A GEOMETRIC COMPARISON OF BRANCHING STRUCTURES IN TENSION AND COMPRESSION VERSUS MINIMAL PATHS

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Summary

Branching structures are based on geometric systems that expand through bifurcation without returning to form closed cells. In this sense, branching structures resemble the structure of trees that branch continually outward. In architectural engineering, these forms can be used either as tension or compression systems. Numerous built examples have been produced since the initial inspiring studies made by Frei Otto in the early 1960's. Form finding techniques based on models have been used in the past to study these forms. Although thread models can be effective in the study of force paths, they cannot distinguish between tension and compression and have no way to take member buckling into account. But buckling does have an influence on appropriate geometry of a compression system. Also, minimal paths (or pseudo minimal paths based on surface tension thread models) have been used to explore possible geometries for branching structures. In this paper, both surface tension thread models dipped in water, and weighted string models are shown in comparison with ideal tension and compression forms found with a computational method based on Genetic Algorithms. The same computational model is used to find geometries with minimal overall member length. Both 2D and 3D geometries are derived.

Keywords: branching structure; tree column; form finding; optimization

1. Background Introduction

Many architects and engineers are familiar with past studies made at the University of Stuttgart under the special research area (SFB 230) with the theme "Natural Structures: lightweight construction in architecture and nature". These studies included work by several institutes including the Institute for Lightweight Structures (IL - Frei Otto), the Institute for Applications of Geodesy to Engineering (IAGB - Klaus Linkwitz) and the Institute of Structural Mechanics (IB - Ekkehard Ramm). In volume four of the SFB 230 report series, "Verzweigungen" (Branchings), different approaches to the modelling of branching structures are described. Matthias Neureither of the IAGB shows a program that generates branching forms based on parameters such as angle, generation (levels), pattern type, etc. [1] Several of the patterns are shown in the article which demonstrate the breadth of geometric possibilities based on various combinations of stated parameters. Figure 1 shows an example of a pattern described using the program. The program is very effective at displaying different configurations of branching structures, but no relation to structural behaviour is intended.

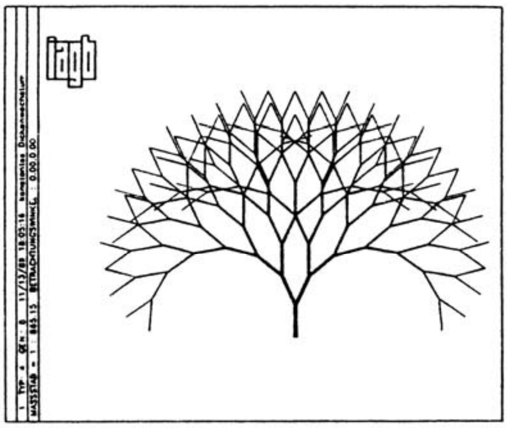


Abb. 6: konstanter Dickenzuwachs (Baumtyp 3) / Verzweigungswinkel 50° / 8 Generationen

Fig. 1 Constant length branching structure by Matthias Neureither, IAGB Stuttgart. [1]

In the same volume, Kai-Uwe Bletzinger showed another computational form finding method using CARAT, developed at the Institute of Structural Mechanics (IB) at the University of Stuttgart. Optimal compression geometries were explored for varying levels of load [2]. The branching columns were modelled as tubular steel structures based on Din18800. Four geometries resulting from Bletzinger's parametric study are shown in Figure 2.

