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MULTIPLE SPECIMEN TESTING AND THE ASSOCIATED FATIGUE STRENGTH RESPONSE

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INTRODUCTION

All well-known fatigue strength and fatigue life distributions predict similar responses in the 10% to 90% range of failure.[1] These distributions differ markedly only at their tails, where existing data are too meager to permit establishing a statistically significant difference between actual and predicted response. Fatigue distributions are therefore argued on the basis of minor differences in goodness-of-fit of the majority of data. Consensus is the Weibull and the extreme value distribution are best suited to describe most fatigue life responses. However, limited data[2,3,4] presently consensus for strength response.

Sufficient data to narrow the selection of possible strength distributions is particularly difficult to generate because strength sample size requirements are more stringent, viz., strength response is quantal. Multiple specimen fatigue testing therefore appears to be the most practical approach to generating the large amount of data required. This paper describes two such tests on miniature fatigue specimens and shows that despite minor experimental difficulties the multiple specimen test approach can be used to generate consistent strength data quite rapidly. These data are not markedly skewed and thus can be described conveniently by the logistic function.

MULTIPLE SPECIMEN COUPONS

Coupons used in both series of tests described herein were sheared from a single 20 gage 1100-H14 aluminum sheet. Assemblies suitable for machining were fabricated by bolting groups of about thirty coupons between 1/4 inch plates. Each assembly was then milled to the dimensions appearing in Figure 1a. The finishing cut was approximately 0.003 inches.

All cantilevers were parallel to the direction of rolling. In this direction 1100-H14 aluminum sheet has an ultimate tensile strength of 17,000 psi, a yield strength of 14,000 psi and an elongation of 9%.[5] Young's modulus (E) is approximately 10^7 psi.

Prior to testing each cantilever was inspected on both sides of the coupon in the vicinity of the 0.031 inch fillets for surface scratches or other blemishes. Only unblemished cantilevers were regarded as specimens in subsequent analyses.

HIGH FREQUENCY TESTS

A 1200 lbv MB electronic shaker was used to conduct forced vibration fatigue tests at approximately 1,000 cps. Figure 1b shows the coupon assembly used in these tests. Spacers were recessed 0.005 inches to avoid fatigue crack initiation enhanced by fretting. Individual allen screws secured each cantilever firmly between the top and bottom support plates. Forty cantilevers per assembly were simultaneously vibrated for 10^6 cycles at a fixed table amplitude. The test was then stopped and specimens with fatigue cracks determined using dye penetrant.

Table 1 lists typical test data. The natural frequency of each cantilever was determined within about 2 cps by varying shaker excitation frequency at low g levels. These data were then used to select a test frequency somewhere in the range $0.96 < f/f_{n,ave} < 0.98$, depending on the approximate proportion of cracked specimens desired. Corresponding test acceleration for each run was selected to produce a uniform value of table excitation amplitude.

Figure 2 summarizes the high frequency data. This response appears symmetrical on (integrated) normal (curve) co-ordinates for stress amplitudes computed using f/f_n and ignoring damping and cyclic dependent effects. Similar forced vibration tests by Bender and Hamm[(4)] also appear symmetrical. In their tests start of failure was indicated by a change in natural frequency.

The logistic function is shown in Figure 2 for comparison. This sigmoidal curve more accurately describes the extreme observations (P = 0 and P = 1) plotted using Berkson's 2N rule.

DISCUSSION OF HIGH FREQUENCY TESTS

Forced vibration fatigue testing of multiple specimens required control of table amplitude rather than specimen amplitude because specimen amplitude varied from cantilever to cantilever despite strict dimensional tolerances. There was therefore no direct control of stress or strain amplitude for individual specimens.

Test time was too short to measure more than a few cantilever tip amplitudes. These measurements, however, did show that tip amplitude changed during testing. In fact, even uncracked specimens displayed some decrease in natural frequency during testing. This change was very small for f/f_n ratios about 0.95 but increased fairly rapidly as f/f_n increased. In a number of cases uncracked specimens tested at f/f ratios about 0.98 experienced larger incremental decreases than specimens cracked at lower values of f/f_n. This slight overlap in incremental change was the primary reason that dye penetrant was used to establish failure for data analyses.

One of the primary objectives of this test was to determine whether stacking of two or more coupons one upon the other (separated by spacers) would give consistent results. It was established in preliminary tests that acoustical effects became pronounced at approximately 1000 cps (f/f_n about .99) when the natural frequencies of any two stacked cantilevers differed by more than 5-10 cps. In this situation the cantilever with the higher natural vibrated with a larger amplitude than the cantilever with the lower natural.... just the opposite to expected behavior. Thus, before commencing the final series of tests, spacer thickness was increased from 0.02 to 0.04

inches and an upper bound of 0.985 was placed on f/f_n . To explore the possibility that smaller acoustical or mechanical coupling effects still influenced these data, a Kolmogorov-Smirnov "d" test was applied to the over-all response divided into two groups: (1) response pertaining only to stacked specimens with natural frequencies that differed by 5 cps or less, and (2) response pertaining to stacked specimens with naturals that differed by more than 5 cps. No difference could be established between these two compared responses (or betwen either group and the over-all response) at the .20 level of significance. It thus appears that several coupons can be stacked one upon the other in future tests to facilitate rapid generation of data. For example, if a datum point corresponding to a probability of failure of 0.01 is desired, this point with a weight Npq of two can be generated in a single test employing ten stacked coupons.

