Reprinted from THE REVIEW OF SCIENTIFIC INSTRUMENTS, Vol. 22, No. 1, 1–29, January, 1951
Printed in U. S. A.

# Radio Telemetry

M. H. NICHOLS AND L. L. RAUCH
*Department of Aeronautical Engineering, University of Michigan, Ann Arbor, Michigan*
(Received September 25, 1950)

## I. Introduction

ALTHOUGH telemetry was developed some years ago for such things as remote metering in the electrical power industry,(R5) the use of a radio link did not become of real importance until the need for remote metering arose in the field of high performance aircraft and rocket development and in the field of atmospheric research. There are essentially two reasons why it is often necessary to resort to the use of radio telemetry, as opposed to airborne recording, for example. One reason is the difficulty of recovery of records from certain types of flights and the other is that in many cases the limitations of space, weight, and operating conditions can be better met by radio telemetry.

There are several features which distinguish radio telemetry from ordinary communication. One is the severe limitation on space and weight and the rigor of the operating conditions, particularly at the transmitter, and another is the high signal-to-noise ratio requirement which results from the precision required in metering. Also in many cases it is necessary to meter quantities which have very slowly varying components, which means that essentially DC response is required. Almost every application of radio telemetry involves some form of multiplexing—i.e., the transmission of several channels of information by the same radio carrier. The high signal-to-noise requirement limits the amount of crosstalk that can be tolerated at any time. In order to simplify reduction of the data, it is generally desirable for each channel to have essentially a linear response. Taken together, these features may place severe requirements on the radio link and *it is important to realize from the start that the characteristics of the radio link are a central factor in the telemeter design.*

As in any other engineering problem, the choice of methods of modulation, multiplexing, and instrumentation to be used in a particular type of telemetry problem usually depends on a number of practical considerations. Among these are antenna requirements and limitations, operating conditions, type of instruments used to convert the quantities to be metered into electrical signals for use in modulation, precision required, number of quantities to be simultaneously metered and their frequency spectrum, radio range, space, weight and power available, expendability, etc. The purpose of this paper is to review and develop the underlying theory of radio telemetry to the point where it can be applied to the choice of method which is to satisfy these requirements and to the formulation of specifications for future developments. No attempt is made to cover techniques. Since there is increasing use of airborne recording and since many of the principles involved in analysis of radio telemetry apply to recording methods, Section VII on recording is included. No attempt will be made in this paper to discuss the problems of propagation, antenna design, etc.

Although work on the material in this paper was started when the authors were employed on an NDRC project at Princeton University, the paper was completed and written under the sponsorship of United States Air Force. This article was planned jointly by both authors, but Sections I through VII and Appendix 1 were prepared principally by M. H. Nichols and Sections VIII through X and Appendices 2 through 5 prepared principally by L. L. Rauch.

## II. Nomenclature

$a_{it}$ = improvement threshold of the $i$th frequency modulated sub-carrier (measured at the video output of the radio receiver).

$AM$ = amplitude modulation.

$A_n$ = ratio of the individual sub-carrier amplitude (from a group of $n$ sinusoidal sub-carriers of equal amplitude) which will cause overmodulation $P_n$ of the time to the largest individual sub-carrier amplitude which will cause no overmodulation at all.

$a_{mi}$ = fully modulated amplitude of the $i$th sub-carrier.

$a_{0i}$ = unmodulated RMS amplitude of $i$th sub-carrier (measured at the video output of the radio receiver).

$\alpha$ = a positive number used in connection with PAM whose reciprocal, $1/\alpha$, gives the fraction of the permissible time that an individual channel is switched on.

$\beta$ = fraction of maximum possible pulse widening.

$C_n$ = cumulative probability function for sum of $n$ sub-carriers.

$D$ = deviation ratio of a frequency-modulated radio link.*

$E$ = information efficiency of modulation multiplex method.

$F$ = number of samples per second per channel of a time division multiplex.

$f$ = information frequency in an individual channel of a multiplex.

* The symbol $D$ is also used in Section VIII, when discussing overmodulation, to denote the overload value.

$F_c$ = video pass band of a radio link.
$f_D$ = maximum frequency deviation of a frequency-modulated (or phase-modulated) radio carrier.
$f_{dh}$ = maximum frequency deviation of the highest frequency sub-carrier.
$f_{di}$ = maximum frequency deviation of $i$th frequency modulated sub-carrier.
$f_h$ = unmodulated frequency of the highest frequency sub-carrier.
$f_i$ = unmodulated frequency of $i$th sub-carrier.
$FM$ = frequency modulation.
$f_m$ = maximum information frequency in a channel of a multiplex.
$f_{mi}$ = maximum information frequency transmitted in $i$th channel.
$F_R$ = cut-off frequency of a recorder.
$g$ = fraction of the time between channels in PPM or PWM allowed for guard space to prevent overlapping of pulses.
$k$ = PAM video band width in units of $nF$.
$K_2$ = a constant characteristic of the modulation of the radio carrier.
$k_2$ = RMS fluctuation noise per unit band width in the video output of the comparison single channel AM link.
$K_{1i}$ = a constant characteristic of the type of modulation of the $i$th sub-carrier.
$m$ = an integer.
$M_{1i}$ = modulation index of the $i$th sub-carrier.
$M_{2i}$ = modulation index of the radio link due to the $i$th sub-carrier.
$N$ = number of binary digits in a PCM system.
$n$ = number of channels in a multiplex.
$PCM$ = pulse code modulation.
$PAM$ = pulse amplitude modulation.
$PM$ = phase modulation.
$p_n$ = probability density function for sum of $n$ sub-carriers.
$P_n$ = probability of the instantaneous sum of $n$ sinusoidal sub-carrier voltages exceeding the voltage $S$ which will fully modulate the radio link.
$PPM$ = pulse position modulation.
$PWM$ = pulse width modulation.
$\Phi_D$ = maximum phase deviation of a phase-modulated radio carrier.
$r$ = number of side band pairs in the video pass band of a PAM multiplex (i.e., video band width in units of $F$).
$R_1$ = signal-to-noise ratio before modulation and after demodulation.
$R_1^*$ = $R_1$ at improvement threshold.
$R_2$ = signal-to-noise ratio after modulation and before demodulation.
$R_2^*$ = $R_2$ at improvement threshold.
$R_{0i}$ = wide band improvement referred to the $i$th channel of a multiplex.
$R_{ti}$ = ratio of the carrier improvement threshold to the $i$th sub-carrier improvement threshold (the latter expressed in terms of corresponding carrier strength).
$S$ = RMS amplitude of the sinusoidal video output of the comparison single channel AM link under condition of full modulation.
$S_t$ = improvement threshold of a frequency- or phase-modulated radio carrier. (Measured at the video output of the fully modulated comparison AM link).
$W_1$ = band width before modulation and after demodulation.
$W_2$ = band width after modulation and before demodulation.

## III. Frequency Division Multiplexing in Radio Telemetry

It is usually not practical to use a separate radio link for each channel if more than one channel of information is to be metered simultaneously. Two important reasons for this are antenna complications and the saving of space, weight, and power which usually can be realized by multiplexing onto a single radio carrier.

There are two general methods of multiplexing in use. One is frequency division (sub-carriers) and the other is time division (commutation). A frequency division system uses a separate sub-carrier for each channel with spaced sub-carrier frequencies. The sub-carriers are mixed linearly and then modulate the carrier. At the receiving end, the sub-carriers are selected out by a linear frequency selective circuit and demodulated. A time division system samples the information in the channels in cyclic serial sequence and puts out a pulse (or pulses) for each channel which is modulated in accordance with the information in each channel. These pulses are then used to modulate the carrier. The sampling process is usually accomplished by some form of commutator either electronic or mechanical. In some cases of slow commutation, the output of the radio receiver is simply recorded by a pen recorder or on a photographic paper and the channels sorted out by inspection. It is also possible to use an automatic apparatus to sort out the channels from the record, demodu late them, and record them. When a large number of channels is sampled at a high rate, a synchronized commutator at the receiver is usually provided.

In specifying the method of multiplexing and modulation used in a system it is customary to work from the individual input channels toward the carrier.[L2] For example, AM-FM means a frequency division multiplex with sub-carriers amplitude modulated in accordance with the information in each channel and with the sub-carriers frequency modulating the carrier; PPM-AM means a time division multiplex with pulses position modulated (in time) in accordance with the information in each channel and with the pulses amplitude modulating the carrier; PAM-FM-FM means that a separate time division multiplex with amplitude modulated pulses is used to modulate the frequency of the sub-carriers of a frequency division multiplex the output of which frequency modulates the carrier. Thus the last group of letters specifies the modulation on the carrier; the preceding groups of letters specify the type of multiplex and modulation of each stage. All three letter groups start with a "P" which means time division multiplex and all two letter groups which precede the final group mean frequency division in the multiplex stage corresponding to the position of the group.

The method of frequency division has had wide use in telephony particularly over wire links.(C2) Its use in radio telemetry is largely restricted to systems having not more than about ten channels. The principal reason for this is crosstalk which results primarily from non-linearity in the radio link. In principle, crosstalk can be corrected for if the characteristics of the radio link and the phases and amplitudes of the sub-carriers are known at all times. However, this is obviously impractical especially when there are more than a few sub-carriers. Because of the precision required in the metering, it is generally necessary to suppress crosstalk to 40 db or more. In frequency division telemetry some form of CW radio transmission is usually used inasmuch as pulse methods are better adapted to time division multiplexing. Due to the particular limitations on operating conditions imposed on radio links for telemetry, as already noted, it is difficult to achieve a high degree of linearity in the radio link. In current telemetry practice, the better radio links have the order of one percent total harmonic distortion in terms of voltage—i.e., of the order of 0.01 percent of the video power appears in the harmonics when the link is modulated to a reasonable level by a single sinusoidal frequency.† When such a link is used to transmit sub-carriers, objectionable crosstalk can result particularly when more than a few sub-carriers are used.

### III-1. CROSSTALK IN FREQUENCY DIVISION

Frequency division crosstalk has been discussed by Bennett(B1) and others in relation to telephony and by Stedman(S5) in relation to radio telemetry. If there is appreciable harmonic distortion in the radio link and if as many as ten or more sub-carriers are used, the important cross modulation frequencies number in the thousands for third harmonic distortion alone.(S5) Thus when many sub-carriers are used, crosstalk tends toward the characteristics of fluctuation noise.

If there are only a few sub-carriers it is frequently possible to choose sub-carrier frequencies so as to eliminate in the receiver frequency selector a large portion of the important cross modulation frequencies—i.e., the ones that occur with large amplitude. It should be emphasized that the cross modulation frequencies occur with amplitudes higher than the corresponding harmonic. For example, in the case of third harmonic distortion, the cross modulation frequencies of the form $f_p \pm f_q \pm f_r$, when $p \neq q \neq r$, occur with amplitudes six times the third harmonic if the amplitudes of all sub-carriers are considered equal.(S5) Also, as pointed out by Stedman,(S5) another effect of third harmonic cross modulation is to increase the amplitude of the fundamental of each sub-carrier by an amount depending on the amplitude of the other channels and the degree of third harmonic distortion. This effect is important in AM sub-carriers but not in FM sub-carriers because of the limiting before demodulation. This effect can be larger than other third harmonic cross modulation effects especially if more than a few sub-carriers are used, and, of course cannot be eliminated by choice of sub-carrier frequencies. Stedman(S5) has shown that this effect can be used as a convenient method of measuring the amount of cross modulation in a frequency division system using AM sub-carriers.

In the case of more than a few channels, it becomes a practical impossibility to eliminate, by choice of sub-carrier frequencies, the large cross modulation terms in a reasonable video band width. In this case the choice of sub-carrier frequencies should depend on other factors such as noise characteristics of the system (see Section III-2) and a radio link which is sufficiently linear must be procured.

In radio links, as well as in other circuits using vacuum tubes, nonlinear effects generally increase as the modulation level is increased. Therefore, there is a certain modulation level (generally called full modulation) which, if exceeded, will result in objectionable crosstalk. Exceeding this level is usually called overmodulation. In order to reduce fluctuation and impulse noise effects, it is desirable to operate the radio link as near full modulation as possible. This is discussed in the next section.

### III-2. NOISE CHARACTERISTICS OF FREQUENCY DIVISION SYSTEMS

In addition to harmonic distortion, discussed in the previous section, the radio link introduces noise into the system. In the interests of maintaining high precision in the metering, it is desirable to keep the individual channel signal-to-noise ratios as high as possible. In comparing noise characteristics (other than crosstalk) of various types of multiplexing, it is convenient to define a wide band improvement ratio, $R_0$, in such a way that it is independent of the carrier signal-to-noise ratio in the radio link.(L2) The wide band improvement ratio, $R_0$, is therefore defined as the ratio of the signal-to-noise ratio of a fully modulated single channel of the multiplex system to the signal-to-noise ratio of a fully modulated one channel system in which the information in the single channel directly amplitude modulates a single channel radio link operating under the same conditions of received carrier power (as is customary the side band

† For an example of the effort required to reduce radio link distortion to a low value see Burrows and Deceno, Proc. Inst. Radio Engrs. 33, 84 (1945).

power is neglected in the comparison radio link) and noise power per unit band width. When computing such things as improvement thresholds, it is convenient to express the radiofrequency noise per unit band width, carrier signal strength, etc., in terms of the output of this single channel AM comparison link. To simplify the expressions, the full modulation RMS video output, $S$, of the radio link carrying the multiplexed signal is taken to be equal to the full modulation RMS video output, $S$, of the comparison link.

At present, most radio telemetry links are operated at VHF or higher. The principal type of noise encountered at these frequencies is fluctuation noise.‡ The analysis in this paper is for fluctuation noise only and, unless modified, the term "noise" will mean fluctuation noise. Fluctuation noise has a constant RMS amplitude *versus* frequency spectrum over the range of the RF pass band of a radio receiver, but the phase distribution is random. However, the RMS amplitude of the noise-frequency spectrum of the output of a radio receiver varies with frequency in a manner which depends on the type of modulation used, although the phase distribution remains random. This results in two important characteristics of the final noise output. For any type of noise with a continuous spectrum, the noise power in a frequency band of small width $\Delta\omega$ is proportional to $\Delta\omega$.[L1] Thus the RMS noise voltage is proportional to $(\Delta\omega)^{\frac{1}{2}}$. Due to the random phase distribution of fluctuation noise, the crest voltage (height of the larger peaks)§ is proportional to the RMS voltage, and therefore to $(\Delta\omega)^{\frac{1}{2}}$. This is not true in the case of impulse noise, where the height of the pulse is proportional to $\Delta\omega$.[L1]

For convenience in setting up Table I which summarizes the fluctuation noise characteristics for frequency division multiplexing, $R_{0i}$ (the wide band improvement, $R_0$, for the $i$th channel) can be written

$$R_{0i}=K_{1i}M_{1i}M_{2i}K_2, \qquad \text{(III-1)}$$

where $K_{1i}$ is a constant characteristic of the type of modulation of the $i$th sub-carrier, $M_{1i}$ is the modulation index of the $i$th sub-carrier, $M_{2i}$ is the modulation index of the radio link due to the $i$th sub-carrier|| and $K_2$ is a constant equal to $1/\sqrt{2}$.¶

TABLE I. Frequency division noise characteristics.

| | First modulation constant | First modulation index | Second modulation index | Sub-carrier improvement threshold | Carrier improvement threshold |
|---|---|---|---|---|---|
| Type | $K_{1i}$ | $M_{1i}$ | $M_{2i}$ | $a_{it}/k_2$ | $S_t/k_2$ |
| AM-AM | 1 | $a_{0i}/S$ | 1 | | |
| FM-AM | $\sqrt{3}$ | $\dfrac{a_{0i}}{S}\dfrac{f_{di}}{f_{mi}}$ | 1 | $4.6(f_{di})^{\frac{1}{2}}$ | |
| AM-FM | 1 | $a_{0i}/S$ | $\dfrac{f_D}{f_i}$ | | $3.2(f_D)^{\frac{1}{2}}$ |
| FM-FM | $\sqrt{3}$ | $\dfrac{a_{0i}}{S}\dfrac{f_{di}}{f_{mi}}$ | $\dfrac{f_D}{f_i}$ | $4.6\dfrac{f_i}{f_D}(f_{di})^{\frac{1}{2}}$ | $3.2(f_D)^{\frac{1}{2}}$ |
| AM-PM | 1 | $a_{0i}/S$ | $\Phi_D$ | | $3.2(f_D)^{\frac{1}{2}}$ |
| FM-PM | $\sqrt{3}$ | $\dfrac{a_{0i}}{S}\dfrac{f_{di}}{f_{mi}}$ | $\Phi_D$ | $\dfrac{4.6}{\Phi_D}(f_{di})^{\frac{1}{2}}$ | $3.2(f_D)^{\frac{1}{2}}$ |

For convenience in the following discussions, Table I gives the values of $K_{1i}$, $M_{1i}$, and $M_{2i}$ for the types of modulation used in frequency division telemetry.** Since in FM and PM, the noise improvement is not realized unless the improvement threshold is exceeded, the sub-carrier and carrier improvement thresholds are also given. The nomenclature in the table is defined in Section II. In Table I it is considered that the FM and PM improvement threshold of the carrier is reached when the amplitude of the unmodulated RF signal equals the crest fluctuation noise in the RF pass band. For this determination, a fluctuation noise crest factor of 4 is used[C4, L1] and the RF pass band is taken as $2.6f_D$ for FM and PM where $f_D$ is the maximum frequency deviation.[K1] The RMS noise per unit band width in the RF pass band is $k_2/\sqrt{2}$ inasmuch as $k_2$ is defined as the RMS noise per unit band width in the video output of the one channel AM system.** The RF improvement threshold is therefore given by

$$\sqrt{2}S=4(k_2/\sqrt{2})(2.6f_D)^{\frac{1}{2}}. \qquad \text{(III-2)}$$

‡ The other type, namely impulse noise, is either man-made or the result of atmospheric disturbances and will not be discussed in this section. In frequency modulation receivers the wide band improvement for impulse noise is twice the deviation ratio as compared to $\sqrt{3}$ times the deviation ratio for fluctuation noise. (See reference C4.) The improvement threshold for impulse noise is somewhat higher than that for fluctuation noise. (See reference C4.)

