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Comment on "Characterization of Interfacial Properties in Fiber-Reinforced Cementitious Composites"

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Modeling the process of pullout of a fiber from a matrix is a fundamental problem that has received great attention among researchers, as indeed shown by the review presented in the paper by Z. Li, B. Mobasher, and S. P. Shah.[1] However, a number of recent studies have also addressed the specific problem of fiber pullout from a cementitious matrix and, as discussed below, contribute significantly to extending the current state of knowledge.[2-6] In these references a fundamental study of the bond stress–slip relationship between steel fibers and cement composites is presented and correlated with the pullout problem. The analysis consists of a primal problem, whereby a complete pullout load versus slip curve can be predicted from an assumed (or experimental) bond stress versus slip relationship, and, a dual problem, in which the bond stress versus slip relationship is obtained from an experimental pullout curve. The solution presented is most general.

The main purpose of this comment is to show first that the model developed by Li *et al.*[1] is a subset of the general model developed by Naaman *et al.*[2-4] and second that, because of simplifying assumptions, it may be quite limited in scope.

I. Particular Solution

The approach used by Li *et al.*[1] to model the fiber pullout process is basically a shear–lag approach, in which the matrix contribution to the axial deformation of the system is neglected. This is a subset of the general solution presented by Naaman *et al.*,[2,4] where the matrix deformation is accounted for. A proof is given next.

The shear–lag equilibrium equation is given by

$$\frac{\partial \sigma_f(x)}{\partial x} = K(u - v) \tag{1}$$

where $\sigma_f(x)$ is the axial stress in the fiber, x the axial distance along the fiber axis, u the local displacement of the fiber, v the local displacement of the matrix, and K the stiffness of the boundary shear–lag layer. Li *et al.*[1] have eliminated the v term from Eq. (1), leading to the solution given by Eq. (9) of their paper. It can be shown that Eq. (9) can be deduced from the general solution presented in Eq. (33) given by Naaman *et al.*[2] (or Eq. (5.50) in Ref. 4), which is reproduced below:

$$Q = 1 + \frac{A_m E_m}{A_f E_f} \tag{2}$$

$$\Delta = \left\{ P(Q-1)u - \frac{t_f u^2}{2}(Q-2) + (P - t_f u)\coth[\lambda(l-u)]\frac{Q-2}{\lambda} - t_f u l \right\} / A_m E_m \tag{3}$$

By simply assuming that E_m tends to infinity (i.e., an infinitely rigid matrix, or zero matrix deformation, as assumed by Li *et al.*[1]), Eq. (33) leads to

$$\frac{Q-1}{A_m E_m} = \frac{Q-2}{A_m E_m} = \frac{1}{A_f E_f} \tag{4}$$

Thus,

$$\Delta = \frac{P - t_f u}{\lambda A_f E_f}\coth[\lambda(l-u)] + \frac{(P - 0.5 t_f u)u}{A_f E_f} \tag{5}$$

which matches Eq. (9) given in their paper, with the following corresponding notation: $u = a$ (debonded length), $l = L$ (embedded length), $t_f = q_f$ (frictional shear flow), $\Delta = U^*$ (fiber end displacement), and $\lambda = \omega = (K/A_f E_f)^{1/2}$.

Another drawback of the rigid matrix assumption is that it may lead to unrealistic simulation of the debonding mechanism along the fiber. Indeed a manipulation of Eq. (1) of their paper and the shear–lag equation (Eq. (1) above) leads to the following results:

$$\sigma_f(x) = \frac{P^*}{A_f}\frac{\sinh(\omega x)}{\sinh(\omega L)} \tag{6}$$

from which one can derive the interfacial shear stress as

$$\tau(x) = \frac{r_f}{2}\frac{\partial \sigma_f(x)}{\partial x} = \frac{P^*}{2\pi r_f}\,\omega\,\frac{\cosh(\omega x)}{\sinh(\omega L)} \tag{7}$$

