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Using Asymmetry to Estimate Potential

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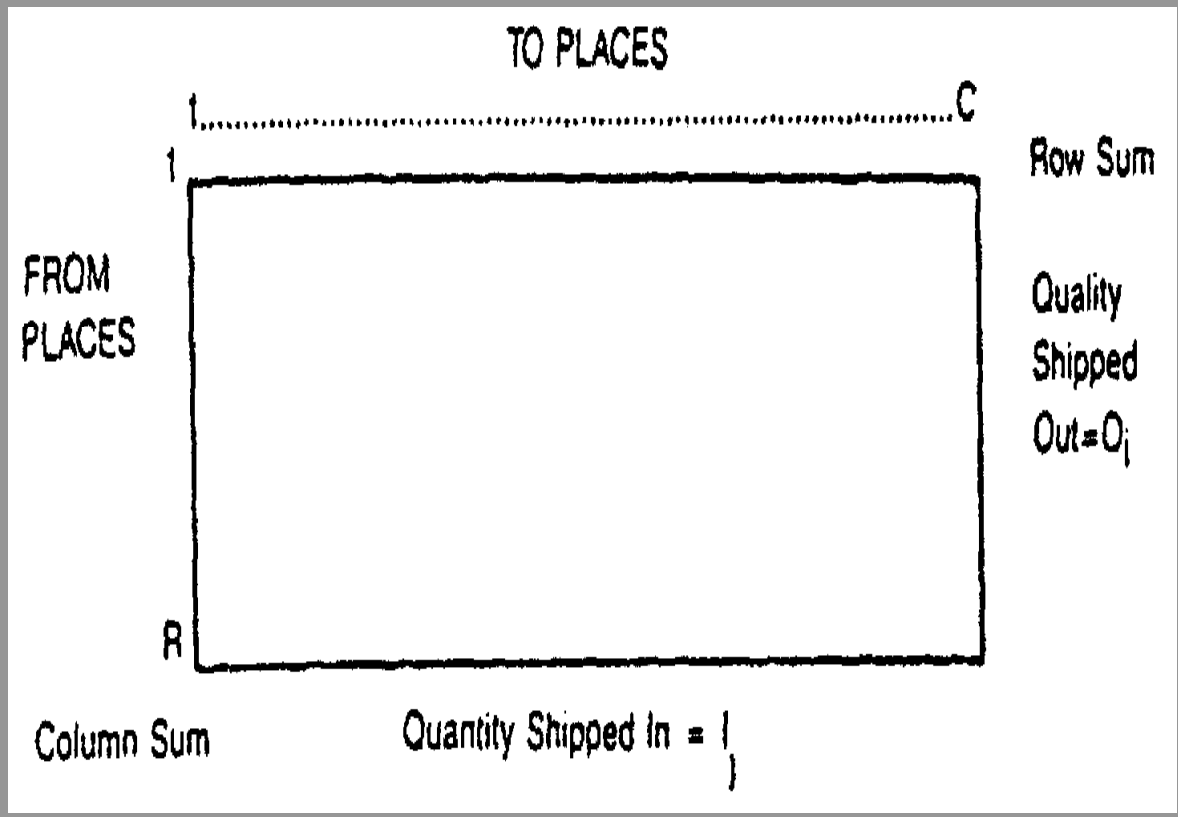
The concern is with complete, square, asymmetric, valued tables, though the procedure may also work with two mode tables.

For this demonstration I have used only small examples.

One example is based on geographic data, the other on journal-to-journal citations.

The form of a movement table M_{ij}

In a non-geographic, network, environment the 'places' are sometimes called 'actors'.



Let M_{ij} represent the movement table, with i rows and j columns. It can be separated into two parts, as follows.

$$M_{ij} = M^{+} + M^{-}$$

where

$$M^{+} = (M_{ij} + M_{ji})/2 \qquad \text{symmetric}$$

$$M^{-} = (M_{ij} - M_{ji})/2 \qquad \text{skew symmetric}$$

The variance can also be computed for each component,

and the degree of asymmetry can be computed.

How the two parts are used

I consider the symmetric component as a type of background.

The real interest is in the asymmetric part.

In the geographic case the position of the places is known.

But if locations are not given then the symmetric part may be used to make an estimate of these positions.

This estimate is made using an ordination, trilateration, or multidimensional scaling algorithm.

The first example uses a 33 by 33 matrix of commuting in the vicinity of Munich, Germany.

The matrix is shown next.

A map of the regions is given in:

D. Fliedner, 1962, "Zyklonale Tendenzen bei Bevölkerungs und Verkehrsbewegungen in Städtischen Bereichen untersucht am Beispiel der Städte Göttingen, München, und Osnabrück", *Neues Archiv für Niedersachsen*, 10:15 (April 4): 277-294, (following p. 285).