§ For a rigorous definition of crest-noise voltage, some distribution function must be assumed for the instantaneous noise voltage. Many writers, for good reason, assume the normal law as a basis for theoretical investigations. This law offers a finite probability for arbitrarily high noise voltages, but in practice nonlinear circuit elements bring the cumulative distribution function to unity for finite values of the instantaneous noise voltage. If we assume the crest voltage is attained when the instantaneous voltage is greater than or equal to four times the RMS value, the normal law provides the result that during a sufficiently long period the crest value will be reached a fraction of the time equal to $63\times10^{-6}$. More can be said if the amplitude *versus* frequency distribution of the noise is taken into account.

|| For a discussion of FM and PM modulation indices, noise improvement, etc., see references (C4, H6).

¶ The value $K_2=1/\sqrt{2}$ results from the appearance of the sub-carrier and both side bands in the video. Thus with AM sub-carriers, for example, the video band width taken up by the $i$th sub-carrier is $2f_{mi}$, where $f_{mi}$= maximum information frequency in the $i$th sub-carrier whereas in the comparison single channel AM system the required video band width is $f_{mi}$. The ratio of the square roots of the band widths gives the factor $K_2=1/\sqrt{2}$. This value also holds for FM and PM.

** In order to simplify calculations in the case of an FM carrier (triangular noise spectrum), the noise amplitude is assumed to be constant over the pass band of a sub-carrier and is taken to be equal to the noise amplitude corresponding to the center of the pass band. Since the sub-carrier pass bands are narrow relative to their center frequencies this is a good approximation.

** See reference (G2), page 249.

Where $S$ = RMS amplitude of the sinusoidal video output of the comparison single channel AM link under condition of full modulation; (assuming peak detection of modulated envelope) the corresponding RMS value of the carrier signal is therefore $S$. The relation for the sub-carriers contains $k_2$ instead of $k_2/\sqrt{2}$ and is given by

$$\sqrt{2}a_{it}=4k_2(2.6f_{di})^{\frac{1}{2}}. \qquad \text{(III-3)}$$

### *III-3. OVERMODULATION VERSUS NOISE*

As pointed out in Section VIII-1, overmodulation causes undesirable crosstalk. On the other hand, Table I shows that for highest signal-to-noise ratio, the sub-carrier amplitudes $a_{0i}$ should be as large as possible. If there is to be no overmodulation at any time, the instantaneous sum of the peak amplitudes of the sub-carriers must never exceed the voltage necessary to fully modulate the radio link. Assuming that the distortion is not too high, this relation can be expressed in terms of the video output voltage as follows:

$$\sum_{i=1}^{n} a_{mi}=S, \qquad \text{(III-4)}$$

where $a_{mi}$ is the fully modulated amplitude of the $i$th sub-carrier and $S$ is the amplitude of the sinusoidal video output of the receiver under condition of full modulation (and is taken to be equal to the video output $S$ of the comparison single channel AM link). If the unmodulated amplitudes $a_{0i}$ of all sub-carriers are equal to $a_0$, and if all channels are fully amplitude modulated —i.e., $a_{mi}=2a_{0i}$—then the maximum unmodulated amplitude $a_0$ is given by

$$a_0=S/2n. \qquad \text{(III-5)}$$

For frequency modulated sub-carriers, the amplitude is independent of modulation so Eq. (III-5) becomes for this case

$$a_0=S/n. \qquad \text{(III-6)}$$

On the other hand, if overmodulation can be tolerated even a very small fraction of time, considerably larger amplitudes than given by (III-4) can be used if the number of channels is the order of ten or greater and provided that the phases of the sub-carriers are random. This last condition is satisfied if separate sub-carrier oscillators are used. The probability, $P_n=1-C_n$, of the instantaneous sum of $n$ sinusoidal sub-carrier voltages exceeding the level $S=D$ for full modulation is calculated in Section VIII. The results are summarized in Fig. 3 in which the permissible overload value $D$ is plotted *versus* $n$, the number of sub-carriers for various values of $P_n$ for the case of $n$ sub-carriers all of amplitude $(2/n)^{\frac{1}{2}}$ (this gives total RMS value unity). The line $D=(2n)^{\frac{1}{2}}$ represents the condition for no overmodulation at any time. The probability $P_n$ may be interpreted as the fraction of the time that the overload level for full modulation is exceeded. From Fig. 3 it can be seen that even if values of $P_n$ as large $10^{-2}$ are permitted, no great advantage is gained unless 10 or more sub-carriers are used but that there is considerable advantage in allowing for $P_n$ values even as small as $10^{-4}$ when the number of sub-carriers becomes large. It is of importance to notice that the value of $D$ is not greatly affected by the value chosen for $P_n$ provided $P_n \ll 1$. In practice, this means that if the amplitudes of all sub-carriers are increased simultaneously the overmodulation will be negligible until a critical region is reached, after which any appreciable increase of the amplitudes of the sub-carriers will result in a large increase in the fraction of the time during which overmodulation occurs.

In order to operate a frequency division telemeter at optimum precision, it is therefore necessary to compromise on the fraction of time of overmodulation so that the sub-carrier amplitudes may be increased. There is no point in being so rigorous in the overmodulation requirement that the signal-to-noise ratio becomes disproportionately small—i.e., that the ratio $a_{0i}/S$ in Table I, Column 2, is disproportionately small.

For purposes of discussion, it is convenient to define the quantity $A_n$ as the ratio of the sub-carrier amplitude which will cause overmodulation $P_n$ of the time to the largest sub-carrier amplitude which will cause no overmodulation at all, assuming all sub-carriers to have the same amplitude. Figure 4 is a plot of $A_n$ *versus* $n$. For larger values of $n$ a very good approximation is $A_n=0.43(n)^{\frac{1}{2}}$. In terms of $A_n$, the first modulation index $M_{1i}$ becomes

$$M_{1i}(AM)=\frac{A_n}{2n}; \quad M_{1i}(FM)=\frac{A_n}{n}\frac{f_{di}}{f_{mi}}. \qquad \text{(III-7)}$$

If $A_n=0.43(n)^{\frac{1}{2}}$, then the first modulation index becomes

$$M_{1i}(AM)=\frac{0.22}{(n)^{\frac{1}{2}}}; \quad M_{1i}(FM)=\frac{0.43}{(n)^{\frac{1}{2}}}\frac{f_{di}}{f_{mi}}. \qquad \text{(III-8)}$$

The above results apply to the case of $n$ sinusoidal sub-carriers of constant amplitude but random phase. The application of the results to AM sub-carriers is somewhat different than the application to FM sub-carriers. For in the FM case, the amplitude remains essentially constant independent of modulation so the above results are independent of modulation and permit the determination of the maximum sub-carrier amplitude once $P_n$ is chosen.

In AM sub-carriers, the above results apply directly to the case of full modulation simultaneously applied to all channels. If the full AM modulation capability of the sub-carrier is used, then the use of the curves gives twice the unmodulated amplitude. For a given $P_n$ value, the condition of all channels simultaneously fully modulated is the most stringent condition because in general all channels will not be fully modulated at once. In order to take this into account, an additional statistical analysis of the data in the channels would have to be made. In this connection, it is sometimes possible, by such things as simply reversing the sign of the data before modulation, to avoid simultaneous high level modulation on all or most of the channels. An example might be adjacent strain gauge stations on structural elements. Stedman[S5] has estimated that upon sufficient operating experience, it may be possible to increase the sub-carrier levels by as much as a factor of 1.5 to 2 above the levels calculated by use of Fig. 3.

### *III-4. EFFECT OF MULTIPATH TRANSMISSION*

Multipath transmission can lead to serious distortion in the case of an FM carrier[C3, G1, M4] and in the case of a fully or nearly fully modulated AM carrier.[G1] In a frequency-division multiplex this results in crosstalk. Aside from certain precautions in the design and operation of the radio link which are discussed in the above references, the multipath effect in radio telemetry can be reduced by the use of directional receiving antennas provided the transmitter is at sufficiently high elevation angle above the horizon. By the use of several receiving stations distributed along the trajectory, the angle of elevation of the transmitter relative to one or more of the receiving stations can usually be kept rather high.

### *III-5. SUB-CARRIER MODULATION*

If a reasonably large deviation ratio can be obtained, frequency modulated sub-carriers have the advantage of noise suppression which, of course, includes crosstalk suppression. Melton[M2] and Coe[C1] have described FM sub-carrier systems in which the sub-carriers, when fully modulated, are deviated $\pm 7.5$ percent of center frequency with satisfactory linearity of response and compactness of equipment at the transmitting end. Thus with a deviation ratio of five, for example, the maximum information frequency in each channel should not be greater than about two percent of the sub-carrier frequency. Therefore, high information frequencies imply high FM sub-carrier frequencies. If an FM carrier is used, the typical triangular noise spectrum must be taken into account. For the noise reduction of the FM sub-carrier is proportional to the deviation ratio but it follows from above that the sub-carrier frequency and hence the noise in the sub-carrier pass band (because of the triangular noise spectrum of the FM carrier) are proportional to the sub-carrier deviation ratio. Thus, in this case there is no net increased noise improvement realized by increasing the sub-carrier deviation ratio if a corresponding increase in sub-carrier frequency is required. However, there is crosstalk improvement provided, of course, that the nonlinear distortion of the radio link does not increase in proportion to the video frequency which is often the case, particularly, if the FM link operates at a low deviation ratio.[R4] On the other hand, if the maximum information frequency is low, the frequency selector (sub-carrier filter) often places a lower limit on the sub-carrier band width so that a high deviation ratio is available without the necessity of running up the sub-carrier frequency. In this case there can be considerable advantage in crosstalk and noise improvement. If the link is AM, then the noise characteristic is rectangular and the net improvement which can be realized is the usual FM sub-carrier improvement regardless of the sub-carrier frequency. The relative noise characteristic of the AM and FM sub-carriers can be readily calculated from Tables I and II.

If in FM-FM, the sub-carrier deviation ratios are proportional to the sub-carrier frequency and if all sub-carriers have the same amplitude, then the individual $R_{0i}$ of all the channels will be the same because of the triangular noise spectrum of the FM radio link. For the same reason, in AM-FM it is necessary to make each sub-carrier amplitude proportional to its frequency (pre-emphasis) in order to have equal $R_{0i}$ for all channels. If in FM-AM and FM-PM the sub-carrier deviation ratio is proportional to sub-carrier frequency, then it is necessary to make each sub-carrier amplitude inversely proportional to the sub-carrier frequency in order to have equal $R_{0i}$ for all channels, etc.

In addition to noise and crosstalk improvement, when they can be realized, FM sub-carriers also have the advantage of an output level independent of the output level of the carrier provided that the individual channel levels are sufficiently high and provided that effective limiters are used. Thus when an AM carrier is used, for example, only mildly effective automatic volume control, if any, is required. Another advantage of FM sub-carriers with large deviation ratio is that the requirements on the frequency selector to avoid interchannel crosstalk are not as rigid—i.e., the attenuation at the neighboring frequencies need not be as great as for AM sub-carriers.

As discussed in Section VI, certain types of instrumentation are better adapted for AM sub-carriers than for FM sub-carriers and conversely. Therefore, it is not always practical to use FM sub-carriers even if the noise and cross talk improvement could be realized because of added complexity and instability, especially at the transmitter end. If the channel contains information which has essentially DC components, then the FM sub-carrier frequency must be sufficiently stabilized. On the other hand, amplitude stability of the FM sub-carrier is only of secondary importance. In the case of AM sub-carriers, the amplitude must be stabilized but the frequency is of secondary importance so long as the sub-carrier with its sidebands does not drift out of the pass band of the frequency selector.

Phase-modulated sub-carriers are generally not used in radio telemetry because in most applications DC response is required. In order to handle DC with phase-modulated sub-carriers, additional channels would be required in order to provide a time base for detection of constant phase shift.

### *III-6. CHOICE OF SUB-CARRIER FREQUENCIES*

Although this topic has been touched on in the previous sections, it is of sufficient importance to be summarized here. In the case of more than several AM sub-carriers on an FM carrier, Stedman[S5] has shown that in the interest of noise improvement the frequencies should be spaced as closely at the low end of the video band as the frequency selectors will permit. This follows from Table I because of the $f_i$ in the denominator of $M_{2i}$. If AM-AM is used, only considerations of instrumentation, cabling to the instruments, restrictions on the radio link band width, etc., need to be taken into account. For FM-AM it is desirable to realize as large a sub-carrier deviation ratio as possible subject to limitations set down in the previous sentence. In the case of FM-FM, crosstalk improvement can be obtained, if the radio link distortion does not increase appreciably with modulating frequency, by increasing the sub-carrier deviation ratio which implies increasing the sub-carrier frequencies. However, no net noise improvement is realized by increasing the deviation ratio if it is necessary to increase the sub-carrier frequencies in proportion because of the triangular noise spectrum of the FM carrier (see Section III-5). It should always be borne in mind that, with an FM link, increasing the sub-carrier frequencies, keeping the carrier deviation ratio constant, increases the carrier improvement threshold. In the case of a PM carrier, the height of the rectangular noise spectrum is independent of the carrier video band width if the phase deviation is held constant. Therefore, in this respect, a PM link is like an AM link except that the PM link has an improvement threshold which, if the phase deviation is constant, increases in proportion to the square root of the video band width (see Table I).

*III-7. CARRIER MODULATION*

Provided that the received carrier signal is sufficiently strong to exceed the improvement threshold, the noise improvement in FM and PM radio links, which can be realized with sufficient frequency or phase deviation, is of importance in increasing the precision of the metering. Also, the video output level of an FM or PM receiver is essentially independent of the received signal strength as long as the limiting threshold is exceeded. There are also practical considerations, particularly at the transmitter end which influence the choice of carrier modulation. In current practice, most of the frequency division systems use an FM radio link. Possibly one of the reasons for the use of FM radio links in small units is that the carrier modulation can be applied at low levels and easily amplified without too much distortion. For small AM units it seems to be most practical to perform the modulation at high level which leads to larger, heavier components and to more distortion unless sufficient feed back is used.

In deciding which type of modulation to use it must always be borne in mind that unless sufficiently strong carrier signals are received—i.e., unless the improvement threshold is exceeded—FM and PM are worse than AM as far as noise is concerned.

*III-8. IMPROVEMENT THRESHOLDS*

The sub-carrier and carrier thresholds are given in Table I. In the case of FM-FM or FM-PM, the carrier and sub-carriers each have improvement thresholds and, as the carrier signal strength diminishes, one or the other thresholds will be reached first depending upon the parameters involved. If FM-FM is used, it follows from Table I that the ratio $R_{ti}$ of the two corresponding carrier strengths is

$$R_{ti}=0.71\left(\frac{f_D}{f_{di}}\right)^{\frac{1}{2}}\frac{f_D}{f_i}\frac{a_{0i}}{S}. \qquad \text{(III-9)}$$

Therefore, if $R_{ti}>1$ for all $i$ sub-carriers than the carrier threshold is the determining factor. If all sub-carrier amplitudes are equal and the $A_n$ of Section VIII is used, Eq. (III-9) becomes for the highest sub-carrier frequency $f_h$

$$R_{th}=0.71DA_n/n(f_D/f_{dh})^{\frac{1}{2}} \qquad \text{(III-10)}$$

where $D=$carrier deviation ratio$=f_D/f_h$ and $f_{dh}$ is the frequency deviation of the highest frequency sub-carrier. Thus there is a lower limit on $D$ above which the carrier threshold governs the system. In a similar way it follows from Table I that for FM-PM

$$R_{ti}=0.71\Phi_D A_n/n(f_D/f_{di})^{\frac{1}{2}}. \qquad \text{(III-11)}$$

## IV. Time Division in Radio Telemetry

Generally speaking, the problem of crosstalk in many channel radio telemetry is handled with greater ease by time division than by frequency division. The reason for this is that in the radio link it is usually easier to obtain sufficient band width to keep the time division crosstalk down than it is to obtain sufficient linearity to keep the frequency division crosstalk down particularly when a large number of channels is required. Numerous types of pulse modulation can be used after the synthesis (that is, after the cyclic sampling of the channels). Three types in use in telemetry are pulse amplitude modulation, pulse width modulation, and pulse position modulation. Pulse code modulation is currently being considered because of its inherently large signal-to-noise ratio and the convenience with which it can be relayed.[O2]

*IV-1. FREQUENCY RESPONSE OF TIME DIVISION*

In time division each channel is sampled in sequence at a repetition rate $F$. The information is carried by some form of pulse modulation. The purpose of this section is to set down the individual channel frequency response in terms of $F$. For this purpose, it is convenient to consider two types of modulation, pulse amplitude and pulse width, inasmuch as the other types are reduced to one of these in the process of recovering the original channel information from the pulse modulation. In this discussion, it is assumed that at the receiver the pulses are sorted out according to channel so that the

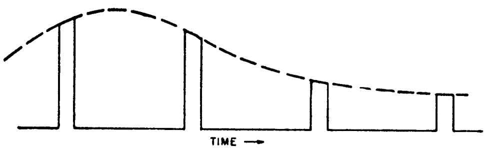



FIG. 1. Schematic wave form of pulse amplitude modulation. The dashed curve represents the wave form before sampling.

information in each channel is carried by a series of modulated pulses. This sorting out may be done by a synchronized commutator or by other automatic methods or it may be done "by hand" by inspection of the recorded pulses of several or all channels on the same channel of a recorder. It is assumed in each case that the sampling of each channel occurs at evenly spaced intervals. If the sampling is done at unevenly spaced intervals(H2) the frequency response is less than the corresponding evenly spaced case. The analysis in the unevenly spaced case is difficult to carry out and no attempt is made in this paper to develop this problem inasmuch as it is about equally feasible to perform the sampling at evenly spaced intervals.(M3, R3)

### *IV-1.1. Direct Recording of Pulse Amplitude Modulation*

The wave form after sorting out is shown schematically in Fig. 1. In some radio telemetry systems in use, this wave form is recorded directly. Let the frequency response of the recorder be such that the heights of the individual pulses recorded are not influenced by the height of any of the other pulses. The usual procedure in handling such a record is to draw a smooth curve through the peaks of the pulses. It has been estimated that if reasonable care is used in smoothing "by eye," at least five to six samples per cycle of information are required in order to keep the uncertainties less than about five percent.

If the individual samples are short in duration, the recording of the output pulses directly by recording galvanometers becomes difficult because of the large pulses of current required. Sometimes cathode-ray recording is used in which case the voltage pulses can be recorded directly. However, as will be seen in Section IV-4, any channel output pass band greater than the information band which can be realized from each channel—i.e., greater than about $F/6$ in this case—reduces the signal-to-noise ratio.

