

In this article we investigate an approach for simultaneously estimating cross-lagged and cross-instantaneous causal influences in two-variable, multiwave, linear panel models introduced by Greenberg et al. This approach requires observations over at least three time points and achieves identification by making assumptions about the consistency of parameter values across the adjacent time intervals 1-2 and 2-3. Our analysis demonstrates that multiwave models can be identified by imposing consistency constraints of this sort. However, this is a useful method only under a very restrictive set of empirical conditions.

Equilibrium and Identification in Linear Panel Models

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Social scientists have been turning with increasing frequency to the analysis of nonrecursive linear models to study complex causal relationships among a set of variables. Yet the stringent identification requirements of these models have sometimes been troublesome. To identify an equation for one of a set of reciprocally related endogenous variables, say X, on the basis of cross-sectional data, one introduces instrumental variables and specifies the partial regression coefficients for the effects of these instruments on X (see, for example, Duncan, 1975; Namboodiri et al., 1975; Hanushek and Jackson, 1977). The source of the trouble is that the analyst may lack the grounds for specifying the values of these coefficients.

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Duncan (1969) demonstrated that the identification problem also arises in two-wave panel models by showing that the most general two-way, two-variable panel model is not identified. Assuming perfect measurement, ignoring common causes of the observed scores X_t and Y_t (where t represents time), and expressing the latter as deviations from their means, this model is

$$X_2 = b_1 X_1 + b_2 Y_1 + b_3 Y_2 + u \quad [1a]$$

$$Y_2 = d_1 Y_1 + d_2 X_1 + d_3 X_2 + v, \quad [1b]$$

where u and v are, respectively, the residuals of X_2 and Y_2 . The six covariances among the observed scores can be expressed in terms of the eight model parameters: three b_i , three d_j , the covariance of X_1 and Y_1 , here denoted by $(X_1 Y_1)$, and the residual covariance (uv). It is impossible to solve uniquely for any of these parameters other than the observed $(X_1 Y_1)$ by manipulating these equations, unless constraints are imposed on the solutions.

Several options for imposing such constraints are available. The most commonly employed of these is to assume that one of the coefficients in each pair (b_2, b_3) and (d_2, d_3) is zero or some other known value. A second option is to fix the ratios of two sets of parameters, such as b_2/b_3 and d_2/d_3 or b_2/d_2 and b_3/d_3 . When appropriate control variables are available, a third option is to use instrumental variable techniques to estimate the seven coefficients (three b_i , three d_i , and uv). This method requires that one make the assumptions described above about the effects of at least one of the instruments on each of the endogenous variables X_2 and Y_2 .

There are times when none of these approaches can be taken. Substantive considerations may dictate that all lagged and instantaneous cross-effects be kept free in the estimation, precluding the first and second options. Yet it might be impossible

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to find a theoretically defensible instrument, excluding the third option as well.

In this article we review and examine critically a fourth option for achieving identification proposed by Greenberg et al (1979).¹ The method differs from more conventional strategies in two ways. First, it requires observations for at least three time points. Second, it achieves identification by making assumptions (1) about the consistency of parameter values rather than about the fixed values of any individual coefficients or else (2) about the ratios of different coefficients. Since this consistency assumption is usually plausible and is in some ways a weaker assumption than those required in the other methods, identification by means of this approach is quite attractive.

IDENTIFICATION BY MEANS OF CONSISTENCY CONSTRAINTS

To illustrate the consistency approach, we consider a three-wave generalization of the model defined by Equations 1a and 1b. In this generalization, shown diagrammatically in Figure 1, X_1 and Y_1 are assumed to influence time 3 variables only through the intervening variables X_2 and Y_2 .² With this assumption, the structural equations are

$$X_2 = b_1X_1 + b_2Y_1 + b_3Y_2 + u_2 \quad [2a]$$

$$X_3 = b_4X_2 + b_5Y_2 + b_6Y_3 + u_3 \quad [2b]$$

$$Y_2 = d_1Y_1 + d_2X_1 + d_3X_2 + v_2 \quad [2c]$$

$$Y_3 = d_4Y_2 + d_5X_2 + d_6X_3 + v_3. \quad [2d]$$

The correlation between time 1 variables is taken into account but not subjected to causal analysis. The error terms u and v are permitted to be correlated cross-sectionally, but not serially.