LOW FREQUENCY TESTS

A modified version of the high frequency coupon was used in the low frequency fatigue tests. All cantilevers were 1 1/8 inches long and each machined assembly was slit along the dashed line in Figure 1a. Thus each low frequency coupon had ten or fewer specimens depending on the number of blemished cantilevers.

Tests on these coupons were conducted at 28 cps using an All-American HA-25T horizontal shaker and the test set up shown schematically in Figure 3. Individual coupons were sandwiched between support plates and tightly secured by an allen screw at each cantilever. The edges of these support plates were recessed 0.010 inches to avoid fretting in the vicinity of possible fatigue cracking. Each coupon was subjected to 10^5 cycles at a fixed range of deflection.

Dynamic table travel was measured with vernier calipers using an optical interference technique similar to the vibrating wedge. Four such measurements were averaged to establish dynamic range. Estimated accuracy is 0.0002 inches. A plot of static table travel versus crank rotation angle facilitated setting the knife edges at the centerline of static range of deflection. Maximum over-all error in knife edge settings is estimated at 0.0007 inches.

Following testing all unbroken specimens cantilevers were dye checked and qualitative data recorded for crack severity. Figure 4 shows the qualitative criteria used. Failed connotes fatigue cracks origniated at both fillets on each side of the coupon and traversed (almost) the entire width of the specimen, but did not propagate to the neutral fiber. Severely cracked infers that fatigue cracks originated at both fillets on each side of the coupon and propagated until at least one-half the specimen width displayed cracking. Figure 5 shows these test results plotted on logistic co-ordinates. Extreme observations are denoted by datum points with arrows.

The range of nominal strain for tightening knife edges was 9% greater than for the untightened or simply-supported condition. Although little difficulty was encountered in uniformly reproducing this 9% figure for either the top or bottom cantilevers, it was quite difficult to achieve uniform tightening from top to bottom. This difficulty represents the major problem associated with these low frequency tests. Considerable preliminary testing was required to achieve fairly uniform tightening. As a final check, the knife edge impressions on the tip of each specimen were examined to confirm that each specimen had been gripped properly.

DISCUSSION OF LOW FREQUENCY TESTS

These tests are remarkably suited to analysis discerning position effect in failure using the binomial distribution. Preliminary tests on coupons with twenty cantilevers (Figure 1a) showed that the top specimens failed more often than the bottom specimens. Thus table motion was corrected by overhauling the bearings and the coupon was cut in two to lower the centroid of the rocking force. Subsequent tests showed that one side failed more often than the other. Accordingly, the fixture was stiffened and bearing collars were added to restrain sidewise table motion. When continued testing finally showed no discernable position effect, the data reported was taken.

The purpose of the failed and severely cracked designations is to describe the progress of fatigue cracking. These responses are subject to error in establishing p observed because it was relatively difficult to distinguish between failed, severely cracked, and cracked in a consistent manner using dye penetrant techniques. Crack propagation data shown in Figure 4 pertain to measurements made on cracked specimens from various coupons tested at different ranges of deflection. The heavy line represents the range of crack length measurements for six to eight specimens.

LEAST-SQUARE ANALYSES OF DATA

Existing strength data were least-squares fitted to the integrated normal curve, the logistic function, the extreme value distribution, and the Weibull distribution, using techniques presented elsewhere(6). Table 2 lists mean-square errors associated with fitting these four distributions to various fatigue strength data. These mean square errors for both high and low frequency tests compare favorably with corresponding errors for single specimen testing. Thus the multiple specimen approach to strength testing appears valid.

Two different methods were used to fit the strength data in Table 2. The first methods avoids the problem of treating extreme observations by simply ignoring data for which p observed is either zero or unity. Accordingly, the remaining data pertain almost exclusively to values of p between 0.05 and 0.95. Corresponding mean square errors are therefore quite similar for each of the four distributions considered (as evidenced by the first three rows of Table 2) and many such sets of strength data are required for meaningful comparison. Considering all eleven sets by Cummings, Stulen, and Schulte on various materials the logistic function displays a smaller mean square error than the weighted integrated normal curve in nine cases. If either distribution is equally valid, the probability that one or the other would display this behavior is only 0.033 + 0.033 = 0.066. There is therefore some engineering justification to preferring the logistic function to the integrated normal curve.