### *IV-1.2. The Low Pass Filter with Pulse Amplitude Modulation*

It can be shown by Fourier analysis(B2) that the frequency spectrum of a pulse amplitude modulated channel with sinusoidal information of frequency $f$ is as indicated in Fig. 2. The spectrum consists of a series of side band pairs several of which are shown. The original input can be obtained *without distortion* by inserting an ideal low pass filter which cuts off at $F/2$ or by an ideal band pass filter which passes the band $mF \pm F/2$ where $m$ is an integer. It is clear that such filters would permit a maximum frequency response of $F/2$. In practice, the maximum frequency is somewhat lower because of the impossibility of making vertical cut-off filters. A conventional low pass filter which transmits with negligible distortion up to $0.4F$ and is cut off sufficiently by $0.6F$ is feasible provided $F$ is not too small (see Section IV-4.4). Such a filter results in a maximum undistorted frequency response of $0.4F$. It is important to note that if at the transmitter end a channel information frequency of greater than $0.4F$ is inserted, it will produce distortion at the receiver because terms of the type $|mF-f| < 0.6F$, where $m$ is an integer, will pass through the filter. If components higher than $0.4F$ are possible, then a low pass filter must be inserted in the channel at the transmitter ahead of the commutator which performs the sampling.

### *IV-1.3. The Pulse Widener with Pulse Amplitude Modulation*

If the pulses of Fig. 1 are widened before they are fed to the recording galvanometer the peak galvanometer current required is considerably lower than in the case of direct pulse recording. If the information is sampled at the transmitter end during a length of time small compared to $1/F$—as is the case if a large number of channels are used—the height of the widened pulse is essentially the instantaneous value of the information at the time of sampling. In practice the pulse in the $i$th channel is widened until the $(i-1)$th channel in the next commutator sequence is turned on at which time the pulse widener of $i$th channel is discharged to zero and is then ready to widen the next pulse in this channel.(K3) Thus in the case of a large number of channels, the pulse is widened to essentially the maximum amount.

If the recording galvanometer has sufficient frequency response to essentially reproduce the rectangular shaped pulses, then pulse widening is of no particular advantage except possibly to reduce the writing speed of the recorder. The advantage of pulse widening as far as reducing the required galvanometer current comes from the use of lower frequency recording galvanometer elements in connection with the widened pulse.

At this point there are essentially two ways of handling the widened pulses. (1) Use a recording galvanometer of lower frequency response than is required to reproduce the sharp corners of the widened pulse but with sufficient frequency response to record the flat top near the middle of the widened pulse independently of the height

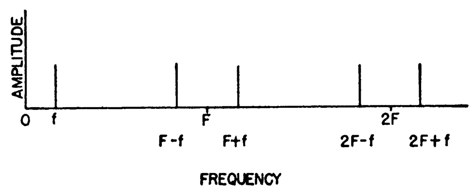



FIG. 2. Frequency spectrum of pulse amplitude modulation in which a sinusoid of frequency $f$ is sampled $F$ times per second. Several side band pairs are included.

TABLE II. Fluctuation noise characteristics of time division telemetry.

| Type | $R_{0i}$ | RMS carrier threshold $S_t/k_2$ | Reference |
|---|---|---|---|
| PAM-AM | $\dfrac{1}{(n)^{\frac{1}{2}}}$ [a] | | (R4) |
| PAM-FM | $\dfrac{\pi f_D}{\alpha n F(r)^{\frac{1}{2}}}$ [b] | $3.2(f_D)^{\frac{1}{2}}$ | (R4) |
| PWM-AM | $\dfrac{1}{n}\left(\dfrac{F_c}{F}\right)^{\frac{1}{2}}$ [c, d] | $4(F_c)^{\frac{1}{2}}$ [c] | Appendix 1 |
| PWM-FM | $\dfrac{\sqrt{6}D}{n}\left(\dfrac{F_c}{F}\right)^{\frac{1}{2}}$ [e, d] | $3.2(f_D)^{\frac{1}{2}}$ [e] | Appendix 1 |
| PPM-AM | $\dfrac{5F_c}{4n^{\frac{3}{2}}F}$ [d] | $4(2nF)^{\frac{1}{2}}$ | Appendix 2 |
| PCM-AM | (See Section IV-5) | $4(nNF)^{\frac{1}{2}}$ | Appendix 3 |

It should be noted that in each case $F_c$ refers to the video band width used in that case.

[a] This is an approximate expression which holds to better than eight percent if $\alpha n \geq 20$ and $r/\alpha n \geq 5/4$.

[b] This is an approximate expression which holds to better than 10 percent if $\alpha n \geq 20$ and $r/\alpha n \geq 1$.

[c] Assumes an average duty cycle of 0.5.

[d] These results are based on the assumption that there is an essentially noise free time reference for each channel. If the channel reference in PWM is obtained by the beginning of the pulse, the $R_{0i}$ above must be divided by $\sqrt{2}$. If the channel reference in PPM is obtained by a second pulse in each channel, then the $R_{0i}$ above must be divided by 2 (not by $\sqrt{2}$ because of the doubling of the duty cycle).

[e] Actually there are two thresholds in this case: the FM carrier threshold and the pulse threshold in the video. It is assumed that the carrier threshold occurs at a higher signal strength than does the pulse threshold.

of the other pulses. After the recording is made a smooth curve is then drawn through the points at the center of the flat top of each pulse. Then as in Section IV-1.1 at least five to six samples per cycle of information are required and a delay of one-half sampling period is introduced into every channel. This introduces the problem of locating the center of the pulses. (2) Use a recording galvanometer (or a low pass filter preceeding the galvanometer) which cuts off ideally at $F/2$ (see Section IV-1.2). This gives no distortion due to components of the type $mF-f$, when $m$ is an integer but introduces the "aperture" effect.(B3, K3) If the widened pulses have flat tops then the aperture effect introduces amplitude distortion of the type $\beta|(\sin\pi\beta f/F)/(\pi\beta f/F)|$ where $\beta$ is the fraction of the maximum possible width $1/F$ to which the pulses are widened.(K3) A linear phase delay $-\pi\beta f/F$ is also introduced. If no compensation is used, if $\beta=1$, and if the attenuation is not to exceed five percent, computation shows that about six samples per cycle of information are required. (It should be noticed again that the method of Section IV-1.2 does not introduce any distortion over the frequency range $0 \leqslant f < F/2$.) If the aperture effect is compensated for by the insertion of a proper network prior to recording,(M1) then pulse widening has the advantage of enhancing the information components over the method of Section IV-1.2 by a factor $\beta n\alpha$ where $n$ is the number of channels and $1/\alpha$ is the fraction of the allotted time, $1/nF$, during which the channel is sampled. If there are DC components present this may be of considerable practical advantage. With suitable compensation and with an ideal low pass filter of cutoff at $F/2$, the frequency response is the same as that of method (IV-1.2) namely, $0 \leqslant f < F/2$.

### *IV-1.4. Pulse Width Modulation*

The type of pulse width modulation most frequently met in radio telemetry is that in which one edge of the pulse is fixed in the time sequence and the other edge varies in accordance with the information; the width of the pulse is proportional to the instantaneous value of the information at the time the variable edge of the pulse occurs. In PWM-AM, the transmitter is turned on full power during the pulses and turned off in between. In PWM-FM the transmitter is fully modulated in the positive direction during the pulses and fully modulated in the negative direction between the pulses, or *vice versa*. In each case the width of the pulses is modulated in accordance with the information. If at this point, the pulse width modulation is converted into pulse amplitude modulation with samples equally spaced, then the results of PAM apply. The other alternative is to pass the width modulated pulses through a low pass filter. This has been discussed by Kretzmer.(K5) In this case frequency components of the type $mF \pm nf$, where $m$ and $n$ are integers, are generated whereas in pulse amplitude modulation only frequency components of the type $mF \pm f$ are generated. Thus in the case of pulse amplitude modulation a low pass filter with vertical cut off at $F/2$ can be used with no distortion as long as no information frequency components greater than $F/2$ are inserted at the transmitting end. However, in pulse width modulation terms of the type $F-nf$ appear with finite amplitudes. Kretzmer(K5) has shown that if the width modulation in the above case does not exceed five percent of the repetition period, terms of the type $F-2f$ enter with about three percent of the input information amplitude and terms of the type $F-3f$ enter with 0.1 percent approximately. Thus under these conditions only two samples per cycle of information are required if several percent distortion can be tolerated.

The above reasoning also applies in many cases to the decoding of pulse position modulation in which the received pulses are used to produce pulse width modulation by such means as "flip-flop" circuits.(K5) Kretzmer(K5) has also pointed out that "symmetrical" pulse width modulation—i.e., both leading and trailing edges are modulated—introduces less distortion than does the unsymmetrical type in which only one edge is modulated.

### *IV-2. CROSSTALK IN TIME DIVISION*

Since in time division only one channel is sampled at a time, the only possibility for crosstalk (aside from such things as improper shielding of the individual channels) is insufficient band pass in the radio link and other common circuits. With a radio link and other common circuits having an infinitely high cut-off frequency (and also with adequate low frequency response) there would be no crosstalk. Nonlinearity in the radio link and other common circuits does not cause crosstalk in time division but results only in a corresponding nonlinearity of response in each individual channel.

The details of required band width, etc., vary somewhat with the type of modulation used and will be discussed in subsequent sections.

### *IV-3. FLUCTUATION NOISE IN TIME DIVISION SYSTEMS*

Table II summarizes the fluctuation noise characteristics of the types of time division which are used in radio telemetry. Because of its unique properties to be discussed later, pulse code modulation appears to have possibilities in telemetry but inasmuch as PCM systems when above threshold, are essentially unaffected by fluctuation noise, the $R_{0i}$ for PCM is not included in the table but quantization noise will be given in Section IV-5. However, the improvement threshold for PCM-AM is given.

Table II lists the wide band improvement $R_{0i}$ of a single channel *versus* the type of modulation. It is assumed that all channels are sampled at evenly spaced intervals and at the same rate, and that the output of each channel is fed through a low pass filter with cut-off at $f_m < F/2$. It is also assumed that the carrier threshold is exceeded and that perfect synchronism between transmitter and receiver commutators is maintained. In the formulae for PPM-AM and PCM-AM it assumed that the pulses have the shape characteristic of the impulse response of the narrowest band width to which they are subjected.[G2] The threshold signal strengths are expressed in terms of $S_t$, the RMS amplitude of the sinusoidal video output of the comparison single channel fully modulated AM link as in Section III-2. In PAM-FM and PWM-FM it is considered that the threshold has been reached when the amplitude of the RF signal equals the crest fluctuation noise in the RF pass band. In PPM-AM, PWM-AM, and PCM-AM the threshold signal is considered to be reached when the pulse amplitude in the video pass band is twice the crest fluctuation noise in the video pass band. In the calculations, a crest factor of 4 is used.[C4, L1]

The minimum acceptable signal-to-noise ratio in the individual channels generally does not occur at the carrier improvement threshold unless the RF band width is adjusted to make this happen. The minimum acceptable signal-to-noise ratio in the individual channels depends on the nature of the measurements to be made and this will usually vary from one experiment to another. However, perhaps a reasonable criterion for minimum acceptable individual channel signal-to-noise ratio is that the noise will exceed 10 percent of the fully modulated individual channel signal only five percent of the time. On the basis of normally distributed noise, this requires that the RMS individual channel signal equals $20/\sqrt{2}$ times the RMS noise in the channel. The minimum carrier signal which will produce this minimum acceptable individual channel signal-to-noise ratio may be expressed in terms of the fully modulated sinusoidal output of RMS amplitude, $S$, from the equivalent AM single channel receiver and in terms of the RMS noise $k_2$ per unit band width in the output of this receiver by multiplying $1/R_{0i}$ by $(20/\sqrt{2})k_2(f_m)^{\frac{1}{2}}$, where $f_m$ is the cut-off frequency of the channel output low pass filter. The second factor is the minimum carrier signal for the AM single channel reference. Once the minimum signal strength is decided on, it is frequently desirable to adjust the improvement threshold to coincide with this value.

The quantities in Table II depend upon the video band width used in each case. The band-width considerations are somewhat different in the different types of modulation and are discussed as follows.

#### *IV-3.1. Band-Width Requirements in PAM*

In Fig. 2 were plotted a few side band pairs which are characteristic of a single channel of a time division system using on-off switching. In PAM the signal handled by the radio link and other common circuits is the superposition, with the proper phases, of such frequency spectra for all individual channels. If the number of side band pairs transmitted is insufficient—i.e., if $F_c$ is not large enough—crosstalk will result. Crosstalk will also result if the gain of the radio link varies appreciably over the pass band (amplitude distortion) or if the phase characteristic departs appreciably from a linear dependence on the frequency (phase distortion). The pass band of the radio link must extend at least to the lowest information frequency in any of the channels. If there is appreciable amplitude or phase distortion at the lower frequencies, crosstalk can take place between channels widely separated in the scanning sequence, whereas when there is distortion at the high frequencies the crosstalk is between adjacent channels.

If DC components are present in the individual channels, which is often the case in radio telemetry and if on-off switching is used, the radio link must have a DC response and DC stability. These requirements are difficult to comply with and can be avoided by "plus-minus" sampling.[B2] For this type of sampling one may imagine that the commutator reverses the polarity of the connections on alternate contacts with each individual channel. In practice this has been accomplished in radio telemetry by using a carrier in each channel which is amplitude modulated by the information in that channel.[R3] The frequency of the carrier is some integral multiple or submultiple of the sampling frequency $F$ and is arranged so that the sampling occurs alternately during positive and negative half-cycles. In this case side band pairs occur at $F/2$, $3F/2$, etc., which carry the information. The side band pair at $F/2$ can be filtered out by an ideal low pass filter which must cut off at $F$ or, in order to eliminate DC drift and other very low frequency noise, by a band pass filter which cuts off at $F$ and at some frequency which is lower than $(F/2)-f_m$

where $f_m$ is the maximum information frequency. By this method the effect of DC drift in the radio link and other common circuits is completely eliminated. The output of the filter is the side band pair about $F/2$ representing amplitude modulation by the information. This is then demodulated in the usual way. Since in this case the energy for the first side band pair comes from the first two terms in the Fourier expansion, the $R_{0i}$ expression in Table III for PAM-FM and PAM-AM is also the correct expression for the case.

Phase or amplitude distortion at the high frequency end of the pass band causes crosstalk between the adjacent channels. The effect of pass bands with varying high frequency cutoffs is discussed in Section IX. Figure 5 gives curves of crosstalk in db between adjacent channels as a function of the high frequency cutoff in units of $nF$ assuming no other amplitude or phase distortion in the radio link. This figure shows that the introduction of "blank" spaces between the channels—i.e., when each channel is sampled a fraction $1/\alpha$ of the time $1/nF$ allotted to it—results in better crosstalk suppression. Some of the curves show definite optimum transmission band widths which suggest the possibility of reducing the required band width by proper shaping of the attenuation characteristic and choice of sampling periods. This is discussed by Bennett[(B2)] and others.[(B4)]

From Table II, it can be seen that, to within the approximation noted therein, $R_{0i}$ for PAM-AM is independent of the band width and independent of $\alpha$ as long as the average RF power is constant. However, it is usually desirable to limit the band width required to keep the crosstalk down by choosing a value of $\alpha$ somewhat greater than 1.

In the case of PAM-FM, $R_{0i}$ and $S_t$ must be considered. Since $DF_c=f_D$ is limited by radio link design considerations and by the magnitude of $S_t$ desired, it is desirable to keep $\alpha(r)^{\frac{1}{2}}$ as small as possible, consistent with sufficient crosstalk suppression, in order to keep $R_{0i}$ large. For example, reference to Fig. 5 shows that if crosstalk is to be kept below 40 db, $\alpha=1.1$, and $r=1.5n$ is a good choice; if below 50 db, $\alpha=1.4$, and $r=3.5n$ is a good choice; and if below 60 db, $\alpha=2$, and $r=3.6n$ with the pass band essentially cut off by $r=4.5n$ is a good choice. The value $\alpha=2$ and $r=3.5n$ will be used in the numerical illustrations to follow. In order to keep $R_{0i}$ as large as possible, it is also necessary to keep $F$ as small as possible. From Section IV-1 it follows that this can best be done by a low pass output filter with cut off as close as possible to $F/2$.

### *IV-3.2. Band-Width Considerations in PWM*

Table II shows that in both PWM-AM and PWM-FM the wide band improvement $R_{0i}$ is proportional to $(F_c)^{\frac{1}{2}}$ but the threshold, $S_t$, is also proportional to $(F_c)^{\frac{1}{2}}$. Therefore improvement in $R_{0i}$ by increasing $F_c$ is always accomplished at the expense of an increased threshold. As compared to PPM-AM, for example, this is a very serious objection to PWM and results because in PWM the only part of the radio signal that carries information is the modulated edge of the pulse; the transmitter power during the flat top of the pulse is therefore wasted. In PWM-FM, $R_{0i}$ can be increased by increasing the deviation ratio $D$ but with a corresponding increase in $S_t$ only proportional to $(D)^{\frac{1}{2}}$. However, the results still do not compare favorably with PPM-AM, for example. Nevertheless, PWM-FM is used in some telemetry applications, presumably because of technical reasons.

The only source of crosstalk, aside from such things as improper shielding of individual channels, coupling of individual channels via the power supply, etc., is overlapping of the pulses. This can be avoided by providing a guard space between pulses. The formulas in Table II do not allow for this space. If the guard space is a fraction $g$ of the maximum space for channel, $1/nF$, then the $R_{0i}$ for PWM-FM must be multiplied by $(1-g)$. If in PWM-AM the average power is kept constant, the $R_{0i}$ must be multiplied by $(1-g)^{\frac{1}{2}}$.