Concentrating on the equations for X_2 and X_3 , we note that there are six regression coefficients to be estimated. By taking

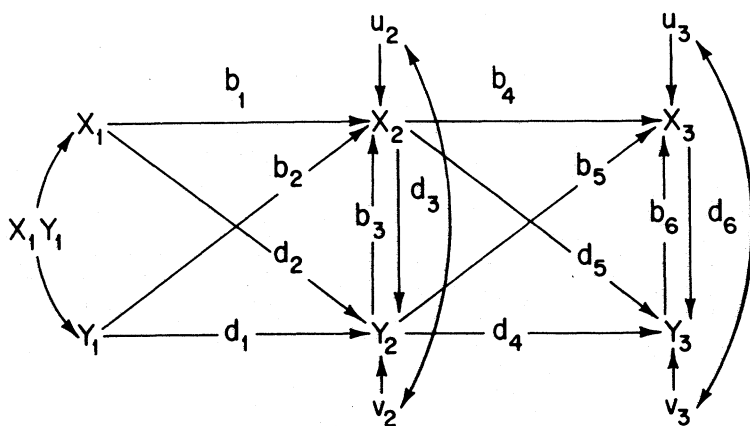


Figure 1

covariances of Equation 2a with X_1 and then with Y_1 , we obtain two normal equations that can be used in the estimation of these parameters. By repeating the procedure for Equation 2b using X_2 and Y_2 , we obtain two additional normal equations.

We cannot similarly use Y_3 to identify Equation 2b because the reciprocal influence between X_3 and Y_3 implies that Y_3 cannot have a vanishing correlation with u_3 .³ For the same reason, neither Y_2 nor Y_3 can be used to help identify Equation 2a.

Seemingly, we should be able to derive an additional two normal equations by taking the covariances of Equation 2b with X_1 and Y_1 , as these variables are not correlated with u_3 . Indeed, one can do this, and the equations so obtained are valid. The trouble is that they contain no new information beyond what is already contained in the other four normal equations. To see why this is so, note that Equations 2a and 2c can be solved to yield "reduced-form" equations that express X_2 and Y_2 in terms of X_1 , Y_1 , and the disturbance terms u_2 and v_2 . It follows that the normal equations derived by using X_1 and Y_1 as instruments will be linear combinations of those derived by using X_2 and Y_2 .

With six parameters and four normal equations that can be used to estimate them, two additional pieces of information are

needed if a unique solution is to be found. In this respect, we are in the same position as when we considered the two-wave, two-variable model. However, we now have an option for identifying the model that we did not have in the two-wave case. Instead of assuming that certain parameters have known numerical values, we can assume that certain effects remain *constant* over the period between the first and third waves, a fairly weak assumption.⁴ In the notation of the model, any two of the following three consistency conditions

$$b_1 = b_4, \quad b_2 = b_5, \quad b_3 = b_6 \quad [3]$$

will reduce the number of independent parameters to be estimated sufficiently to allow the remaining parameters to be identified. If all three conditions are imposed simultaneously, the model will be overidentified.

To see that the imposition of two of these constraints does indeed enable one to obtain unique solutions for the coefficients, suppose that $b_2 = b_5$ and $b_3 = b_6$. The four normal equations become

$$(X_1 X_2) = b_1 + b_2(X_1 Y_1) + b_3(X_1 Y_2) \quad [4a]$$

$$(Y_1 X_2) = b_1(X_1 Y_1) + b_2 + b_3(Y_1 Y_2) \quad [4b]$$

$$(X_2 X_3) = b_2(X_2 Y_2) + b_3(X_2 Y_3) + b_4 \quad [4c]$$

$$(Y_2 X_3) = b_2 + b_3(Y_2 Y_3) + b_4(X_2 Y_2). \quad [4d]$$

Solutions for the parameters b_1 , b_2 , b_3 , and b_4 take the form

$$b_j = \det(B_j) / \det(B), \quad [5]$$

where B is the coefficient matrix of the four normal equations [4a-4d], B_j is the matrix derived from B by replacing the j^{th} column of B with the column of moments $[(X_1 X_2), (Y_1 X_2), (X_2 X_3), (Y_2 X_3)]$, and \det symbolizes the determinant. The solutions given by Equation 5 are unique as long as $\det(B)$ is nonzero.

