{i}(x)-\mathbf{P}_{s-j-slot}{}^{i}(x)\right)\cdot\left(\mathbf{P}_{s-j-hole}{}^{i}(z)-\mathbf{P}_{s-i-slot}{}^{i}(z)\right)=0;\ \forall j=1\cdots q;\ \forall i=1\cdots r\ .$$

Appendix II: Geometry of the parts (small car vertices)

	Vertex location (x / z) in mm	
Part 1	External Vertices	(0 / 0); (500 / 0); (550 / 250); (1070 / 250); (1070 / 800); (100 / 450); (50 / 350)
Part 2	External Vertices	(1070 / 0); (2300 / 0); (2300 / 1260); (1740 / 1260); (1070 / 800)
	Internal Vertices	(1270 / 120); (2200 / 120); (2200 / 1160); (1790 / 1160); (1270 / 800)
Part 3	External Vertices	(2300 / 0); (3550 / 0); (3550 / 740); (3190 / 1260); (2300 / 1260)
	Internal Vertices	(2400 / 120); (3450 / 120); (3450 / 640); (3090 / 1160); (2400 / 1160)
Part 4	External Vertices	(3550 / 250); (4000 / 250); (4050 / 0); (4500 / 0); (4430 / 650); (3550 / 740)

Appendix III: Location of the measurement points (small car)

Measurement point	Position (x/z) in mm	Measurement point	Position (x/z) in mm
M1	100 / 450	M11	1500 / 100
M2	1070 / 800	M12	2150 / 1260
M3	1100 / 300	M13	3550 / 700
M4	1100 / 600	M14	3350 / 250
M5	1360 / 940	M15	3300 / 100
M6	2000 / 1200	M16	2500 / 100
M7	2300 / 1200	M17	3800 / 685.8
M8	2200 / 1000	M18	4430 / 650
M9	2200 / 400	M19	4050 / 0
M10	2000 / 100		

Appendix IV: Optimal location of the locators (dedicated lines)

Non-Aligned PLPs (x / z) in mm			
	Small	Medium	Large
P1	868.5 / 402.6	975.6 / 435.4	828.2 / 412.9
P2	93.8 / 305.3	54.2 / 257.0	205.9 / 260.7
P3	1103.7 / 414.0	1167.6 / 369.3	1238 / 408.5
P4	2257.6 / 823.7	2397.5 / 868.7	2508.2 / 905.7
P5	3091.2 / 55.4	3136.5 / 56.5	2666.2 / 1342.1
P6	3516.3 / 477.9	3537.5 / 1006.3	3839.9 / 851.3
P7	4327.4 / 81.1	3822.2 / 512.7	4124.9 / 666.3
P8	4002.4 / 53.0	4587.6 / 539.8	4926.6 / 365.2

Aligned PLPs (x / z) in mm			
	Small	Medium	Large
P1	459 / 546.5	529 / 572	1079 /554
P2	721 / 546.5	908 / 572	386 / 553
P3	1240 / 546.3	1305 / 572	1320 /554
P4	2269 / 546.5	2407 / 572	2546 / 554
P5	2331 / 546.3	2483 / 572	2658 / 554
P6	3503 / 546.5	3727 / 572	3911 / 554
P7	3767 / 46.5	3938 / 572	4168 / 554
P8	4174 / 46.4	4497 / 72	4884 / 554

Appendix V: Optimal location of the locators (Reconfigurable line)

Location of the PLPs (x / z) in mm			
	Small	Medium	Large
P1	608.7 / 480.8	579.4 / 416.4	1063.2 / 437.2
P2	328.6 / 90.9	177.3 / 154.1	538.2 / 429.1
P3	1115.7 / 95.5	1189 / 605.5	1391.7 / 944.5
P4	2270 / 211.9	2231.8 / 97.3	2480.8 / 1344.2
P5	3336.6 / 79.6	2497.1 / 615.9	2806.7 / 1353.4
P6	3256 / 997.9	3257.5 / 94.8	3684.7 / 1072.5

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CHAPTER 5
CONCLUSIONS AND FUTURE WORK

5.1 Conclusions

The research on synthesizing products, processes and control in multistation reconfigurable assembly lines originates from the existence of real engineering problems in assembly systems and the maturity of reconfigurable assembly systems (tools and control). In such a system, programmable tools (PTs) are often used to achieve the flexibility and reconfigurability of the assembly system for different models in a product family. Meanwhile, the precision of PT can also serve as an actuation device, which provides the capability for dimensional control and quality improvements for a given product. However, there is a lack of methodologies to effectively improve dimensional quality with PTs in reconfigurable assembly systems, especially considering the "multistation" nature of an assembly system. This dissertation is aimed to fill that gap, and presents a comprehensive framework to model and analyze reconfigurable multistation assembly systems. The developed methodologies enable an effective improvement of dimensional quality through proper design of products and process, and the use of active control to compensate errors as they happen along the process.