It should be noted from Table II that $R_{0i}$ for PWM-FM contains the deviation ratio $D$ times $(F_c)^{\frac{1}{2}}$. Under condition of constant RF band width, it would at first appear desirable, in order to keep $R_{0i}$ large, to make $F_c$ small and $D$ correspondingly large. However, the increase in transient decay time would require a large guard space in order to keep the pulses from overlapping thus causing crosstalk. The guard space must be at least $1/F_c$ seconds[(G2)]—i.e., $1-g\leqslant(F_c-nF)/F_c$. If the equality sign is taken, it turns out that with constant RF band width, $W_2$, $R_{0i}$ has a maximum value when $F_c=3nF$ given by

$$R_{0i}=0.4W_2/n^{\frac{3}{2}}F. \tag{IV-1}$$

### *IV-3.3. Band-Width Considerations in PPM-AM*

In this case it can be seen from Table II that $R_{0i}$ is proportional to the video band width $F_c$ and that $S_t$ is independent of $F_c$. In order to keep $R_{0i}$ large, $F_c$ should be large provided that the transmitted pulses have the shape characteristic of the impulse response for a band width of $F_c$. Thus as $F_c$ is increased, the peak power must also be increased to keep the average power constant. This in addition to other factors, limits the band width because generally larger equipment is required the larger the peak power. It should be mentioned that $F_c$ should not be any larger than the effective band width to which the pulses are subjected. For example, if a modulated power oscillator is used for transmitting the pulses at around 300 megacycles, it is difficult to obtain efficient operation with an RF band width greater than about 4 megacycles;[(R1)] thus in this case the maximum $F_c$ is about 2 megacycles. It should also be mentioned that the PPM-AM formulae in Table II do not allow for a guard space between channels to prevent overlapping of the pulses. If the guard space is a fraction $g$ of the maximum total space $1/nF$, then the $R_{0i}$ must be multiplied of $(1-g)$. In order to prevent the pulse from a given channel from encroaching on the guard space it may be necessary to provide limiters in the individual information channels prior to commutation at the synthesizer. It is clear that under these conditions there is no possibility of crosstalk in a PPM system except from such things as improper shielding of channels.

### *IV-3.4. Band Width for PCM*

In this case it is only necessary to provide sufficient video band width to keep the individual pulses from overlapping appreciably. If the video band width is taken to be 1/2 the reciprocal of the pulse length, the required video band width is $\frac{nNF}{2}$ since $nNF$ pulses per second must be allowed for.

## *IV-4. FLUCTUATION NOISE CHARACTERISTICS OF SYSTEMS WHICH DO NOT USE THE LOW PASS OUTPUT FILTER*

As previously pointed out, the results in Table II are based on the case of a low pass filter in the output of each channel with cutoff $f_m < F/2$. In numerous telemeter systems now in use, the low pass filter principle is not used with the result of generally poorer fluctuation noise characteristics. Several examples are discussed as follows.

### *IV-4.1. Recording Pulse Amplitude Modulation with High Frequency Response*

If the recording is done without decommutation—i.e., if all channels are recorded in natural time sequence on a single recorder channel—and if a smooth curve is to be drawn through the peak of each pulse of an individual channel (see Section IV-1.1) then the frequency response must be sufficient to prevent the height of any pulse from depending on the height of any other pulse. In this case the frequency response of the recording instrument must extend at least up to about $4nF$[(G2)] in order not to have more than about one percent cross talk. If the channels are decommutated and the pulses in each channel recorded on a separate recorder or channel, (see Section IV-1.1), and if there are many channels, then in order to have the height of each pulse independent of the other pulses the frequency response of the recorder must extend up to at least about $4F$ if the distortion is to be kept within several percent.* The record in this case would consist of rather broad pulses through the peaks of which a smooth curve must be drawn. As stated in Section IV-1.1 it is estimated that at least five or six samples per cycle of information are required if reasonable care is exercised in smoothing "by eye."

If the recorder frequency, $F_R$, is sufficiently high to essentially reproduce the pulses—i.e., equal to the video band width, $F_c$, which should be at least about $4nF$—then the RMS noise on top of the pulses is just the video noise. In this case it is easy to show that the wide band improvement $R_{0i}$ for PAM-AM is $(\alpha f_m/F_R)^{1/2}$ and for PAM-FM is $\sqrt{3}D(f_m/F_R)^{1/2}$. On the other hand, if the recorder cut off frequency is low—but sufficiently high to reproduce the decommutated pulses without appreciable overlapping, that is about $4F$ or greater—the recorder tends to integrate the noise and the results are approximately the same as those of pulse integration—see Appendix 4.

Thus, in either case, the signal-to-noise ratios are no better than with the low pass filter. In fact, since roughly six samples per sample of information are required if the smoothing is done "by eye," whereas in the case of a low pass filter as few as 2.5 samples per cycle are required with high quality filters, Table II shows that the signal-to-noise ratio in the low pass filter case can be several times higher than for the case of recording without filters.

### *IV-4.2. Recording Pulse Width Modulation and Pulse Position Modulation with High Frequency Response*

The method considered here is to record the pulse plus noise directly so that its width or position can be subsequently measured. In principle, the width is then plotted as a function of time and a smooth curve drawn through the points. In practice, the width of the pulse may be recorded by intensity modulating a cathode-ray beam so as to give a line whose length is equal to the length of the pulse plus noise. Pulse position may be recorded by intensity modulation which produces a dot when the pulse (plus noise) occurs.†† In this type of recording the frequency response of the recorder, $F_R$, should be equal to the video response $F_c$. Then by reference to Appendix 1 it is easy to show that

for PWM-AM, $R_{0i} = (\sqrt{2}/nF)(F_R f_m)^{1/2}$,

for PWM-FM, $R_{0i} = (2\sqrt{3}D/nF)(F_R f_m)^{1/2}$, (IV-2)

and

for PPM-AM, $R_{0i} = (5/2\sqrt{2})[F_R(f_m)^{1/2}/(nF)^{3/2}]$.

Reference to Table II will show that the wide band improvements in the corresponding cases with low pass output filter are better than those of Eq. (IV-2). This is especially so when it is considered that about $6f_m$ samples are required without the low pass filter and as low as about $2.5f_m$ with the filter.

It should be pointed out that PWM or PPM can be converted into PAM at the receiver and handled by the PAM methods discussed above or vice versa.

### *IV-4.3. The Effect of Pulse Integration*

Consider the individual channel pulses of Fig. 1. When noise is present, it will be superimposed on top of the pulses. Suppose that by using an integrating circuit,

* The value of $4F$ is arrived at by considering that the channel pulse is of short duration so that the problem reduces to a $\delta$-function impulse through a low pass filter with cutoff at $F_R$. The well-known response is $\sin 2\pi F_R t/2\pi F_R t$.

†† If the pulses plus noise are first demodulated, as by a trigger circuit, and then fed to the recorder still in the form of time modulation, $F_R$ may be somewhat less than $F_c$. In this case the corresponding $R_{0i}$ is given by Eqs. (IV-2) with $F_R$ replaced by $F_c$ if such effects as width of the recording trace, etc., are neglected.

TABLE III. Comparison of systems of Table II with equal radiofrequency band width.[a]

| Ratio | $R_{0i}$ | $S_t$ |
|---|---|---|
| $\frac{\text{PAM-AM}}{\text{PPM-AM}}$ | $\frac{0.8nF}{F_c}$ | |
| $\frac{\text{PAM-FM}}{\text{PPM-AM}}$ | 0.5 | $0.5\left(\frac{F_c}{nF}\right)^{\frac{1}{2}}$ |
| $\frac{\text{PCM-AM}}{\text{PPM-AM}}$ | | $(N/2)^{\frac{1}{2}}$ |
| $\frac{\text{PWM-AM}}{\text{PPM-AM}}$ | $0.8\left(\frac{nF}{F_c}\right)^{\frac{1}{2}}$ | $\left(\frac{F_c}{2nF}\right)^{\frac{1}{2}}$ |
| $\frac{\text{PWM-FM}}{\text{PPM-AM}}$ | 0.6[b] | $0.5\left(\frac{F_c}{nF}\right)^{\frac{1}{2}}$ |

[a] In this table $F_c$ refers to the video band width used for the PPM-AM.
[b] Equation (IV-1) was used to compute this ratio.

such as a resistance and capacitance in series with $RC \gg 1/\alpha nF$, the top of the pulse is averaged during the interval of time the pulse is on. It is shown in Appendix 4 that the result is exactly the same as using a low pass filter with cutoff at $F/2$. Integration with or without pulse widening can be applied to high frequency recording with the resultant decrease of noise by the above factor. It should be pointed out here that the use of pulse widening does not alter the signal-to-noise ratio since both information and noise components are enhanced by the same amount.

If the output of the pulse integrator is recorded without the smoothing action of a low pass filter as individual samples, then about $6f_m$ samples per record are required as compared to as low as $2.5f_m$ in the case of a low pass output filter.

### *IV-4.4. General Considerations*

The above results indicate that the use of a low pass filter results in a better signal-to-noise ratio than does high frequency recording. The use of a low pass filter gives a smoothed record which is usually a great convenience and which requires fewer samples per cycle of information which in turn usually leads to simpler equipment.

At low sampling rates, a low pass filter constructed of passive components with cutoff lower than a few tens of cycles per second has been generally impractical to construct. By the application of new principles one of the authors (LLR) has found that it is practical to construct low pass electric filters with relatively sharp cutoff at frequencies of several cycles per second or considerably less.

## *IV-5. PULSE CODE MODULATION FOR TELEMETRY*

Pulse code modulation is essentially independent of fluctuation noise when above threshold.[O2] There is, however, a quantization noise resulting from the use of a finite number of digits. The RMS full modulation signal to RMS quantization noise ratio for a binary code of $N$ digits is shown by Bennett[B3] [see Eq. (1.3) of Bennett's paper] to be

$$R = (3/2)^{\frac{1}{2}} 2^N. \tag{IV-3}$$

Thus for a seven digit code, $N=7$ and $R=150$ regardless of the RF signal strength as long as it is above threshold. It should be realized that $R$ applies to full modulation. At low modulation, the signal-to-noise ratio may be small. For example, at one-tenth full modulation with $N=7$, $R=15$ regardless of signal strength. In the other systems of Table II or Table I, the full modulation signal-to-noise ratio can be large for high signal strengths and consequently the signal-to-noise ratio at low channel modulation levels can also be high. This effect should be taken into account in the choice of $N$. In some cases a binary code with $N=10$ may be required on this account. The quantity $R$ can be made essentially constant independent of modulation level by logarithmic quantization.

It is likely that the simplest way to handle PCM at the receiver is to convert to PAM and use a low pass filter in the output of each channel. The results of Section IV-1.2 show that the maximum undistorted frequency response is $F/2$.

In applications where it is necessary to relay the radio signals from the telemeter transmitter by radio links, PCM has great advantages because it is only necessary to reshape the pulses and retransmit. This process does not change the noise level because the signal is not decoded until the end of the relay chain. With the other types of multiplexing, the noise (and also crosstalk in frequency division) is generally increased each time the signal is relayed. In case digital computers are to be used on received data, there is some advantage in having the individual channel data already in digital code. This form is also convenient in some forms of data storage.

No papers on pulse code radio telemetry in the unclassified journals have come to the authors' attention. Coding tubes have been constructed[S2] but they require a relatively high level input which means that, with much telemetry instrumentation such as resistance strain gauges, more stages of amplification are required. The coding can also be done by coding circuits.[G3,B5] To be of practical use in radio telemetry, these circuits must be sufficiently stable and must also often times handle DC components unless such things as plus-minus sampling are used. (See Section IV-3.1.)

## *IV-6. COMPARISON OF FLUCTUATION NOISE CHARACTERISTICS OF THE SYSTEMS IN TABLE II ON THE BASIS OF EQUAL RADIO FREQUENCY BAND WIDTH*

Since PCM is essentially independent of fluctuation noise when above improvement threshold, only the threshold will be compared in this case. Inasmuch as single side band transmission is usually not used in radio telemetry, the RF band width will be taken as twice the video band width $F_c$ for PPM-AM and PCM-AM. For

PAM-FM, the RF band width is taken as $2.6f_D$[K1], $\alpha=2$, and $F_c=3.5nF$. For PCM-AM a seven digit binary code will be used for computing the improvement threshold (see Section IV-5). For Table III, PPM-AM is used as a basis of comparison; other ratios can be obtained by multiplying or dividing the appropriate ratios given in the table. The same number of channels $n$, and the same sampling rate, $F$, are used in each case. In each case a low pass output filter with cutoff at $f_m$ is used. In the radiofrequency region where it is feasible to use all types listed in Table III, $F_c$ is generally larger than $nF$ by sometimes as much as ten or more. Under this condition it is clear that PPM-AM and PCM-AM (with sufficiently large $N$) give better noise performance in both $R_{0i}$ and $S_t$. PAM-FM compares favorably as far as $R_{0i}$ is concerned but has a larger $S_t$. PWM compares unfavorably in $S_t$; the reason for this is that the transmitter power is essentially wasted during the flat top portion of the pulse.

### *IV-7. IMPULSE NOISE IN TIME DIVISION MULTIPLEXING*

It is not the purpose of this paper to discuss in detail the effects of impulse noise; however, it has seemed to be desirable to include some brief remarks on impulse noise at this point. In considering impulse noise it should be born in mind that radio telemetry in flight testing is generally carried out under special conditions in which it is usually possible to suppress most of the man-made impulse noise at least during the flight. Also the testing is usually carried out during periods of settled weather so that atmospheric impulse noise is at a minimum. If the impulse noise cannot be eliminated, its effect can be reduced by various methods of pulse discrimination all of which decrease the signal to fluctuation noise by three to six db or more.

#### *IV-7.1. Impulse Noise in PAM*

Assuming for the moment that the impulse noise does not affect the synchronism of the commutators, it is clear that a single impulse will in general affect only one channel and its duration will depend upon the cut-off frequency of the individual channel low pass filter. If the commutators are synchronized by a "master" pulse which occurs in the beginning of each sampling sequence, it is possible to discriminate against impulse noise affecting synchronization by such things as making this pulse considerably longer than the minimum pulse length characteristic of the cut-off frequency $F_c$ or by coding it by using a train of several short pulses. In the case that a switching pulse is used for each channel[R3] the discrimination becomes more difficult because the switching pulse cannot be any longer than the off time—i.e., any longer than $(1-1/\alpha)/nF$ seconds. It is possible to make use of circuits having large fly wheel effect (low damping) to override the effect of impulse noise on the synchronizing pulses provided that the impulse noise does not occur too frequently.[R3]

#### *IV-7.2. Impulse Noise in PPM*

Unless discriminated against, strong impulse noise in this case can trigger the circuits just the same as an information pulse. If the impulse noise does not interfere with the synchronizing, a single noise pulse will generally affect only one channel. However, if the information pulses are also used for switching the receiver commutator, as is often done,[M3] a single noise impulse can also affect all the following channels in the commutation sequence.

If the receiver commutator sequence is initiated by a master pulse, it is generally feasible to make this pulse longer than the minimum length characteristic of the video cut-off frequency $F_c$ or to code the pulse. However, if the average power is left constant, it is not compatible with maximum fluctuation noise improvement to make the individual information pulses longer or to code them because this would reduce the pulse height and therefore decrease the signal-to-fluctuation noise ratio. This effect is not serious in the case of the master pulse since it occurs only once each sequence.

Impulse noise considerations in PWM are essentially the same as in PPM except that it is conceivable that some form of pulse width discrimination could be used—i.e., a pulse could be discriminated against if it is shorter than the minimum pulse width used in the carrier.

#### *IV-7.3. Impulse Noise in PCM*

The effect here of strong impulse noise is similar to that in PPM in that a strong impulse can either add or subtract an information pulse.

### *IV-8. SOME CONSIDERATIONS INVOLVED IN THE CHOICE OF TYPE OF CARRIER MODULATION IN TIME DIVISION TELEMETRY*

The choice of type of modulation in any frequency region usually depends upon a compromise among numerous considerations such as antenna and propagation problems, frequency allocations and band widths, required range, power available, space and weight limitations, minimum acceptable signal-to-noise ratio, etc. It is not within the scope of this paper to give a thorough discussion of this problem but a few brief remarks are included in this section.

The video frequency should extend up to about $3.5nF$ in PAM in order to suppress crosstalk. (See Section IV-3.1.) In the other pulse systems, sufficient video band width must also be provided to prevent crosstalk (see Sections IV-3.2–IV-3.4). The video requirement puts a lower limit on carrier frequency for efficient operation of the power stage of the transmitter, etc. In some cases of telemetry from small vehicles, it has been possible to use effectively all or a portion of the vehicle as an antenna.[C1] This requires the use of wavelengths of the order of magnitude of the size of the vehicle or the portion of the vehicle used and often results in a broad antenna pattern as well as a reasonably broad frequency band. A broad antenna pattern is often desirable to insure reception of signals over a wide range of orientation of the vehicle relative to the receiving antenna. Since it is desirable to use as simple and compact transmitting apparatus as is possible, modulated power oscillators are often used for PPM systems. For efficient operation, the video band width is limited; for example, at 300 megacycles a video band width greater than about 2 megacycles is difficult to obtain.[R1] In the high UHF or microwave regions greater band width can be obtained efficiently but the antenna problem at these wavelengths is often troublesome because the antenna must generally be on the surface of the vehicle facing the receiving station in order to obtain satisfactory reception. In some cases this can be alleviated somewhat by using several receiving stations at different locations with respect to the trajectory.

Multipath transmission must also be considered. In PAM, multipath transmission can cause distortion in individual channels but in order to cause crosstalk, the multipath delay must exceed $(1-1/\alpha)1/nF$ seconds. In PPM, multipath transmission can have essentially the same effect as strong impulse noise. If the signal originates near the horizon, multipath effect is apt to be more pronounced and is more difficult to eliminate by such things as direction sensitive receiving antenna, etc., than if the signal originates at larger elevation angle relative to the receiver. Again the use of several receiving stations distributed along the trajectory can alleviate this problem.

Another consideration is that as the frequency goes up, simple antennas such as a half-wave dipole supply a signal voltage to the receiver which is inversely proportional to the frequency. This is because the receiving antenna grows smaller and intercepts less of the energy flux available at the receiving location. A receiving antenna with large intercepting area at high frequency is usually very directional and this is not always desirable for telemetering work because of pointing problems. See for example reference (T1), Chapter 14.

In frequency regions where it is feasible, as far as technical considerations are concerned, to use any of the systems in Table II, such as around 200 megacycles if $nF$ is not much greater than $10^4$ per second, Table III indicates that as far as $R_{0i}$ is concerned, PAM-FM, PPM-AM, and PCM-AM are good choices. A somewhat larger threshold is required for PAM-FM and for PCM-AM than for PPM-AM. When considering different frequency ranges, it should be borne in mind that the receiver noise figure increases with increasing frequency over the range of about 50 megacycles to 1000 megacycles above which the noise figure is about constant. The best obtainable noise figure (power ratio) at 50 megacycles is about 2 and at 1000 megacycles about 10.[T1] Thus, everything else being equal, the individual channel signal-to-noise ratio would decrease by about a factor of $1/(5)^{\frac{1}{2}}$ corresponding to this noise figure change.