The second method is more expedient. It is based on the notion that extreme observations must be used in analysis because these data are not only as valid as the other data, but are situated in the primary area of interest. Thus Berkson's 2N rule is used to plot at least one extreme observation on each tail (when available and of sufficient weight) and the corresponding least-squares analysis treats the total collection of datum points. Results of this approach are typified by the mean-square errors listed in the bottom six rows of Table 2. Further support for the logistic function is derived from noting that inclusion of extreme observations seldom affects the mean-square error for the logistic function, but often affects the errors for the other three distributions.

P-S-N SURFACE

The logistic function can be coupled with the S-N curve to describe the P-S-N surface analytically. First the logistic function

$$P = \frac{1}{1 + e^{-(\alpha+\beta S)}} \tag{1}$$

is rewritten as

$$\ln\left(\frac{P}{1-P}\right) = \alpha + \beta S = \beta\left(\frac{\alpha}{\beta} + S\right) \tag{2}$$

Then using the condition that $S = S_{50}$ when $P = 0.50$ to adduce that $S_{50} = -\alpha/\beta$, Equation (2) can be interpreted as

$$\ln\left(\frac{P}{1-P}\right) = \beta(S - S_{50}) \tag{3}$$

The logistic parameters in Equation (3) are β, the slope of the linear transform on logistic paper and S_{50} , the median strength for a given fatigue life. In turn, substituting S_{50} from a S-N curve of the form $S_{50} - S_N = A\,N^m$ yields the expression:

$$\ln\left(\frac{P}{1-P}\right) = \beta(S - S_N - A\,N^m) \tag{4}$$

Figure 6 compares slices through this analytical surface to the response reported by Bender and Hamm.

CONCLUSIONS

1. Despite experimental difficulties with multiple specimen testing, this approach appears to be the answer to generating large amounts of fatigue strength data. The primary problem remaining is to establish a more accurate approach to measuring and describing fatigue cracking.

2. Strength response does not appear to be markedly skewed. It therefore can be approximated by either the logistic function or the integrated normal curve, apparently somewhat more accurately by the former.

3. The logistic function can be coupled with the S-N curve to develop an explicit expression for the P-S-N surface. This expression should suffice to describe most responses in probability ranges remote to the tails of the distribution.

