

# THE ASYMPTOTIC STRUCTURE OF SPACE-TIME <sup>1</sup>

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## ABSTRACT

Astronomical observations strongly suggest that our universe is now accelerating and contains a substantial admixture of dark vacuum energy. Using numerical simulations to study this newly consolidated cosmological model (with a constant density of dark energy), we show that astronomical structures freeze out in the near future and that the density profiles of dark matter halos approach the same general form. Every dark matter halo grows asymptotically isolated and thereby becomes the center of its own island universe. Each of these isolated regions of space-time approaches a universal geometry and we calculate the corresponding form of the space-time metric.

**Introduction.** The basic cosmological parameters that describe our universe have now been measured with compelling precision. Recent measurements of the cosmic microwave background radiation indicate that the universe is spatially flat [1]. Complementary measurements of the redshift-distance relation using Type Ia supernovae strongly suggest that the universe is now accelerating [2]. Taken together, the current astronomical data argue for a cosmological model with matter density  $\Omega_{m,0} = 0.3$ , dark vacuum energy density  $\Omega_{v,0} = 0.7$ , curvature constant  $k = 0$ , and Hubble constant  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . Although the time dependence of the dark energy has not been fully determined, the current data are consistent with the vacuum energy density being temporally constant, as this work assumes.

This newly consolidated cosmological model represents a milestone in our understanding of the universe. The large scale space-time of the universe is now known and its corresponding metric can be specified. In the absence of structure formation, the universe is homogeneous and isotropic, and

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the space-time would be described by the maximally symmetric Robertson-Walker metric [3]. Since the universe does contain gravitationally collapsed structures, however, the metric that describes space-time is one step more complicated — it must include the contribution from the structures.

If the universe is already starting to accelerate, as observations indicate, then structure formation is virtually finished. In the relatively near future, the universe will approach a state of exponential expansion and growing cosmological perturbations will freeze out on all scales. Existing structures will grow isolated. Because the parameters of our universe are now relatively well known, this future evolution of cosmological structure can now be predicted with a high degree of confidence. Several recent papers have begun to explore the possible future effects of vacuum energy density [4–6], and demonstrate that the universe will indeed break up into a collection of “island universes”, each containing one gravitational bound structure.

In this essay, we present the results of a recent series of numerical simulations that describe the evolution of structure in a universe dominated by dark vacuum energy (with  $\Omega_{v,0} = 0.7$  at the present epoch). These numerical experiments show that each gravitationally bound halo structure grows isolated and that its density profile always approaches the same general form. After describing the numerical simulations in greater detail and specifying the form of this density profile, we construct the metric for each isolated patch of space-time. Each island universe attains the same geometry and we find the universal form for the metric that describes these patches of space-time.

**Numerical Simulations.** As part of a more comprehensive study of structure formation in the future of an accelerating universe, we have performed a series of numerical simulations [7]. This set of cosmological simulations used the GADGET numerical package [8] and was run on an Intel parallel cluster (at U. Michigan Center for Academic Computing). The simulations were set up using a standard suite of initial conditions starting at scale factor  $a = 0.05$  [9], and were evolved forward into the future until the scale factor had grown to  $a = 100$ . The cosmology was chosen to have the standard parameters described above, with  $\Omega_{m,0} = 0.3$ ,  $\Omega_{v,0} = 0.7$ , and  $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$ . All of the work reported here uses this choice of cosmological parameters.

The simulations followed the evolution of a cubic, periodic region with comoving linear size 366 Mpc. Only the evolution of the dark matter was computed and we only obtain information about dark matter halos at relatively large spatial scales. The numerical resolution was set by using

$128^3$  dark matter particles, each with an effective mass of  $9.57 \times 10^{11} M_\odot$ . The force resolution had a constant value of 285 kpc. With this force and mass resolution, the inner workings of the galaxy formation process are not well-resolved, but larger scale structures — the dark matter halos containing most of the mass — are well characterized.

