# J. M. POWERS and R. G. CRAIG

School of Dentistry, University of Michigan, Ann Arbor, Michigan 48104, USA

The frictional behavior of natural fluorapatite single crystals under sliding was evaluated. Strain rate did not influence the coefficient of friction. Low and high regimes of friction were related to the amount of penetration; higher values of friction were associated with deeper penetration.

It is a convenient approximation to regard the frictional forces between two solids as made up of two components: the force required to shear adhering junctions in the region of surface contact and the force required to deform the underlying bulk material.<sup>1</sup> With an elastic solid, the deformation component may be due mainly to an elastic hysteresis loss; but if plastic flow occurs, the deformation component will represent the force required to produce bulk deformation or to drag surface irregularities through the solid and to plow out a track.<sup>2</sup>

Although the coefficient of friction is not necessarily indicative of wear phenomena, it is traditional to attempt to correlate it with wear data. Steijn,3 in his examination of polycrystalline and single crystal copper, observed that the plowing component of friction was increased by indentors of smaller diameter and by higher loads for a given indentor. Bowden and Brookes<sup>4</sup> reported for MgO single crystals that depth of penetration (from static indentation tests) was related directly to the magnitude of the coefficient of friction, with deeper penetration correlating with higher friction. Both low and high friction regimes were observed depending on the load and slider design used.

This paper describes the frictional behavior of natural fluorapatite single crystals under sliding and attempts to correlate this behavior with wear data reported in part one of this investigation.<sup>5</sup> Specifically, the variables friction force and coefficient of friction are examined.

## Materials and Methods

Diamond sliders of known geometry were slid across the basal surfaces of natural fluorapatite single crystals in a dry environment. Measurements were made based on a series of three, one-traversal passes on each of two crystals for a given condition. The effect of crystallographic direction on frictional properties was not examined in this initial study.

Two dependent variables (friction force and coefficient of friction) were evaluated with respect to four factors (specimen, slider speed, load, and slider design) by means of a factorial design with replications. Analysis of variance<sup>6</sup> and multiple comparisons<sup>7</sup> were used to estimate the effects of factors and their interactions. Factors examined in this experimental design are shown in Table 1.

Fluorapatite single crystals\* were polished and given a surface treatment as described in part one of this investigation.<sup>5</sup> The apparatus used for scratching the surface of a specimen and measuring the friction force consisted of mechanisms that can be categorized as follows: surface grinder, loading jig, friction transducer, diamond sliders, and sample holder. With the exception of the friction transducer and its accompanying circuitry, these mechanisms have been described in detail.<sup>5</sup>

A soft brass bar was machined with parallel flats on two opposite sides to accept four, 500 ohm strain gauges<sup>†</sup> to form a full-

This investigation was supported by USPHS Training Grant DE-00181 and Research Grant DE-02415 from the National Institute of Dental Research, National Institutes of Health, Bethesda, Md.

This paper was presented in part at the 49th general session of the IADR, Chicago, Ill, March 19, 1971. Received for publication March 15, 1971.

<sup>\*</sup> Southwest Scientific Co., Box 10, Hamilton, Mont.

<sup>†</sup> EA-09-125AS-500 Strain Gage, Micromeasurements, Romulus, Mich.

Factor	Levels	Comments				
Specimen	2	2 crystals (3 replication per crystal)				
Slider speed	2	0.025 and 0.076 cm/second				
Load	5	10, 25, 50, 100, and 500 gm $(A_1$ through $A_5$ , respectively)				
Slider design	5	75° cone, 0.018 cm radius 75° cone, 0.064 cm radius 104° cone, 0.005-0.008 cm radius 123° cone, 0.005-0.008 cm radius 143° cone, 0.005-0.008 cm radius ( $B_1$ through $B_5$ , respectively)				

