THREE- AND FOUR-PERSON GAMES

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Game theory concerns certain formalized models of conflicts of interest. Each set of interests is ascribed to an actor called a player. The player acts by choosing among a set of available alternative decisions. The alternatives available at a given choice point depend on the situation arising in the course of the process (called a "play of a game"). These situations, in turn, are defined by the rules of the game and by the choices made by the several players up to that phase of the game. Certain situations are defined as terminal. When they occur, a play of the game is over. Depending on the particular situation that ends the game (called an "outcome"), certain positive or negative payoffs are awarded to each of the players.

GAME THEORY

In the theory of games the players are assumed to be "rational." As used here, the word has a strictly technical meaning. A rational player is defined as one who makes his choices so as to maximize his payoff (at times, the statistically expected payoff) in the outcome of the game. Moreover, in making his decisions, the rational player takes into account all the information that the rules of the game make possible for him to have. Also he assumes that all of the other players are likewise rational; that is, are governed by the same sort of considerations. It is important to keep in mind that according to this definition a player is neither benevolent nor malevolent vis-à-vis the other players. That is to say, he tries to maximize only his own payoff without regard for the payoffs of others, except to the extent that the projected payoffs of others give him information as to how others are likely to play. The rational player is neither gratified nor peeved by the winnings or losses of other players.

In games with two players (two-person games), the interests of the two may or may not be diametrically opposed. They are so opposed if the sum of the payoffs to the two players is the same whatever be the outcome of the game; for, in that case, the more one player wins, the less the other can win (or the more he must lose). Such games are called constant-sum. In games that are not constant-sum, the interests of the two players may not be diametrically opposed, because some of the outcomes may be preferred to other outcomes by *both* players. In the extreme case where the two players have the same preference order for the outcomes, there is actually no conflict of interest. Consequently, such games are of no theoretical interest in game theory. In general, however, the interests of two players in a nonconstant-sum game will be *partially* opposed. This will be so if some outcomes are preferred to others by both players, but the *preference order* for some outcomes is opposite.

The existence of coincident interests (preference by both for some outcomes over others) may induce the two players to form a coalition; that is, to act jointly in order to ensure an outcome of the game that is preferred by both to other outcomes. The question of how to decide among *these* preferred outcomes (where the interests of the players may be in conflict) is one of the central questions in the theory of two-person nonconstant-sum games where the rules allow the formation of a coalition.

The assumption of rationality suggests that, if joint action can serve the interests of both players, they will form a coalition if given the opportunity to do so. As an example, consider the "game" between a prospective buyer and the seller of a house. The possible outcomes of this game are the various amounts of money for which the house can be sold. In addition, there is one other outcome: no sale. Clearly, any outcome with a larger sale price is preferred by the seller, and one with a smaller price is preferred by the buyer. Thus, the interests of the two players are opposed with regard to the outcomes representing the transaction. Nevertheless, both the buyer and the seller may prefer every sales price in a certain range to the "no sale" outcome. In this context, the realization of the sale is a consequence of a "coalition" between the buyer and the seller, entered into so as to prevent the outcome not preferred by either (no sale).

In two-person games there is only one coalition that may or may not occur, namely the coalition between the two players. In games with more than two players, called n-person games, the number of possibilities is, of course, larger. For instance, in a game with three players, the following partitions of the players into coalitions may occur:

(a) every man for himself;[1] (b) players 1 and 2 in coalition against 3; (c) players 1 and 3 against 2; (d) players 2 and 3 against 1; (e) the Grand Coalition including all three players.

As the number of players increases, the number of ways of partitioning them into coalitions increases much more rapidly. The reader can verify that there are eighteen possible partitions of four players (including "every man for himself" and the Grand Coalition). With five players, forty-two different partitions are possible.

It seems, therefore, that two separate questions arise in connection with games with more than two players. The first question is analogous to the question regarding the "solution" of the two-person game: If each of the n players of a given game is rational, what will be the final distribution of payoffs among them? The second question, specifically relevant to the n-person game, is: How will the n rational players in a given n-person game organize themselves into coalitions, assuming that the rules of the game allow coalitions? Finally, the two questions can be combined into a single one: Given a particular coalition structure, how will the payoffs accruing to each coalition be apportioned among its members?

Note that the "apportionment of payoffs" makes sense only if the payoffs are in a transferable commodity, such as money. This need not, of course, always be the case. Conflicts of interest can occur when payoffs are not transferable; for instance, when prizes are power, competitive status, objects with personal sentimental value, and so forth. N-person game theory can be extended also to such situations, providing the payoffs can be defined as quantities (utilities) with regard to individual players. However, for the most part it is assumed in the theory of games with more than two players that the payoffs are in a transferable commodity, so that a coalition may play with the view of maximizing the sum of the payoffs to its members and later may face the problem of apportioning the total among themselves.