*IDENTIFICATION PROBLEMS OF THE
CONSISTENCY APPROACH*

By direct computation of the determinants in Equation 5 we find for the model under consideration,

$$\begin{aligned} \det(B) = & [(X_2 X_3) - (X_1 X_2)] + [(X_1 Y_1)(X_2 Y_1) \\ & - (X_2 Y_2)(X_3 Y_2)] + [(X_1 X_2)(X_2 Y_2)^2 \\ & - (X_2 X_3)(X_1 Y_1)^2] + [(X_2 Y_2)(X_3 Y_2)(X_1 Y_1)^2 \\ & - (X_1 Y_1)(X_2 Y_1)(X_2 Y_2)^2] \end{aligned} \quad [6]$$

and

$$\begin{aligned} \det(B_1) = & [(Y_1 Y_2)(Y_2 X_3) - (Y_2 Y_3)(Y_1 X_2)] \\ & + (X_1 X_2) [(X_1 Y_1)(Y_2 Y_3) - (X_2 Y_2)(Y_1 Y_2)] \\ & + (X_1 Y_2) [(Y_1 X_2)(X_2 Y_2) - (Y_2 X_3)(X_1 Y_1)] \\ & + (Y_1 Y_2) [(X_2 X_3) - (X_1 X_2)] \\ & + (X_1 Y_1) [(X_1 X_2)(X_2 Y_3) - (X_2 X_3)(X_1 Y_2)] \\ & + (Y_1 X_2) [(X_1 Y_2) - (X_2 Y_3)]. \end{aligned} \quad [7]$$

The other three $\det(B_j)$ have the same general form as Equation 7.

These five determinants are made up of sums of terms, each of which is a difference of covariances or products of covariances that vary in time but not in time lag. It follows that all the determinants will vanish when the system described by these equations is in equilibrium. By equilibrium, we mean that the observed moments do not depend on time, only on the lag between variables. At equilibrium then, $(X_1 X_2) = (X_2 X_3) = (X_t X_{t+1})$, $(X_1 Y_2) = (X_2 Y_3) = (X_t Y_{t+1})$, and so on.⁵ Systems of causal relations are not always in equilibrium, but when they are, Equation 5 is not

defined, and the consistency approach breaks down. Unique solutions for Equations 4a-4d cannot be obtained, and as a result, Equations 2a-2d are underidentified.

Since the consistency approach depends critically on whether the system is in equilibrium, it is important to know two things: first, under what conditions equilibrium will be reached; and, second, how the parameter estimates behave as the system approaches equilibrium.

APPROACH TO EQUILIBRIUM

The likelihood of finding a system that meets these requirements is a function of two factors: (1) the frequency with which new sets of structural relations are known to take hold of this system in the real world, shifting it from an old equilibrium or an old pattern of change to a new pattern (which may or may not be approaching equilibrium) and (2) the speed with which this new set of structural relations approaches equilibrium.