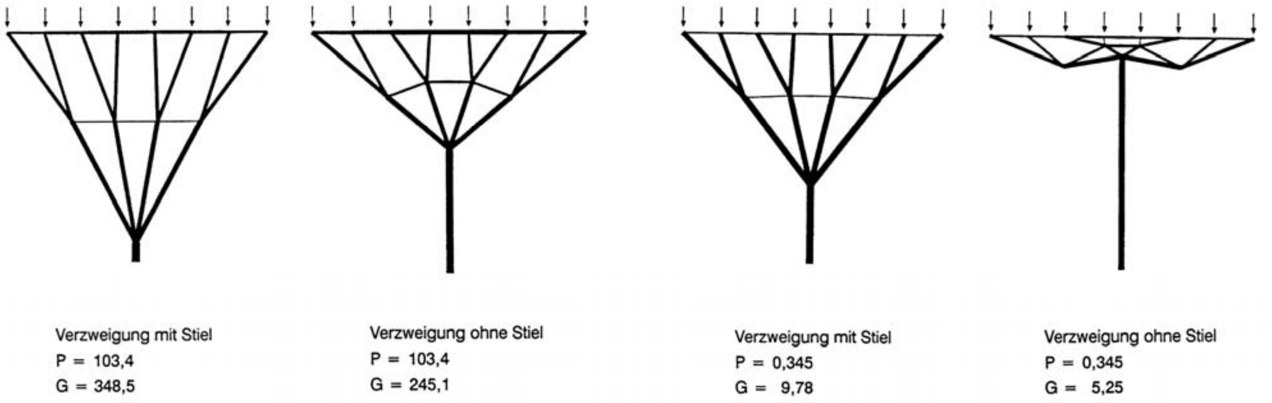


Fig. 2 Four branching compression structures based on DIN 18800 found using CARAT. [2] P = load level, G = weight.

Taking a different approach to form finding, models have been used to explore structural form. Marek Kolodziejczyk at the IL in Stuttgart used string models dipped in water to find a pseudo minimal path forms produced by the water surface tension on the strings [3]. The actual form is influenced by the amount of slack in the strings (usually between 5% and 10%). In this way, both 2D and 3D forms can be studied. Figure 3 shows forms derived from 2D models. Figure 4 exhibits an example of a 3D tree structure produced by Kolodziejczyk.

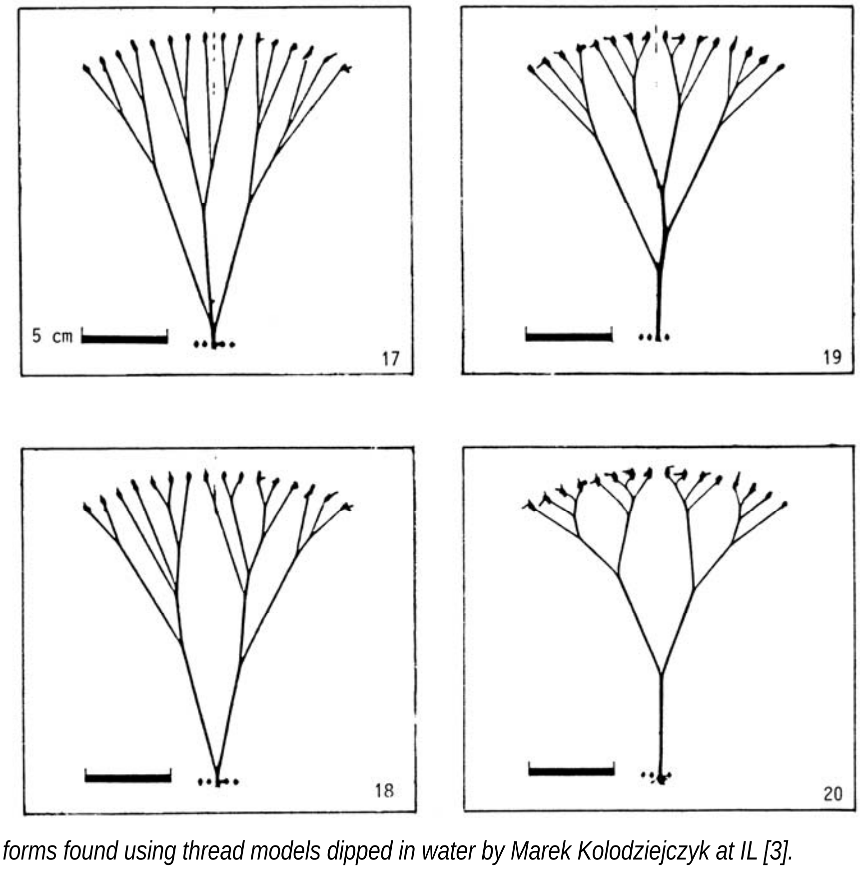


Fig. 3 2D branching forms found using thread models dipped in water by Marek Kolodziejczyk at IL [3].