The question of the joint payoffs accruing to coalitions can be answered unambiguously if the n-person game is constant-sum and if the n players partition themselves into just two coalitions. The definition of the constant-sum n-person game is analogous to that of the two-person constant-sum game. Regardless of how a coalition and a countercoalition form, the sum of the payoffs to the two coalitions remains the same. It follows that, if the players are in two coalitions, the resulting game is essentially a two-person constant-sum game, and its known solution answers the question of how the total payoff is to be apportioned between the two coalitions.[2] Put in another way, we can assert the following: Whatever coalition of k of the n players forms, it can assure for itself some minimum payoff. This is so even if the game is not constant-sum, for the coalition can always play in such a way as to expect the "worst," namely that the remaining players in a countercoalition will attempt to minimize the joint payoff of the first coalition. This minimum guaranteed payoff is the value of the game to the coalition that can be sure to get it.

Much of n-person game theory is concerned with matters that depend only on the respective values of the game to each of its possible coalitions. Consequently, an n-person game can be *defined* by listing the value to each of its coalitions. This listing is called the *characteristic function* of the game. To make the idea clear, consider the following extremely simple game.

Players A, B, and C are to divide a dollar among them. The rules specify that the dollar is divided by majority vote. Clearly, a coalition of any two players can appropriate the dollar. Our questions pertaining to this game are the following:

(1) Which coalitions will form?

(2) How much will the coalitions [that form] get?

(3) How will the dollar finally be apportioned among A, B, and C?

Let us consider all possible subsets of the set (A, B, C). These are (A), (B), (C), (A, B), (A, C), (B, C), (A, B, C), and (ϕ). The last symbol represents the "empty" subset, one without members. It is included to make our arguments complete. In general, a set containing n members contains 2^n subsets, including the empty subset and the "grand subset" (itself).

Now, it is clear that no coalition of a single player can be *sure* of getting anything more than zero, since, if the other two players form a coalition, they can get the whole dollar by awarding it to themselves by majority vote. We symbolize this by writing

$$v(A) = 0; v(B) = 0; v(C) = 0. \qquad [1]$$

Here the symbol v( ) stands for the value of the game to the coalition in parentheses.

Next, we note that any coalition of two players can assure itself a payoff of one dollar. We write accordingly,

$$v(AB) = 1; v(AC) = 1; v(BC) = 1. \qquad [2]$$

Finally, all three players already have the dollar in their possession, and, of course, the coalition (ϕ), devoid of members, gets no payoff. In symbols,

$$v(ABC) = 1; v(\phi) = 0. \qquad [3]$$

Equations (1), (2), and (3) constitute the *characteristic function* of this game.

Recall that a coalition implies a joint action of its members. It stands to reason that a coalition of rational players can assure itself at least as large a total payoff as the sum of the payoffs that its members can assure for themselves individually; that is, everyone playing against all others. Clearly, the members of the coalition can do everything that the individual members can do separately. And, in addition, they may be able to do better by properly coordinating their decisions. Accordingly, if S is a set of players in coalition, and T is another set, and if the two sets have no members in common, we can write

$$v(S \cup T) \geqslant v(S) + v(T), \qquad [4]$$

where $(S \cup T)$ denotes the set in which the members of S and of T have combined.

Next, we can without loss of generality assume that in any game every player can assure for himself at least 0. For if, in an actual game, the assured payoff of some player is negative, we can simply pay him something for playing the game. If the amount we pay him does not depend on how he plays the game, his decisions in the course of the game should not be affected by that "bonus." In this way, by compensating every player (in advance) if the value of the game to him is negative, and by exacting a fee from every player for whom the value of the game is positive, we can make $v(i) = 0$ for every player i without changing the nature of the game itself. This is done merely in the interest of simplifying the analysis. In fact, the bonuses can be retracted and the fees refunded, again without affecting the strategic considerations of the game.

From what has been said, we can assume the following inequalities in any characteristic function of a game:

$$\begin{aligned} &v(i) = 0 \quad (i = 1, 2 \ldots n) \\ &\text{If } S \subset T, \text{ then } v(S) \leqslant v(T), \end{aligned} \qquad [5]$$

where $S \subset T$ indicates that all members of the subset S are also members of subset T.