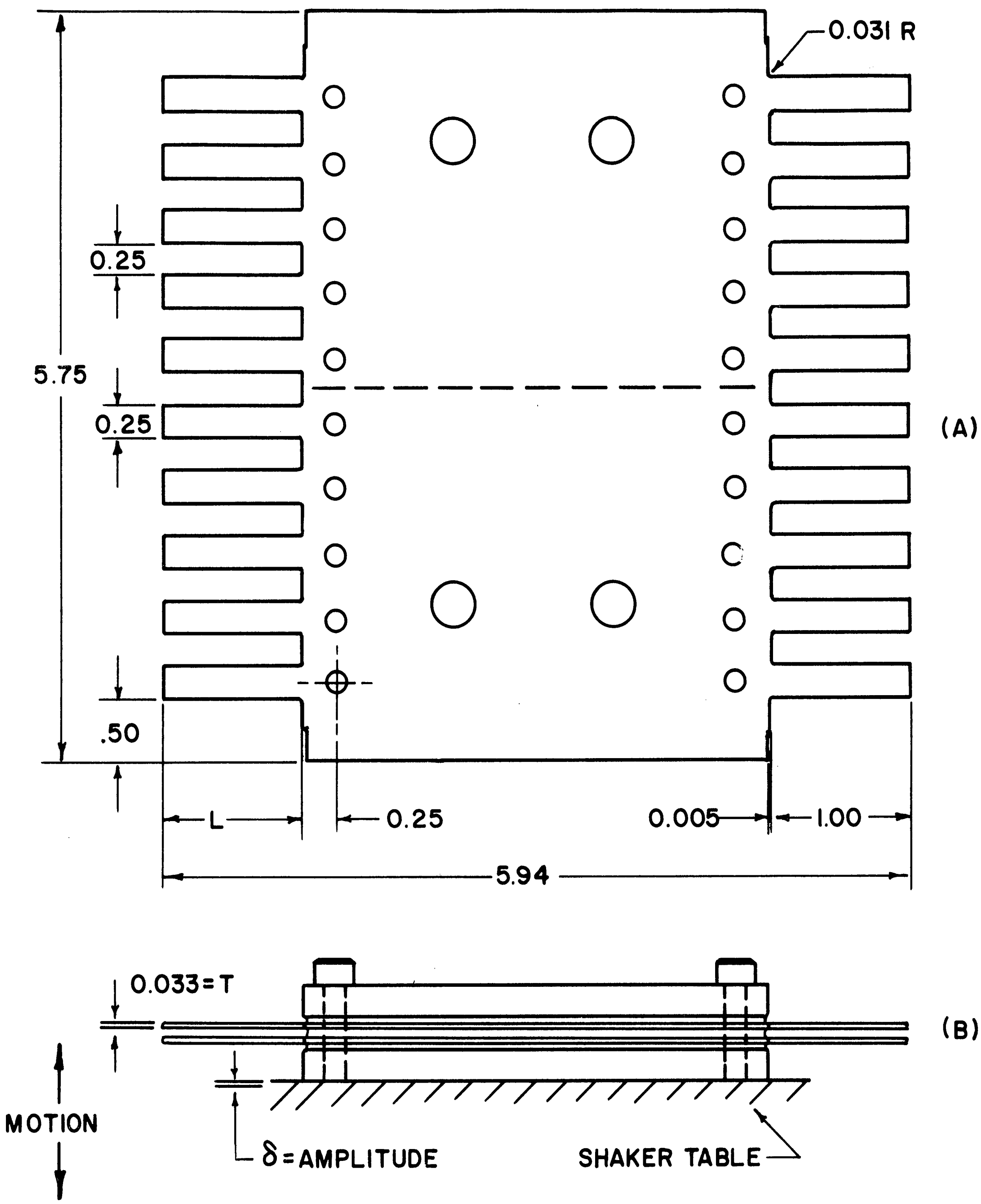

Figure 1. (A) Multiple Specimen Coupon.
(B) High Frequency Test Coupon Assembly.

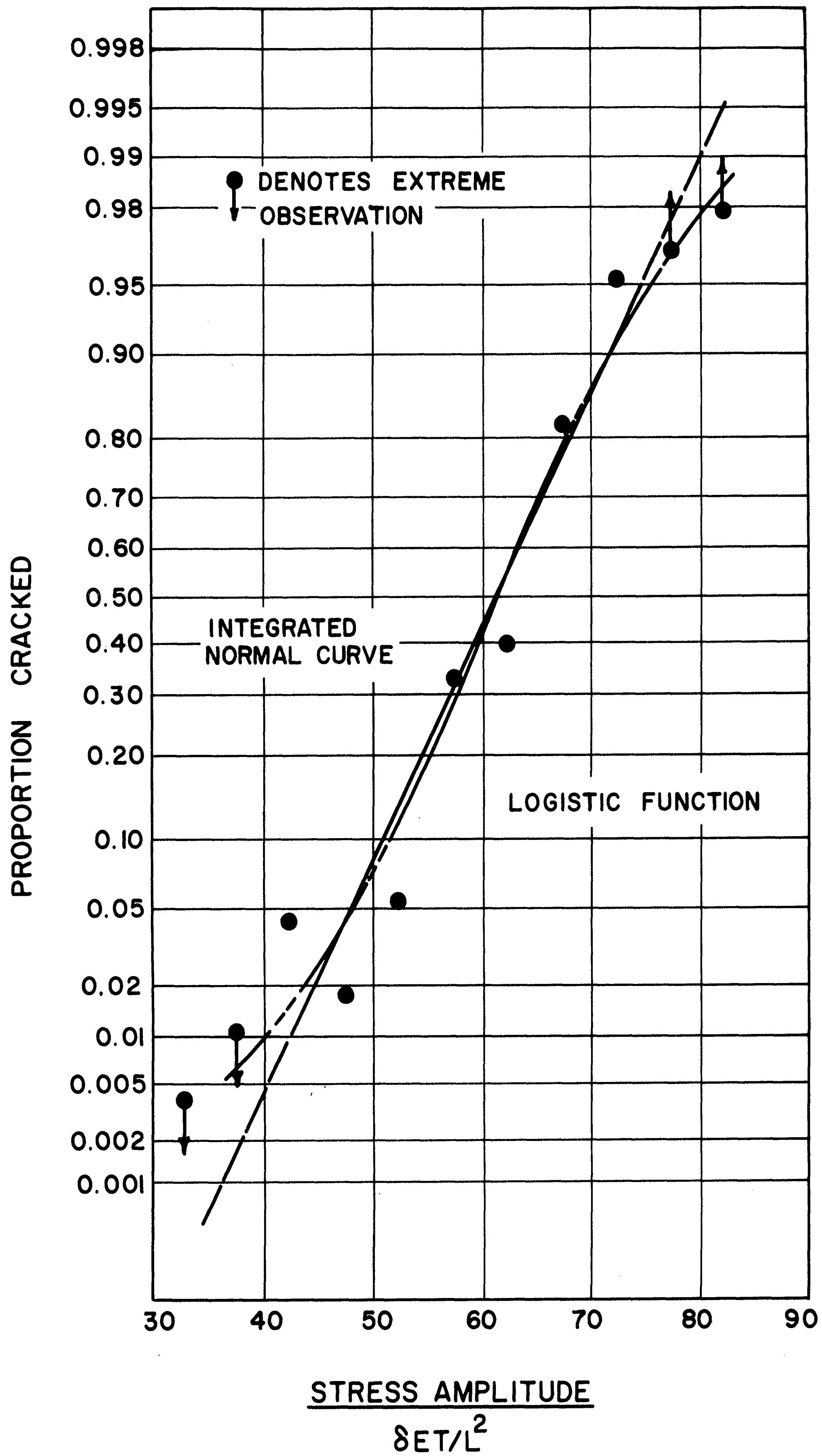

Figure 2. High Frequency Test Data (See Appendix A).

