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Hierarchical multilevel optimization for reliability target allocation in probabilistic design of decomposed systems

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Analytical target cascading (ATC) is a methodology for translating system-level design targets to subsystem and component design specifications in hierarchically decomposed optimal system design problems. In previous work we extended the ATC formulation to account for uncertainties, where the bounds of the probabilistic design constraints were chosen arbitrarily and held fixed during the ATC process. In this work, we extend the probabilistic ATC formulation to include reliability targets in the vector of cascaded quantities. In this manner, we quantify the optimality-reliability tradeoffs for each element of the decomposed system and compute the probabilistic constraint bounds required to satisfy the overall system reliability target.

1 Introduction

Design optimization of complex engineering systems can often be accomplished only by decomposition. The system is partitioned into subsystems, the subsystems are partitioned into components, the components into parts, and so on. The outcome of the decomposition process is a multilevel hierarchy of system-constituent elements. Analytical target cascading (ATC), is a methodology for solving hierarchically decomposed multilevel optimal design problems. In ATC, system design targets are cascaded to subsystems and components using the model-based hierarchy. An optimization problem is formulated and solved for each element to minimize deviations of local responses from propagated targets. Solving the subproblems using appropriate coordination strategies yields overall system optimality and consistency. Several case studies have demonstrated the usefulness of ATC, e.g., on automotive engineering applications [1,2].

In recent work, we extended the deterministic ATC formulation to probabilistic design in order to account for uncertainties [3]. Uncertain quantities were modeled as random variables, and we used the means of the random variables as optimization variables assuming known standard deviation. The ATC optimization problems were reformulated as reliability-based design optimization (RBDO) problems. In RBDO, reliability is defined as the statistical estimate of satisfying probabilistically formulated design constraints. Although reliability is nowadays considered to be a system design criterion of primary importance, its target value is prescribed more or less arbitrarily as opposed to being assigned optimally. RBDO methods require the specification of reliability targets R_{t_j} for each of the M probabilistic design constraints:

$$\min_{\mu_{\mathbf{X}}} f(\mu_{\mathbf{X}}) \text{ subject to } P[g_j(\mathbf{X}) \leq 0] = R_j \geq R_{t_j} \; j = 1, 2, \ldots, M,$$

where $P[\cdot]$ denotes probability measure and R denotes reliability. One recently reported RBDO method considers an aggregated system reliability constraint that allows flexibility regarding the satisfaction of the probabilistic design constraints as long as the overall system reliability target is met [4]. This formulation considers the probabilistic optimal system design problem as a series system since the violation of just one of these constraints would correspond to a system failure. However, it does not consider decomposition-based system design optimization.

Reliability allocation in decomposed systems has been studied for parallel-series systems [5,6]. A parallel-series system consists of n subsystems arranged in series, each of which is comprised by m_i parallel components, where $i = 1, 2, \ldots, n$. Given reliability target values R_{t_i} for each subsystem i, a cost minimization problem was formulated to determine the optimal number of components $m_i \; \forall i$ and the required component reliability values $R_{t_{ij}}$, where $j = 1, 2, \ldots, m_i$ (redundancy and reliability allocation). Cost functions that are monotonic with respect to reliability were used, and it was shown that all parallel components within a subsystem must have identical reliability. These studies, however, are not concerned with design optimization. Furthermore, the system decomposition structure is limited to that of a parallel-series system.

2 Reliability target allocation

In this work, we extend the probabilistic ATC formulation presented in [3] to conduct reliability target allocation, i.e., to cascade reliability target values to each component of the decomposed system. In a decomposed system, the reliability of each element (subsystem, component, etc.) depends not only on its ability to satisfy the probabilistic design constraints, but also on the reliability of "children" elements. Consider the generic element j at level i of the hierarchy shown in Figure 1. The reliability of the element $R_{E_{ij}}$ is a function of its "design constraint" reliability $R_{C_{ij}}$ and the children reliability values $R_{E_{(i+1)k}}$, $k = 1, 2, \ldots, n_{ij}$, where n_{ij} is the number of children elements. We exploit this functional dependency to include reliability

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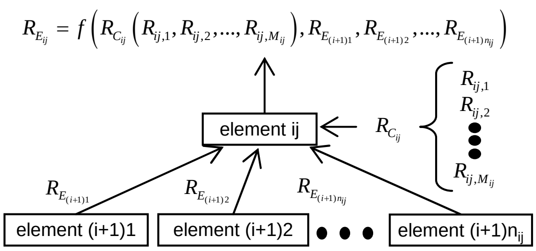