In other words, as new members join a coalition, they cannot make smaller the *total* payoff that the coalition can guarantee itself. However, this does not mean that each new member necessarily makes possible a larger *per capita* share of the total payoff. Thus, a new member will not necessarily be "welcomed" into any coalition; he might be an "extra mouth to feed," as it were.

In particular, suppose we are dealing with a constant-sum game "normalized" so that the value of the game is zero to each individual player. Then the value to any n - 1 players must be the same as the value to all n players (the Grand Coalition). We can expect that in such a game at least one player will be left out of a coalition; in other words, that the Grand Coalition will not form. Returning to our divide-the-dollar game (a constant-sum game), we can expect that if all players are rational, two will join against the third and appropriate the whole dollar.

Now the question arises as to which of the players will be left out, and also how the dollar will be divided by the two in the coalition. In real-life situations similar to the one depicted, various factors may decide the outcome. One of the three players may be "disliked" by the other two, or the two in coalition may have been attracted to each other. Real-life coalitions may be guided by political, ideological, or purely personal considerations extraneous to the actual game. Such considerations fall outside the scope of game theory unless the payoffs associated with the factors in question can be specified, in which case a game considerably more complex than the original one may result. In all cases, in game theory the players are assumed to be "psychologically equal"; none is a better bargainer or a more astute strategist than another. The outcome of the game is supposed to be a result of perfectly rational decisions by perfectly rational players whose rationality has already been defined in terms of utilizing all available information with a view of maximizing one's own payoff. Under these circumstances, one must conclude that it is not possible to single out any of the three players in a constant-sum three-person game as the one who gets left out. All we can say is that some one of the three is likely to be in that position.

It is tempting to conclude also that the two players in coalition will divide the dollar equally, again because of the complete symmetry of the situation. Nevertheless, if we analyze the bargaining process more closely, we shall see that the matter is not quite that simple.

Suppose that two of the players, A and B, have decided to exclude C and to divide the dollar equally between them. However, the deal has not yet been closed, and C has the opportunity to make offers either to A or B. In fact, C can make a very attractive offer to A. He can promise him $.70 of the dollar if A forsakes B and joins with C. Note that both A and C stand to gain in this arrangement. A will get $.70 instead of $.50, which he gets as B's partner. C will get $.30 instead of nothing. Should A accept C's offer? Attractive as it is, acceptance is fraught with danger. For, if A accepts, B, who now stands to get nothing, can offer C, say, $.50 if he leaves A to join with B. We see that both B and C stand to gain in this new arrangement; moreover, they can enforce it, being a majority. A question now arises: is there a division of the dollar which is somehow "stable" in the sense that disturbances of this arrangement will not benefit any potential new coalition that can effect a new arrangement beneficial to itself? The theory of n-person games give one unequivocal answer to this question. If the game is constant-sum—that is, if the sum of the values to any two coalitions comprising all of the players is the same—then *no* apportionment of the payoff accruing to the Grand Coalition can satisfy all potential coalitions, and, moreover, there will always be some disgruntled set of players that can effect another arrangement more satisfactory to itself by forming a coalition, thus breaking up the Grand Coalition.

It stands to reason, however, that in an actual situation that can be depicted by a constant-sum game (like, say, the divide-the-dollar game), rational players will *somehow* apportion the total payoff that they can jointly get. This raises the question of whether there might not be some rational principle of apportionment that by-passes the ability of disgruntled players to get their way.

A number of such principles have been suggested. Before examining them, we must point out that the above-mentioned fundamental instability of the n-person constant-sum game does not characterize *all* n-person games. Some non-constant-sum games have *cores.* In these games apportionments can be singled out that can satisfy all potential coalitions in the sense that the members of any potential coalition cannot improve their joint payoff by breaking away from the Grand Coalition. The proposed principles of apportionment, however, do not distinguish between constant-sum and nonconstant-sum games. One such principle is the so-called Shapley Value of an n-person game.

SOME PRINCIPLES

SHAPLEY VALUE

Imagine the Grand Coalition being built up one by one, as new members are recruited into it. Recall that an n-person game can be so formulated that, as a new member joins a coalition, he will certainly not make the value of the game to the new coalition smaller, and possibly he will make it larger. That is to say, each new member adds some non-negative increment to the value of the game to the coalition that he joins. Therefore, as a player joins a coalition, he can demand a portion of the eventual total payoff *up to* the amount of the increment that he has brought in by joining. This is not to say that he can get it, but he can reasonably demand it. Now, what a player contributes to the coalition he joins depends on which coalition he joins; that is, on the order in which the players build up the Grand Coalition. Assume that all possible orders are equally likely. If there are n players, there are $n! = 1 \times 2 \times 2 \times \ldots n$ possible orders in which the Grand Coalition can be built up. Averaging a player's contributions over the n! ways, we obtain the statistical expectation of his payoff. The set of these expectations (whose sum can be shown to be equal to the value of the game to the Grand Coalition) is the Shapley Value of the game.