A geographic example

Munich Commuting 1939

Between 33 districts of known location

VON WOHNSITZ IN STADTBEZIRK ..... / NACH ARBEITSSTÄTTE IN STADTBEZIRK	1 NO-Altstadt	2 SO-Altstadt	3 SW-Altstadt	4 NW-Altstadt	5 Universität/Viertel	6 Techn.-Hochschulviertel	7 SW-Schwabing	8 Viertel nw. Hauptbahnhof	9 Viertel südl. Hauptbahnhof	10 Viertel nö. Südbahnhof	11 Viertel sw. Isar u. Südfriedhof	12 Viertel sw. Museumsinsel u. Altstadt	13 Viertel südl. Engl. Garten	14 N-Haidhausen	15 Mittel-Haidhausen	16 SW-Haidhausen	17 Giesing	18 Harlaching u. Menterschwaige	19 Unter- u. Mitter-Sendling	20 Viertel westl. Theresienwiese	21 O-Neuhausen	22 O-Schwabing u. Freimann	23 Gern u. W-Neuhausen	24 Ober-Sendling, Solln u. Thalkirchen	25 Laim	26 W-Schwabing	27 N-Schwabing u. Milbertshofen	28 Moosach	29 Bogenhausen, Daglfing etc.	30 Ramersdorf u. Perlach	31 Berg am Laim u. Trudering	32 Allach u. Feldmoching	33 Pasing u. Menzing	Insgesamt	
1 NO-Altstadt		174	151	435	212	157	48	119	247	74	86	100	192	52	52	47	79	26	70	30	76	91	22	112	23	27	86	24	42	12	29	8	7	2910	1
2 SO-Altstadt	428		335	482	215	153	51	176	414	164	176	170	170	59	48	68	123	57	102	76	87	124	34	232	49	46	128	40	58	16	31	18	15	4345	2
3 SW-Altstadt	209	149		455	114	133	43	143	350	107	83	60	73	35	30	42	58	24	85	39	59	75	29	128	26	38	100	31	48	22	19	10	8	2821	3
4 NW-Altstadt	104	32	55		77	76	13	46	92	25	21	23	41	5	6	7	21	6	20	14	11	22	5	32	12	8	48	6	15	7	1	6	3	860	4
5 Universität/Viertel	549	165	242	998		669	139	284	587	121	110	74	367	61	38	60	99	26	90	72	172	393	59	187	61	207	335	39	95	20	39	22	9	6387	5
6 Techn.-Hochschul-Viertel	308	122	252	765	516		180	697	784	140	124	77	220	66	44	53	86	36	116	89	326	230	111	235	72	160	373	70	51	13	26	41	18	6401	6
7 SW-Schwabing	441	180	348	955	1063	981		625	878	197	121	94	365	106	66	70	133	53	141	112	497	438	95	274	94	386	709	95	110	37	39	53	28	9762	7
8 Viertel nw. Hauptbahnhof	245	123	207	643	405	504	159		880	158	116	66	185	81	57	66	103	40	124	171	469	191	204	262	100	107	536	76	58	26	29	36	72	6253	8
9 Viertel südl. Hauptbahnhof	338	190	429	843	368	426	88	439		350	162	91	194	62	77	77	109	36	188	197	179	145	69	302	141	78	270	33	67	26	21	61	39	6095	9
10 Viertel nö. Südbahnhof	426	566	483	874	394	362	87	447	1312		405	149	241	115	95	186	258	117	432	193	190	196	87	746	125	80	541	46	73	41	45	58	37	9005	10
11 Viertel sw. Isar u. Südfriedhof	553	510	546	1049	485	472	97	456	984	726		347	392	168	131	272	360	151	362	144	228	231	65	740	125	86	422	56	94	91	62	59	34	10478	11
12 Viertel sw. Museumsinsel u. Altstadt	548	415	354	847	370	308	88	256	653	203	332		548	134	123	225	145	82	193	110	132	181	64	289	65	59	258	32	63	69	61	40	22	7049	12
13 Viertel südl. Engl. Garten	836	221	270	1157	526	379	65	266	495	146	153	193		150	107	129	108	44	63	77	115	244	40	149	29	52	220	21	110	37	35	51	19	6485	13
14 N-Haidhausen	508	318	286	795	448	375	108	360	637	204	192	140	741		463	304	295	92	172	85	152	247	62	349	75	81	332	47	348	184	313	38	20	8787	14
15 Mittel-Haidhausen	509	344	331	747	448	401	69	443	726	248	283	199	476	658		461	436	121	903	77	184	226	55	442	110	79	368	61	212	213	397	43	32	9602	15
16 SW-Haidhausen	625	531	478	946	442	431	84	456	843	377	469	322	441	304	399		562	244	257	109	211	272	77	511	128	95	391	58	135	190	188	49	21	10654	16
17 Giesing	380	374	284	582	319	337	111	306	678	353	381	198	257	276	209	375		885	233	111	120	200	44	506	73	55	294	46	77	212	90	33	17	7816	17
18 Harlaching u. Menterschwaige	647	567	522	1013	568	639	99	430	1028	570	642	266	478	303	214	525	1830		407	123	216	286	47	766	120	89	415	86	148	176	38	42	28	12792	18
19 Unter- u. Mitter-Sendling	592	447	571	1178	591	692	65	690	1556	908	422	162	419	234	99	184	540	114		280	254	259	98	2165	310	111	477	82	82	155	52	50	77	13694	19
20 Viertel westl. Theresienwiese	278	216	296	606	281	403	86	722	1756	284	235	81	208	107	73	94	185	59	484		228	172	136	609	415	54	402	64	54	42	40	68	51	8789	20
21 O-Neuhausen	305	166	234	664	390	510	73	987	805	171	185	53	233	114	59	81	109	40	117	150		263	281	266	164	123	427	192	42	32	27	105	37	7325	21
22 O-Schwabing u. Freimann	575	239	327	1042	1053	725	121	428	713	204	166	66	537	189	60	104	102	57	143	71	265		85	284	92	444	1221	127	168	32	55	173	29	9901	22
23 Gern u. W-Neuhausen	559	282	340	1147	616	893	105	1186	1824	226	148	103	397	126	54	81	119	30	169	217	635	561		379	614	156	455	243	86	44	39	121	54	12007	23
24 Ober-Sendling, Solln u. Thalkirchen	290	212	201	647	373	418	69	291	372	396	237	88	178	150	62	100	185	112	717	151	113	182	46		107	39	161	47	71	51	31	16	58	6089	24
25 Laim	318	342	457	1115	567	799	84	946	2312	371	303	147	347	206	84	120	201	74	598	799	424	440	269	897		118	508	153	81	90	52	139	199	13760	25
26 W-Schwabing	494	228	361	1230	1188	1039	278	530	871	211	155	95	519	117	71	67	118	54	124	112	414	1140	100	295	107		1220	79	125	53	41	81	114	11556	26
27 N-Schwabing u. Milbertshofen	212	142	160	405	419	346	93	229	582	133	80	38	235	83	29	63	70	34	99	65	247	802	47	162	65	280		91	90	36	22	51	30	5828	27
28 Moosach	378	272	295	743	499	726	117	639	950	204	129	68	309	129	56	71	134	44	188	135	1074	439	470	401	240	157	1054		82	40	29	175	52	10516	28
29 Bogenhausen, Daglfing etc.	366	169	165	680	452	379	41	190	437	124	101	87	753	321	199	143	101	51	105	52	116	297	24	147	43	79	224	48		76	234	28	14	6246	29
30 Ramersdorf u. Perlach	329	240	248	518	319	317	49	210	480	155	187	130	316	252	419	222	347	78	144	49	123	215	26	273	58	46	214	43	165		262	27	21	6482	30
31 Berg am Laim u. Trudering	300	268	236	457	347	330	55	245	537	221	206	144	337	396	859	258	301	110	197	53	149	286	54	328	92	77	261	63	259	339		28	20	7793	31
32 Allach u. Feldmoching	103	65	95	201	149	178	74	204	389	96	44	54	78	10	17	36	26	33	58	45	200	230	100	123	93	59	1169	320	36	44	12		220	4501	32
33 Pasing u. Menzing	307	195	243	380	356	602	67	521	1255	172	107	69	251	90	54	58	73	67	342	238	215	265	151	564	529	66	184	90	38	33	41	284		8081	33
Insgesamt	13260	8864	9822	24963	14450	15160	2901	14167	26445	7999	6501	3954	10258	5119	4534	4629	6616	2597	6743	4246	7881	9343	3054	13139	4355	3550	13419	2509	5182	2461	2456	2014	1274	260440	

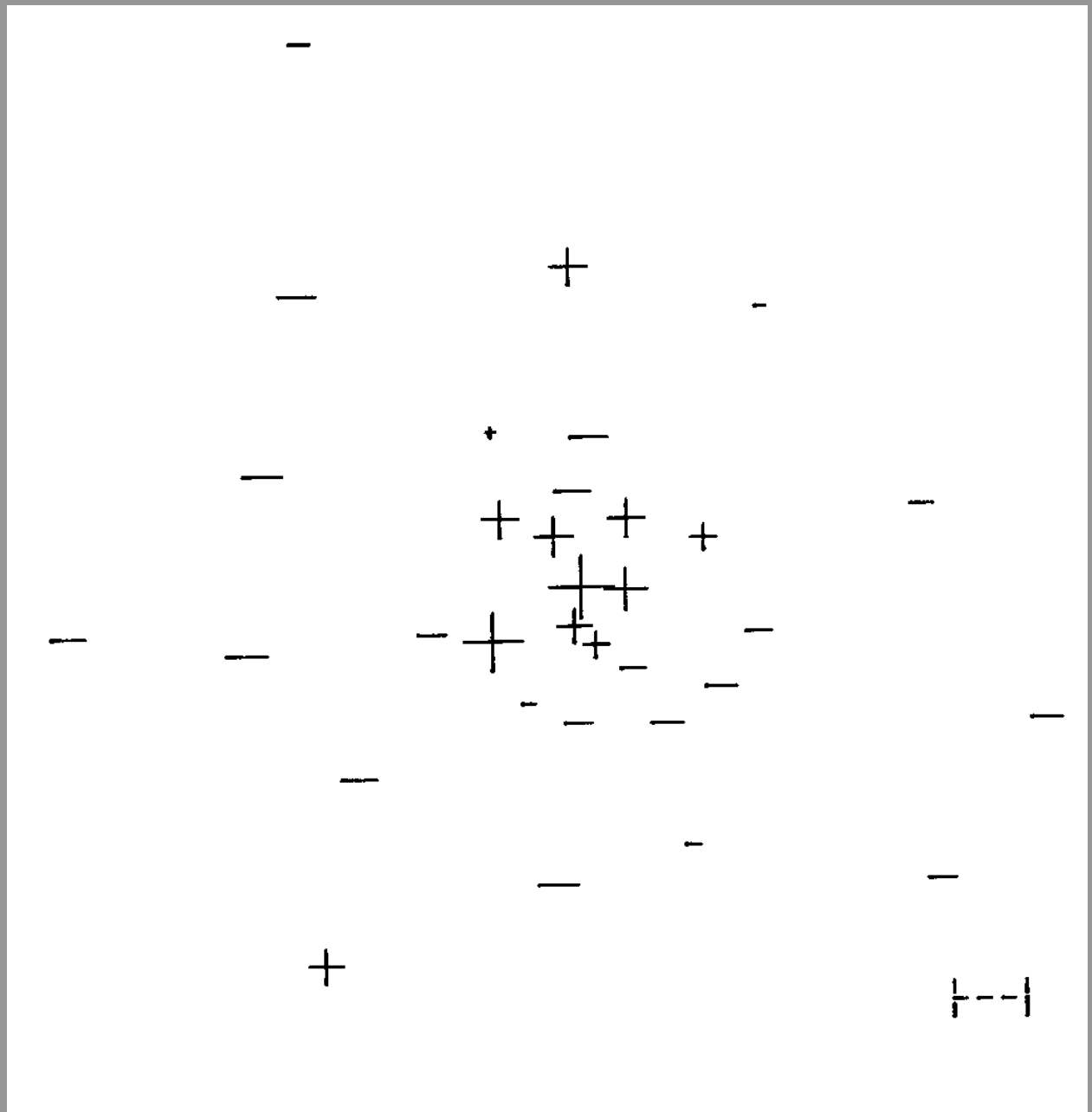
Adding across the table, the column marginals give the outsums (a.k.a. outdegree). Summing down the rows gives the insums (a.k.a indegree).