The first of these influences will, of course, vary from one substantive situation to another. It is possible to make some general statements, though, about the speed with which equilibrium is approached, for systems that *do* approach equilibrium. Consider the structural equations

$$X_t = b_1 X_{t-1} + b_2 Y_{t-1} + b_3 Y_t + u_t,$$

$$Y_t = d_1 Y_{t-1} + d_2 X_{t-1} + d_3 X_t + v_t,$$

and for the sake of simplicity, assume that u_t and v_t are uncorrelated. It is a tedious but straightforward exercise in algebra to show that⁶

$$\begin{aligned} (X_t Y_t) = & [(b_3 + d_3 + b_1 d_2 + b_2 d_1) \\ & + (b_1 d_1 + b_2 d_2) (X_{t-1} Y_{t-1})] / (1 + b_3 d_3). \end{aligned}$$

This equation has the general form

$$P_t = h + gP_{t-1}. \quad [8]$$

The most general solution of this recursion formula is (Goldberg, 1958: 63-67):

$$\begin{aligned} P_t &= h/(1 - g) + [P_0 - h/(1 - g)]g^t & (g \neq 1) \\ &= P_0 + ht. & (g = 1) \end{aligned}$$

When the parameter

$$g = (b_1d_1 + b_2d_2)/(1 + b_3d_3) \quad [9]$$

is greater than one in magnitude, or equal to one, the solution does not approach equilibrium, but explodes as t increases without limit.⁷ Thus the solution converges toward the equilibrium value

$$(X_\infty Y_\infty) = (b_3 + d_3 + b_1d_2 + b_2d_1)/(1 + b_3d_3 - b_1d_1 - b_2d_2)$$

only if the absolute value of g is strictly less than 1.

It follows directly from the recursion relation from $P_t = (X_t Y_t)$ that

$$\Delta(X_t Y_t) = g\Delta(X_{t-1} Y_{t-1}). \quad [10]$$

We can see from this expression just how g affects the approach to equilibrium. When g is small, each increment will be much smaller than the preceding increment, and the system will approach equilibrium rapidly, while if g is close to but less than 1, the system will approach equilibrium slowly. It follows from Equation 9 that the approach to equilibrium will be slow when the product of the stability coefficients b_1d_1 is large, when the cross-lagged effects of b_2 and d_2 are the same sign and large, and when the cross-instantaneous effects b_3 and d_3 are the

opposite sign and large. Approach to equilibrium will be fastest when the product of the stability coefficients is small, when the cross-lagged effects are the opposite sign and large, and when the cross-instantaneous effects are the same sign and large.

By iterating Equation 10, we see that $\Delta(X_t Y_t) = g^t \Delta(X_1 Y_1)$. To get an idea of what this means for the approach to equilibrium, we consider two models. In the first, the parameters are $b_1 = .3$, $b_2 = .2$, $b_3 = -.2$, $d_1 = .3$, $d_2 = -.2$, and $d_3 = .2$; we assume that the initial cross-sectional correlation is $(X_1 X_1) = -.9$. For this model, g is approximately .05, and it is apparent that after one time unit, $(X_t Y_t)$ will be very close to its equilibrium value of 0. In the second model, $b_1 = .9$, $b_2 = .1$, $b_3 = -.1$, $d_1 = .9$, $d_2 = .1$, and $d_3 = .1$, and we again assume the initial cross-sectional correlation to be $-.9$. Here g is approximately .81, and the approach to equilibrium will be slow; after 10 time units have elapsed, the cross-sectional correlation will be .717, still some distance from the equilibrium correlation .947. More generally, the time T in which the time-dependent term in the expression for P_t in Equation 8 is reduced in magnitude to a given fraction of its value at time 0 is proportional to the reciprocal of the natural logarithm of g .

It can be shown without difficulty that the parameter g , which governs the approach of the correlation $(X_t Y_t)$ to equilibrium, also governs the approach to equilibrium of the correlations $(X_t X_{t-1})$, $(Y_t Y_{t-1})$, $(X_t Y_{t-1})$, and $(Y_t X_{t-1})$; thus the results summarized above for the correlation $(X_t Y_{t-1})$ hold equally well for the entire system of equations.

BEHAVIOR OF THE PARAMETER ESTIMATES AS EQUILIBRIUM IS APPROACHED

We have demonstrated so far that Equation 5 yields unbiased estimates of b_i and d_j when the matrix B is not in equilibrium. When equilibrium is reached, though, the model parameters are not identified. But what about the transition? Intuitively, we know that the difference terms making up the estimator approach

zero as the system approaches equilibrium. Yet this does not occur abruptly. As the system comes closer and closer to equilibrium the parameter estimates become more and more inefficient.