TABLE 1 EXPERIMENTAL DESIGN FACTORS

bridge, cantilever beam type of frictional force transducer. The lower end of the bar was machined to hold, by means of a set screw, the shaft of the slider being used. Means for accurate positioning of the flat perpendicular to the direction of sliding, as well as for positioning the tip of the slider a half inch from the end of the bar were provided. On deflection of the beam during sliding, the signal caused by the resulting bridge unbalance was amplified\* and recorded as the friction force on a servo Y-T recorder.† A precision dc null voltmeter‡ was used intermediate to the recorder and amplifier to serve as a range expander and to reduce the noise to signal ratio. Linear calibration of the transducer was accomplished by a dead weight procedure. The transducer and a schematic drawing of the accompanying circuitry are shown in Figures 1 and 2, respectively.

The value of the friction force recorded for a run was that value most representative of the force-time curve for the given condition. The coefficient of friction (f) was derived in terms of the measured friction force and the load by the following equation: f = force/load.

T Model EUW-20A, Heath Company, Benton Harbor, Mich. ‡ Model 419A, Hewlett-Packard Corp., Palo Alto,

<sup>‡</sup> Model 419A, Hewlett-Packard Corp., Palo Alto, Calif.

§ The usual symbol for the coefficient of friction is  $\mu$ . The symbol *f* is used here in order not to confuse the coefficient of friction with the symbol for microns ( $\mu$ ).



FIG 1.-Strain gauge transducer.

<sup>\*</sup> Indicator Model 300C with Type 80 Strain Gage Plug-in Unit, Daytronic Corp., Dayton, Ohio. † Model EUW-20A, Heath Company, Benton Harbor,



FIG 2.--Schematic drawing of transducer and its accompanying circuitry.

## Results

The effects of specimen, slider speed, load, and slider design on the variable, friction force, were evaluated by means of analysis of variance and multiple comparisons. The effect of specimen and slider speed on the friction force was found not to be significant, whereas the effects of load and slider design were significant. Yet, on comparison of the force data with the corresponding coefficient of friction data, the coefficient of friction was probably more reliable in discriminating among differences. For this reason the friction force data will not be discussed, although ranking of this data with respect to constant load and constant slider design are presented in Tables 2 and 3, respectively, for informational purposes.

The effect of specimen on the coefficient of friction was examined at two levels (two crystals) for five slider designs and five loads with three replications per cell. Unless otherwise indicated, the factorial designs analyzed will have in common a slider speed of 0.025 cm/second and six replications per cell. The null hypothesis  $(H_o)$  that there were no differences between the effects at two levels of specimen was evaluated.

The main effect of crystals on the coefficient of friction was not significant ( $F = 0.01 < F_{(1,60)} = 4.00$ ) at the 0.05 level. It was deduced that there were no differences between two levels of specimen, ie,  $H_o$  was accepted; consequently, the data from three replications for the analysis of load and slider design factors were used. The coeffi-

Slider Design	Ranking (units of gm)								
$\overline{B_1}$	Load	$A_1$	$A_2$	$A_3$	A <sub>4</sub>	$A_5$			
1	Mean	1.70	5.08	10.9	23.5	(138.0)			
$B_2$	Load	$\overline{A_1}$	$\overline{A_2}$	$A_3$	$A_4$	$A_5$			
	Mean	1.70	3.96	7.00	20.0	(90.4)			
$B_3$	Load	$\overline{A}_1$	$\overline{A_2}$	$A_3$	$A_{\mathbf{A}}$	$A_5$			
Ū	Mean	3.48	7.04	19.0	57.0	(274.0)			
$B_4$	Load	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$			
-	Mean	3.78	9.79	22.8	43.5	(221.0)			
$B_5$	Load	$A_1$	$A_2$	$A_3$	$A_{4}$	$A_5$			
5	Mean	2.08	5.58	19.8	37.2	(177.0)			

 TABLE 2

 Results of Duncan's New Multiple Range Test for Load Differences

Note: Dependent variable, friction force. Underscore indicates no significant difference at the 95% level.