The ‘sending’ places (rows) are known as ‘sources’, and are shown on the map as negative signs.

The ‘receiving’ places (columns) are the ‘sinks’ and are shown as plus signs.

The size of the symbol represents the magnitude of the movement volume.

Munich Commuting (1939)



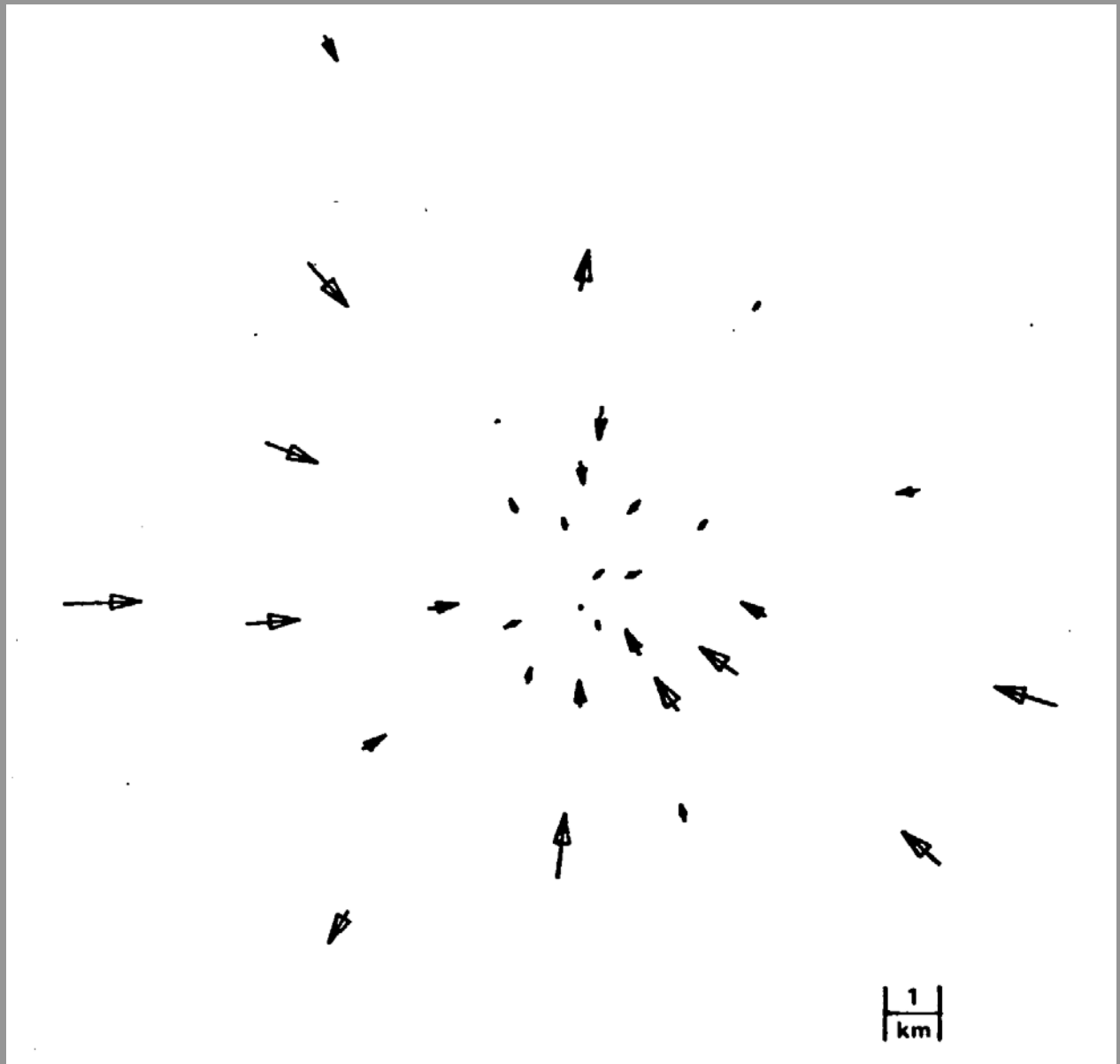
The movement from source to sink can be computed to show the direction and magnitude of the movement.

The computation is based on the asymmetry of the movement table.

Small directed vectors represent this movement on the next map.

Munich Commuting

Displacement vectors

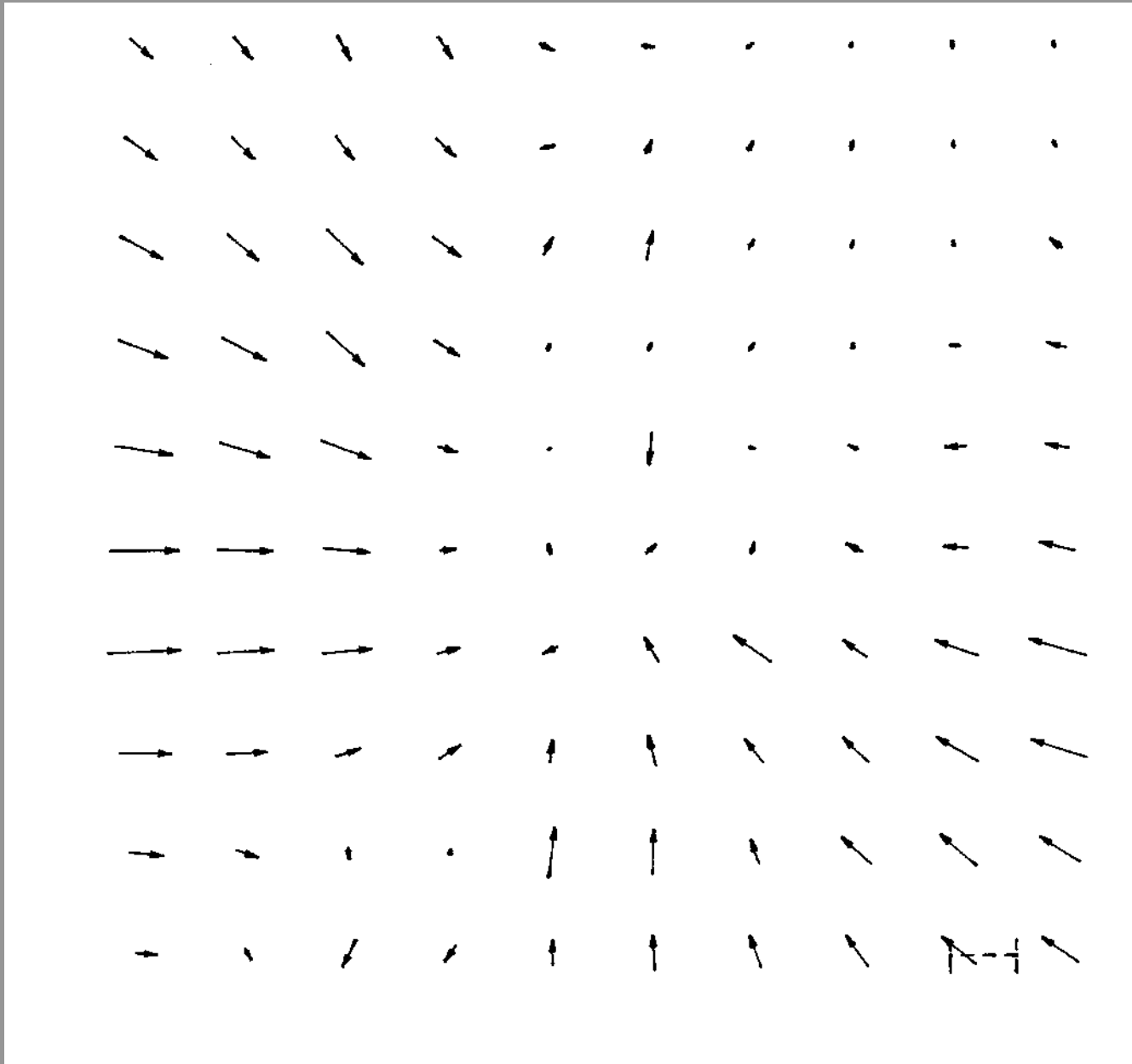


An interpolation is then performed to obtain a vector field from the isolated individual vectors.