### *IV-9. TRIPLE MODULATION IN RADIO TELEMETRY*

Generally speaking, greater flexibility in radio telemetry can often be obtained by the use of triple modulation. For example, consider a frequency division or time division telemeter in which one or more channels carries a time division multiplex. Such a telemeter might provide a number of channels capable of handling information frequencies up to several hundred cycles per second and several other channels each carrying as many as thirty channels of PAM time division each sampled several times a second.[C1]

All of the considerations of multiplexing developed in the previous sections are applicable to triple modulation. Noise formulas can be assembled for triple modulation by simply multiplying together the appropriate factors from Tables I and II. For example, consider a PAM-FM-FM system in which pulse amplitude modulation frequency modulates a subcarrier which frequency modulates a carrier.[C1] The $R_{0i}$ for this case can be assembled with the help of Tables I and II.

$$R_{0i}(\text{PAM-FM-FM}) = \left[\frac{\pi(r)^{\frac{1}{2}}}{\alpha n}\right]\left(\frac{a_{0i}}{S}\frac{f_{di}}{f_{mi}}\right)\left(\frac{f_D}{f_i}\right). \quad \text{(IV-4)}$$

In triple modulation systems in which a channel of a time or frequency division system is time division multiplexed with PAM, the sampling rate is usually rather low. Since low pass filters have not previously been available for low frequencies, methods of the type described in Sections IV-1.1 and IV-1.3 are used. However, in the future, it will likely be advantageous to use the low pass filter method (see Section IV-4.4).

It might be mentioned that in some special cases simultaneous modulation—for example, simultaneous amplitude modulation and frequency modulation of the same carrier—have been used but no such systems for radio telemetry have been described in the regular journals.

### *IV-10. MISCELLANEOUS REMARKS ON TIME DIVISION MULTIPLEXING*

Television has been used for telemetering data from aircraft[F2] and may be classed as time division. The procedure has been to televise instrument indications, flashing of lights, etc. Because of the waste space between instruments, on the dials of the instruments, etc., it is clear that a very large amount of useless information is transmitted along with the useful information when an instrument panel is televised. In this sense, television is a very inefficient telemeter and results in poorer performance, larger power, space and weight requirements, etc. It is certainly true that television is useful if it is necessary to transmit pictures of such things as cockpit instruments, pilot reactions, etc., to an observing station during flight or in cases in which recovery of photographic records is not feasible.

Instead of generating the time scale for time division multiplex telemetry in the airborne equipment, it is possible to use pulses received from an external transmitter, such as a radar, to switch the airborne commutator.(O1) In this way the apparatus serves as a radar beacon and telemeter with the telemeter information carried by pulses interspaced with the beacon range pulses. This could result in a reduction of airborne equipment but has the disadvantage of being dependent on the receipt of the radar pulse.†††

All the time division multiplex radio telemeter systems described in the literature make use of mechanical commutators or electronic commutators using standard tubes arranged so as to perform a gating function.(C1,M2,M3,R3) However, several types of commutator tubes which perform the gating function for all channels in one envelope have been described.(G5,S3,S4) As described, these tubes are generally not suitable for airborne radio telemetry because of insufficient ruggedness and the complexity of auxiliary equipment such as channel preamplifiers, etc. It is conceivable, however, that if suitable commutator tubes were developed, a considerable saving in space, weight, power, and complexity could be realized in radio telemetry together with increased reliability.

## V. Comparison of Frequency Division and Time Division Multiplexing in Radio Telemetry‡‡

The comparison of the two types of multiplexing should be based on noise characteristics, crosstalk, complexity, reliability, etc. The type of instrumentation, see Section VI, should also be considered. For example, variable inductance instrumentation can be used very effectively with a FM frequency division multiplex with few parts.

### *V-1. COMPARISON OF CROSSTALK AND FLUCTUATION NOISE*

It has already been stated that in radio links suitable for many channel radio telemetry from aircraft and rockets it is generally easier to provide sufficient band width to keep time division crosstalk down than it is to provide sufficient linearity to keep frequency division crosstalk down. If FM sub-carriers with large deviation ratios are used, the crosstalk can usually be reduced so that more sub-carriers of this type can be handled with a prescribed crosstalk suppression than can be handled with lower deviation ratio frequency modulated sub-carriers or with amplitude modulated sub-carriers. In order to get a number of wide deviation ratio sub-carriers into a reasonable frequency band, the maximum information frequencies in the individual channels must be limited.

The fluctuation noise comparison can be made with the aid of Tables I and II. As an example, the following numerical comparison will be made. Since in many cases, the crosstalk requirements will limit the number of frequency division multiplex channels to about ten, a comparison will be made between an FM-FM ten-channel frequency division multiplex and a ten-channel PPM-AM time division multiplex on the basis of equal RF band width. (This should not imply that in practice the number of channels in time division is limited to ten. As the number of channels is increased, the advantages of time division over frequency division, as far as crosstalk is concerned, become greater.) In the FM-FM system the calculations will be based on a channel with deviation ratio 5 and a center frequency of $10^4$ (see the second paragraph of Section III-5). The value of $A_n$ from Fig. 2 is taken as 1.5. In the PPM-AM system the maximum channel information frequency, $f_m$, is taken as 100 cps and the sampling rate $F$ is taken as $2.5f_m=250$ per second. The radiofrequency band width for the FM-FM is taken as $2.6f_D$(K1) where $f_D$ is the full modulation frequency deviation; hence $F_c=1.3f_D$, where $F_c$ is the video band width of the PPM-AM system. Then

$$\frac{R_{0i}(\text{FM-FM})}{R_{0i}(\text{PPM-AM})}=0.4. \tag{V-1}$$

The corresponding ratio of improvement thresholds is $10^{-2}(W_2/2)^{\frac{1}{2}}$ where $W_2$ is the RF band width. In practice, it may be difficult to achieve with sufficient linearity as large an RF band width on FM-FM as is feasible on PPM-AM.

### *V-2. COMPARISON OF COMPLEXITY AND RELIABILITY*

One measure of complexity is the number of tubes required per channel. The literature provides a comparison of an (AM-FM) strain gauge channel(F1) with a (PAM-FM) strain gauge channel.(R3) The AM-FM multiplex (14 channels) requires three tubes per channel exclusive of the radio transmitter and the PAM-FM multiplex (20 channels) requires four tubes per channel exclusive of radio transmitter.

Except for overloading the radio link and such things as common power supplies, the individual channels of a frequency division multiplex are essentially independent —i.e., the functioning of any one channel is independent of the functioning of any other channel. In a time division multiplex, all channels depend upon the functioning (and synchronizing) of the commutators. In some cases(M3,R3) the commutator is made up of a chain of trigger circuits, one for each channel. In case any of these circuits fails, all the following channels in the sequence may be affected. Thus in these types of time division there appears to be more chance for failure than in frequency division. In cases of sufficiently slow time division it is sometimes feasible to record essentially the video output so that if synchronism of commutators is lost, a record is still obtained although laborious to reduce.

††† It might also be mentioned that frequency division can be used in a doppler radar channel in which case the radio carrier is filtered out to give the doppler shift.

‡‡ For a qualitative comparison, see reference (H3).

TABLE IV. Tabulation of telemeter systems described in detail in the literature.

| Reference | Type | Number of channels | Number of samples per second per channel | Frequency response per channel | Number of tubes per channel[a] | Carrier frequency in megacycles | Remarks |
|---|---|---|---|---|---|---|---|
| (R3) | PAM-FM | 18 | 952 | 200 cps | 4 | Not specified | Includes amplifiers with sufficient gain for strain gauge instrumentation. |
| (C6) | PAM-FM | 16 | 6400 | 2240 cps | 1.7 | 200 | The system may be altered to provide 32 or 64 channels at sampling rates of 3200 and 1600, respectively. |
| (F1) | AM-FM | 14 | | 200 cps | 3 | 70 | Includes amplifiers with sufficient gain for strain gauge instrumentation. |
| (H2) | PPM-AM | 23 | 190 | | 2+ | 1000 | Voltage input range 0 to 5 volts. Channels sampled at irregular intervals depending on information in previous channels. |
| (M3) | PPM-AM | 30 | 312 | | 2+ | 1025 | Voltage input range 0 to 5 volts. |
| (M2) | FM-FM | 6 | | 60 cps | 1 or 3[b] | Not specified | |

[a] Exclusive of radio transmitter.
[b] Depending upon type of instrumentation.

At this point it seems appropriate to list in Table IV the telemeter systems which have been described in detail in the literature. It is to be noted that the largest number of frequency division channels listed is 14;[(F1)] in this case the paper[(F1)] does not include a quantitative discussion of the crosstalk in the system nor does the paper state the modulation level at which it was possible to operate the FM transmitter.

## VI. Instrumentation for Radio Telemetry

It is necessary to convert the quantity to be measured into an electrical signal which eventually modulates the radio link. In this paper, the apparatus which performs this conversion is called an instrument.

Since the principal use of radio telemetry is for flight testing of aircraft and rockets and for upper atmosphere research, the remarks in this section are confined to these fields. Because of the limitations in space, weight, and power and the extremes of temperature, vibration, etc., which are often imposed on the entire airborne apparatus, it is desirable to choose a method of instrumentation which will meet the requirements with a minimum of complications such as high gain amplification, sensitivity to vibration, etc. It is not the purpose of this paper to enumerate or discuss the various methods and types of instrumentation in use in radio telemetry. However, in the interest of completeness, the following remarks on the types of instrumentation used with the different types of multiplexing are included. The reader should not infer from the small space given to instrumentation in this paper that the authors consider the problem of instrumentation for telemetry of secondary importance. In fact, it is the authors' opinion that too little attention has been given to instrumentation and, as a result, the instrumentation development has in some cases lagged the other more electronic developments. For convenience in discussion, the instrumentation is divided into several groups as follows.§§

### *VI-1. RESISTANCE WIRE STRAIN GAUGE TYPE OF INSTRUMENTATION*

The resistance wire strain gauge is an instrument whose resistance changes upon elongation of the resistance wires which make up the gauge.[(R14)] It is inherent, then, that the fractional change in resistance is small—about two percent maximum. This is a decided disadvantage but its other advantages in the measurement of strain such as in aircraft structure frequently outweigh this disadvantage. Unbonded strain gauges are also used in pressure gauges, accelerometers, rate of turn indicators, etc.[(F1,R10,R6,M5)] The resistance wire strain gauge is usually used in an AC bridge circuit. The output of the bridge is thus amplitude modulated by the information. If the bridge is fed by a sub-carrier oscillator of fixed frequency in a frequency division system, the result is an amplitude modulated sub-carrier. In a time division system it is convenient to excite the strain gauge bridge by an alternating voltage whose frequency

§§ For a general discussion of instrumentation, some of which is applicable to radio telemetry, see Roberts (reference R14). For a qualitative discussion of instruments in telemetry and aircraft control see Kiebert (reference K4), and Andresen (reference A1).

is locked in with the transmitter commutator so that the amplified output of the bridge is sampled at the crest.[R3] Another application similar to the strain gauge is the resistance thermometer[F1] which can be handled in the same way.

### *VI-2. VARIABLE INDUCTANCE TYPE INSTRUMENTATION*

In the cases of pressure and acceleration measurements, the variable inductance type of instruments have an advantage over the strain gauge type in that a much larger fractional change in inductance can be obtained.[C1,M2,P2] For example, $\pm 20$ percent change is feasible.[P2] The variable inductance instrument can be used in a bridge circuit or it can be used directly as the inductance in an LC frequency modulated sub-carrier oscillator.[C1,M2,P2] In general, the variable inductance strain gauge is much less convenient to use than is the variable resistance type described in the previous section.[P2,H1] The variable inductance principle has also been applied to shaft position indication in connection with controls, pointer indications, etc.[C1,M2,P2,S1,R7,R8]

### *VI-3. POTENTIOMETER TYPE INSTRUMENTATION*

Shaft position measurement can often times be carried out by connecting a simple potentiometer onto the shaft.[C1,M2,R3] Special low torque potentiometers have been used in connection with regular aircraft cockpit instruments, indicating pressure gauges, etc.[C1,R9,R11] The potentiometer can be connected across a sub-carrier oscillator thus providing an amplitude modulated sub-carrier,[F1] connected as a rheostat in the phase shift network of a phase shift oscillator providing a frequency modulated sub-carrier,[C1,M2] or connected across a voltage source to provide a variable voltage for modulating a time division multiplex[R3] or for frequency modulating a sub-carrier oscillator as in the next section.

### *VI-4. VOLTAGE TYPE INSTRUMENTS*

The voltage output of a potentiometer, as described in the previous section, or other source of voltage has been used to frequency modulate a sub-carrier.[C1,M2] Many other instruments such as Geiger counters, ionization chambers, ionization pressure gauges, Pirani gauges, etc., provide a voltage output which can be used to modulate a sub-carrier or a time division multiplex. The principle of magnetic amplification[C1] has also been found useful in telemetering currents such as from thermocouples.

### *VI-5. MISCELLANEOUS REMARKS ON INSTRUMENTS*

The design and application of accelerometers has been discussed by Weiss.[W2] A vacuum tube acceleration pick up has been developed by Ramberg.[R2,R12] For a brief history and discussion of application of magnetic amplifiers see Greene,[G4] Logan,[L4] etc. A transistor oscillator circuit for frequency modulated sub-carriers has been described by Lehan.[L3] In order to obtain higher precision and output torque, servo type null instruments have been developed.[D1,E2]

## VII. Recording in Radio Telemetry

It is usually necessary to obtain records of the received information in permanent or semipermanent form during the flight. There are many ways of recording such as by photographing cathode-ray tube displays, photographing electric meters, pen and photographic recording galvanometers, magnetic tape and wire recorders, etc. The pen recorders (or equivalent) have an advantage of requiring no photographic development but they are limited in frequency response and require relatively high driving power and sometimes under field conditions the pens cause trouble by clogging. The magnetic tape recorders are used mostly for recording FM sub-carriers or the entire FM sub-carrier multiplex; the records are subsequently played back, demodulated, and recorded in one of the above forms. In this type of recording, special care has to be taken to correct for "wow," and if a sub-carrier multiplex is recorded, special attention has to be given to linearity. When a large number of channels are telemetered it is usually desirable to record as many as possible on the same film or paper so as to have a common time scale; if more than one film or paper is required, it is generally necessary to record a time scale on both records in some convenient form so that the records may be aligned in time with minimum effort.

The process of reducing records is frequently long and expensive in both manpower and delay. The usual procedure is to measure the displacement of the individual channel record from a base line, apply an instrument calibration curve, and then plot as a function of time or set down the data in tabular form. There are many aids to this process of various degrees of automatic form which are immediately apparent. For example, in order of increasing departure, these might be: (1) visual aids for record reduction; (2) a stylus manipulated by an operator so as to follow the record of a single channel having mechanical (or electrical) linkages which automatically inject a correction from a calibration cam and then record in tabular, punched card, or plotted form; (3) automatic electromechanical or electrical injection of calibration curves during the flight with recording by conventional means and/or during flight coding and recording in digital form. The recording in digital form may be played back in order to punch cards, print the data in tabular form, or fed into a digital computer. In the case of recording in digital form, the remarks on quantization noise in Section IV-5 are applicable.

In the authors' opinion, at least at this stage of development of telemetry, a continuous recording of all channels as a function of time is desirable regardless of other types of recording, such as in digital form, automatic injection of calibration curves, etc., which may be carried out simultaneously. The reason for this is that there is a need for editing of the record to check on such things as fading of the signal, crosstalk, and other imperfections which are easily discernable when the channel records as a continuous function of time are examined simultaneously.

### *VII-1. AIRBORNE RECORDING*

Airborne recording by such means as recording galvanometers, photography of instruments, etc., has been extensively used in flight test work.(B7,C5,E1,L6,P1,S7,W1) It is the authors' opinion that, wherever airborne recording is satisfactory under flight conditions and recovery of records after flight is feasible, airborne recording should be used in place of radio telemetry. However, there are many cases in which the recovery of records is a problem or in which operating conditions and space and weight requirements can be more effectively met by radio telemetry. In certain cases in which space and weight are available, it has been found desirable to use both airborne recording and radio telemetry in order to increase the probability of obtaining a record and in order to permit checking the two sources of data against each other.

Recent developments in compact magnetic tape or wire recording have reduced the size and increased the ruggedness of airborne recording equipment.(B6,K2) If a quantitative record as a function of time is desired, it is best to modulate a carrier; frequency modulation seems to be the most satisfactory. If a record of the time occurrence of pulses, the height of which is unimportant, is desired, it is possible to modulate the tape or wire directly.

## VIII. Crosstalk from Overload in Frequency-Division Multiplex Radio Links

As an approximation we shall assume the nonlinear distortion to be of a simple type where the distortion is a function only of the instantaneous signal at the input of the radio link.|| || For sufficiently small signals the distortion will be negligible. Let us assume it has been determined that the distortion of the input signal $f(t)$ is negligible when $|f(t)|\leqslant D$ where $D$ is a positive constant. Let us also assume the worst possible condition in which the output signal $g(t)$ is given by $g=f$ for $|f|\leqslant D$, $g=D$ for $f>D$, and $G=-D$ for $f<-D$. That is, the characteristic is perfectly linear with unit gain between $-D$ and $+D$ and the radio link completely limits signals outside of this range. Some equipments have characteristics approximating this, particularly when inverse feedback is used and the final amplifier stage overloads.

The distortion that results when $|f|>D$ may be thought of as an error signal of value $g-f$ introduced into the output of the radio link. Let the input signal $f(t)$ be the linear sum of a number of sinusoidal sub-carrier signals, the sum of whose amplitudes is just a little greater than $D$. Then it is clear that during most of the time $|f(t)|\leqslant D$ provided there are only random phase relations between the sinusoidal components. Only occasionally will the relative phases of the components be such that $|f(t)|$ becomes nearly as large as the sum of the amplitudes and so exceeds $D$.

Thus the error signal will be zero most of the time with occasional negative or positive pulses of short duration. The frequency spectrum of these pulses will generally be wide enough to cover the sub-carrier pass bands uniformly.