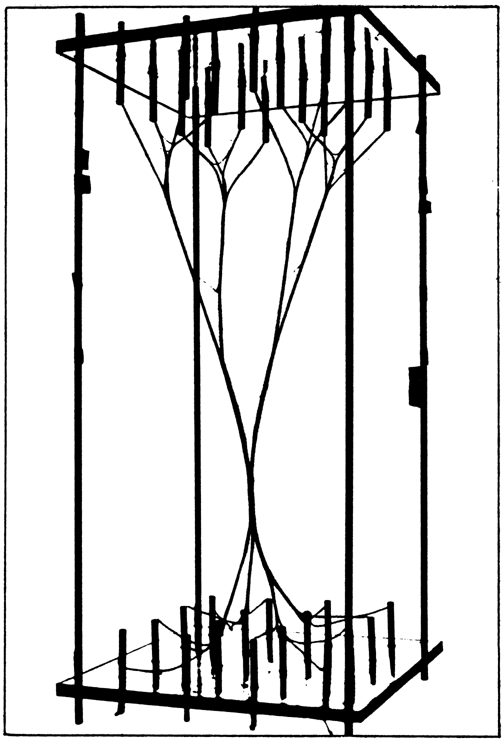


Fig. 4 3D thread model by Marek Kolodziejczyk at IL [IL Archive]

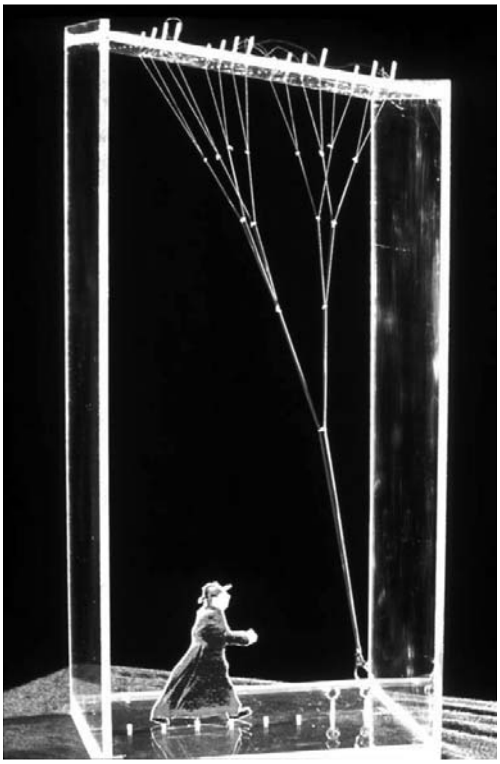


Fig. 5 3D thread model using beaded strings [IL Archive]

Another modelling technique developed at IL uses dry strings connected with beads. The beads are held in place simply by friction against the strings. The strings can be fixed in a frame or tensioned with suspended weights. These models allow much more control over the described form than do the models dipped in water. The beads allow positioning and repositioning of the nodes and the length of threads can also be adjusted in the frame. Figure 5 shows a study model produce by Jürgen Hennicke at IL.

2. Exploring Branching Structures with Genetic Algorithms

The program used in the form exploration carried out for this paper is called an Intelligent Genetic Design Tool (IGDT) and uses Genetic Algorithms (GA's) to search a design space for geometries and topologies optimized for given structural and geometric objectives [4]. GA's are stochastic search methods based on principles of genetic reproduction and evolution. Progressive generations of solutions are 'bred' using operators of crossover, mutation and selection applied to numerically coded 'chromosomes'. An overview of the mechanics of the method was given in a previous IASS paper [5]. At this point the structural analysis component is limited to 2D finite element truss members, however the 3D geometries can be explored without a structural analysis, for example to find minimal length geometries.

Three different parameters were compared in the exploration of 2D branching geometries. The first two are based on a structural analysis and member sizing based on the AISC steel code (buckling included). The form of the branching structure was chosen based on least weight. The other parameter used to determine form was the minimal length geometry (also called minimal paths or minimal ways).

The compression members were designed as hollow pipe sections following the AISC-ASD American steel code. A yield strength of Fy=248 MPa (A-36 steel) was used. The tension members were sized as simple bars with an allowable stress of 0.6 Fy.