As an example, consider a somewhat modified divide-the-dollar game. Players A, B, and C are now stockholders. A controls fifty percent of the stock, B, forty-nine percent, and C, one percent. They are to divide a dollar dividend. According to conventional rules governing the apportionment of dividends, A should get \$.50, B, \$.49, and C, \$.01. If, however, the decision on how to divide the stock is adopted by majority vote, the situation becomes a three-person game. Let us see what the Shapley Value of this game turns out to be. There are six ways in which the Grand Coalition can form; namely (in the order of the joining players):

ABC, BCA, CAB, CBA, BAC, ACB.

No single player has a majority. However, coalitions (AB) and (AC) do form majorities, since jointly each pair controls more than fifty percent of the stock. Consider the player who "swings" the majority; that is, turns the coalition he joins into a majority coalition (he is called the *pivot*). In four cases out of six, this player is A. B is the pivot in one case, and C in one case. The Shapley Value of this game therefore awards \$.66 2/3 to A, \$.16 2/3 to B, and \$.16 2/3 to C. The apportionment does not reflect the proportions of the stock owned by the three, but it does reflect the "real" relative voting power.

If each of the three has one vote, then, of course, any pair would be a majority. Then each player could have an equal chance of being the pivot and so would receive an equal share. It should be emphasized, however, that the equal division of the dollar is not a "stable" solution of the game in the sense of satisfying every potential coalition. As long as coalitions can form, there is nothing to prevent any pair of players from joining together and freezing the third one out. Thus, the Shapley Value is not due to an "equilibrium of forces," such as can be ascribed to the solution of the two-person constant-sum game. It is rather an "arbitrated solution," based on an equity principle of sorts. An equity principle need not involve an "equilibrium of forces." Nor need it be egalitarian. The Shapley Value equity principle does reflect the unequal "power" of the three stockholders. But it also awards a positive payoff to each player, in spite of the fact that either player in coalition with the strongest player (A) can exclude the third.

BARGAINING SET

Another apportionment that takes the relative power of the players into account introduces the concept of the Bargaining Set. In this approach, the formation of the Grand Coalition is *not* assumed. The theory of the Bargaining Set has nothing to say about how the players will partition themselves into coalitions, not even that a coalition and a countercoalition will form (which is assumed in the classic theory of at least the constant-sum n-person game). In deriving the Bargaining Set of a game, it is assumed that the players have already partitioned themselves in some specified way. The Bargaining Set is then defined in relation to each of the possible partitions.

We have seen that in the three-person game five different coalition structures are possible; namely

(1) Every man for himself.

(2) A and B versus C.

(3) A and C versus B.

(4) B and C versus A.

(5) The Grand Coalition.

The game is defined, as usual, by its characteristic function; that is, by designating the payoff that will accrue to each of the possible coalitions. To fix ideas, suppose the characteristic function of a game is

$$\begin{aligned} &v(A) = 0; v(B) = 0; v(C) = 0. \\ &v(AB) = 60; v(AC) = 70; v(BC) = 90. \\ &v(ABC) = 100. \end{aligned} \tag{6}$$

Note that the game is not constant-sum because, if two players join in a coalition against the third, the three do not get a joint payoff of 100, while if they join in the Grand Coalition, they do. Now, it is *collectively* rational for the players to form the Grand Coalition, because whatever any pair can get for themselves, they can get somewhat more if they induce the third player to join. It is also in the third player's interest to join if he is offered anything at all, because otherwise he gets nothing. Nevertheless, no apportionment of the 100 that accrues to the Grand Coalition is stable. To see this, observe that, in order to satisfy A and B, they must jointly get 60. Similarly, A and C must jointly get 70, while B and C must jointly get 90. Thus we have

$$\begin{aligned} A + B &= 60 \\ A + C &= 70 \\ B + C &= 90 \end{aligned} \tag{7}$$

Consequently, $2(A + B + C) = 220; A + B + C = 110.$

But there is only 100 to be apportioned among the members of the Grand Coalition. Hence there is no apportionment that will satisfy every potential coalition. In view of this instability, we cannot be sure that the Grand Coalition will form.