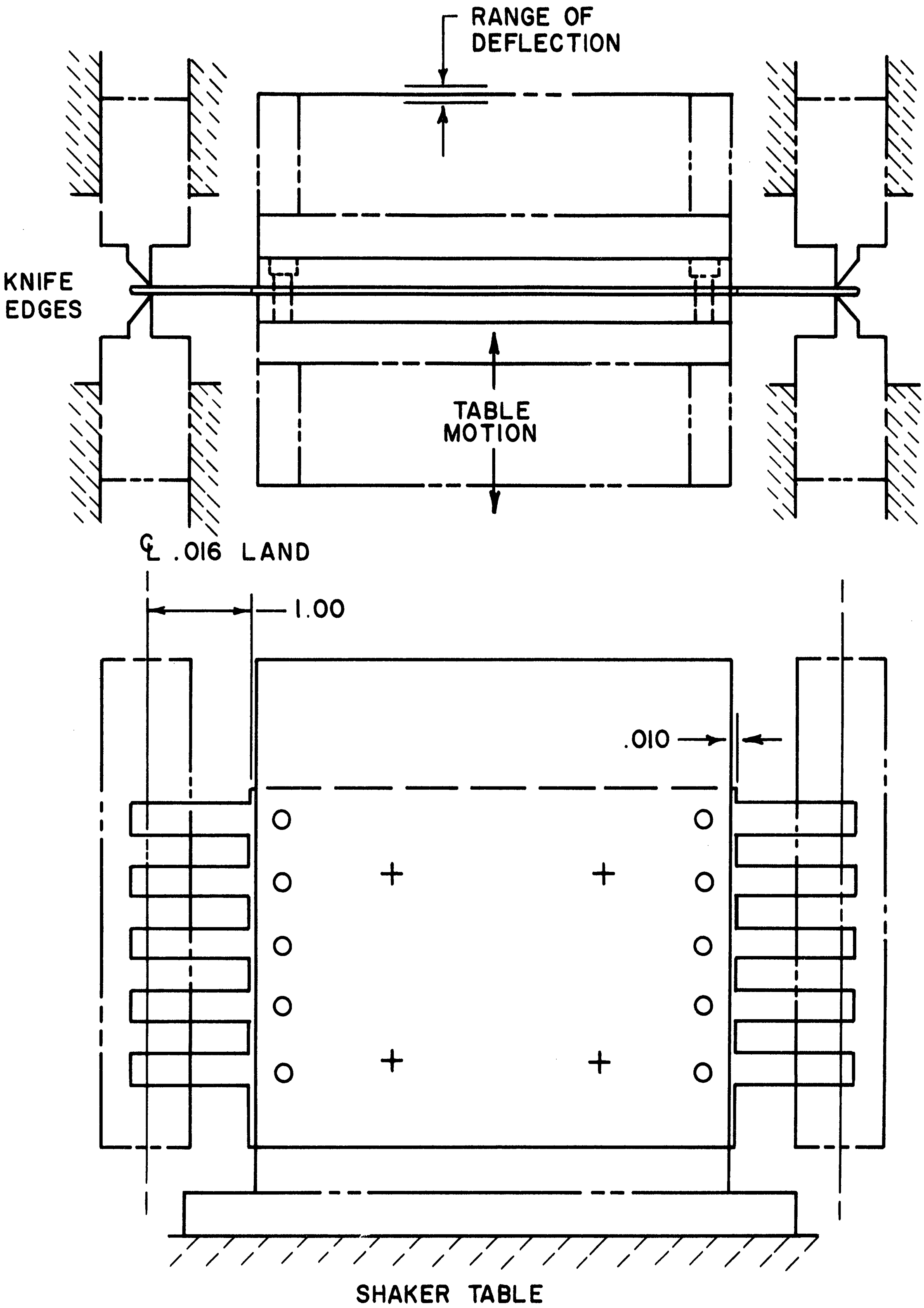

Figure 3. Low Frequency Test Set Up.