### *VIII-1. CRITERION FOR TOLERABLE CROSSTALK*

A criterion proposed by Hollbrook and Dixon(H5) for maintaining crosstalk at acceptably low levels is that the error signal shall be non-zero during an average of not more than $10^{-3}$ of the time. This has been verified experimentally for certain conditions.¶¶

This criterion leads to definite relations giving the permissible amplitude of the sub-carriers as a function of the number $n$ of sub-carriers and the overload value $D$. To obtain such a relation it is first necessary to know the probability density function $p_n$ for a signal consisting of the sum $x$ of $n$ sinusoidal components with random phase relations. For a single sinusoidal component of unit RMS value this is(L5)

$$p_1(x)=\frac{1}{\pi[1-(x^2/2)]^{\frac{1}{2}}},\quad x^2<2,$$
$$p_1(x)=0,\qquad x^2\geq 2. \qquad \text{(VIII-1)}$$

|| || The concept of overload in an amplifier or radio link is not a simple one. What is usually meant by overloading is that the device ceases to act as a linear four-terminal network. By a linear four-terminal network we mean a device which operates linearly upon an input function of time $f(t)$ to give an output function of time $g(t)$. That is, the superposition theorem holds. Mathematically this is expressed by $g(t)=\int_{-\infty}^{\infty}f(\tau)h(t-\tau)d\tau$ where $h(\tau)$ determines the linear operation and $h(\tau)=0$ for $\tau<0$. Accordingly, the deviation from linear response is usually not expressible as a simple plot of instantaneous output as a function of instantaneous input which deviates from linearity for sufficiently great inputs. For example, in the case of sinusoidal inputs the amount of nonlinear distortion is not only a function of the instantaneous input, but also often a function of the frequency. In the case of a pulse the nonlinear distortion may depend on the rise time and duration (frequency spectrum) as well as on the height. It is well known in the case of frequency modulation radio links that with sinusoidal modulation of a given amplitude the nonlinear distortion increases as the modulating frequency is increased.(R3) This effect is emphasized even more in phase-modulation radio links and is present to a certain extent in amplitude-modulation radio links. It is true that the nonlinear response often originates somewhere in the circuit as a simple nonlinear function of voltage or current independent of time (FM and PM radio links are exceptions). However, after linear networks are placed on both sides of the simple nonlinear element we have the general situation described above. This frequency-dependent nonlinear distortion in radio links depends very much on the circuit adjustment and design of a particular unit and there are no general rules describing it for particular methods of modulation, etc.

¶¶ See reference (H5) page 634: "Experiments have been conducted on a number of different multi-channel amplifiers, each loaded by various numbers of active channels all at the same volume."

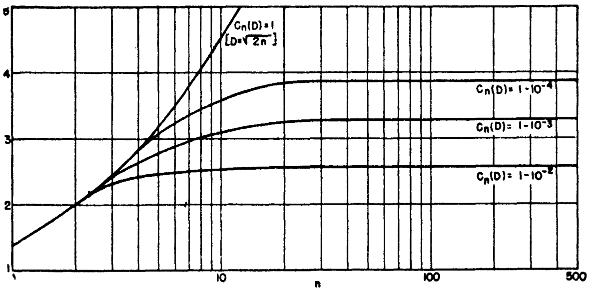



FIG. 3. Overload value $D$ exceeded by $n$ sinusoidal subcarriers of total RMS value unity [each sub-carrier of amplitude $(2/n)^{\frac{1}{2}}$] during a fraction of the time $1-C_n(D)$ equal to 0, $10^{-4}$, $10^{-3}$, and $10^{-2}$.

That is, the probability of the instantaneous value falling between $x$ and $x+dx$ is $p_1(x)dx$.

For $n>1$ the formulas for $p_n(x)$ rapidly become very complicated and for $n>2$ $p_n(x)$ is not expressible in terms of the elementary functions. However it can be shown that[L5]

$$p_n(x)=\frac{1}{\pi}\int_0^\infty \cos\left(\frac{n}{2}\right)^{\frac{1}{2}} xtJ_0{}^n(t)dt, \quad \text{(VIII-2)}$$

where $J_0(t)$ is the zero-order Bessel function and the amplitudes of the sinusoidal components are equal to $(2/n)^{\frac{1}{2}}$ so that the resultant sum has unit RMS value. The probability that $|x|\leqslant D$ is

$$C_n(D)=\int_{-D}^{D} p_n(x)dx. \quad \text{(VIII-3)}$$

Substituting from (VIII-2) gives, after integration with respect to $x$,

$$C_n(D)=\frac{2}{\pi}\left(\frac{2}{n}\right)^{\frac{1}{2}}\int_0^\infty \frac{\sin(n/2)^{\frac{1}{2}}Dt}{t}J_0{}^n(t)dt. \quad \text{(VIII-4)}$$

Formulas are available for the case of unequal amplitudes of the sub-carrier components,[L5] but we shall limit this discussion to the case of equal amplitudes.

In accordance with a well-known theorem of mathematical statistics[H4] the probability density function approaches the normal form as the number of sinusoidal components becomes large. That is

$$\lim_{n\to\infty} p_n(x)=\frac{1}{(2\pi)^{\frac{1}{2}}}\exp\left(-\frac{x^2}{2}\right)=p_\infty(x) \quad \text{(VIII-5)}$$

where the signal has unit RMS value. The crosstalk criterion determining $D$ becomes

$$C_\infty(D)=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\int_0^D \exp\left(-\frac{x^2}{2}\right)dx=1-10^{-3}. \quad \text{(VIII-6)}$$

Reference to a table for the error integral provides the result $D=3.29$. Thus the overload value is 3.29 times the RMS value of the signal provided the number of components is sufficiently great.

It has been shown that the normal form is a fairly good approximation for $p_n(x)$ when $n>10$ and it grows better as $n$ increases.[L5] Figure 3 is a plot of $D$ as a function of $n$ for $1-C_n(D)=0$, $10^{-4}$, $10^{-3}$, and $10^{-2}$. For $n\leqslant 10$ the curves have been calculated by using the proper non-normal forms for $p_n(x)$.*** The curves for $10^{-4}$ and $10^{-2}$ are included to show that the overload value $D$ is not very sensitive to the magnitude used for the crosstalk criterion. The solution of $1-C_n(D)=0$ corresponding to no overload is of course $D=(2n)^{\frac{1}{2}}$.

In order to better visualize the improvement offered by the crosstalk criterion over the simple requirement of no overload Fig. 4 is a plot of the sub-carrier amplitude improvement ratio $A_n$ obtained by use of the overload criterion. $A_n$ is the ratio of the sub-carrier amplitude permitted by the overload criterion to the sub-carrier amplitude permitted by no overload. Figure 4 is obtained from Fig. 3 by dividing the ordinants of the curve for $C_n(D)=1$ by the ordinants of the curve for $C_n(D)=1-10^{-3}$.

### *VIII-2. JUSTIFICATION OF THE CRITERION*

As far as the authors know there has been no theoretical justification of the above criterion for crosstalk due to overload ($C_n=1-10^{-3}$). We shall attempt to obtain an approximate formula giving the ratio of the RMS crosstalk disturbance to the desired signal in any channel of a sub-carrier multiplex system when the number of channels is large. That is, we assume

$$p_n(x)=\frac{1}{(2\pi)^{\frac{1}{2}}}\exp\left(-\frac{x^2}{2}\right).$$

The mean value $M$ of the error signal is

$$M=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\int_D^\infty \exp\left(-\frac{x^2}{2}\right)(x-D)dx$$

$$=2.79\times 10^{-4} \quad \text{(VIII-7)}$$

for $D=3.29$.

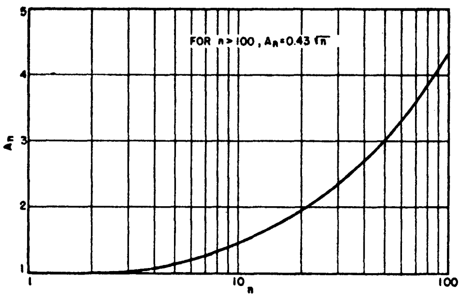



FIG. 4. The sub-carrier amplitude improvement ratio $A_n$ as a function of $n$ for $1-C_n=10^{-3}=P_n$, where $A_n$ is the ratio of the largest sub-carrier amplitude which will cause overmodulation $P_n$ of the time to the largest sub-carrier amplitude which will cause no overmodulation at all, assuming all sub-carriers to have the same amplitude.

*** The authors are indebted to Professor W. C. Johnson of Princeton University for the calculations for small $n$ involving the non-normal forms.

Now the expected number of maxima per second of the absolute value of the total signal exceeding the value $D$ is approximately

$$(\tfrac{3}{5})^{\frac{1}{2}} \exp\left(-\frac{D^2}{2}\right) f_V = 3.47\times 10^{-3} f_V, \quad \text{(VIII-8)}$$

where $f_V$ is the upper frequency limit of the low pass band occupied by the sub-carrier channels.(R13) This is very nearly the expected number of pulses per second in the error signal and is based upon the approximation that the energy of the sub-carrier channels lying in the frequency range up to $f_V$ is distributed uniformly throughout the frequency range. Dividing this into $M$ provides the result that the mean area of the pulses composing the error signal is $0.0804/f_V$.

Since we do not know the distribution function for the area of the pulses let us make the approximation that the pulses are of equal areas and equal to the above mean area. As previously pointed out most of the pulses are short compared to the envelope rise time of the channel band-pass filter. This means that the envelope of the pulses at the output of a filter will have the shape characteristic of the impulse response of the filter, whatever the shape of the short pulse at the input. Also the area of the output pulse envelope will be the same as the area of the input pulse.

In the case of an "ideal band-pass filter" with the $i$th sub-carrier located at the center, the shape of the output envelope of the pulse is given by(G2)

$$h(t) = a\frac{\sin 2\pi f_{fi}t}{2\pi f_{fi}t},$$

where $f_{fi}$ is half the band width of the $i$th filter and provided the pulse is small relative to the sub-carrier. This has a mean value $a/2f_{fi}$ and an RMS value $a/(2f_{fi})^{\frac{1}{2}}$. Thus the RMS value is $(2f_{fi})^{\frac{1}{2}}$ times the mean value. Multiplying this by the mean area of the pulses we obtain for the RMS value of each pulse disturbance in the $i$th channel

$$0.114\frac{(f_{fi})^{\frac{1}{2}}}{f_V}.$$

Since the individual disturbances occur on the average at the rate of $0.00347f_V$ per second, the RMS value of the total crosstalk disturbance in the $i$th channel is

$$0.00672\left(\frac{f_{fi}}{f_V}\right)^{\frac{1}{2}}. \quad \text{(VIII-9)}$$

In the case of AM sub-carriers of equal band width we have approximately

$$\left(\frac{f_{fi}}{f_V}\right)^{\frac{1}{2}} = \frac{1}{(2n)^{\frac{1}{2}}} \quad \text{(VIII-10)}$$

assuming no guard space between channels. The RMS value of each sub-carrier is $1/(n)^{\frac{1}{2}}$ so the ratio of the RMS crosstalk disturbance to the RMS sub-carrier signal is 0.00475 or about one-half of one percent of full modulation. Thus the crosstalk criterion appears to have a reasonable theoretical basis.

Additional reasoning based upon the non-normal distributions for small numbers of channels shows that the criterion results in a smaller crosstalk disturbance when the number of channels is small. Thus the criterion can probably be relaxed somewhat when the number of channels is small.

It should be pointed out that the above result was obtained on the assumption of complete saturation of the signal for absolute values greater than $D$. In practice this saturation is not complete so that the above result should be considered an upper limit for the crosstalk disturbance.

## IX. Crosstalk Due to Restricted Bandwidth in Pulse-Amplitude Modulation Multiplex Systems

Crosstalk as a function of band width in PAM multiplex systems has been studied by Bennett.(B2) Results are obtained by studying the Fourier series representation of the periodic PAM multiplex signal. In this section we extend the calculations in Bennett's paper using a different method. An approach which appears to lend itself better to calculation is by means of the nonperiodic transient response due to the transfer characteristic of the restricted band width. Except in cases where the number of channels is so small or the band width so restricted that the transient response is prolonged beyond an entire frame period of $1/F$, the transient approach must give the same answer as the periodic approach.**

We assume an ideal low pass filter with cut-off frequency $knF$ and no phase delay.§§§ This is equivalent in the periodic approach to neglecting all frequency components above $knF$. It is easy to calculate that a pulse of unit height beginning at $t=0$ and ending at $t=1/\alpha nF$ comes out of the filter with a functional form $f(t)$ where

$$f(t) = \frac{1}{\pi}\left\{Si(2\pi knFt) - Si\left[2\pi knF\left(t - \frac{1}{\alpha nF}\right)\right]\right\} \quad \text{(IX-1)}$$

where

$$Si(t) = \int_0^t \frac{\sin x}{x}dx.$$

We assume the measurement of the pulses is by means of area as when low-pass filters or pulse integration is used.‡‡‡ The area of the wanted signal in any channel at

** Actually the transient approach gives the same answer as the periodic approach under all conditions provided we include the transient disturbances from *all* of the preceding samples of the crosstalking channel instead of from the first preceding sample alone.

§§§ For other transfer characteristics see reference (M6).

‡‡‡ See Appendix 4.

its maximum value is

$$\gamma_0=\int_0^{1/\alpha nF} f(t)dt$$

and the area of the crosstalk signal in the $m$th following channel corresponding to maximum signal is ($m=1$ for adjacent channel)

$$\gamma_m=\int_{m/nF}^{m/nF+1/\alpha nF} f(t)dt. \qquad \text{(IX-2)}$$

Thus the ratio of the maximum crosstalk in the $m$th following channel to the maximum signal in that channel is

$$\rho_m=\frac{\gamma_m}{\gamma_0}. \qquad \text{(IX-3)}$$

Substituting for $f(t)$ in (IX-2) and changing variables and limits gives

$$\gamma_m=\frac{1}{\pi}\left[\int_0^{(m\alpha+1)/\alpha nF}+\int_0^{(m\alpha-1)/\alpha nF}-2\int_0^{m/nF}\right]Si(2\pi knFt)dt.$$

If we define

$$Sii(t)=\int_0^t Si(x)dx$$

then

$$\gamma_m=\frac{1}{2\pi^2knF}\left\{Sii\left[2\pi k\left(m+\frac{1}{\alpha}\right)\right]+Sii\left[2\pi k\left(m-\frac{1}{\alpha}\right)\right]-2Sii(2\pi km)\right\}. \qquad \text{(IX-4)}$$

For convenience in calculation we use

$$\gamma_m^*=2\pi^2knF\gamma_m$$

and

$$\rho_m=\frac{\gamma_m^*}{\gamma_0^*}. \qquad \text{(IX-5)}$$

For $\alpha=\infty$ this reduces to

$$\rho_m=\frac{\sin 2\pi km}{2\pi km}. \qquad \text{(IX-6)}$$

A table of $Sii(x)$ has been computed*** and used to calculate the data in Figs. 5a through 5e for the adjacent channel crosstalk ratio $\rho_1$. For convenience the crosstalk ratio is expressed in decibels as crosstalk attenuation. The solid points indicate in-phase crosstalk and the open points out-of-phase crosstalk. It should be noted that at each change of phase there is a value of $k$ giving no crosstalk indicated by a vertical dashed line. The frequent changes of phase are due to the ringing of the assumed ideal low pass filter. Due to the simple form of $\rho_1$ for $\alpha=\infty$ in (IX-6) it is not presented as a figure. However for reference the function $1/2\pi k$ is plotted in terms of attenuation in Fig. 5a (in addition to $\rho_1$ for $\alpha=1$). This is the locus of the attenuation minima of $\rho_1$ for $\alpha=\infty$. The points of infinite attenuation occur at half-integer values of $k$. Calculations of $\rho_1$ for $\alpha=20$ show it to be the same as for $\alpha=\infty$ within 2 db for $k\leq 6$. Calculations show $\rho_1$ for $\alpha=10$ also to be closely the same as for $\alpha=\infty$ until $k\geqslant 4$. When $k=6$ the $\alpha=10$ value of $\rho_1$ has become approximately 9 db greater than the $\alpha=\infty$ value of $\rho_1$.

The general characteristics of the crosstalk attenuation plots are as follows. As $\alpha$ increases above unity, the attenuation plot rises very rapidly along its entire length until at $\alpha=1.11$ the average rise is approximately 10 db. The rise for $5\leq k\leq 6$ is considerably greater than the average. As $\alpha$ increases to 1.43 the plot falls at small values of $k$ and does not change significantly for larger values of $k$. As $\alpha$ is increased to 2.0 the plot falls slightly for small values of $k$ and rises for $5\leq k\leq 6$. As $\alpha$ increases to 2.86 there is no significant change for $k\leq 6$. Eventually as $\alpha$ is increased the plot falls below the $\alpha=1$ level toward the $\alpha=\infty$ curve of Fig. 5a. However, no matter how large a finite value of $\alpha$ is chosen the plot will always rise considerably above the $\alpha=\infty$ level for sufficiently large $k$.

### X. Information Efficiency of Pulse-Time Modulation and Multiplex Methods

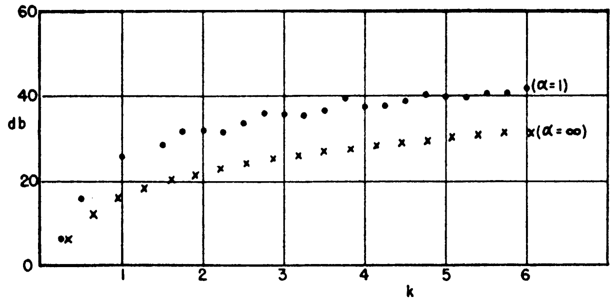
The information capacity of a transmission channel depends upon the band width and signal-to-noise ratio available. The information capacity $I$ in bits per second is given by

$$I=W\log_2(1+R^2) \qquad \text{(X-1)}$$

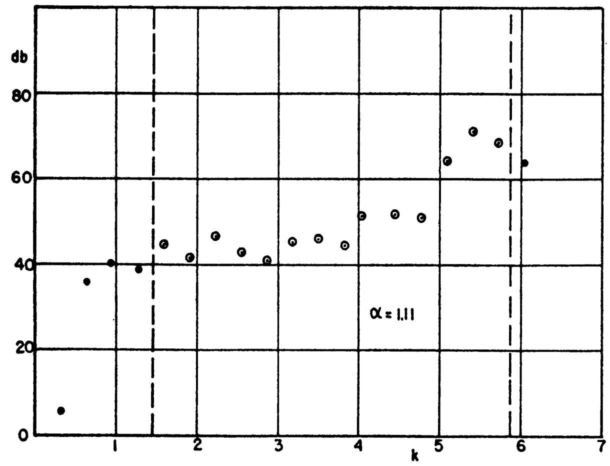
where $W$ is the band width and $R$ is the RMS signal-to-noise ratio.[S6] A bit is the basic two-valued unit of information often represented by a binary digit as in PCM. Equation (X-1) holds only for noise with a white frequency spectrum and a Gaussian probability distribution.