Hundreds of dark matter halos form within the volume of the universe studied by the simulations. The evolution from high redshift to the present follows the now-standard scenario. Most of the structure in the universe is already in place by the present epoch with  $a = 1$ . As the universe evolves into the future, the structures grow more defined and more isolated. As the accelerating universe continues to expand, bound structures separate rapidly from each other [5–7]. In the long term, existing cosmic structures remain bound but grow isolated, as illustrated by Figure 1. A large cluster will become effectively isolated in about 120 Gyr, whereas a smaller structure (like our Local Group) will grow isolated in about 180 Gyr. These structures will be embedded within an accelerating universe with a constant horizon scale, where the horizon distance  $r_H$  is given by

$$r_H = \chi^{-1} = \frac{c}{H_0} \frac{2}{\pi} \left( \frac{15}{\Omega_{v,0}} \right)^{1/2} \approx 12,600 \text{ Mpc}. \quad (1)$$

This horizon distance  $r_H$  is not the same as the particle horizon, but rather is essentially the Hubble radius. The distance scale  $r_H$  provides an effective “boundary for microphysics” within the much larger space-time of the universe [3]. The acceleration of the universe effectively divides our present-day space-time into many smaller “island universes”. For this discussion, we consider the center of each dark matter halo to lie at the center of its own island universe. As we show next, these dark matter halos develop density profiles with a universal form in both time and mass (for our chosen cosmology).

**Generic Form for the Density Profile.** Numerical simulations indicate that cosmic structures, from galaxies to clusters, tend to develop the same basic form for the density profiles of their dark matter halos [7,10]. As a result, every island universe will attain the same geometry for its space-time. In order to estimate the geometry of these space-times, we must first estimate the (nearly universal) form for the density profile of the dark matter halos.

Using the results from our numerical simulations, we have constructed a composite dark matter halo from the 50 largest halos produced by one realization of the simulation. These 50 halos are normalized so that the mean interior density has the same value at the spatial scale  $r_{200}$  (the

radius at which the enclosed density is 200 times the critical density). With this normalization, the individual dark matter halos show relatively little dispersion (with a mean of about 35 percent) and hence the composite average is well defined. The profiles are close to being spherically symmetric (this point is discussed in Refs. [7,11]) so we consider density distributions that depend only on radius. The composite profiles are shown in Figure 2 for varying cosmological epochs, starting from the present (top curve) and extending to  $a = 100$  (bottom curve). Notice how the density profiles display the same characteristic form over a wide range of epochs, with each subsequent profile being a stretched version of the previous one. This fact that dark matter halos tend to approach a universal form has been noted earlier [10], although the previous composite profiles were more limited in spatial extent and did not match smoothly onto the background universe.

The density profile at every cosmological epoch can be fit with a spherical density profile of the form

$$\rho(r) = \frac{\rho_0}{r/r_S[1 + (r/r_S)^p]^{3/2}} [1 + r/r_\infty]^{1+3p/2}. \quad (2)$$

This profile describes the basic radial dependence of dark matter halos in the inner regions and matches smoothly onto the background density of the universe at large radii. Using the parameters  $r_S = 0.50 r_{200}$  and  $p = 1.8$ , the above functional form provides a good fit to the numerically determined density profiles for all epochs. In order to match the profile onto the background density of the universe, the remaining parameter  $r_\infty$  must scale according to  $r_\infty = r_{\infty(0)} a^{6/(3p+2)}$ , where the present-day value  $r_{\infty(0)} = 4.7 r_{200}$ . The resulting fits to the density profiles are shown as the dashed curves in Figure 2. This relatively simple function (eq. [2]) applies over a factor of 10 in halo mass scale, and fits the numerically calculated density profiles over nearly 5 decades in radial scale, 11 decades in density, and a factor of 100 in the scale factor  $a$ . Over this range, the RMS departure of the fitted functions (eq. [2]) from the composite averages is 0.13 in  $\log_{10} \rho$  (which corresponds to differences of  $\sim 35\%$  in  $\rho$ ).