2.1 Compression Forms

In determining the ideal shape of systems loaded in compression, the level of the load plays a roll in the optimal geometry. With higher load levels the most efficient member lengths are longer. This can be observed in the following examples. With lower level loads, smaller sections are required and member lengths must be shorter to maintain a good slenderness ratio against Euler buckling. Figures 6 through 8 show four levels of compressive load and the shape appropriate to each. In Figure 6 the load is applied upward to the bottom node with the top nodes held as reactions. In Figures 7 and 8 the load is applied downward on the top four nodes with the reaction on the bottom node. The structures in Figures 7 and 8 have two members added for stability that are not needed when the system is loaded from below and fixed above. The range of load is deliberately taken between extremes of low and high to show the full pattern of geometry response. The line weight in the illustrations is scaled to the pipe diameters having the required area in order to give a better comparison of member weight. For the lowest level (far left) the nodes shift to find the geometry with minimal member length (minimal path) since the force is of secondary importance. Member area is not allowed to diminish to zero. Notice the progressive lengthening of the upper 'branches' as the load increases.

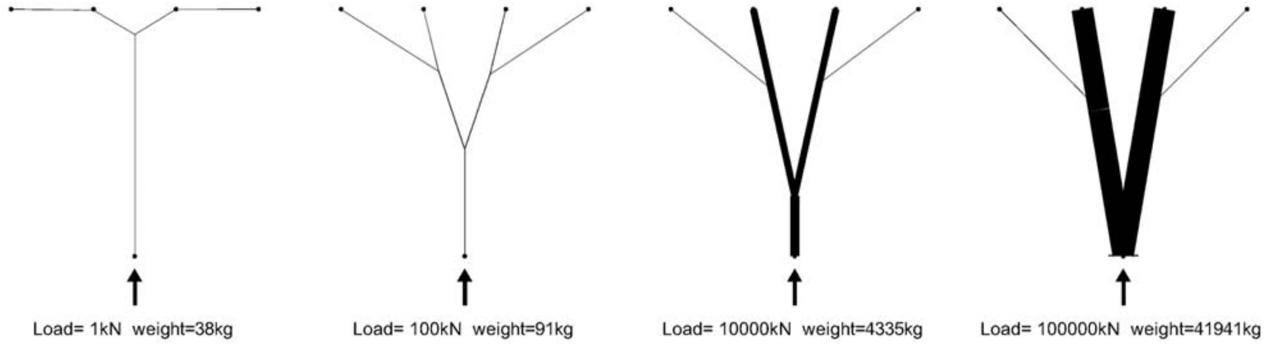


Fig. 6 Branching columns loaded from below in compression using four levels of loading.

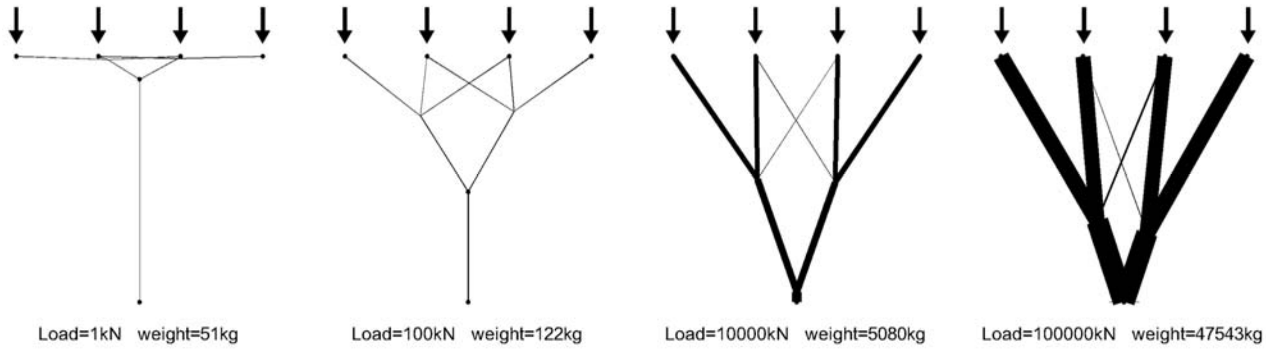
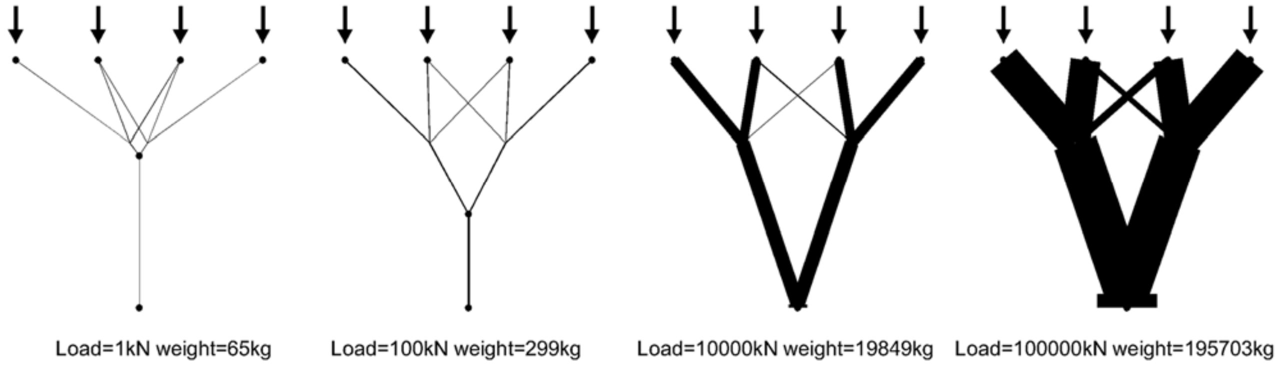