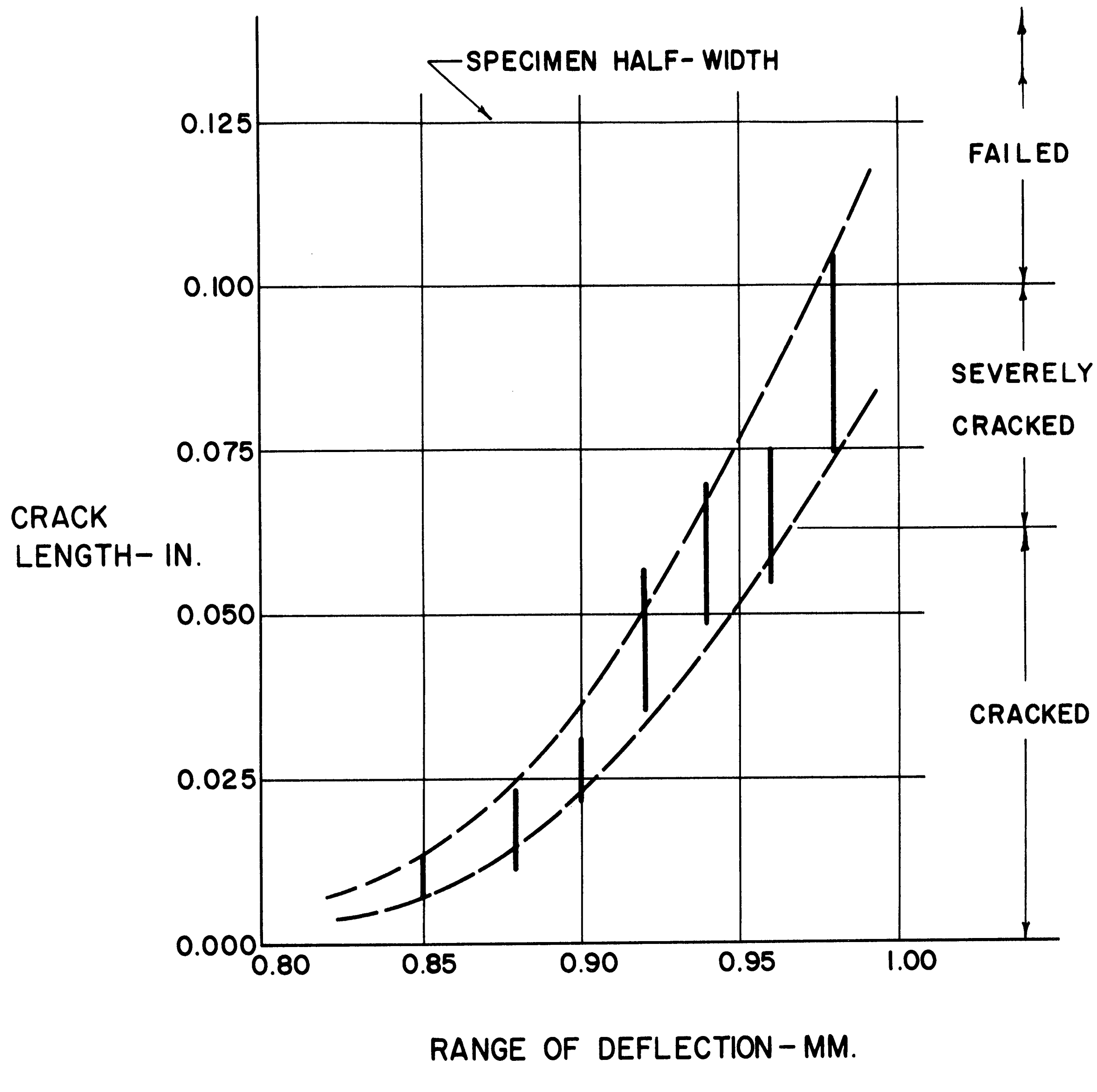

Figure 4. Crack Length Criteria and Data.