To ground this intuitive reasoning, we carried out a series of simulations. In each simulation we began with a "true" structural model, generated the observed correlation matrix among the variables in the model over a very long period of time,⁸ and then attempted to recover the underlying structure from this matrix at different points as the system approached equilibrium.

Table 1 shows the parameters for 36 simulated systems. Four sets of stability coefficients (b_1 , d_1) were used. For each of these, nine sets of cross-coefficients (b_2 , b_3 , d_2 , d_3) were used. Together, these cover a range of stabilities from .3 to .7 and a range of cross-coefficients from -.4 to .4.

Beginning with the arbitrary assumption that $(X_1X_1) = -.9$, three-wave correlation matrices for times 1-3, 2-4, 3-5, 4-6, and 5-7 were analyzed for each of the 36 models. The LISREL IV program (Jöreskog and Sörbom, 1977) was used to impose the consistency conditions given in Equation 3 and to obtain maximum-likelihood estimates of the underlying model parameters and their standard errors.

The LISREL solutions clearly demonstrated two results anticipated from analytic investigation: (1) that the estimator will reproduce the underlying parameters without bias as long as the system is not extremely close to equilibrium and (2) that the standard errors of the parameter estimates increase dramatically as we move from the times 1-3 matrix to the equilibrium matrix.

Table 2 illustrates these features with a detailed example from one of the simulated models. The value of g in this model is approximately .11. The first row presents the parameters of the underlying model. The remaining rows display the parameter estimates and their standard errors for each successive three-wave correlation matrix. (Standard errors were computed on the assumption that $n = 1000$.) The final column contains the value of the exogenous correlation (X_tY_t), with $t = 1, 2, 3, 4, 5$.

TABLE 1
Values of the Stability and Cross-Effect Coefficients Used in the
Thirty-Six (four stabilities by nine cross-effects) Simulations

<u>Stabilities</u>	b_1	d_1		
1	.3	.3		
2	.5	.5		
3	.7	.7		
4	.3	.7		
<u>Cross-Effects</u>	b_2	b_3	d_2	d_3
1	.1	.1	.1	.1
2	.1	-.1	-.1	.1
3	.1	.1	.1	-.1
4	.2	.2	.2	.2
5	.2	-.2	-.2	.2
6	.2	.2	.2	-.2
7	.4	.4	.4	.4
8	.4	-.4	-.4	.4
9	.4	.4	.4	-.4

We see that the true score parameters are reproduced exactly for this model in the times 1-3 and 2-4 matrices. However, in subsequent iterations, as the correlation ($X_t Y_t$) gets close to its equilibrium value, the estimates become wildly discrepant from

TABLE 2
Detailed Results of Applying the Consistency Approach
to the First Five Iterations of a Simulated Matrix¹

	b_1	b_2	b_3	d_1	d_2	d_3	$x_t y_t$
True Score Parameters	.5	.4	-.4	.5	-.4	.4	
Parameter Estimates Times 1-3	.500	.400	-.400	.500	-.400	.400	.900
Standard Errors	.028	.057	.092	.023	.040	.059	.004
Parameter Estimates Times 2-4	.500	.400	-.400	.500	-.400	.400	-.096
Standard Errors	.128	.419	.735	.123	.400	.703	.037
Parameter Estimates Times 3-5	.520	.335	-.287	.519	-.336	.287	-.010
Standard Errors	x	x	x	x	x	x	x
Parameter Estimates Times 4-6	.620	.004	.296	.620	-.004	-.296	.007
Standard Errors	x	x	x	x	x	x	x
Parameter Estimates Times 5-7	.620	.004	.296	.620	-.004	-.296	.000
Standard Errors	x	x	x	x	x	x	x

1. The model is $(X_1 Y_1) = -.9$, $b_1 = .5$, $b_2 = .4$, $b_3 = -.4$, $d_1 = .5$, $d_2 = -.4$, $d_3 = .4$.
 x indicates that standard error exceeds 1.0.

the true values. This happens even though the model is formally identified in all five iterations.