Load	Ranking (units of gm)								
<i>A</i> <sub>1</sub>	Slider design	$B_2$	$B_1$	B <sub>5</sub>	<i>B</i> <sub>3</sub>	<i>B</i> <sub>4</sub>			
	Mean	1.70	1.70	2.08	3.48	3.78			
$A_2$	Slider design	B <sub>2</sub>	<i>B</i> <sub>1</sub>	В <sub>5</sub>	<i>B</i> <sub>3</sub>	B <sub>4</sub>			
	Mean	3.96	5.08	5.58	7.04	9.79			
$A_3$	Slider design	$B_2$	<i>B</i> <sub>1</sub>	<i>B</i> <sub>3</sub>	<i>B</i> <sub>5</sub>	<i>B</i> <sub>4</sub>			
	Mean	7.00	10.9	19.0	19.8	22.8			
$A_4$	Slider design	$B_2$	<i>B</i> <sub>1</sub>	B <sub>5</sub>	B <sub>4</sub>	<i>B</i> <sub>3</sub>			
	Mean	20.0	23.5	37.2	43.5	57.0			
$A_5$	Slider design	<i>B</i> <sub>2</sub>	B <sub>1</sub>	B <sub>5</sub>	$B_4$	<i>B</i> <sub>3</sub>			
	Mean	(90.4)	(138.0)	(177.0)	(221.0)	(274.0)			

 TABLE 3

 Results of Duncan's New Multiple Range Test

 for Slider Design Differences

Note: Dependent variable, friction force. Underscore indicates no significant difference at the 95% level.

cient of variation for the data before being combined was 7.5%. The first and second order interactions were not significant (p < 0.05).

The effect of slider speed on the coefficient of friction was examined at two levels (0.025 and 0.076 cm/second) for five slider designs at a 500 gm load. The main effect of speed was not significant ( $F = 0.02 < F_{(1,40)} = 4.08$ ). Likewise, the first order interaction was not significant at the 0.05 level. The data for the two speeds, therefore, were combined to yield 12 replications per cell for subsequent analysis of slider design. The coefficient of variation for the aforementioned data (six replications) was 7.7%. By combining the data for the two levels of speed, this value decreased to 5.2%.

The main effect of load on the coefficient of friction was significant (F = 65.2 >

 $F_{(3, 60)} = 6.17$ ) at the 0.001 level. Higher loads resulted in higher values of the coefficient of friction. The first order interaction was significant, indicating that load and slider design were not independent. The coefficient of variation for this data was 5.0%.

The loads were ranked with respect to constant slider design as shown in Table 4. The values for the 500 gm load are included for comparison, but were not ranked statistically. With the exception of slider design  $B_3$ , there was no significant difference (p < 0.05) observed between the 10 and 25 gm loads. Little distinction could be made among any of the loads for slider designs  $B_1$  and  $B_2$ . Values for the 500 gm load probably were equal to those for the 100 gm level for designs  $B_3$  through  $B_5$ .

The effect of slider design on the coefficient of friction was examined at five levels

Slider	Ranking								
Design	(units of gm/gm)								
<i>B</i> <sub>1</sub>	Load	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$			
-	Mean	0.17	0.20	0.22	0.24	(0.28)			
<i>B</i> .,	Load	$A_3$	$\overline{A_2}$	$A_1$	$A_4$	$A_5$			
-	Mean	0.14	0.16	0.17	0.20	(0.18)			
$B_3$	Load	$A_2$	$\overline{A_1}$	$A_3$	$A_4$	$A_5$			
.,	Mean	0.28	0.35	0.38	0.57	(0.55)			
$B_4$	Load	$A_1$	$\overline{A_2}$	$A_3$	$A_4$	$A_5$			
-	Mean	0.38	0.39	0.46	0.46	(0.44)			
$B_5$	Load	$\overline{A_1}$	$A_2$	$\overline{A_4}$	$A_3$	$A_5$			
U U	Mean	0.21	0.22	0.35	0.40	(Ŏ.35)			

 TABLE 4

 Results of Duncan's New Multiple Range Test for Load Differences

Note: Dependent variable, coefficient of friction. Underscore indicates no significant difference at the 95% level.