Fig. 1 Reliability estimation for element j at level i of the hierarchy

target allocation in the ATC formulation. ATC operates by formulating and solving an optimization problem for each element in the hierarchy that minimizes discrepancies between what upper-level elements "want" and what lower-level elements "can." Similarly, if design variables are shared among some elements at the same level, their consistency is coordinated by their common parent element at the level above. The responses of element j at level i are $\mathbf{Z}_{ij} = \mathbf{f}_{ij}(\mathbf{Z}_{(i+1)1}, \ldots, \mathbf{Z}_{(i+1)n_{ij}}, \mathbf{X}_{ij}, \mathbf{Y}_{ij})$, where $\mathbf{Z}_{(i+1)1}, \ldots, \mathbf{Z}_{(i+1)n_{ij}}$ denote the element's children responses, $\mathbf{X}_{ij}$ represent local design variables, and $\mathbf{Y}_{ij}$ denote local shared design variables (i.e., design variables that this element shares with other elements at the same level). The ATC problem for that element is formulated as

$$\begin{gathered}
\min \|\mu_{\mathbf{Z}_{ij}}(\mu_{\mathbf{Z}_{(i+1)1}}, \ldots, \mu_{\mathbf{Z}_{(i+1)n_{ij}}}, \mu_{\mathbf{X}_{ij}}, \mu_{\mathbf{Y}_{ij}}) - \mu^u_{\mathbf{Z}_{ij}}\|_2^2 + \textstyle\sum_{k=1}^{n_{ij}} \|\mu_{\mathbf{Z}_{(i+1)k}} - \mu^l_{\mathbf{Z}_{(i+1)k}}\|_2^2 + \qquad (1)\\
(R_{Eij}(R_{C_{ij}}(R_{ij,1}, \ldots, R_{ij,M_{ij}}), R_{E_{(i+1)1}}, \ldots, R_{E_{(i+1)n_{ij}}}) - R^u_{E_{ij}})^2 + \textstyle\sum_{k=1}^{n_{ij}} (R_{E_{(i+1)k}} - R^l_{E_{(i+1)k}})^2 + \\
\|\mu_{\mathbf{Y}_{ij}} - \mu^u_{\mathbf{Y}_{ij}}\|_2^2 + \textstyle\sum_{k=1}^{n_{ij}} \|\mu_{\mathbf{Y}_{(i+1)k}} - \mu^l_{\mathbf{Y}_{(i+1)k}}\|_2^2 \\
\text{w.r.t. } \mu_{\mathbf{Z}_{(i+1)1}}, \ldots, \mu_{\mathbf{Z}_{(i+1)n_{ij}}}, \mu_{\mathbf{X}_{ij}}, \mu_{\mathbf{Y}_{ij}}, \mu_{\mathbf{Y}_{(i+1)1}}, \ldots, \mu_{\mathbf{Y}_{(i+1)n_{ij}}}, R_{ij,1}, \ldots, R_{ij,M_{ij}}, R_{(i+1)1}, \ldots, R_{(i+1)n_{ij}} \\
\text{subject to } P[g_{ij_p}(\mathbf{Z}_{ij}, \mathbf{X}_{ij}, \mathbf{Y}_{ij}) \leq 0] \geq R_{ij,p},\ p = 1, 2, \ldots, M_{ij} \\
\left(1 - \left(\textstyle\sum_{p=1}^{M_{ij}} P_{f_p} - \sum_{p,q=1}^{M_{ij}} \max_{q<p} P_{f_{pq}}\right)\right) \textstyle\prod_{k=1}^{n_{ij}} R_{E_{(i+1)k}} \geq R^u_{E_{ij}},
\end{gathered}$$

where coordinating variables for the shared design variables of the children are denoted by $\mathbf{Y}_{(i+1)1}, \ldots, \mathbf{Y}_{(i+1)n_{ij}}$ and local design constraint functions are represented by $\mathbf{g}_{ij}$. Superscripts u (l) are used to denote response and shared variable values that have been obtained at the parent (children) problem(s), and have been cascaded down (passed up) as design targets (consistency parameters). After all the element's problem parameters have been updated using the solutions obtained at the parent- and children-problems, the above problem is solved to update the parameters of the parent- and children-problems. All element subproblems are solved iteratively according to an appropriate coordination strategy until all tolerance optimization variables do not change after iterations. We adopt the series system RBDO method of [4] to solve the RBDO problem for each element of the hierarchy and estimate its reliability. In this method, the optimizer determines not only the optimal values of the original optimization variables $\mu_{\mathbf{X}}$, but also the optimal reliability value for each probabilistic design constraint. Given a component reliability target, the optimizer allocates reliability target values among the failure failure modes (a failure mode being the violation of a probabilistic design constraint). Reliability allocation and optimal design are thus conducted simultaneously. Note that it is possible that the ATC process will converge without meeting reliability (or other performance) targets. The ATC process does not guarantee that top-level targets will be met; it guarantees that design specifications and reliability targets will be allocated to all elements of the decomposed system so that the design of the latter is consistent while trying to meet the pre-specified targets as close as possible. We have applied this methodology successfully to a bi-level example problem with two subsystems that was presented at the conference but is omitted here due to space limitations.

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