We also see that the standard errors increase substantially from the first to second iteration, even though the parameter estimates themselves remain constant. The increase is particularly dramatic for the cross-coefficients, where standard errors change from between .057 and .092 in the first iteration to between .400 and .735 in the second. While all standard errors are less than half their associated parameters in the first iteration, they all exceed their parameters by the second iteration. By the third iteration, all standard errors exceed 1.0.

The results for the other 35 models considered are very much like those for this illustrative case. In most cases, the parameter estimates perfectly reproduce the true parameter values initially, but deteriorate as equilibrium is approached. Variations among the models appear primarily in the speed with which the deterioration occurs.

TABLE 3
Number of Iterations (from a maximum of five) for Which All
Model Parameters are at Least Twice Their Standard Errors*

Stabilities				
	$b_1=.3$ $d_1=.3$	$b_1=.5$ $d_1=.5$	$b_1=.7$ $d_1=.7$	$b_1=.3$ $d_1=.7$
Cross Effects				
Magnitude = .1				
$b_2 \ b_3 \ d_2 \ d_3$				
+ + + +	0	0	0	0
+ - - +	0	0	0	0
+ + + -	0	0	0	0
Magnitude = .2				
$b_2 \ b_3 \ d_2 \ d_3$				
+ + + +	0	0	0**	2**
+ - - +	0	0	0	0
+ + + -	0	0	2	0
Magnitude = .4				
$b_2 \ b_3 \ d_2 \ d_3$				
+ + + +	0**	0**	0**	0**
+ - - +	1	1	1	0
+ + + -	1	2	3	3

*Standard errors are based on the assumption that $n = 1000$.

**Indicates that the model exploded (that is, it was not approaching equilibrium) during the interactive calculation of the correlation matrix.

Table 3 presents summary results for all 36 models, recording the number of iterations for which standard errors of all parameters are less than half the parameter values.⁹ These results are discouraging. They suggest that except for systems that approach equilibrium only very slowly, and that have moderately large cross-effects, parameter estimates will be too imprecise for the consistency approach to be of practical value. This will be true even when observations are collected immediately after an external shock has placed the system very far from equilibrium. In less optimal conditions, the situation would be even worse.¹⁰

Only where the stability coefficients are quite high, then, will the consistency approach be practically useful. And even here its usefulness will be limited to times that are fairly close to the time when the new set of structural equations first took hold. For most problems it will not be possible to obtain observations that meet this criterion, and identification via this approach will not be possible.

CONCLUSION

In practice, we have no way short of estimating the consistency model to determine if the observed data are too close to equilibrium, or moving too rapidly toward equilibrium, for the approach to be used.¹¹ The simulations show quite clearly, though, that the approach can be used in practice only under a very restricted set of circumstances. Consequently, the researcher who believes that it is necessary to distinguish empirically between lagged and instantaneous cross-coefficients in a panel model¹² should, whenever possible, consider the availability of theoretically justifiable instrumental variables. When these are available, it is possible to identify all three b_i 's and all three d_j 's even when the system is at equilibrium. Furthermore, the standard errors of these estimates will almost certainly be lower than those parameters estimated via the consistency approach, even when the latter can be used.

NOTES

1. Land and Felson (1978) have described a method for "partial identification" which is not reviewed here. In this approach, bounds are placed on the parameter estimates by imposing theoretically generated inequality constraints on the parameters.

2. When this assumption cannot be made, two other approaches are possible: First, if X_1 is assumed to influence both X_3 and Y_3 , the consistency assumptions obviously will not hold over the two time intervals 1-2 and 2-3. However, they would still hold over the intervals 2-3 and 3-4, controlling X_1 and X_2 in the respective prediction equations. Therefore, the same approach can be used in a case of this sort, but more than three waves of data are required. Second, if X_1 influences X_3 or Y_3 , but not both, the model can be identified by using X_1 as an instrumental variable.