This is done to simplify the mathematical integration needed to obtain the forcing function.

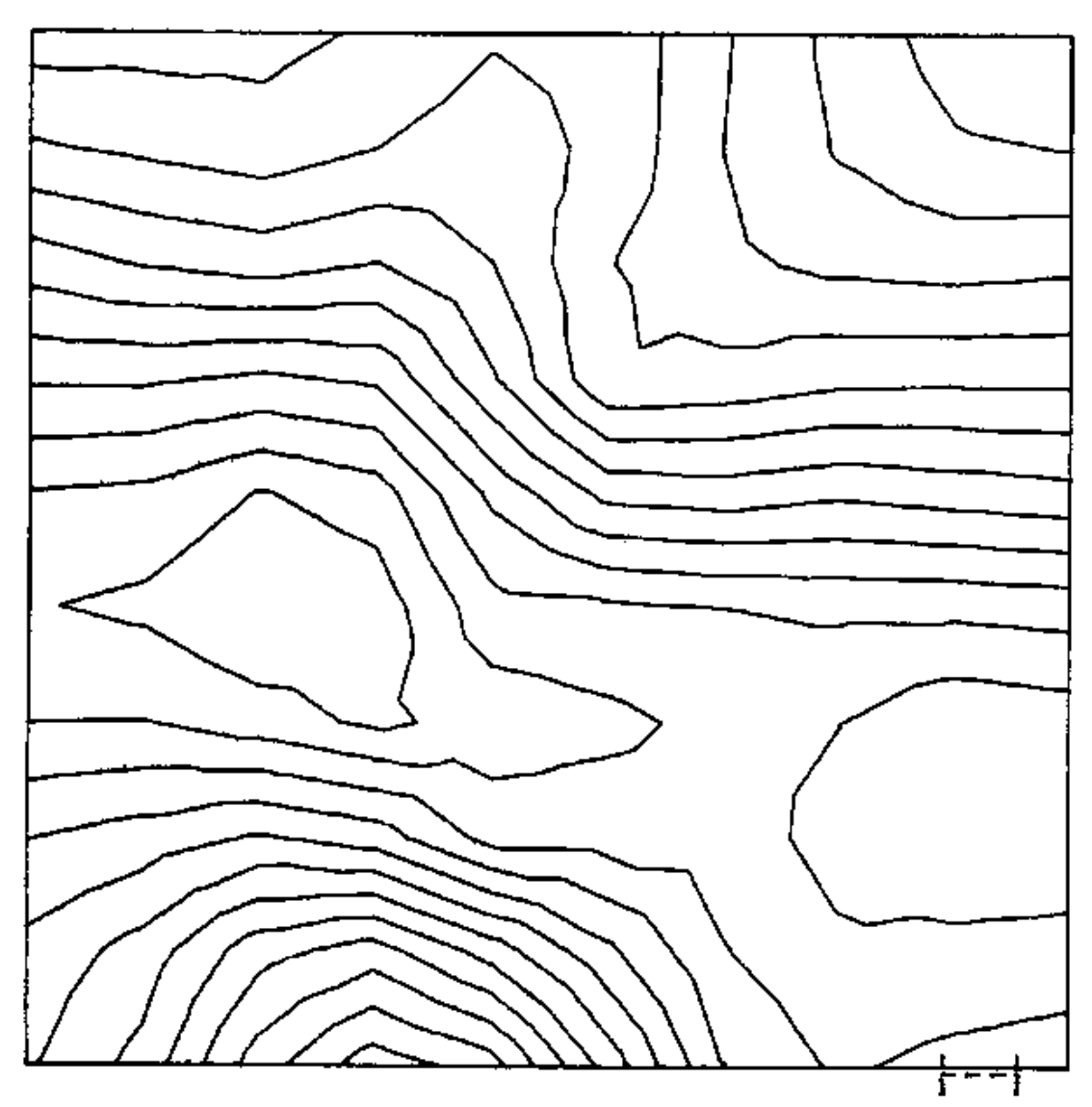
Interpolated Field

of displacement vectors



Computed Potential

based on the displacement vectors



The computed potential should have the vector field as its gradient.

This is a hypothesis that can be tested.

The base level of the potential is determined only up to a constant of integration.

The vector field, to be a gradient field, must be curl free. This can also be tested.

The attempt is now made to apply these ideas in a social space.

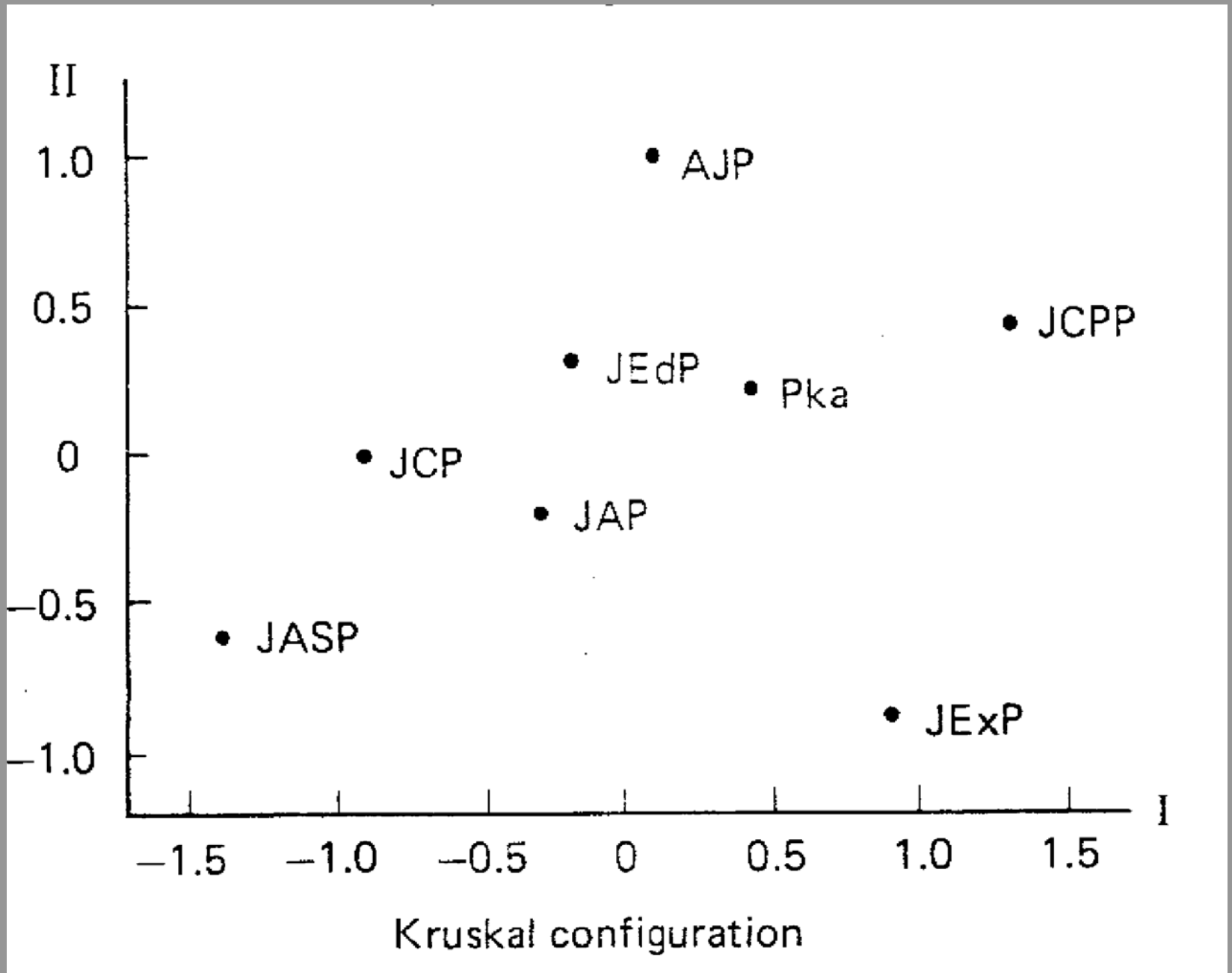
This can be considered a development of Lewin's *Topological Psychology* or his *Field Theory in the Social Sciences*.