**Asymptotic Form for the Metric.** Using the specified form (eq. [2]) for the density profile, we can now determine the line element  $ds^2$  for the space-time within the horizon distance  $r_H$  [12]. The center of the coordinate system is taken to be at the center of the cluster (or galaxy) and the mass distribution is assumed to be spherically symmetric. We begin by writing the line element in

the form

$$ds^2 = -\left(1 - A(r) - \chi^2 r^2\right) dt^2 + \left(1 - B(r) - \chi^2 r^2\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (3)$$

where we have explicitly separated out the contribution due to the cosmological constant, which is set by the parameter  $\chi^2 \equiv (2\pi^3/45)^{1/2} \Lambda^2/M_{\text{pl}}$  (where the energy scale  $\Lambda \approx 0.0003$  eV for  $\Omega_{\text{v},0} = 0.7$ ). In an “empty” universe containing only vacuum energy, the line element would have the above form with  $A = 0 = B$ . Because of the vacuum contribution, the metric contains an outer horizon at  $r_H = \chi^{-1}$ . This outer horizon supports the emission of radiation through a Hawking-like mechanism [13] and hence the future universe will be filled with a nearly thermal bath of radiation with temperature  $T \sim \chi \sim 10^{-33}$  eV and characteristic wavelength  $\lambda \sim r_H \sim 12,600$  Mpc. This radiation will become the dominant background radiation field after about one trillion years. The functions  $A(r)$  and  $B(r)$  take into account additional curvature due to the mass distribution, which has a density profile given by equation [2].

If we adopt units in which  $c = 1$  (and hence  $G = M_{\text{pl}}^{-2}$ ), the function  $B(r)$  can be written in the form

$$B(r) = 2G \frac{m(r)}{r} = 8\pi G \frac{1}{r} \int_0^r \rho(\tilde{r}) \tilde{r}^2 d\tilde{r}, \quad (4)$$

where the density profile  $\rho(r)$  is given by equation [2]. Since we are interested in the asymptotic form for the metric, we can consider late times for which the scale  $r_\infty$  is stretched beyond the horizon  $r_H$ . In this limit, the function  $B(r)$  can be simplified to the form

$$B(r) = 4\pi G \rho_0 r_S^2 \frac{1}{\xi} \int_0^\xi \frac{x dx}{(1+x^p)^{3/2}} \equiv \eta_0 \beta(\xi). \quad (5)$$

In the second equality, we have defined the parameter  $\eta_0 = 4\pi G \rho_0 r_S^2$  which sets the “strength” of the curvature and the dimensionless function  $\beta(\xi)$  which specifies the radial dependence of the metric coefficient (where  $\xi = r/r_S$ ). For typical values, the strength parameter  $\eta_0 \approx 10^{-6}$ , indicating that the departure from flatness is relatively small. The resulting function  $\beta(\xi)$  is shown in Figure 3.

The function  $A(r)$  is related to the usual gravitational potential  $\Phi$  through the definition  $e^{2\Phi} \equiv 1 - A(r)$  [12], where the potential is defined through the source equation

$$\frac{d\Phi}{dr} = \frac{G[m(r) + 4\pi r^3 p]}{r(r - 2Gm)}. \quad (6)$$

In this setting, the mass is dominated by collisionless dark matter particles and the pressure  $p$  is negligible. Furthermore, the potential is small so that we can use the approximation  $e^{2\Phi} \approx 1 + 2\Phi$

and hence  $A(r) = -2\Phi(r)$ , with  $\Phi$  given by the integral of equation [6]. As a result, the function  $A(r)$  can be written in the form

$$A(r) = A_\infty - \int_0^r \frac{2Gm(r)}{r} \frac{dr/r}{1 - 2Gm(r)/r} = \eta_0 \left[ \alpha_\infty - \int_0^\xi \frac{\beta(\xi)}{1 - \beta(\xi)} \frac{d\xi}{\xi} \right] \equiv \eta_0 \alpha(\xi), \quad (7)$$

where  $\eta_0$  and  $\beta(\xi)$  are as defined previously. We have also defined an analogous dimensionless function such that  $A(r) = \eta_0 \alpha(\xi)$ . The quantity  $A_\infty$  and its dimensionless counterpart  $\alpha_\infty$  are defined so that the potential  $\Phi$  vanishes at spatial infinity [12]. As before, the dimensionless parameter  $\eta_0 = 4\pi G \rho_0 r_S^2 \approx 10^{-6}$  sets the level of the curvature. The resulting function  $\alpha(\xi)$  is shown in Figure 3. This completes the specification of the metric.