From (X-1) it is clear that transmission channels with different band widths and signal-to-noise ratios can have the same information capacity. For $I$ constant and $R$ large compared to unity we have the result that when the band width $W$ is increased by the factor $k$ the signal-to-noise ratio $R$ becomes $R^{1/k}$. Many of the modulation methods in current use take advantage to some extent of this possibility of exchanging band width for signal-to-noise ratio. The original information consists of a signal with a certain band width and required signal-to-noise ratio. The signal-to-noise ratio of the transmission channel turns out to be less than that required by the original information signal. So the information signal

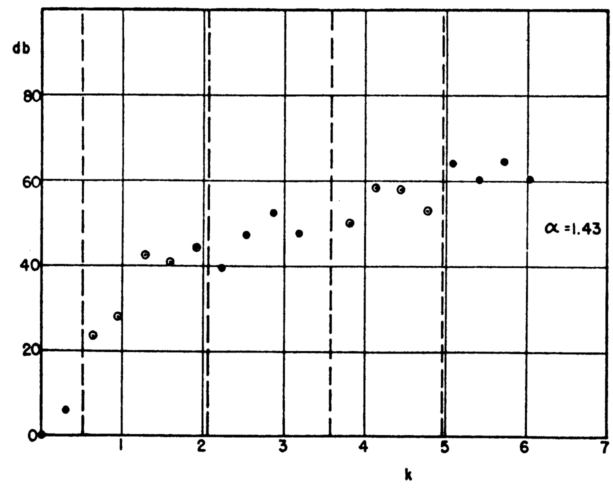
*** Copies of this table are available from LLR.



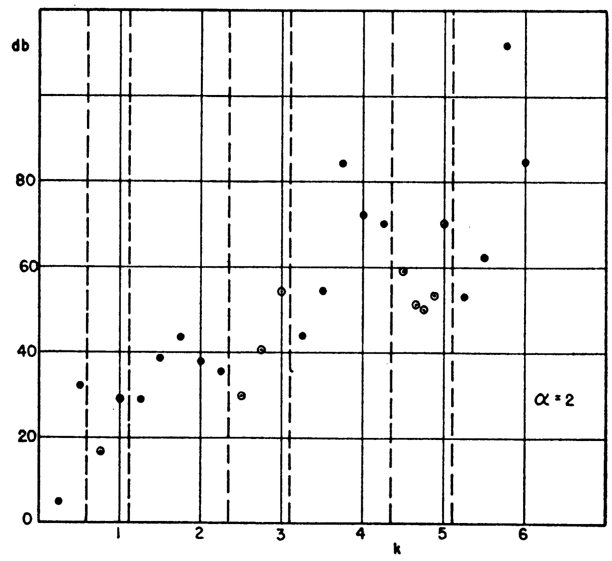

(a)



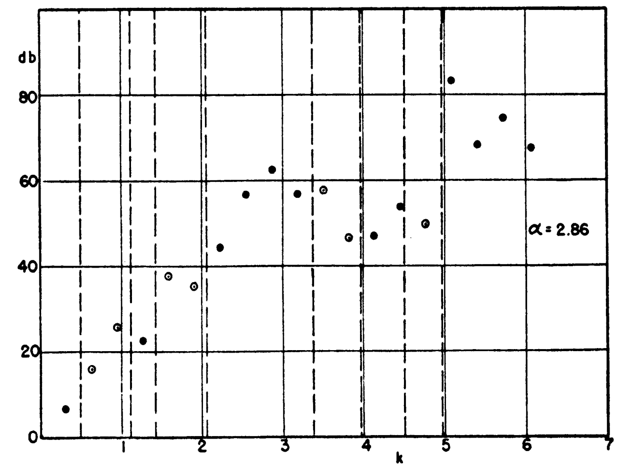

(b)




(c)




(d)




(e)

FIG. 5. Crosstalk attenuation in decibels as a function of frequency in units of $nF$ for various values of alpha as follows: (a) $\alpha=1$ and $\infty$, (b) $\alpha=1.11$, (c) $\alpha=1.43$, (d) $\alpha=2$, (e) $\alpha=2.86$. Solid points represent in phase crosstalk and circles out-of-phase crosstalk. The dashed vertical lines (drawn in by inspection) separate regions of in phase crosstalk from regions of out-of-phase crosstalk where the attenuation is infinite.

is subjected to a nonlinear operation (modulation) which results in a new information signal containing the same information, but requiring a larger band width and smaller signal-to-noise ratio. At the other end of the transmission channel the original information signal can be recovered by use of the inverse nonlinear transformation (demodulation). Sometimes it is not desirable to recover the original information signal and the transformed signal is used directly as in PCM data storage.

None of the so-called noise improvement modulation methods in current use take full advantage of the band width and signal-to-noise exchange offered by (X-1). The various modulation methods require a larger band width than required by (X-1) for the signal-to-noise ratio reduction obtained. Another way of saying this is that the information capacity of the transformed (modulated) signal channel must be greater than the information capacity of the original signal channel. Frequency modulation is very poor in this respect while pulse-code modulation is very good though not perfect.

To compare the various modulation methods Table VI gives the information efficiency for each method in terms of the desired channel signal-to-noise ratio $R_1$. Information efficiency is the information capacity of the original signal channel divided by the information capacity of the modulated signal channel. It is clear that the ideal information efficiency is 1.0. From (X-1) it follows that in considering multiplex methods, it is proper to add the information capacity of several

TABLE V[a].

| Modulation method | Before modulation (after demodulation) Band width $W_1$ | Signal-to-noise $R_1$ | Signal-to-noise at threshold $R_1^*$ | After modulation (before demodulation) Band width $W_2$ | Signal-to-noise $R_2$ | Signal-to-noise at threshold $R_2^*$ |
|---|---|---|---|---|---|---|
| PAM-AM | $nf_m$ | $\frac{S}{k_2}\cdot\frac{1}{(nf_m)^{1/2}}$ | — | $16nf_m$ | $\frac{S}{k_2}\cdot\frac{1}{2(2nf_m)^{1/2}}$ | — |
| PAM-FM | $nf_m$ | $\frac{S}{k_2}\cdot\frac{\pi f_D}{8n^{3/2}f_m^{3/2}}$ | $1.26\frac{f_D^{3/2}}{n^{3/2}f_m^{3/2}}$ | $2.6f_D$ | $\frac{S}{k_2}\cdot\frac{1}{(1.3f_D)^{1/2}}$ | 2.8 |
| PWM-AM | $nf_m$ | $\frac{S}{k_2}\cdot\frac{(F_c)^{1/2}}{\sqrt{2}nf_m}$ | $2.83\frac{F_c}{nf_m}$ | $2F_c$ | $\frac{S}{k_2}\cdot\frac{1}{(F_c)^{1/2}}$ | 4 |
| PWM-FM | $nf_m$ | $\frac{S}{k_2}\cdot\frac{\sqrt{3}f_D}{nf_m(F_c)^{1/2}}$ | $4.50\frac{f_D(D)^{1/2}}{nf_m}$ | $2.6f_D$ | $\frac{S}{k_2}\cdot\frac{1}{(1.3f_D)^{1/2}}$ | 2.8 |
| PPM-AM | $nf_m$ | $\frac{S}{k_2}\cdot\frac{5F_c}{8n^{3/2}f_m^{3/2}}$ | $3.54\frac{F_c}{nf_m}$ | $2F_c$ | $\frac{S}{k_2}\cdot\frac{1}{(F_c)^{1/2}}$ | $8\left(\frac{nf_m}{F_c}\right)^{1/2}$ |
| PCM-AM | $nf_m$ | $\sqrt{\frac{3}{2}}\cdot 2^N$ | $1.22\cdot 2^N$ | $2Nnf_m$ | $\frac{S}{k_2}\cdot\frac{1}{(nNf_m)^{1/2}}$ | 5.65 |
| FM-FM[b] | $nf_m$ | $\frac{S}{k_2}\cdot 2.3\sqrt{\frac{3}{2}}\cdot\frac{f_D}{n^{3/2}f_m^{3/2}}$ | $6.05\frac{f_D^{3/2}}{n^{3/2}f_m^{3/2}}$ | $2.6f_D$ | $\frac{S}{k_2}\cdot\frac{1}{(1.3f_D)^{1/2}}$ | 2.8 |

[a] In constructing this table from Table II we have used the relations $\alpha=2$, $F=2f_m$, $F_c=4nF$, $r=4n$, and $F_D=DF_c$.
[b] In the FM-FM case, it is assumed that the carrier threshold governs. The following relations are used for the center channel : $a_{0i}/S=2.32/(n)^{3/2}$, $f_{mi}=f_m$, $f_i=f_D/2D$, and $f_{di}=f_D/2nD$.

channels with the same signal-to-noise ratio by adding the band widths of the channels.

As an intermediate step to Table VI, Table V gives the band widths $W_1$ and $W_2$ and signal-to-noise ratios $R_1$ and $R_2$ before and after modulation (after and before demodulation) based on the data of Table II. The FM-FM case from Table I is included for comparison purposes.

The information efficiency $E$ is given by

$$E=\frac{W_1\log(1+R_1^2)}{W_2\log(1+R_2^2)} \tag{X-2}$$

where the common base of the two logarithms may be as desired. In calculations we shall use the base 10. Now $R_1>R_2$ in all cases where noise improvement is obtained including PAM-AM. Therefore, the maximum value of $E$ is obtained in all cases when the common factor $S/k_2$ is made as small as possible. In all cases except PAM-AM this is when $S/k_2$ has the improvement threshold value as given in Table II.

The values of $R_1$ and $R_2$ obtained by using the improvement threshold values of $S/k_2$ are denoted respectively by $R_1^*$ and $R_2^*$. These are also listed in Table V.

In Table VI the information efficiencies are given in terms of the signal-to-noise ratio $R_1$ of the original information transmission channel. Except in the case of PAM-AM, it is assumed that each system is operating at its improvement threshold.

It should be noted that where an AM carrier is indicated it has been assumed that double sideband transmission is used. If single sideband transmission is used instead, the information efficiency will be increased by a factor 2.

TABLE VI.

| | |
|---|---|
| PAM-AM | $\frac{1}{16}\frac{\log(1+R_1^2)}{\log\left(1+\frac{R_1^2}{8}\right)}$ |
| PAM-FM | $\frac{0.48}{R_1^{2/3}}\log(1+R_1^2)$ |
| PWM-AM | $\frac{1.15}{R_1}\log(1+R_1^2)$ |
| PWM-FM | $\frac{1.83(D)^{1/2}}{R_1}\log(1+R_1^2)$ |
| PPM-AM | $\frac{1.77}{R_1}\frac{\log(1+R_1^2)}{\log\left(1+\frac{226}{R_1}\right)}$ |
| PCM-AM | $0.10\cdot\frac{\log(1+R_1^2)}{\log R_1-0.088}$ |
| FM-FM | $\frac{2.86}{R_1^{2/3}}\log(1+R_1^2)$ |

## APPENDIX 1

### *WIDE BAND IMPROVEMENT FOR PWM-AM AND PWM-FM*

In PWM-AM the radio transmitter is turned full on for a length of time proportional to the instantaneous value of the signal. It is assumed that the duration of the pulse is measured from a precise time reference (i.e. no noise on the time reference) to the point when the voltage has decayed to one-half its full on value (i.e. half-height slicing). It is also assumed that the sampling takes place at regular intervals. In deriving the wide band gain for this method, the theorem proved in Appendix 5 is used. The RMS noise in the video is $k_2(F)_c^{1/2}$. From reference (G2) page 75, the slope of a step function at the half-height is $2hF_c$ where $h$ is the height of the step function. Therefore the RMS time fluctuation at the half-height point due to the video noise is $k_2(F)_c^{1/2}/2hF_c$. If the duty factor of the transmitter is assumed to be $\frac{1}{2}$ then $h=\sqrt{2}\sqrt{2}S$. If the peak to peak variation of the pulse width due to the information modulation is $1/nF$, the corresponding RMS value is $1/\sqrt{2}2nF$. Therefore, the video signal-to-noise ratio may be written

$$\frac{S(2F_c)^{1/2}}{nk_2F}. \tag{1-1}$$

In applying the theorem of Appendix 5, the pulse widths may be considered as an instantaneous time series, in which case the signal-to-noise ratio after passage through a low pass filter with cutoff at $f_m$ is

$$\frac{S(2F_c)^{1/2}}{nk_2F}\left(\frac{F/2}{f_m}\right)^{1/2}. \tag{1-2}$$

Division by the signal-to-noise ratio, $S/k_2(f_m)^{1/2}$ of the comparison[1a] single channel AM link gives the wide band improvement $R_{0i}$ for PWM-AM.

$$R_{0i}=1/n(F_c/F)^{1/2}. \tag{1-3}$$

If it is considered that the improvement threshold occurs when the crest fluctuation noise in the video equals half the pulse height—i.e., when

$$4k_2(F_c)^{1/2}=\frac{\sqrt{2}\sqrt{2}S}{2}$$

the improvement threshold is

$$S/k_2=4(F_c)^{1/2}. \tag{1-4}$$

In PWM-FM, the FM carrier is switched from full modulation to one side of center frequency to full modulation the other side for a length of time proportional to the instantaneous value of the signal. Proceeding as in the case of PWM-AM, taking into account the triangular FM noise spectrum, the $R_{0i}$ turns out to be

$$R_{0i}=\frac{(6)^{1/2}D}{n}\frac{F_c}{F} \tag{1-5}$$

where $D$ is the carrier deviation ratio. If it is assumed that the carrier FM improvement threshold governs, then the improvement threshold is (see Section III-2)

$$S/k_2=3.2(f_D)^{1/2}. \tag{1-6}$$

In the above derivations it has been assumed that the instantaneous sampling takes place at regularly spaced intervals whereas in practice a triangular timing voltage is usually used which displaces the sampling time in accordance with the voltage of the signal.[(K5)] In the case of many channels the correction for this is small.

## APPENDIX 2

### *WIDE BAND IMPROVEMENT AND IMPROVEMENT THRESHOLD FOR PPM-AM*

Goldman[2a] has obtained an expression for the wide band improvement of PPM-AM. If the shorter rise time of an impulse is used instead of the rise time of a step, the expression for the wide band improvement $R_{0i}$ becomes[2b]

$$R_{0i}=\frac{5F_c}{4n^{1/2}F}. \tag{2-1}$$

The response of an ideal low pass filter of band width $F_c$ to an impulse at $t=0$ is, neglecting time delay,

$$a\frac{\sin 2\pi F_c t}{2\pi F_c t} \tag{2-2}$$

where $a$ is the maximum height of the impulse response. To obtain the improvement threshold we calculate the RMS value of a PPM signal consisting of these impulses occurring at the rate of $nF$ per second. The RMS value of a single impulse is

$$a\left(\int_{-\infty}^{\infty}\frac{\sin^2 2\pi F_c t}{4\pi^2 F_c^2 t^2}dt\right)^{1/2}=\frac{a}{(2F_c)^{1/2}}. \tag{2-3}$$

For $nF$ impulses per second this becomes

$$S=a(nF/2F_c)^{1/2} \tag{2-4}$$

where $S$ is the RMS video signal.

At the improvement threshold the impulse height $a$ should be 2 times the peak noise which we assume to be 4 times the RMS noise. Thus

$$a=S(2F_c/nF)^{1/2}=8k_2(F_c)^{1/2}. \tag{2-5}$$

So at the improvement threshold the RMS signal-to-noise per unit band width ratio in the video channel is

$$\frac{S}{k_2}=4\sqrt{2}(nF)^{1/2}. \tag{2-6}$$

Since double sideband AM transmission is assumed, we have for the RF channel an RMS signal-to-noise per unit band-width ratio of

$$4(nF)^{1/2}. \tag{2-7}$$

## APPENDIX 3

### *IMPROVEMENT THRESHOLD FOR PCM-AM*

The RMS signal-to-quantization noise ratio in the channels is[(B3)]

$$\sqrt{\tfrac{3}{2}}\cdot 2^N. \tag{3-1}$$

To obtain the improvement threshold[3a] we note that on the average approximately half of the possible $NnF$ pulses per second do not occur. We assume the pulses to be of width $1/NnF$ with a rise time of $1/NnF$ since crosstalk is not important. Thus the height of the pulses is $\sqrt{2}S$ and the video band width is $NnF/2$.[(G2)] Equating the pulse height to 2 times the peak noise gives

$$\sqrt{2}S=8k_2\left(\frac{NnF}{2}\right)^{1/2} \tag{3-2}$$

$$\frac{S}{k_2}=4(NnF)^{1/2} \tag{3-3}$$

for the improvement threshold in the video channel. For the RF channel this is

$$2\sqrt{2}(NnF)^{1/2}. \tag{3-4}$$

## APPENDIX 4

### *INTEGRATION OF PAM SAMPLE PULSES*

It is clear that if the integrated value of each sample pulse is used rather than an instantaneous sample taken at some time during the pulse, the fluctuation noise will be reduced. In the following we determine exactly the extent of this noise reduction due to pulse integration.

[1a] See reference (62) page 249.
[2a] See reference (G2) pages 280–283.

[2b] L. L. Rauch, "Fluctuation Noise in Pulse-Position Multiplex Systems," 1946; privately distributed paper prepared for Melpar, Incorporated, and the Applied Science Corporation of Princeton.
[3a] L. L. Rauch, "Noise in Pulse-Code Multiplex Systems," 1946; privately distributed paper prepared for the Applied Science corporation of Princeton.

The result is that the noise reduction due to pulse integration is exactly that obtained by passing the sample pulses of a given channel through a low pass filter of cut-off frequency $F/2$. This result clarifies the twofold action of the low pass filter. It first produces a noise reduction due to its integrating action on the pulses. Its second action is then the smoothing of the integrated pulses. In agreement with the above, if a low pass filter is preceded by pulse integration there is no improvement over the low pass filter used alone.