Fig. 7 Branching columns loaded from above in compression using four levels of loading.

The program also allows axial components of nodes to be constrained against movement (held at a preset location). In Figure 8 the positions of the upper pair of interior nodes were held in place vertically. This might be useful when required by architectural considerations beyond structural performance. The effect of lengthening of members now can be seen below the constrained nodes.

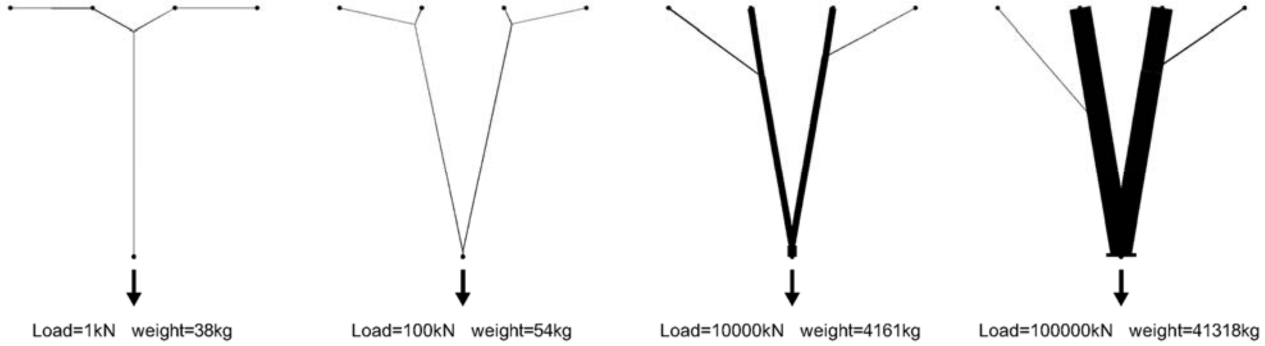


Load=1kN weight=65kg Load=100kN weight=299kg Load=10000kN weight=19849kg Load=100000kN weight=195703kg

Fig. 8 Branching columns loaded from above in compression using four levels of loading. Upper two nodes constrained vertically.