**Summary.** In this essay, we have constructed the asymptotic form of the metric that describes space-time in our cosmological future. Using numerical simulations, we have demonstrated that individual gravitationally bound structures will become isolated in the near future and thereby become their own “island universes” (Figure 1). Each of these gravitationally bound entities — dark matter halos — will attain a characteristic form for its density distribution (see Figure 2 and equation [2]). Finally, each bound structure will live at the center of its own island universe, and the metric of the surrounding space-time can be described by a line element of the form

$$ds^2 = -\left(1 - \eta_0 \alpha(\xi) - \chi^2 r^2\right) dt^2 + \left(1 - \eta_0 \beta(\xi) - \chi^2 r^2\right)^{-1} dr^2 + r^2 d\Omega^2, \quad (8)$$

where  $\eta_0 = 4\pi G \rho_0 r_S^2$ ,  $\xi = r/r_S$ , and where  $\alpha(\xi)$  and  $\beta(\xi)$  are shown in Figure 3. Astronomical entities (planets, stars, and galaxies) living within the universe will continue to evolve over much longer time scales [4,14], but space-time itself can be described by equation [8] for the vast majority of the total life of the universe.

The idea that some type of dark energy could affect the expansion of the universe dates back to Einstein’s original introduction of a cosmological constant. Although this idea has been called Einstein’s greatest blunder, the currently observed cosmic acceleration suggests that this concept may become one of Einstein’s greatest legacies. The motivation for the cosmological constant was to keep the cosmos static. In a twist of irony, the observed dark vacuum energy does not make the universe static, but rather drives it to expand at an accelerating rate. But even though the universe expands and changes, and its constituent astrophysical objects age, this essay shows that the local space-time metric does approach a “static” asymptotic form (eq. [8]).

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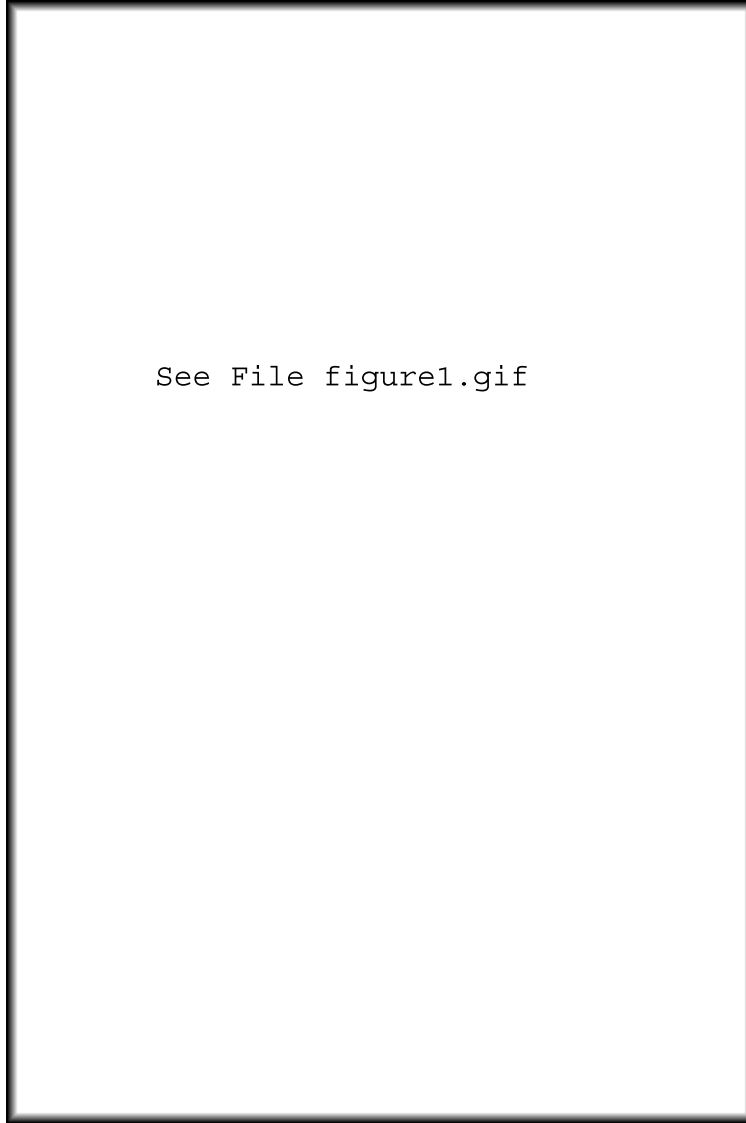