The data represent citations between a small set of psychological journals. Larger citation tables are now also available.

Citations among psychology journals
Coombs et al 1970
Data from 1964

	AJP	*JASP*	*JAP*	*JCPP*	*JCP*	*JEdP*	*JExP*	*Pka*	*Total*
American Journal of Psychology	119	8	4	21	0	1	85	2	240
Journal of Abnormal and Social Psychology	32	510	16	11	73	9	119	4	774
Journal of Applied Psychology	2	8	84	1	7	8	16	10	136
Journal of Comparative and Physiological Psychology	35	8	0	533	0	1	126	1	704
Journal of Consulting Psychology	6	116	11	1	225	7	12	7	385
Journal of Educational Psychology	4	9	7	0	3	52	27	5	107
Journal of Experimental Psychology	125	19	6	70	0	0	586	15	821
Psychometrika	2	5	5	0	13	2	13	58	98
Total	325	683	133	637	321	80	984	102	3,265

In Journal Space



Journal to Journal Citations

To / From									X	Y	Net Flow
AJP	119	8	4	21	0	1	85	2	125	910	-85
JASP	32	510	16	11	73	9	19	4	-1382	-644	91
JAP	2	8	84	1	7	8	16	10	-261	-237	3
JCPP	35	8	0	533	0	1	126	1	1302	366	67
JCP	6	116	11	1	225	7	12	7	-924	-2	64
JEdP	4	9	7	0	3	52	27	5	-180	324	27
JExP	125	19	6	70	0	0	586	15	904	-924	-163
Pka	2	5	5	0	13	2	13	58	416	207	-4

AJP Am J of Psychology

JASP J of Abnormal & Social Psychology

JAP J of Applied Psychology

JCPP J of Comparative & Physiological Psychology

JCP J of Consulting Psychology

JEdP J of Educational Psychology

JexP J of Experimental Psychology

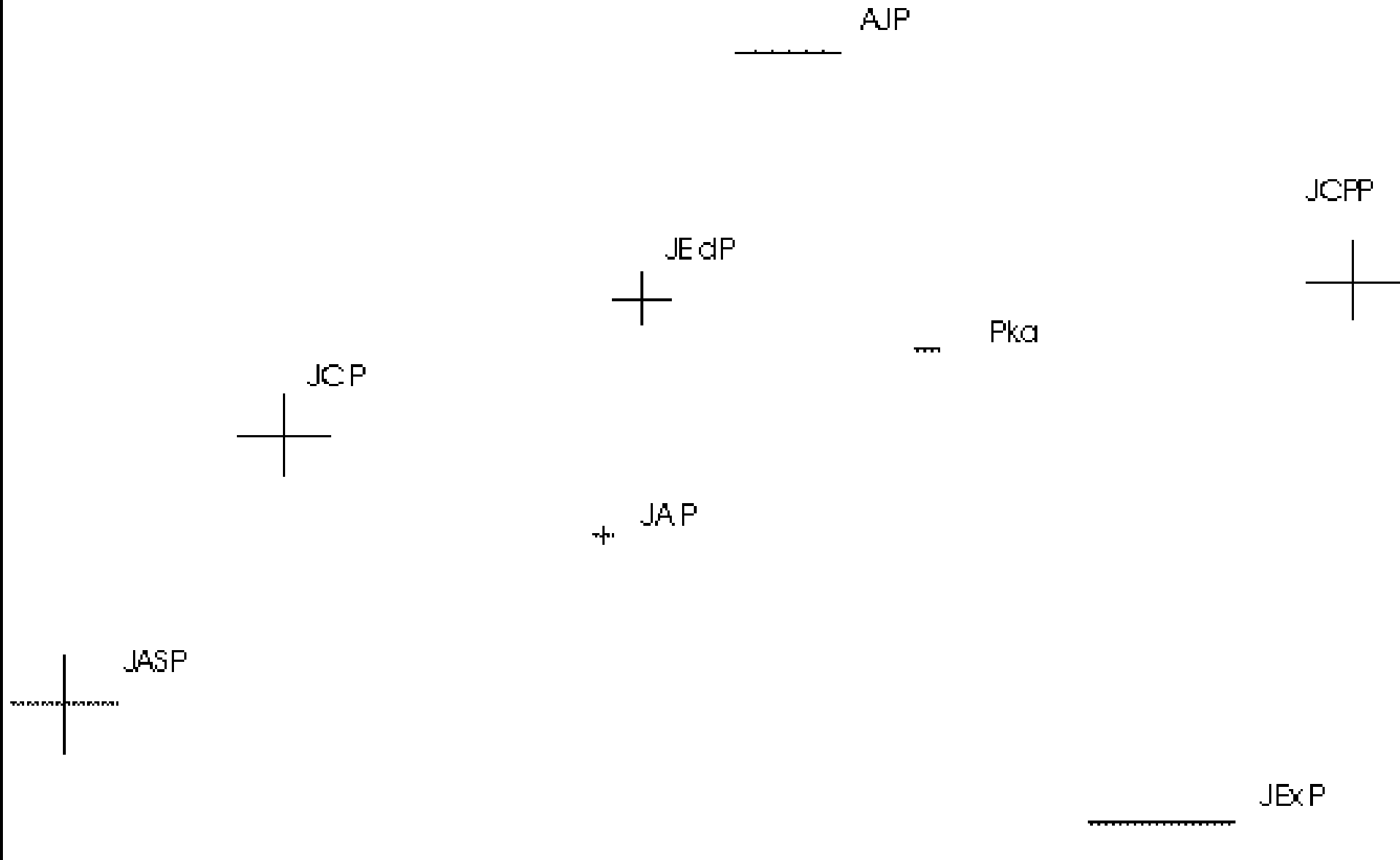
Pka Pyschometrika

C. Coombs, J. Dawes, A Twersky, 1970, *Mathematical Psychology*, Prentice Hall, Engelwood Cliffs, NY, Pages 73-75

The table gives the being-cited journal across the columns. But the information can be considered to move from that journal to the citing journal.

Therefore the transpose is used to produce the source to sink map.