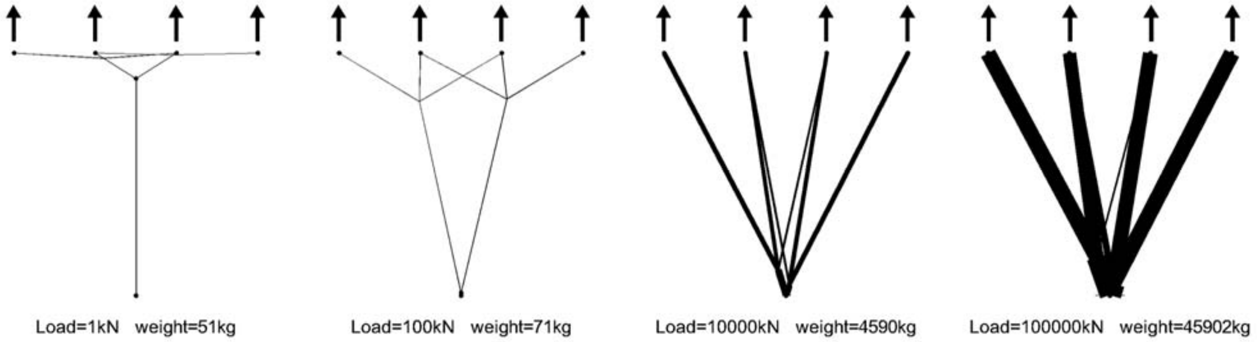
2.2 Tensile Forms

Figures 9 through 11 show forms derived from a tensile loading. The same 4 load levels were chosen that were used in the compression study. In the case of tension, the lower vertical stem disappears except for the very low load level. This can be seen when comparing the 100kN loading in compression versus tension. Even at the 10000kN level there remains a short lower stem member in compression which is missing in the tension forms. The low level load (far left) is again the minimal path form and is similar to the compression examples. With the branching nodes positioned vertically (Figure 11) the form is not affected by load level.



Load=1kN weight=38kg Load=100kN weight=54kg Load=10000kN weight=4161kg Load=100000kN weight=41318kg

Fig. 9 Branching columns loaded from below in tension using four levels of loading.



Load=1kN weight=51kg Load=100kN weight=71kg Load=10000kN weight=4590kg Load=100000kN weight=45902kg

Fig. 10 Branching columns loaded from above in tension using four levels of loading.

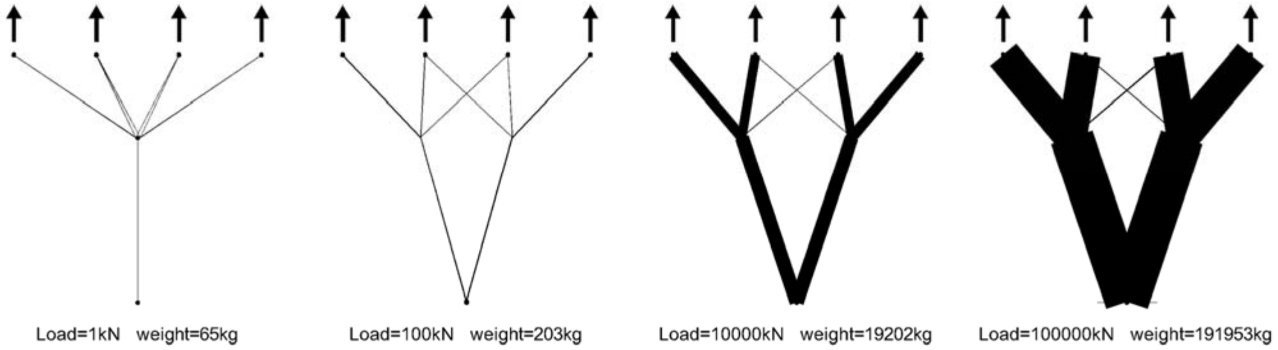


Fig. 11 Branching columns loaded from above in tension using four levels of loading. Upper two nodes constrained vertically.

2.3 Minimal Path Forms

As already noted above, the minimal load case from the examples above, (far left structure in Figures 6-11) follows the geometry with the minimal path between the points. With no constrains on the interior points, (Figures 6, 7, 9 and 10) the form matches that found experimentally with a soap film model [6] (see Figure 12).

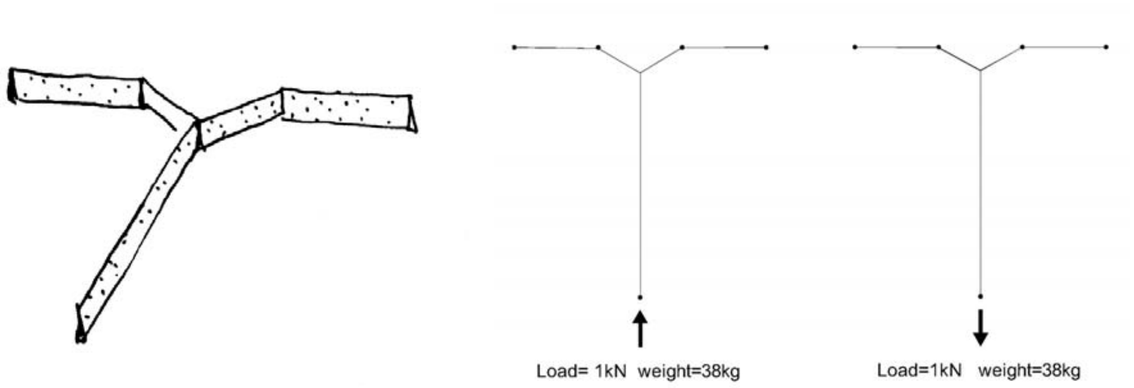


Fig. 12 Comparison of soap film form [6] with forms of low load level branching structures from Figures 6 and 9.