Fig. 1.— Results of numerical simulations of structure formation in an accelerating universe with a constant density of dark vacuum energy. Top panel shows a portion of the universe at the present epoch when the scale factor  $a = 1$  (cosmic age 14 Gyr). The box in the upper panel shows the region that expands to become the picture in the center panel, which shows a portion of the universe at a future epoch when  $a = 11.4$  (cosmic age 54 Gyr). The box in the center panel expands to become the picture shown in the bottom panel when the scale factor  $a = 100$  (cosmic age 92 Gyr). By this future epoch, the dark matter halo in the center of the bottom panel has grown effectively isolated.



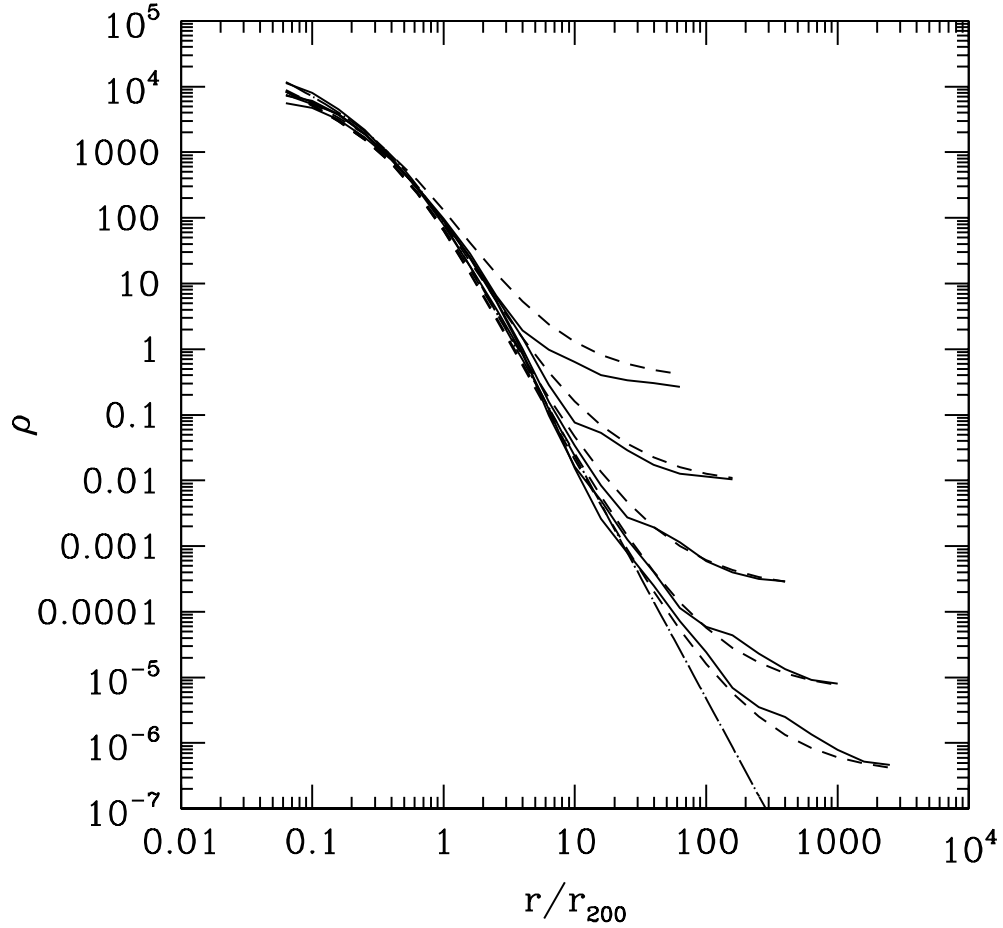


Fig. 2.— The density profile for dark matter halos. Each curve shows the average of the 50 largest dark matter halos in the numerical simulation