Journal Sources and Sinks



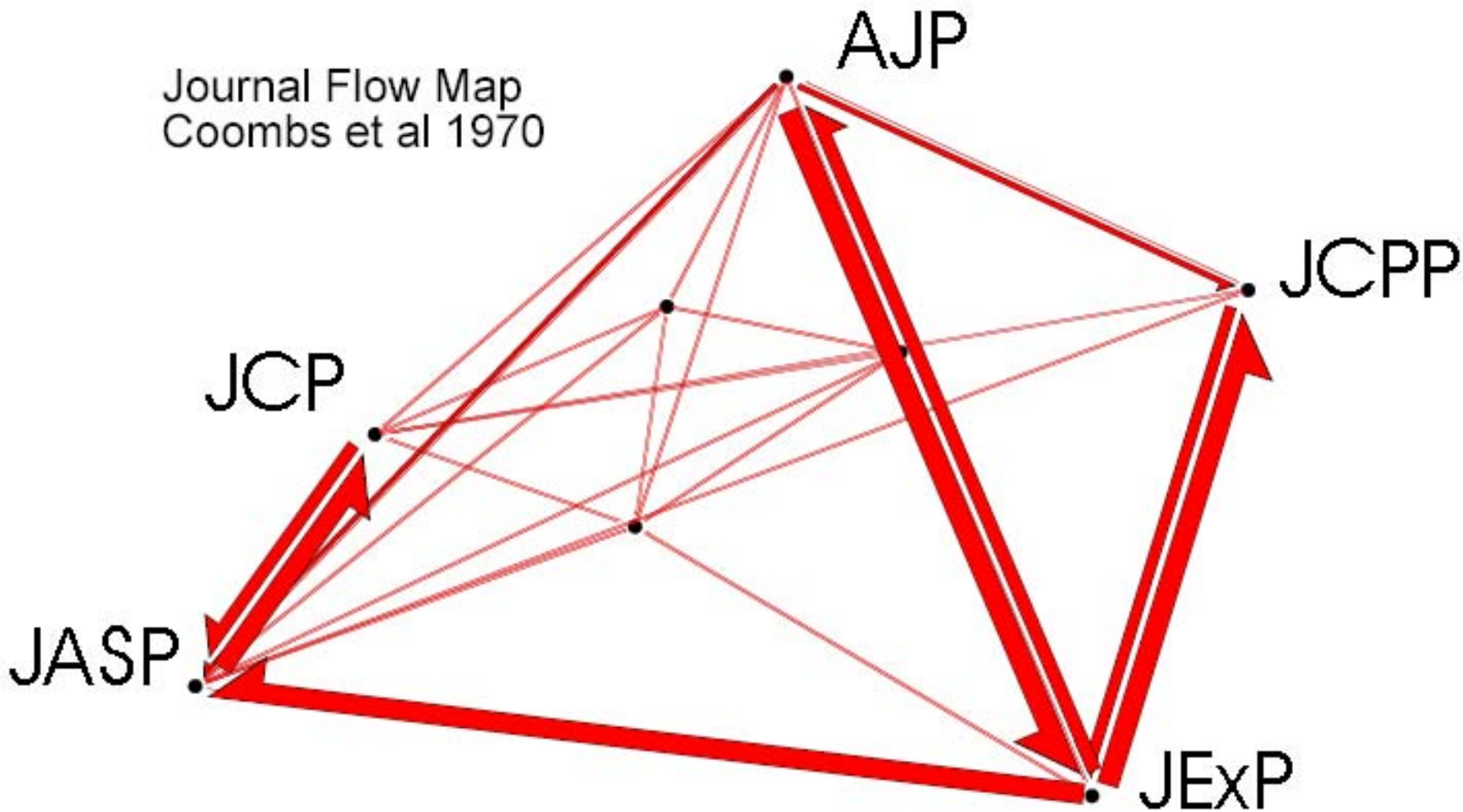
We now have an assignment problem. How to get 163 citations from JExP, 85 from AJP, & 4 from Pka to the 5 receiving journals. There are obviously many possibilities

One solution is to use the "Transportation Problem" (Koopmans, Kantorovich, ~1949): Minimize M..d.., subject to $M._J = O_I$, $M_I.= I_J$, $M_{IJ} >= 0$, given the distances computed from the coordinates and using the simplex method for the solution.

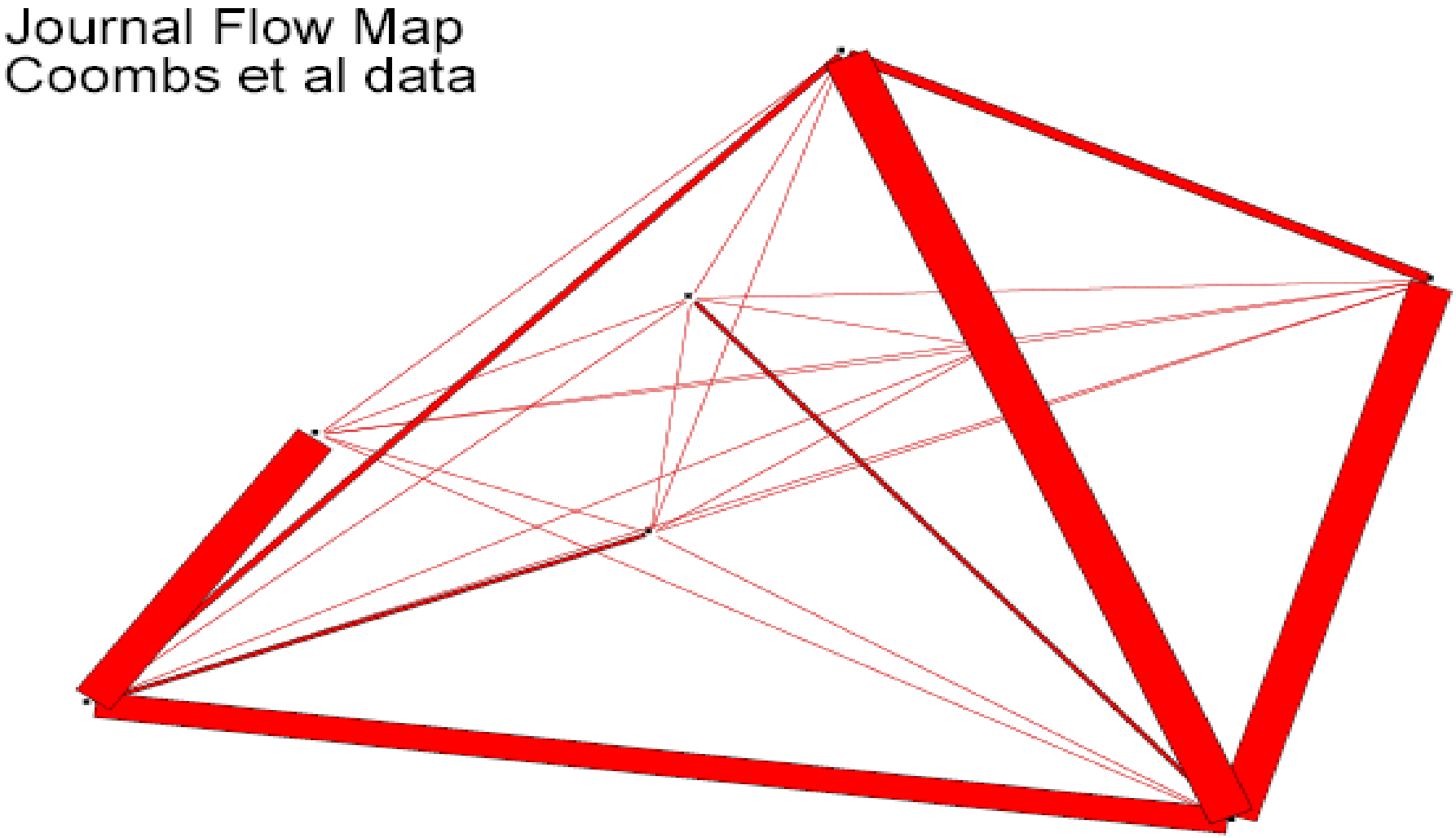
A more realistic solution is given by the quadratic transportation problem: Minimize M^2..d.., subject to the same constraints.

Both of these solutions result in discrete answers, and 'shadow prices'. We are looking for a spatially continuous solution that allows vectors and streamlines, in order to determine spatial flow fields and a continuous potential.