The major achievements of this dissertation can be summarized in four parts:

1. *Development of a state space model of stream of variation for controlled multistation assembly systems*

The objective of this study was to develop a model to describe the variation and its propagation in a multistation manufacturing system by including the use of control to compensate deviations on a part-by-part basis. The model incorporates in-line pre-assembly measurements and a feedforward control approach with multistation variation propagation models in a state space format. The proposed model creates a basis for developing novel techniques to analyze and design controlled multistation processes, which will help to improve quality, responsiveness and reduce cost.

2. *Design of a feedforward control strategy for quality enhancement*

The objective of this design was to develop a part-by-part deviation control technique for multistation manufacturing systems that includes product/processes limitations and requirements, which are included as constraints into the controller derivation. This inclusion will result in optimal control actions that accounts for process's limitations and product requirements. The proposed constrained controller was applied to a simulation of an automotive panel assembly. As a result of using this controller, the dimensional quality can be significantly improved (by more than 45 % in the present case) compared with not using control, as well as with the use of a controller that does not incorporate constraints (by more than 4 % in the presented case).

3. *Optimal actuator placement in multistation assembly processes*

The objective of this work was to evaluate the efficiency of an actuator network and to determine the optimal selection and distribution of actuators in a multistation assembly process to cost-effectively improve dimensional quality. The important elements in this framework are described and derived for a multistation assembly system. These include four relationships or models including: cost function, controlled SoV model, dimensional quality function and controllability indices. Those four relationships were incorporated into a multiobjective combinatorial optimization problem resulting in Pareto sets (trading cost and variation), which can be used to properly select and distribute actuators in a multistation assembly system. Controllability indices, defined similar to the controllability index in the control theory, provide bounds for the necessary number of actuators to ensure controllability at the system and station levels. The use of these bounds helps to reduce the search space for solutions that effectively improve quality, because they prevent analyzing conditions where redundant or unnecessary actuators are used. Simulations indicates that an optimal distribution of actuators leads to enhance quality by more than 86 % compared with not using control, and ratify that the use of any extra imperfect actuators (actuators redundancy) leads to an increase in variation instead of reducing it compared with the use of the appropriate number of actuators.

4. *Product family fixture layout design*

The objectives of this study were two, first to study the effect that assembly flexibility (by means of allowing the assembly of different products in the same reconfigurable line) has on dimensional quality; and second, to develop a methodology for a robust distribution of fixtures used to assembly a product family in a reconfigurable line. Based on the state space model of the assembly of the different products of the family, a product family-variation-index was developed. It was proposed to minimize this index with consideration of products and processes constraints. The solution of a reconfigurable line was benchmarked against the solutions of dedicated production lines (one for each product in the family) and showed that the worst possible deviation of a product family is 31 % larger than the one for dedicated lines. This difference reflects the existence of tradeoff between production flexibility and dimensional quality.

5.2 Future Work

The future work described in this section contributes to the synthesis of products, processes, control and monitoring of reconfigurable assembly systems. The proposed future works are:

1. *Optimal Sensors Placement*

The sensor distribution plays an important role for both automatic deviation control and statistical process control (monitoring and diagnosis) because they mainly determine the capability to detect and identify errors in the process. Preliminary work has been done on the sensor placement for diagnosis in multistation assembly processes (Ding *et al.,* 2003; Liu *et al.,* 2005). However, no work has been done on the sensor placement for deviation control in multistation processes and its combination with diagnosis. It is important to combine both objectives because while control compensates deviations, their root cause should be identified. The sensor placement problem can be approached from two perspectives: station level and part level. The station level is focused on determining the appropriate stations along the process, where having measurements will simultaneously maximize the capability to control and diagnose errors. The part level is a more detailed one, and it is focused on determining the appropriate features of the parts that should be measured in order to improve the estimation of the part deviation.

2. *Integration of SPC and Automated Process Control (APC) for monitoring feedforward controlled processes*

    Related with the previous proposed research topic is the development of methods for rapid and precise diagnosis of errors in controlled multistation manufacturing processes. The use of control will tend to hide the existence of errors and deviations in the process if measurements are taken at the final station. Therefore, it is necessary to change the strategy to monitor not only post-control measurements, but also pre-control measurement and control signal, to extract sufficient information to perform an adequate diagnosis. The combination of SPC and APC in multistation manufacturing processes can be very beneficial. The use of the information obtained from SPC techniques, such as detection of mean shifts, can be incorporated into the APC to determine better control actions (in upstream stations of the place where the mean shift was detected).

3. *Tolerance allocation for controlled multistation assembly systems*

    The use of control for variation reduction in multistation assembly processes should be considered in the allocation of tolerances of product and processes. The use of control allows designers to define wider tolerances on parts and tools while ensuring that the process capability is adequate (e.g., C_p>1.3). The use of wider tolerances will result in cost reductions.

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