Although the IGDT program currently lacks finite element capability in 3D, it can perform dimensional operations in 3D. Figure13 shows perspective and elevation views of a minimal length path for a 3D branching structure with no constrains on interior nodes. These 3D paths are more difficult to find using soap film models than the 2D paths. In fact, the device produced at IL to study minimal paths is limited to 2D [7]. Wetted thread models do not precisely find minimal paths because the thread lengths need to be chosen before the model is dipped. The form found in wetted thread models is determined by the lengths of the threads and the surface tension between the threads. The surface tension is generally not high enough to pull the threads into a minimal path.

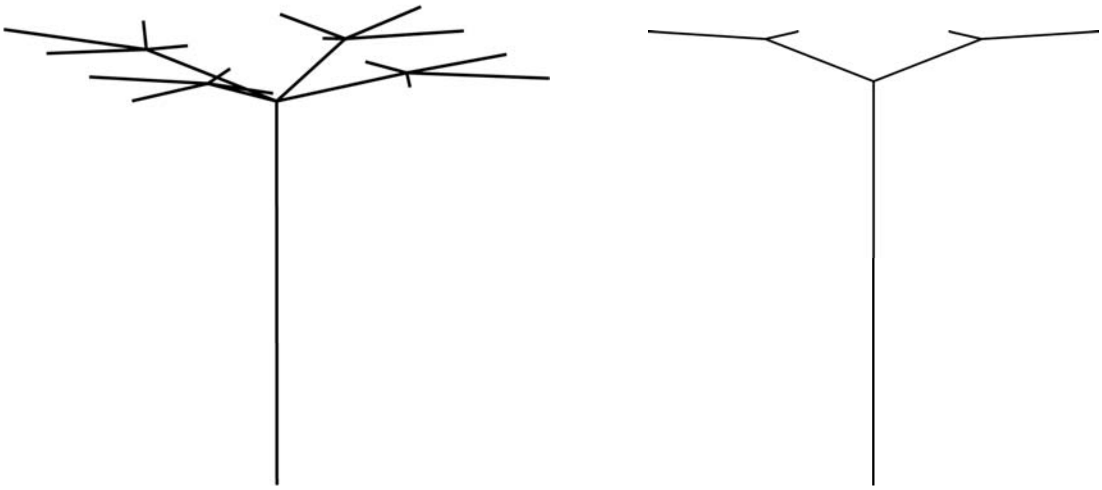


Fig. 13 Perspective and elevation views of a minimal length geometry for a simple 3D branching structure.

Using the IGDT minimal length geometries can also be found placing constraints on selected coordinates of selected nodes. In Figure 14, the vertical coordinates of the internal nodes were constrained and the two horizontal coordinates were determined.

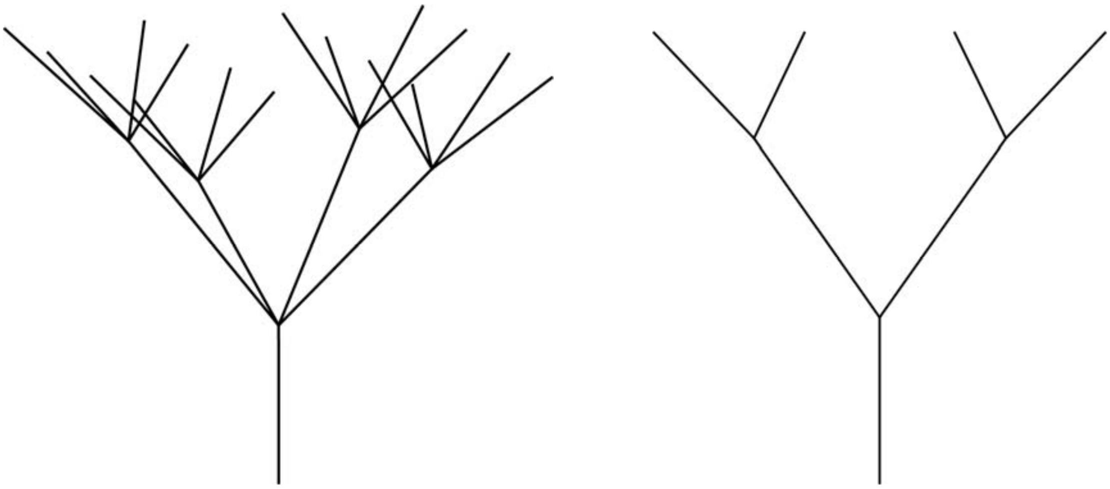


Fig. 14 Perspective and elevation views of a minimal length geometry for a simple 3D branching structure with set height of nodes.