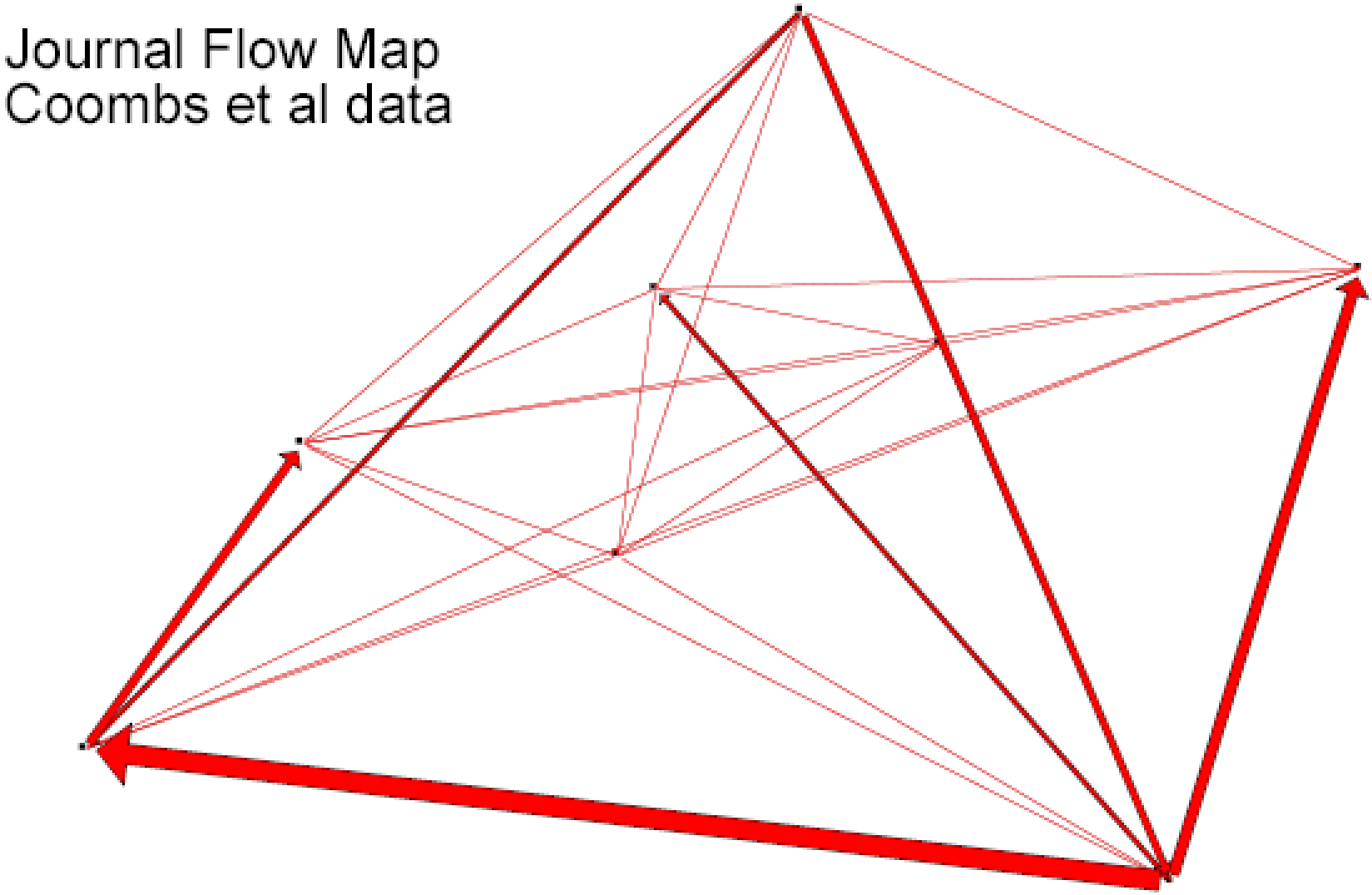
The observed two-way flow between the journals



The total flow between the journals



The net flow between the journals



The next step is to compute the displacements between the cited journals.

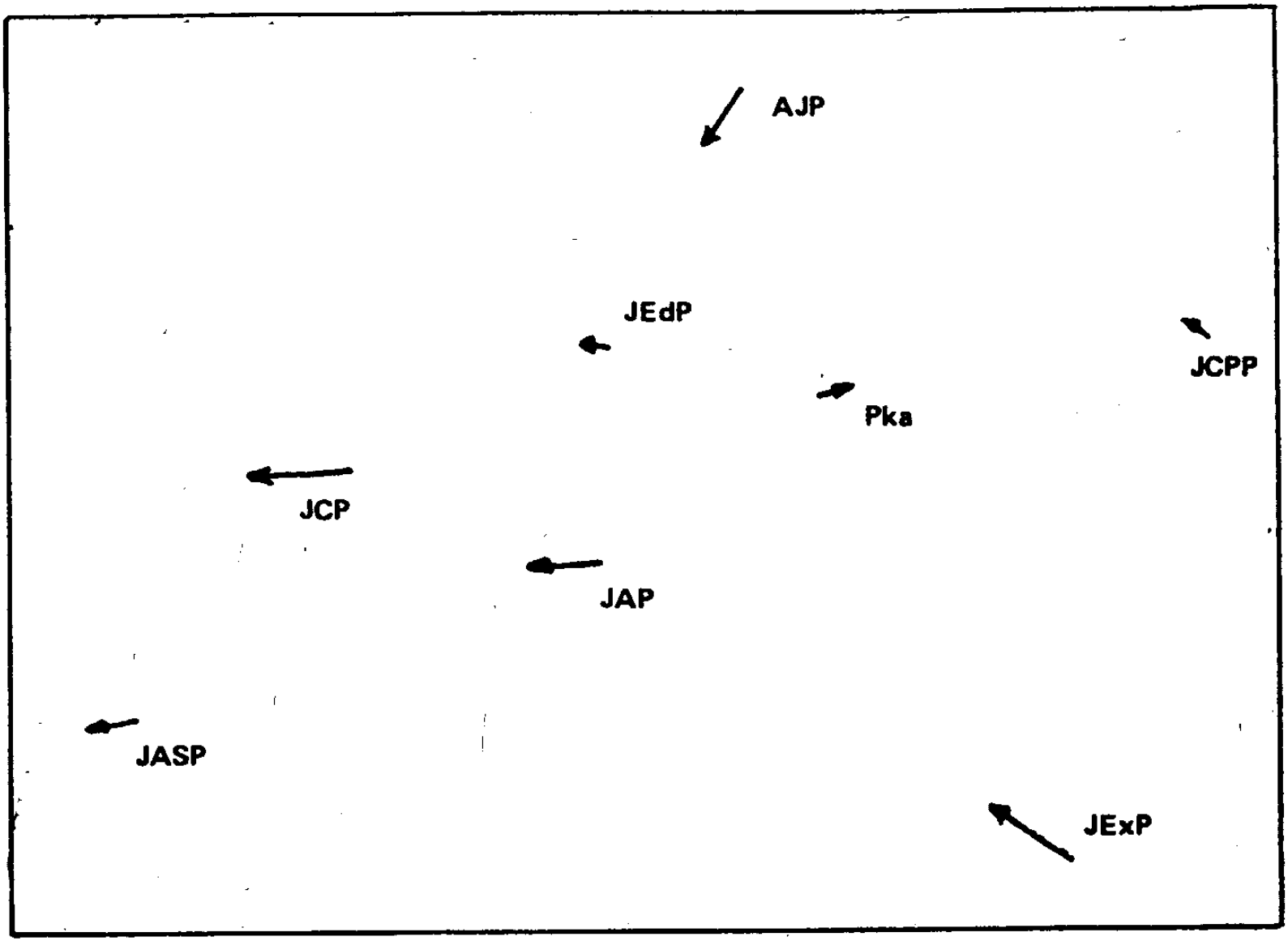
This is based on the asymmetry of the citations table.

The fundamental idea being that there exists a ‘wind’ making movement easier in some directions.

The mathematical details are given in a published paper.