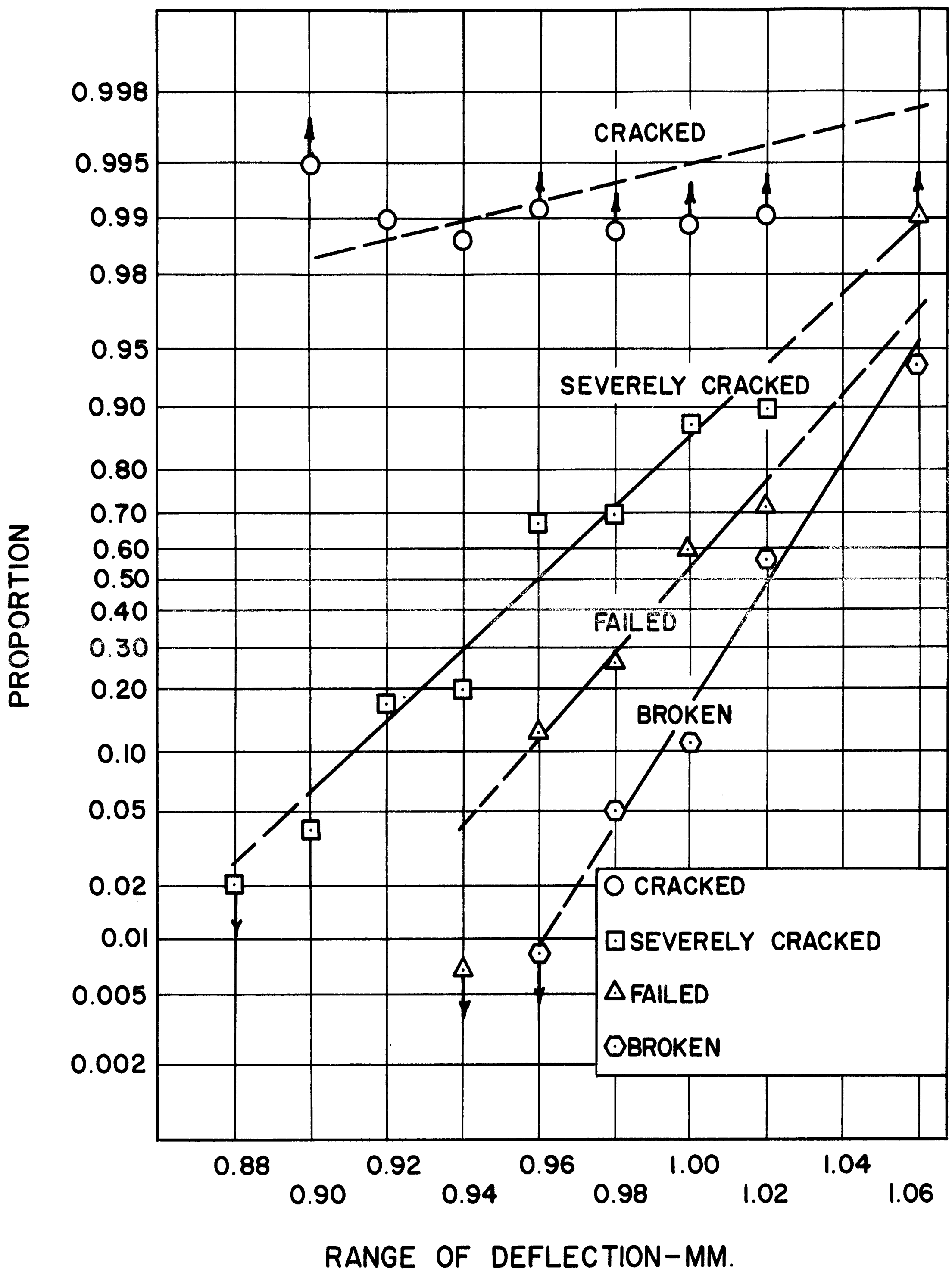

Figure 5. Low Frequency Test Data (See Appendix B).