Before proceeding with the derivation it is necessary to define what we mean by "RMS value of the pulse signal." This is the standard deviation (in the statistical sense) of the time series whose terms are the values associated with each sample pulse. These values may be the integrals of samples of finite duration or they may be the heights of instantaneous samples. The integrals of samples of finite duration are always normalized by dividing by the length of the sample. It immediately follows from the above definition that the RMS value of the instantaneous samples of a continuous noise signal is exactly equal to the RMS value of the continuous noise signal.

It is interesting to note that the inverse relation is also true, although in a more limited sense. If a time series with a regular repetition rate $F$ is passed through a low pass filter with cut-off frequency $F/2$, the RMS value of the continuous signal at the filter output is exactly equal to the RMS value of the time series. This is proved in Appendix 5.

Let us assume a continuous fluctuation noise signal $g(t)$ with a spectrum $s(\omega)$ of random phase $\theta = \arctan(I[s(\omega)]/R[s(\omega)])$ where $I$ and $R$, respectively, denote the imaginary and real parts of $s(\omega)$. The correlation properties of the noise signal will determine the RMS value per unit band width $|s(\omega)|$ as a function of frequency where $|s(\omega)|$ denotes the magnitude of the complex valued spectrum function $s(\omega)$. The $m$-th term of the time series corresponding to the integrated sample is

$$\alpha nF\int_{(m/F)-(1/\alpha nF)}^{m/F} g(t)dt. \tag{4-1}$$

The same time series is obtained as instantaneous samples of the function $G(t)$ taken at the rate $F$ where

$$G(t) = \alpha nF\int_{t-(1/\alpha nF)}^{t} g(\tau)d\tau. \tag{4-2}$$

Thus to evaluate the RMS value of the time series consisting of the integrated pulses it is, by the preceding arguments, only necessary to obtain the RMS value of $G(t)$.

We may write

$$G(t) = G_1(t) - G_2(t) = G_1(t) - G_1\left(t - \frac{1}{\alpha nF}\right) \tag{4-3}$$

where

$$G_1(t) = \alpha nF\int_c^t g(\tau)d\tau$$

$$G_2(t) = \alpha nF\int_c^{t-(1/\alpha nF)} g(\tau)d\tau$$

and $c$ is any constant. Now the spectrum $S(\omega)$ of $G(t)$ is the difference of the spectrum $S_1(\omega)$ of $G_1(t)$ and the spectrum $S_2(\omega)$ of $G_2(t)$. That is

$$S(\omega) = S_1(\omega) - S_2(\omega). \tag{4-4}$$

Integration of a time function multiplies its spectrum by the factor $1/i\omega (i^2 = -1)$. Thus

$$S_1(\omega) = \alpha nF\frac{s(\omega)}{i\omega}. \tag{4-5}$$

We recall that

$$G_2(t) = G_1\left(t - \frac{1}{\alpha nF}\right).$$

When a time function is delayed by a time $1/\alpha nF$, its spectrum is multiplied by the factor $\exp(-i\omega/\alpha nF)$. Thus

$$S_2(\omega) = S_1(\omega)\exp\left(-\frac{i\omega}{\alpha nF}\right) = \alpha nF\frac{s(\omega)}{i\omega}\exp\left(-\frac{i\omega}{\alpha nF}\right). \tag{4-6}$$

Finally

$$S(\omega) = \alpha nF\frac{1-\exp[-(i\omega/nF)]}{i\omega}s(\omega). \tag{4-7}$$

It is now evident that $G(t)$ is obtained from $g(t)$ by the transfer function

$$T(\omega) = \alpha nF\frac{1-\exp(-i\omega/\alpha nF)}{i\omega}. \tag{4-8}$$

Straightforward calculation gives

$$T(\omega) = \left|\frac{\sin(\omega/2\alpha nF)}{\omega/2\alpha nF}\right|\exp[-(i\omega/2\alpha nF)]. \tag{4-9}$$

Thus the amplitude factor is

$$\left|\frac{\sin(\omega/2\alpha nF)}{(\omega/2\alpha nF)}\right|$$

and there is a simple time delay of

$$1/2\alpha nF$$

which is one-half of a sample period.

Now the RMS value of $G(t)$ (RMS value of integrated pulses) is given by

$$\text{RMS-IP} = \left[\int_0^{2\pi rF}|S(\omega)|^2 d\omega\right]^{\frac{1}{2}} \tag{4-10}$$

where

$$|S(\omega)| = |T(\omega)|\cdot|s(\omega)|$$

and $2\pi rF$ is the angular frequency of the upper limit of the radio link video pass band.

Changing from angular frequency $\omega$ to cyclic frequency $f$ we have for the case of an FM radio link

$$\text{RMS-IP(FM noise)} = \left[\int_0^{rF}|T(2\pi f)|^2 k_1^2 f^2 df\right]^{\frac{1}{2}} \tag{4-11}$$

and for the case of an AM radio link

$$\text{RMS-IP(AM noise)} = \left[\int_0^{rF}|T(2\pi f)|^2 k_2^2 df\right]^{\frac{1}{2}}. \tag{4-12}$$

Treating the FM case first we obtain

$$\begin{aligned}\text{RMS-IP(FM noise)} &= \frac{k_1\alpha nF}{\pi}\left(\int_0^{rF}\sin^2\frac{\pi f}{\alpha nF}df\right)^{\frac{1}{2}}\\ &= k_1\left(\frac{\alpha nF}{\pi}\right)^{\frac{1}{2}}\left(\frac{\pi r}{2\alpha n} - \frac{1}{4}\sin\frac{2\pi r}{\alpha n}\right)^{\frac{1}{2}}.\end{aligned} \tag{4-13}$$

In most practical work we may assume $r \geq \alpha n$ as in reference (R4). Under these conditions neglect of the trigonometric term under the radical will cause less than 10 percent error in the expression. The result is

$$\text{RMS-IP(FM noise)} = k_1\frac{\alpha nF^{\frac{1}{2}}r^{\frac{1}{2}}}{\sqrt{2\pi}}. \tag{4-14}$$

With signal of RMS value $S$ this provides exactly the same signal-to-noise ratio as the low pass filter. [See Eq. (7) in reference (R4) with $f_{\max} = F/2$.]

The AM case gives

$$\begin{aligned}\text{RMS-IP (AM noise)} &= \frac{k_2\alpha nF}{\pi}\left(\int_0^{rF}\frac{1}{f^2}\sin^2\frac{\pi f}{\alpha nF}df\right)^{\frac{1}{2}}\\ &= k_2\left(\frac{\alpha nF}{\pi}\right)^{\frac{1}{2}}\left(\int_0^{\pi r/\alpha n}\frac{\sin^2 y}{y^2}dy\right)^{\frac{1}{2}}.\end{aligned} \tag{4-15}$$

Now

$$\int_0^\infty \frac{\sin^2 y}{y^2}dy = \frac{\pi}{2}.$$

If we again assume $r \geq \alpha n$ then

$$\int_0^\pi \frac{\sin^2 y}{y^2}dy \leq \int_0^{\pi r/\alpha n}\frac{\sin^2 y}{y^2}dy \leq \frac{\pi}{2}.$$

Now

$$\int_0^\pi \frac{\sin^2 y}{y^2}dy = 0.85\frac{\pi}{2}$$

so the radical term in the expression is equal to $(\pi/2)^{\frac{1}{2}}$ with an error of not more than eight percent. The result is

$$\text{RMS-IP(AM noise)} = k_2\left(\frac{\alpha n F}{2}\right)^{\frac{1}{2}}. \qquad (4\text{-}16)$$

With a signal of RMS value $S$ this provides exactly the same signal-to-noise ratio as the low pass filter [see Eq. (11) in reference (R4) with $f_{\max} = F/2$].

Using the above procedure and the method developed in reference (R4), it can be proved that the noise reduction obtained by a low pass filter of band width $F/2$ is the same as the noise reduction obtained by pulse integration for any fluctuation noise spectrum.

## APPENDIX 5

### *RMS VALUE OF A SMOOTHED TIME SERIES*

Let us assume a time series occurring in regular time sequence at the rate $F$. We first show that such a time series can always be the result of taking instantaneous samples of some continuous signal containing no frequency components above $F/2$. It is clear that if we do not so restrict the band width of the continuous signal then there is always a "wide band" continuous signal whose samples give rise to the time series.

Now the spectrum of the sampled signal arising from this wide band continuous signal will be, according to Eq. (2) of reference (R4), a series of carriers[5a] located at frequencies $mF(m=1, 2, \cdots)$, with AM sidebands above and below each carrier according to the spectrum of the wideband continuous signal. The sideband components for each carrier corresponding to a given signal component have equal amplitudes. A little observation will show that although sideband components may be located much farther from their carrier than $F/2$ the components within $F/2$ of each carrier, many of which may belong to other carriers, are arranged symetrically about the carrier in an AM fashion and are exactly the same as the components located within $F/2$ of every other carrier. In the light of reference (R4) this means that the sample signal (time series) can equally well arise from a restricted band width continuous signal whose spectrum is exactly the sideband components within $F/2$ of any carrier.

Thus we have shown that the time series can be considered as arising from sampling a continuous signal containing no frequency components above $F/2$. It will be noted that the above argument breaks down for wide samples because the corresponding sideband components for different carriers may have different amplitudes. This is because a sequence of wide samples contain much more information (lost in the smoothing process) than the time series formed from instantaneous samples.

It is well known[(R4)] that when a signal containing no components above $F/2$ is sampled at a rate $F$, and the samples passed through a low pass filter with cutoff $F/2$, the signal out of the low pass filter is exactly the original signal except for any time delay introduced. Now when we smooth the time series by a low pass filter with cutoff $F/2$ the signal out of the filter is exactly the restricted bandwidth signal of the preceeding paragraph whose samples give rise to the time series. As pointed out in Appendix 4 the continuous signal giving rise to the time series has exactly the RMS value (standard deviation) of the time series. Therefore, the smoothed time series, using a low pass filter of cutoff $F/2$, has exactly the RMS value of the time series. If the time series is smoothed with a low pass filter of cutoff $f<F/2$ then RMS value of the smoothed signal may be less than that of the time series depending upon the spectral distribution. If this is uniform (white) the RMS value is $(2f/F)^{\frac{1}{2}}$ times that of the time series.

[5a] The carriers may or may not be suppressed according to whether there is not or is a DC component in the continuous signal.

## Reference List


(A1) J. Andresen, Instruments **23**, 347 (1950).
(B1) W. R. Bennett, Bell System Tech. J. **19**, 587 (1940).
(B2) W. R. Bennett, Bell System Tech. J. **20**, 199 (1941).
(B3) W. R. Bennett, Bell System Tech. J. **27**, 446 (1948).
(B4) W. P. Boothroyd and E. M. Creamer, Elec. Eng. **68**, 583 (1949).
(B5) H. S. Black and J. O. Edson, Elec. Eng. **66**, 1123 (1947).
(B6) Buhler, Freeman, and Saunders, U. S. Air Force (Air Tech. Intell.) Tech. Data Digest **13**, 11 (August 15, 1948).
(B7) W. G. Brombacher, Instruments **20**, 700 (1947).
(C1) J. C. Coe, Proc. Inst. Radio Engrs. **36**, 1404 (1948).
(C2) E. H. Colpitts and O. B. Blackwell, Trans. AIEE **40**, 205
(C3) M. S. Corrington, Proc. Inst. Radic Engrs. **33**, 878 (1945).
(C4) M. G. Crosby, Proc. Inst. Radio Engrs. **25**, 472 (1937).
(C5) R. E. Conover, Instruments **23**, 445 (1950).
(C6) Chisholm, Buckley, and Farnell, Proc. Inst. Radio Engrs. **39**, 36 (1951).
(D1) C. A. Dyer, Elec. Eng. **69**, 643 (1950).
(E1) S. B. Elggren, Instruments **18**, 2 (1945).
(E2) J. F. Engelberger and H. W. Kretsch, Elec. Eng. **69**, 642 (1950).
(F1) C. L. Frederick, Elec. Eng. **65** (Supplement), 861 (1946).
(F2) C. L. Frederick, Soc. Exp. Stress Analysis Proc. **4**, No. 2, 103 (1947).
(G1) I. H. Gerks, Proc. Inst. Radio Engrs. **37**, 1272 (1949).
(G2) S. Goldman, *Frequency Analysis, Modulation and Noise* (McGraw-Hill Book Company, Inc., New York, 1948), Chapter IV.
(G3) W. M. Goodell, Bell System Tech. J. **26**, 395 (1947).
(G4) W. E. Greene, Electronics **21**, 124 (September, 1947).
(G5) Grieg, Glauber, and Moskowitz, Proc. Inst. Radio Engrs. **35**, 1251 (1947).
(H1) C. M. Hathaway and E. S. Lee, Mech. Eng. **59**, 653 (1937).
(H2) Heeren, Hoeppner, Kauke, Lichtman, and Shifflett, Electronics **20**, 100 (March, 1947) and Electronics **20**, 124 (April, 1947).
(H3) G. L. Hinckley, Electronic Eng. **21**, 209 (1949).
(H4) P. G. Hoel, *Introduction to Mathematical Statistics* (John Wiley and Sons, Inc., New York, 1947) Theorem II, Chapter IV.
(H5) B. D. Holbrook and J. T. Dixon, Bell System Tech. J. **18**, 624 (1939).
(H6) A. Hund, *Frequency Modulation* (McGraw-Hill Book Company, Inc., New York, 1942).
(J1) A. E. Jabitz, Elec. Mfg. 77 (February, 1950).
(K1) O. E. Kell, Wireless Eng. **18**, 6, 56 (1941).
(K2) L. G. Killian, Electronic Ind. and Electronic Instrumentation **2**, 3 (April, 1948).
(K3) S. C. Kleene, Proc. Inst. Radio Engrs. **35**, 1049 (1947).
(K4) M. Kiebert, Aeronautical Conference, London, 1947, page 241 (Royal Aeronautical Society, 1948).
(K5) E. R. Kretzmer, Proc. Inst. Radio Engrs. **35**, 1230 (1947).
(L1) V. D. Landon, Proc. Inst. Radio Engrs. **24**, 1514 (1936).
(L2) V. D. Landon, RCA Rev. **9**, 287 (1948).
(L3) F. W. Lehan, Electronics **22**, 90 (August, 1949).
(L4) F. G. Logan, Electronics **21**, 104 (October, 1948).
(L5) R. D. Lord, Phil. Mag. **39**, 66 (1948).
(L6) Leiss, Nitchie, and Underhill, Instruments **20**, 709 (1947).
(M1) L. A. Meacham and E. Peterson, Bell System Tech. J. **27**, 1 (1948).
(M2) G. H. Melton, Electronics **21**, 106 (December, 1948); see also Mayo-Wells, Instruments **23**, 717 (1950).
(M3) J. T. Mengel, Instruments **23**, 70 (1950); see also N. R. Best, Electronics **23**, 82 (August, 1950).
(M4) S. T. Meyers, Proc. Inst. Radio Engrs. **34**, 256 (1946).
(M5) H. Matheson and M. Eden, Rev. Sci. Instr. **19**, 502 (1948).
(M6) Moskowitz, Diven, and Feit, Proc. Inst. Radio Engrs. **38**, 1330 (1950).
(O1) M. O'Day, Elec. Eng. **69**, 642 (1950).
(O2) Oliver, Pierce, and Shannon, Proc. Inst. Radio Engrs. **36**, 1324 (1948).

(P1) J. P. Paine, Instruments **20**, 30 (1947).
(P2) W. H. Pickering, Soc. Exp. Stress Analysis Proc. 4, No. 2, 74 (1947).
(R1) Radio Research Laboratory Staff, Harvard University, *Very High Frequency Techniques* (McGraw-Hill Book Company, Inc., New York, 1947), Vol. I, page 392.
(R2) W. Ramberg, Elec. Eng. **66**, 555 (1947).
(R3) L. L. Rauch, Electronics **20**, 114 (February, 1947).
(R4) L. L. Rauch, Proc. Inst. Radio Engrs. **35**, 1230 (1947).
(R5) *Resumé of Telemetering Practice Up to 1941.* AIEE Committee on Automatic Stations and Instruments and Measurements, Trans. AIEE **60**, 1411 (1941).
(R6) Rev. Sci. Instr. **16**, 158 (1945).
(R7) Rev. Sci. Instr. **18**, 376 (1947).
(R8) Rev. Sci. Instr. **18**, 804 (1947).
(R9) Rev. Sci. Instr. **18**, 859 (1947).
(R10) Rev. Sci. Instr. **19**, 373 (1948).
(R11) Rev. Sci. Instr. **20**, 328 (1949).
(R12) Rev. Sci. Instr. **20**, 626 (1949).
(R13) S. O. Rice, Bell System Tech. J. **24**, 46 (1945).
(R14) H. C. Roberts, *Mechanical Measurements by Electrical Methods.* (The Instruments Publishing Company, Pittsburgh, 1946.) This material is also published in Instruments **17**, pages 196, 260, 334, 398, 475, 534, 603, 668, 742 (1944); **18**, pages 10, 83, 150, 225, 299, 389, 462, 542, 616, 685, 882 (1945).
(S1) H. Schaevitz, Soc. Exp. Stress Analysis Proc. 4, No. 2, 79 (1947); see also W. D. MacGeorge, Instruments **23**, 610 (1950).
(S2) R. W. Sears, Bell System Tech. J. **27**, 44 (1948).
(S3) A. M. Skellett, Bell System Tech. J. **23**, 190 (1944).
(S4) A. M. Skellett, Proc. Inst. Radio Engrs. **36**, 1354 (1948).
(S5) C. K. Stedman, Proc. Inst. Radio Engrs. **36**, 36 (1948).
(S6) C. E. Shannon, Bell System Tech. J. **27**, 623 (1948).
(S7) H. Sostman, Instruments **23**, 37 (1950).
(T1) F. E. Terman, *Radio Engineering* (McGraw-Hill Book Company, New York, 1947), page 770.
(W1) D. K. Warner, Instruments **22**, 314 (1949).
(W2) D. E. Weiss, Soc. Exp. Stress Analysis Proc. 4, No. 2, 89 (1947).

(R5.1) AIEE Report on Telemetering, Supervisory Control and Associated Circuits, Sept. 1948.

(R5.2) Proceedings of the joint conference on telemetering sponsored by the AIEE and the National Telemetering Forum, AIEE Special Publication No. S-41, Aug., 1950.