2

Load	Ranking (units of gm/gm)							
$A_1$	Slider design	<i>B</i> <sub>2</sub>	B <sub>1</sub>			B <sub>4</sub>		
	Mean	0.17		0.21	0.35	0.38		
$A_2$	Slider design	$B_2$	$B_1$	$B_5$	$B_3$	$B_4$		
	Mean	0.16	0.20	0.22	0.28	0.39		
$A_3$	Slider design	$B_2$	$\overline{B_1}$	$\overline{B_3}$	$B_{5}$	$B_4$		
5	Mean	0.14	0.22	0.38	0.40	0.46		
$A_4$	Slider design	$B_2$	$B_1$	$\overline{B_5}$	$\overline{B_4}$	$B_3$		
-	Mean	0.20	0.24	0.35	0.46	0.57		
$A_5$	Slider design	$\overline{B_2}$	$B_1$	$B_{5}$	$B_4$	$B_3$		
	Mean	(0.18)	(0.28)	(0.35)	(0.44)	(0.55)		

 TABLE 5

 Results of Duncan's New Multiple Range Test

 for Slider Design Differences

Note: Dependent variable, coefficient of friction. Underscore indicates no significant difference at the 95% level.

 $(B_1 \text{ through } B_5)$  for five loads. The main effect of slider design on the coefficient of friction was significant  $(F = 227 > F_{(4, 60)} = 5.31)$  at the 0.001 level.

The designs were ranked with respect to constant load for the coefficient of friction as shown in Table 5. The values for the 500 gm load are included and were ranked based on the data obtained from the slider speed analysis. The means for slider designs  $B_3$  through  $B_5$  were significant statistically (p < 0.05) at loads of 100 and 500 gm, and were ranked in order of decreasing conical angle; the sharper angle was of higher magnitude. With the exception of the 50 and 500 gm loads, no significant difference (p < 0.05) was observed between designs  $B_1$  and  $B_2$ . In general, the larger diameter slider designs had lower values of the coefficient of friction.

#### Discussion

It was assumed that the effect of crystallographic direction on frictional properties was less important than other factors. The low F values found when comparisons of two crystals with three replications (of arbitrary sliding direction) were made imply that this assumption was valid for the friction force and coefficient of friction. Although Bowden and Brookes<sup>4</sup> have observed for MgO single crystals that a critical amount of subsurface deformation was necessary for anisotropic friction, it should be emphasized that the slip and cleavage systems of MgO and fluorapatite are quite different. Studies on single crystals of sapphire (hexagonal) have yielded conflicting results with respect to frictional anisotropy on the basal surface.<sup>8,9</sup>

The purpose of the polishing and surface treatment procedure<sup>5</sup> was to standardize the crystals, since it is well established that frictional behavior responds to environmental and polishing effects.<sup>3,10</sup> The data reported herein may be considered valid only for the particular surface condition produced, although trends may be predictable.

In contrast to those variables related to the quantitative measure of wear,<sup>5</sup> no strain rate effect was observed for friction. No effect on friction would be predicted if indeed friction were related more to the initiation of cracks rather than to their propagation over the range of speeds studied. This suggests that the strain rate effect observed for the wear variables is the result of the propagation of surface and subsurface cracks.

As previously discussed,<sup>5</sup> the use of analysis of variance and multiple comparisons requires the assumption of equal variances; their power as statistical tools decreases as this assumption becomes less of a reality. With the friction force data, the assumption of equal variances was not as valid as it was with the coefficient of friction data. It was thought, therefore, that more reliable conclusions could be drawn from the coefficient of friction data for the analysis of load and slider design effects.