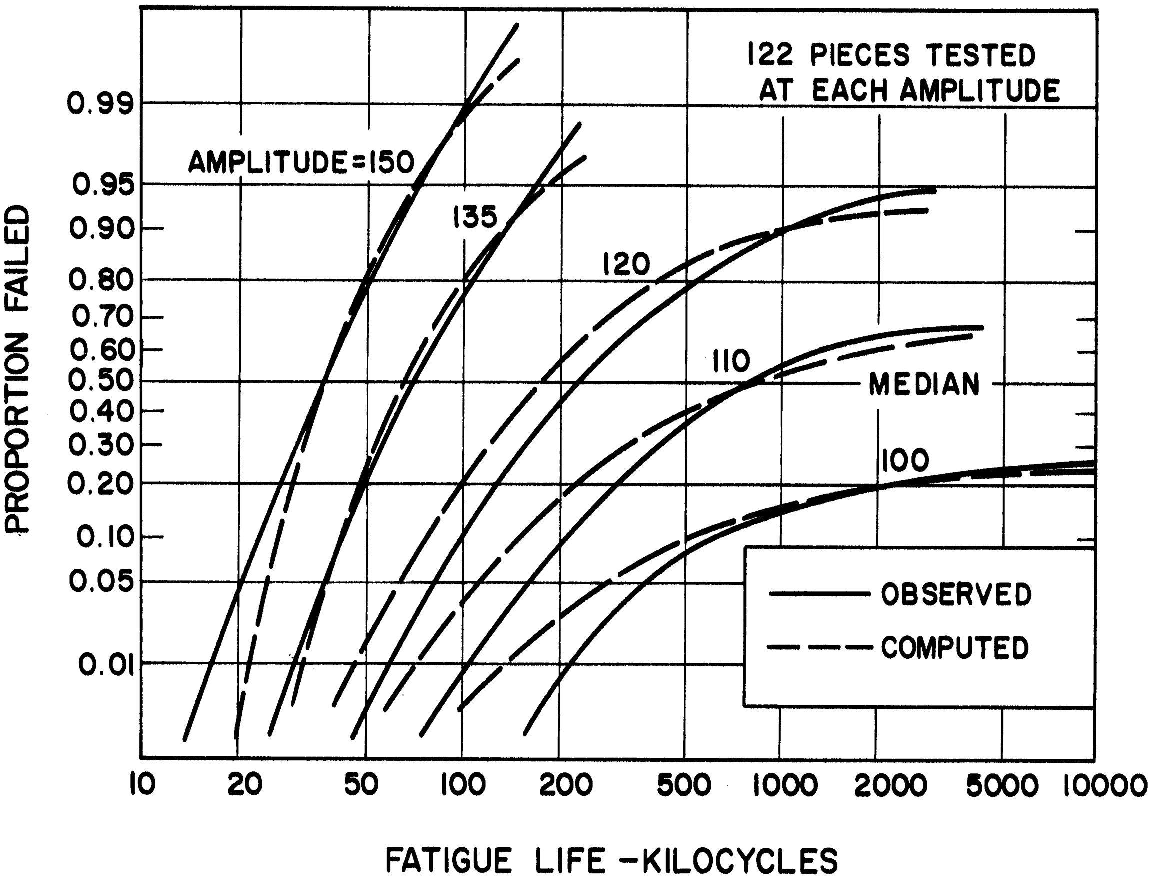

Figure 6. Comparison of Response Versus Prediction for Data by Bender and Hamm.

TABLE 1

Typical Test Data. Natural Frequencies of Individual Cantilevers, With Cracking Data.

Cantilever Number	Front Top	Front Bottom	Back Top	Back Bottom
1	1054-X	1043-X	1060-0	1052-0
2	1049-X	1040-X	1060-0	1052-0
3	1049-X	1042-X	1060-0	1049-X
4	1054-0	1042-X	1062-0	1052-0
5	1049-X	1043-X	1061-0	1049-X
6	1049-X	1044-X	1058-0	1049-X
7	1048-X	1046-X	1058-0	1049-X
8	1049-X	1046-X	1058-0	1050-X
9	1051-X	1051-X	1058-0	1049-X
10	1058-0	1056-0	1065-0	1058-0

(Cantilever Position: Front / Back)

Excitation Frequency = 1029.8 cps

X = Cracked, 0 = Uncracked

TABLE 2

Mean-Square Error Analysis of Fatigue Strength Data

Investigator and Test	Integrated Normal Curve	Logistic Function	Extreme Value Distribution	Weibull Distribution
Cummings, Stulen and Schulte				
4330	1.145	0.872	0.794	0.983
4340	0.511	0.484	1.472	0.789
4350	2.107	1.308	0.930	0.791
Bender and Hamm	2.075	0.333	6.605	-----
Cazaud	1.740	0.200	4.256	-----
Present Tests:				
Low Frequency				
Broken	1.175	0.831	6.719	3.240
Failed	5.667	1.192	12.551	-----
Severely Cracked	5.264	1.882	13.849	-----
High Frequency	3.858	1.733	2.540	-----

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2. Cummings, H. N., Stulen, F. B., and Schulte, W. C., "Investigation of Materials Fatigue Problems," WADC: TR-56-611, 1957, (ASTIA: AD-134539.)

3. Cazaud, R., See Chapter by F. Bastentaire in the Proceedings of Symposium on Fatigue in Aircraft Structures, Edited by W. Barrois and E. L. Ripley, Pergamon Press, 1963.

4. Bender, A., and Hamm, A., The Application of Probability Paper to Life or Fatigue Testing, Engineering Department Paper of Delco-Remy Division, General Motors Corporation.

5. __________, Metals Handbook, ASM, 1948.

6. Soni, A. H., and Little, R. E., Fatigue-Limit Analyses and Design of Fatigue Experiments, Tenth Conference on Design of Experiments in Army Research Development and Testing, November, 1964.

APPENDIX A

High Frequency Test Data

Stress Amplitude $\delta Et/L^2$	Number Specimens Tested	Number Specimens Cracked
30-35	125	0
35-40	40	0
40-45	43	2
45-50	56	1
50-55	53	3
55-60	67	22
60-65	45	18
65-70	44	37
70-75	42	40
75-80	15	15
80-85	23	23

APPENDIX B

Low Frequency Test Data

		Cummulative			
Range of Deflection MM.	Number Specimens Tested	Number Broken	Number Failed	Number Severely Cracked	Number Cracked
1.06	50	47	50	--	--
1.02	57	32	41	51	57
1.00	45	5	27	39	45
.98	43	2	11	30	43
.96	57	0	7	38	57
.94	74	0	0	14	73
.92	96	0	0	16	95
.90	101	0	0	4	101
.88	25	0	0	0	25
.85	19	0	0	0	19