Substituting $x = (L,0)$ in Eq. (7) to compute the shear stress at both ends of the fiber, we get

$$\tau_{face} = \frac{P^*}{2\pi r_f}\,\omega\coth(\omega L) \tag{8}$$

$$\tau_{emb} = \frac{P^*}{2\pi r_f}\,\omega\coth(\omega L)\frac{1}{\cosh(\omega L)} = \tau_{face}\frac{1}{\cosh(\omega L)} \tag{9}$$

where $\cosh(\omega L) \geq 1$ for all positive values of ωL. Therefore, $\tau_{emb} < \tau_{face}$, implying that debonding will always initiate at the point where the fiber penetrates the matrix. This result is not always correct. Indeed, an analytical study by Leong and Li[7] has reported that, for the ratio $\alpha = V_f E_f / E_c > 0.5$, debonding will start at the embedded end of the fiber. This also agrees well with the analytical solution presented by Naaman *et al.*,[2,4] where τ_{emb} can exceed τ_{face} if

$$P\lambda(Ae^{\lambda l} - Be^{-\lambda l}) > P\lambda(A - B) \tag{10}$$

or

$$\frac{1}{Q} > 0.5 \tag{11a}$$

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Table I. Predictive Results Computed Using Both Models

Sample†	P/Δ (N/mm)	P^*_{peak} (N)	U^*_{peak} (mm)	a (mm)	τ_y (MPa) Li *et al.*[1]	τ_y (MPa) Naaman *et al.*[2,4]	τ_f (MPa) Li *et al.*[1]	τ_f (MPa) Naaman *et al.*[2,4]
H2SF	7875	59.6	0.02	22.3	1.72	1.75	1.70	1.53
H2SL	12075	98.3	0.04	19.5	3.36	9.58	3.20	1.78

†Fiber length = 25 mm (1 in.) and fiber diameter = 0.5 mm (0.019 in.).

where

$$\frac{1}{Q} = \frac{A_f E_f}{A_f E_f + A_m E_m} = \alpha \tag{11b}$$

II. Other Limitations

Similar to the assumptions made by Naaman *et al.*,[2,4] the model presented by Li *et al.*[1] characterizes the bond stress–slip relationship by three basic parameters: τ_y, τ_f, and K. However, it has been observed from an extensive investigation of pullout tests[4] that the frictional shear stress at the fiber–matrix interface is a function of the local slip. To model such an observation, Naaman *et al.*[2,4] introduced a decay factor in their model, thus providing a solution more general than that presented in the paper by Li *et al.*[1]

Furthermore, Li *et al.*[1] state that ω has to be computed first from their Eq. (9) for $a = 0$, then substituted into their Eq. (14) along with P^*_{peak} and U^*_{peak} to solve for the debonded length, a. However, a restriction is made on U^*_{peak}, leaving a gap in the procedure, namely, should U^*_{peak} exceed U^*_{crit}, U^*_{peak} is to be limited to U^*_{crit}. Should a different fiber be investigated, a different numerical procedure is needed to solve directly for the three variables; for a similar case, Newton's algorithm for nonlinear systems has been followed by Naaman *et al.*[3,4] and has led to good results.

The usefulness of a model is measured by how well it can simulate and predict experimentally observed results. Predictive results using both the model presented by Li *et al.*[1] and by Naaman *et al.*,[2,4] are compared in Table I, using experimental data taken from Ref. 3. Only two specimens are used for illustration, a specimen (H2SF) where a steel fiber is pulled out from a plain cementitious matrix, and another, similar, specimen (H2SL) for which a latex emulsion was added to the matrix to improve the bond. Two observations can be made from Table I: (1) the rigid-matrix model presented by Li *et al.*[1] leads to slightly larger values of τ_y because matrix deformations are neglected, and (2) in the case of specimen H2SL, where $U^*_{peak} > U^*_{crit}$, the model is insensitive to increased values of τ_y. Indeed, the presence of latex leads to a maximum shear of about 9.58 MPa (1390 psi), as supported by experiments, whereas the model of Li *et al.*[1] predicts only 3.36 MPa (488 psi).

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