The coefficient of friction was low in value ( $\overline{f} = 0.20$ ) and relatively insensitive to loads over the range studied for slider designs  $B_1$  and  $B_2$ , and for slider designs  $B_3$  through  $B_5$  at the lower loads. In these in-

stances the penetration was observed to be relatively low, ranging from 0.04 to 0.64 micrometers ( $\mu$ m).<sup>5</sup> Under such conditions friction probably represents the combined effects of adhesion and deformation components. With low penetration, a plastic deformation mechanism may be able to accommodate the strain (energy) resulting from sliding. Where additional strain has to be accommodated, a cleavage mechanism probably would relieve this strain by yielding a limited amount of tensile cracking and correspondingly higher values of friction.

High values of friction ( $\overline{f} = 0.45$ ) were observed for slider designs  $B_3$  through  $B_5$ at the higher loads studied, with sharper angles vielding higher values of friction. In these instances penetration was in the range of 1.0 to 25 µm.5 Higher friction values were associated with higher values of penetration, although the increase was not of the magnitude suggested by Bowden and Brookes<sup>4</sup> for MgO single crystals where the deformation was primarily plastic. In the present study, it is suggested that cleavage and chevron formation (crystallographically nonspecific fracture) are more likely to be the dominating mechanisms of strain release than is plastic deformation under conditions of deep penetration for fluorapatite single crystals. This may be due in part to the limited number of operable slip systems on the basal plane of a hexagonal crystal,<sup>9</sup> as contrasted to the number of slip systems available in MgO.4

#### Conclusions

The frictional behavior of natural fluorapatite single crystals under sliding was evaluated by an examination of the variables friction force and coefficient of friction. Trends observed from these data were related to trends observed from quantitative wear data.

The coefficient of friction data were more reliable in discriminating among differences in load and slider design than the friction force data.

No strain rate effect was observed for the coefficient of friction under the conditions examined. This suggested that friction was more related to crack initiation, whereas the variables measuring wear were more related to crack propagation.

Low values of friction ( $\overline{f} = 0.20$ ) were observed with low values of penetration (< 0.65 µm). This suggested that friction under these conditions was composed of an adhesive component and a deformation component. Plastic deformation probably was the primary mechanism of strain energy release under these conditions, although some cracking indicated that a cleavage mechanism was initiated.

High values of friction ( $\overline{f} = 0.45$ ) were observed with high values of penetration (1.0 to 25 µm). Under these conditions cleavage and chevron formation were suggested as the major mechanisms of strain energy release.

### References

- 1. BOWDEN, F.P., and TABOR, D.: The Friction and Lubrication of Solids, Oxford: Oxford University Press, part I, 1954.
- Oxford University Press, part I, 1954.
  2. GREENWOOD, J.A., and TABOR D.: The Friction of Hard Sliders on Lubricated Rubber—The Importance of Deformation Losses, Proc Phys Soc 71:989-1001, 1958.
- 3. STEIJN, R.P.: Friction and Wear of Single Crystals, Wear 7:48-66, 1964.
- 4. BOWDEN, F.P., and BROOKES, C.A.: Frictional Anisotropy in Non-metallic Crystals, *Proc R Soc (London) A* **295:**244-258, 1966.
- 5. POWERS, J.M., and CRAIG, R.G.: Wear of Fluorapatite Single Crystals: I. A Method for Quantitative Evaluation of Wear, J Dent Res 51:168-176, 1972.
- 6. DALBY, J., programmer: BMD08V— Analysis of Variance, Statistical Research Laboratory, University of Michigan, Ann Arbor, 1968.
- 7. DUNCAN, D.B.: Multiple Range and Multiple F-test, *Biometrics* 11:1-42, 1955.
- 8. BUCKLEY, D.H.: Frictional Characteristics in Vacuum of Single and Polycrystalline Aluminum Oxide in Contact with Themselves and with Various Metals, *ASLE-Trans* 11:134-145, 1967.
- 9. STEIJN, R.P.: On the Wear of Sapphire, J Appl Phys 32:1951-1958, 1961.
- BUCKLEY, D.H.: Effect of Surface Active Media on Friction, Deformation and Fracture of Calcium Fluoride, NASA TN D-5580, 1969.