W. Tobler, 1976,“Spatial Interaction Patterns”, *J. of Environmental Systems*, VI(4):271-301

Displacement between Journal Citations

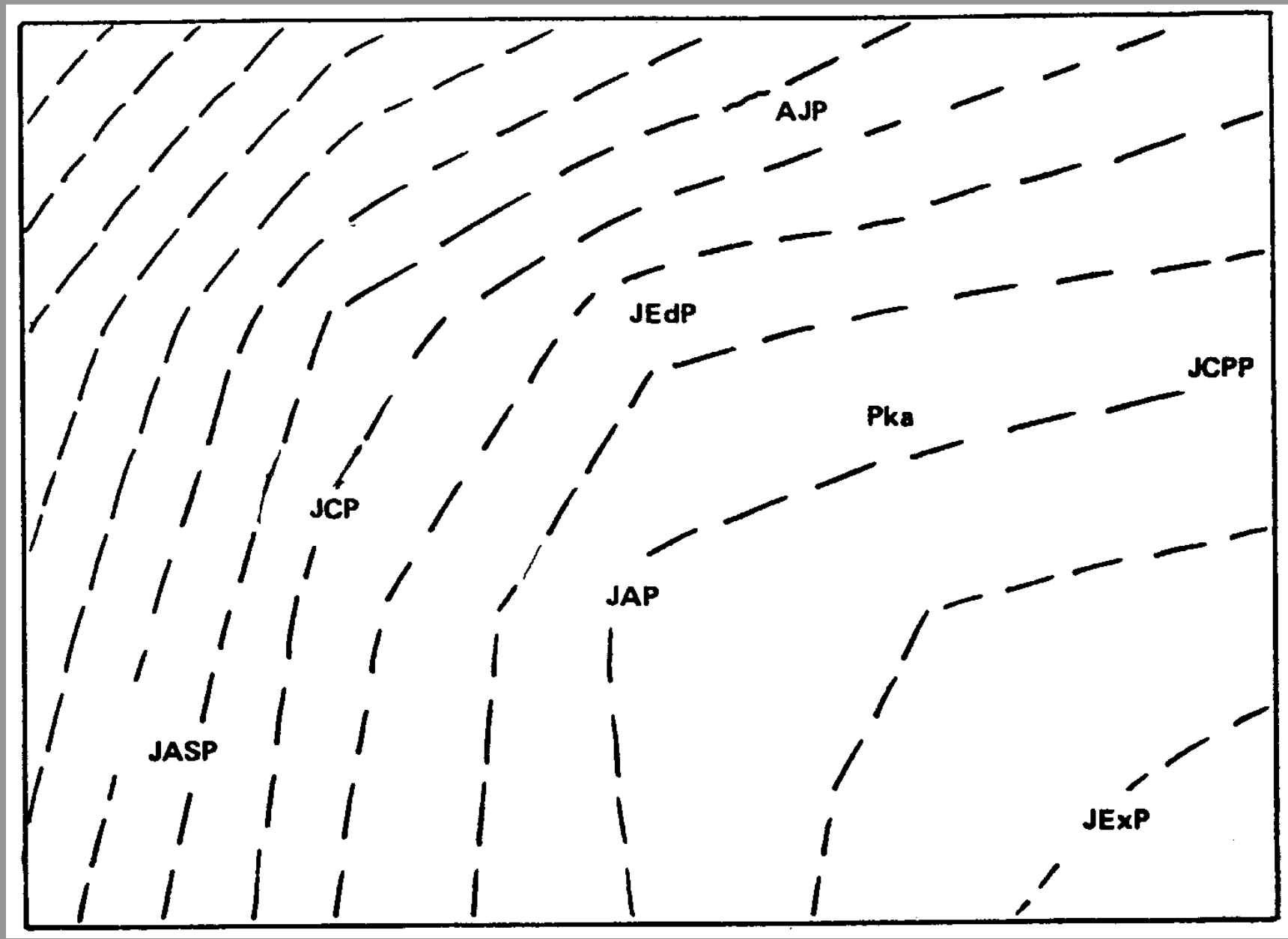


Then the potential is computed by integration.

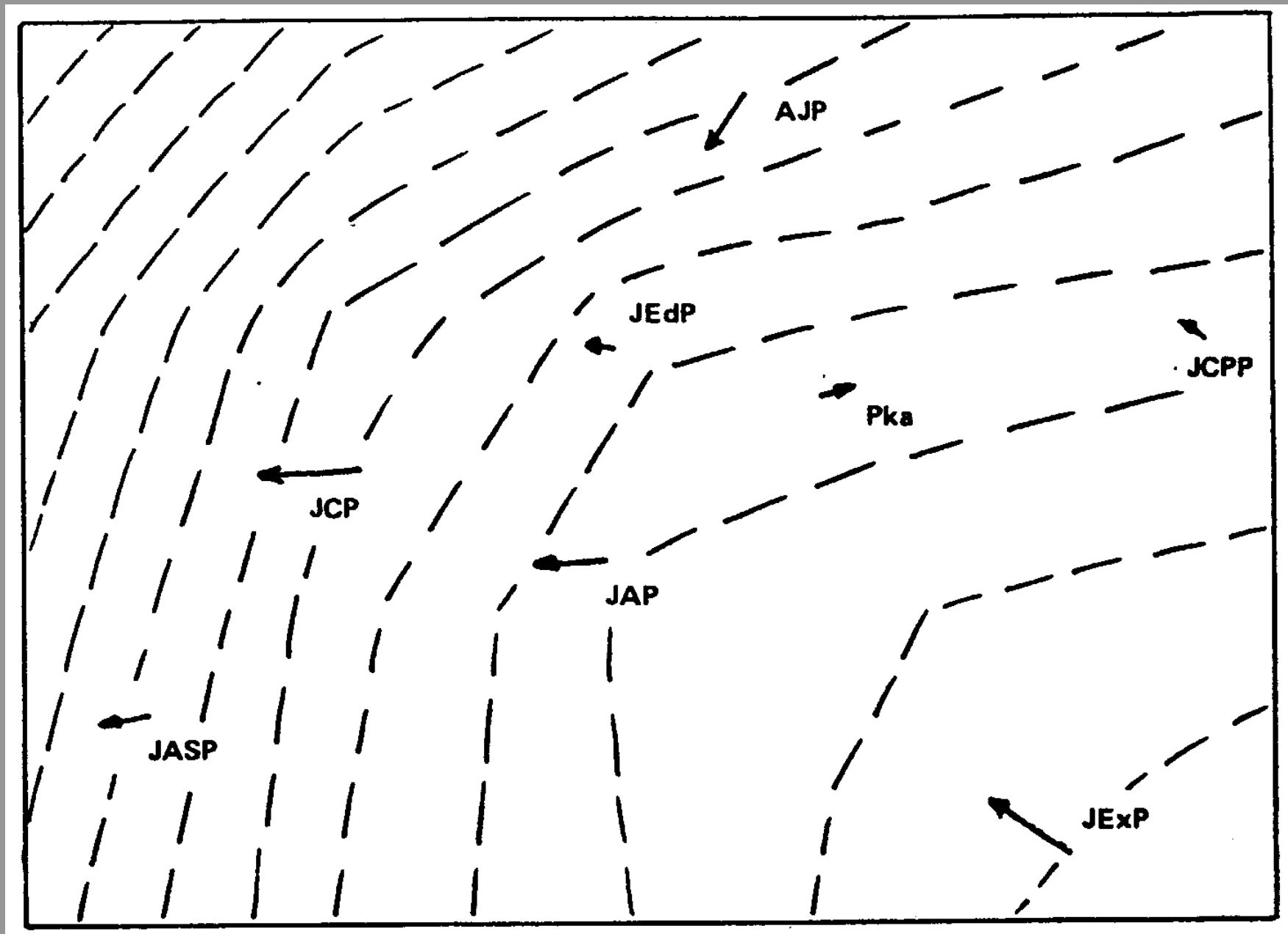
This potential should be such that its gradient coincides with the displacement vectors.

It may be necessary to use an iteration to obtain this result.

Journal Potential Function



Flow and Potential between Psychological Journals



Some questions

Suppose a new psychological journal were started. Where should it be inserted into in this space?

Does it make sense to treat journal citations as being located in a continuous two-dimensional social space?

Can other social data be treated in a similar fashion, for example social mobility tables?

And more general network data?

CONCLUSION

I have given some speculative thoughts on how one might represent network relations with vectors fields and scalar potentials in a continuous social space.

Still needed are error estimates.

Your comments are desired.

Thank you for your attention.

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C. Coombs, J. Dawes, A. Twersky, 1970, *Mathematical Psychology*, Prentice Hall, Englewood Cliffs, NY.

D. Fliedner, 1962, "Zyklonale Tendenzen bei Bevölkerungs und Verkehrsbewegungen in Städtischen Bereichen untersucht am Beispiel der Städte Göttingen, München, und Osnabrück", *Neues Archiv für Niedersachsen*, 10:15 (April 4): 277-294 (Table 2, p. 281, map following p. 285).

K. Lewin, 1936, *Principles of Topological Psychology*, McGraw Hill, New York

K. Lewin, 1951, *Field Theory in the Social Sciences*, Harper, New York.

W. Tobler, 1976, "Spatial Interaction Patterns", *J. of Environmental Systems*, VI (4) 1976/77, pp. 271-301.

W. Tobler, 1981, "A Model of Geographic Movement", *Geographical Analysis*, 13 (1): 1-20.

W. Tobler, 1996, "A Graphical Introduction to Surveying Adjustment", *Cartographica*, 33-42.

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