3. Conclusion

This paper has given a review of techniques used to explore branching structures. Traditional physical modelling techniques using thread models or soap films are compared to solutions found using a computational exploration tool based on Genetic Algorithms. The computational method has the advantage of being able to take member buckling into account, and thereby can make the distinction between forms suited for tension and compression. Finally, geometries associated with minimal length systems were derived, and compared to tension and compression forms. The computational tool also had the ability to selectively constrain nodal coordinates in the system. Examples of this effect were shown in 2D and 3D geometries.

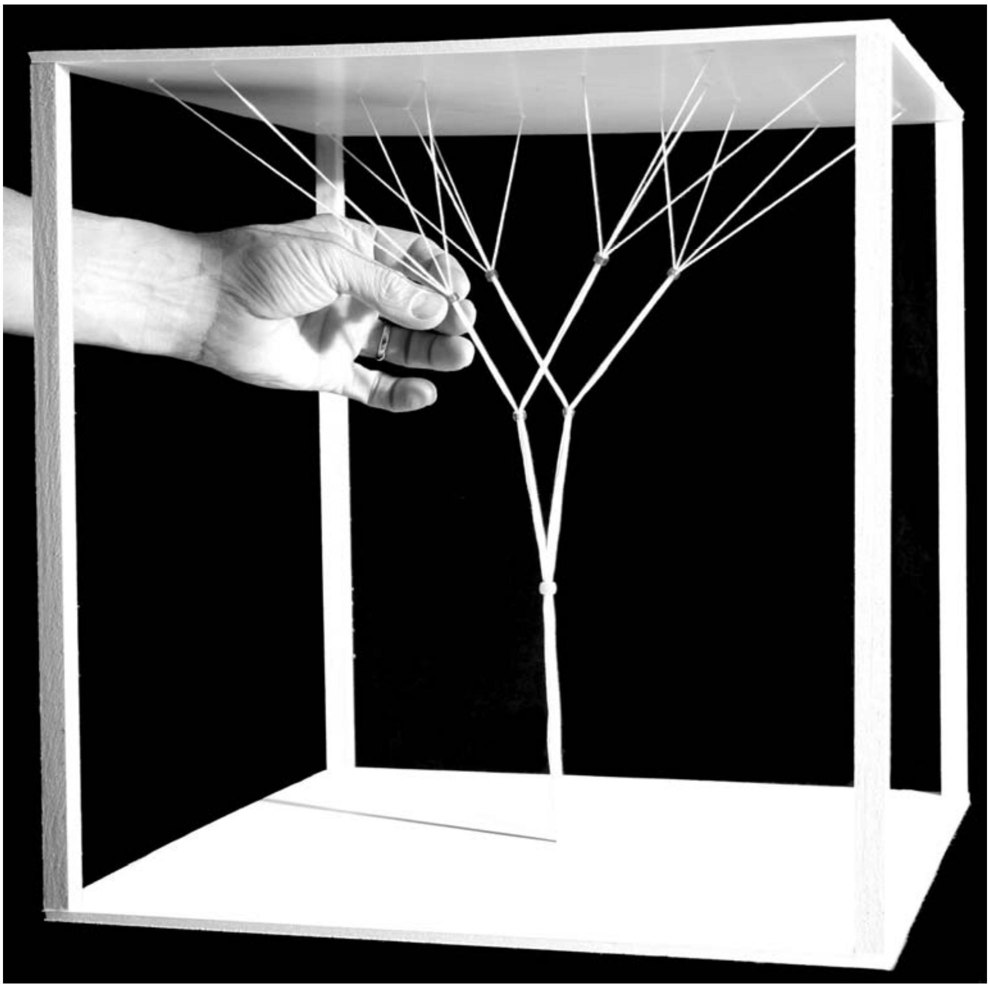


Fig. 15 Thread model with adjustable nodes used to study geometry.

4. References

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