3. Note that in Equation 2d, Y_3 contains a term in X_3 . If the expression for X_3 given in Equation 2b is substituted into Equation 2d, it will be found that Y_3 is proportional to u_3 . Hence it cannot be uncorrelated with u_3 .

4. In fact, any relationship that determined some parameters in terms of others would reduce the number of parameters to be estimated. Our discussion will be limited to this case, which is the simplest and easiest for purposes of computation given the limitations of existing computer programs; however, the generalization poses no conceptual difficulties.

5. In some of the literature, a system is defined to be in equilibrium only if each *variable* remains at a constant value or is stable under small displacements. This is a more stringent definition: Some systems that are in equilibrium under our definition will not be stable under this second definition. For further discussion, see Goldberg (1958: 169-184).

6. The derivation of this equation is given in a technical appendix available from the authors on request. A more general discussion of equilibrium time can be found in Heise (1975: 227-231).

7. Some structural systems have parameters such that covariances do not eventually stabilize at finite equilibrium values. Instead, they oscillate without any damping or explode. Oscillation without damping occurs when $g = -1$. Here each increment ΔX_t is followed by an increment ΔX_{t+1} of equal magnitude but opposite sign. Explosion, corresponding to values of g that are greater than or equal to 1, or strictly less than -1, implies values of ΔX_t that increase in magnitude without any finite limit. A situation of this kind cannot prevail indefinitely. When it is encountered, one may infer that the equations are being extrapolated beyond their range of validity. Either the correct equation contains nonlinear terms that prevent covariances from growing geometrically or the parameters themselves do not remain constant over time. Rather, the system responds to a variable or a relationship that has increased beyond a certain point by changing the structural parameters in a manner that reduces the magnitude of the variable or relationship.

8. The recursion relations given in the technical appendix (see Note 6) were used to generate sets of three-wave correlation matrices for the variables $X_t, Y_t, X_{t+1}, Y_{t+1}, X_{t+2}, Y_{t+2}$ ($t = 1, 2, 3, \dots$). We continued generating matrices for a particular model until correlations in the 4×4 submatrix $X_t, Y_t, X_{t+1}, Y_{t+1}$ were identical to three decimal places to the corresponding elements in the 4×4 submatrix $X_{t+1}, Y_{t+1}, X_{t+2}, Y_{t+2}$. If more than five three-wave matrices were generated by this procedure for a particular model, we submitted only the first five and the final matrices to analysis.

9. Detailed results of these simulations are available from the authors on request.
10. Even more discouraging is the fact that these results ignore measurement error. In more realistic applications, the difference terms in Equation 5 will become dominated by error as the true score covariances approach equilibrium.
11. Lest our finding that structural equations typically reach equilibrium very rapidly be taken as an endorsement of static analyses or equilibrium models of society, we point out that the conclusion is a theoretical one, not an empirical one. It is based on the premises that (1) the system of variables under study is isolated from external shocks and (2) the system approaches equilibrium. Neither of these assumptions is necessarily valid empirically.
12. The importance of distinguishing instantaneous from lagged cross-coefficients would seem to be greatest in situations where the signs of the two effects are opposite, for in this case the two terms will tend to sum to zero. However, it can be important to distinguish the two effects even when they are of the same sign. Suppose that an instantaneous effect is present but a lagged effect is not, and the estimation is carried out on the assumption that the cross-effect is lagged but not instantaneous. It can be shown (Greenberg and Kessler, 1981) that in this circumstance, the estimate of the cross-coefficient can have the wrong sign. One can determine quickly whether this possibility need be of concern by carrying out the estimation on the assumption that only lagged effects are present, and again on the assumption that only instantaneous effects are present. If the signs of the estimates are the same, the sign will be unbiased under either assumption, though the magnitude of the parameter may be in error in the model that is misspecified. If the signs of the estimates are opposite, one will be wrong. In the absence of a priori information about the correct lag, a multiwave model will have to be considered to avoid the risk of estimating a cross-coefficient with the wrong sign.

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