for a given time, ranging from the present epoch  $a = 1$  (top curve) to  $a = 100$  (bottom curve). The numerically determined results (averaged together) are shown as the solid curves. The dashed curves show the fits to the numerical results obtained from the analytic density profile of equation [2]. The dot-dashed curve shows the asymptotic form of the density profile (in the limit  $t, a \rightarrow \infty$ ).

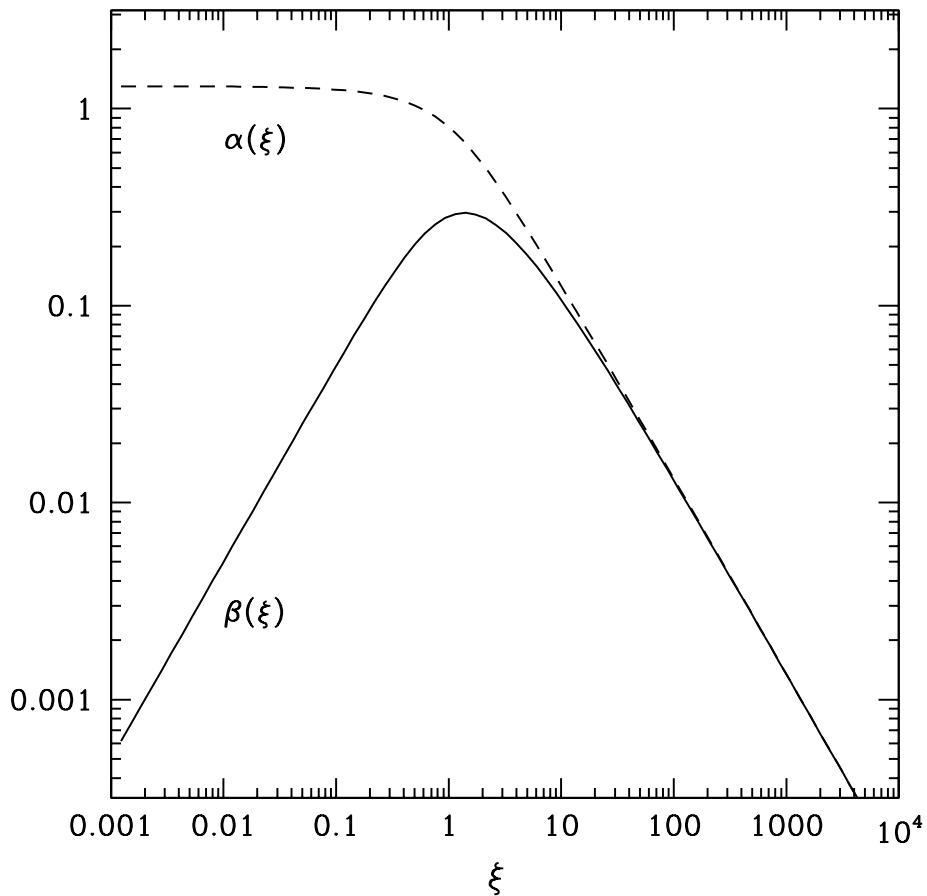


Fig. 3.— The dimensionless functions  $\alpha(\xi)$  and  $\beta(\xi)$  appearing in the asymptotic form of the space-time metric of equation [8]. The functions are plotted versus the dimensionless radial coordinate  $\xi = r/r_S$  (see text). These functions, in conjunction with equation [8], specify the line element for the majority of the life of the universe.

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