Assume, then, that B and C have formed a coalition. Assume further that they have provisionally decided to split

90 equally. Neither B nor C has reason to be completely satisfied with this arrangement. For instance, B can say to C: "I think I should get more than 45. If you don't give me more, I shall leave you and join A. I can offer A any amount up to 15. Since he and I can get 60, any amount less than 15 accepted by him will leave me more than 45, which is what I get in the present arrangement with you." We say that B has an *objection* against his partner C. Note, however, that C can offer a powerful counterargument: "Yes, you can offer A up to 15 and still come out ahead. But I can offer him up to 25 and still come out better than I am in the present arrangement with you, because he and I can get 70. It stands to reason that I can induce A to join me more easily than you can get him to join you." Because of this counterargument, we say that B's objection against C is *not justifiable.* On the other hand, C's objection against B is justifiable, because B cannot top C's offer to A.

An apportionment of payoffs among the members of a coalition is called *balanced* if no player has a justifiable objection against any of his coalition partners. It is easy to see that the apportionment of 90 between B and C will be balanced if B gets 40 and C gets 50. For, in that case, each can offer A up to 20 but no more. Similarly, the apportionment of 70 between A and C will be balanced if A gets 20 and C gets 50; the apportionment of 60 between A and B will be stable if A gets 20 and B gets 40.

Suppose the Grand Coalition forms. Then it is easy to see that the apportionment (50/3, 110/3, 140/3) is balanced. For A can offer B up to 130/3 to induce him to exclude C, but so can C, to induce B to exclude A. Similarly, B can offer A up to 70/3 to induce him to exclude C, but C can match this offer in return for excluding A.

An apportionment of payoffs among the members of the Grand Coalition is called an *imputation.* At times the Bargaining Set associated with the Grand Coalition consists of a single imputation. In some games, however, the

Bargaining Set may consist of many imputations. This is the case if the game has a *core,* to be defined below.

From the foregoing it can be seen that the Shapley Value and the Bargaining Set provide answers of sorts to the questions we posed. The Shapley Value, on the basis of a certain rationale, prescribes a definite portion of the total payoff to members of the Grand Coalition. The Bargaining Set singles out apportionments of payoffs among coalition members that "stabilize" the particular coalitions. These answers are based on certain assumptions of "rationality," but these assumptions do not by any means involve everything that the concept of rationality might involve. And they include assumptions that are not necessarily involved in other possible definitions of "rationality." For instance, the Shapley Value of a game can be said to be a solution only if "rational" players do not act as they might act in their own individual interest, such as by breaking away from the Grand Coalition in order to get more than the Shapley Value allots them. The concept of the Bargaining Set, on the other hand, admits the possibility of the Grand Coalition not forming in spite of the fact that "collectively rational" players certainly cannot lose by forming it and may gain by doing so.

What makes it possible for the Shapley Value and the Bargaining Set to single out certain payoff apportionments as rational are *additional* assumptions beyond those which initially define individual and collective rationality.

In the case of the Shapley Value, one additional assumption is that collectively rational players will take due account of their relative power and resist the temptation to go after immediate greater gains by breaking away from the Grand Coalition. Presumably, if they do so, the new coalition becomes vulnerable to tempting offers from outsiders.

In the case of the Bargaining Set, an additional assumption is that the players have already partitioned themselves into coalitions. How this was done is not specified.

We have mentioned the *core* of the game (see p. 200). A game has a core if there are imputations which make it unprofitable for any subset of players to break away from the Grand Coalition. Consider the game with characteristic function identical to the preceding one (Eq. 6) except that v(ABC) = 150. Here there are imputations in which every pair gets more than its value as specified by the characteristic function. Then all such imputations are stable in the sense of the Bargaining Set, because no player can induce another to exclude the third. Any pair breaking away from the Grand Coalition will get less than in the Grand Coalition.

One other concept, especially relevant to four-person constant-sum games, deserves attention. All such games can be shown to have so-called Quota solutions, in which each pair gets exactly its value. One might suppose that the imputations of the Quota could be considered as solutions of four-person constant-sum games.

TESTING THE THEORY

We have discussed a number of concepts proposed as solutions of n-person games, in particular, of three- and four-person games. Each of these solutions can be viewed as partial answers to the principal questions posed in n-person game theory:

(1) How will payoffs be finally apportioned among rational players?
(2) How will rational players partition themselves into coalitions?

The answers are only partial because the answer to the first question assumes that the total payoff available to the Grand Coalition will actually be achieved. In *that* case, the concept of the core provides the answer to the first question: If the game has a core, we can expect that the rational players will arrive at one of the imputations in the core. The concept of Shapley Value provides a somewhat more specific answer. It is also a more general answer because it applies also to games without cores. Nevertheless, both of these answers by-pass the second question, related to the first. Or rather, these answers involve a tacit assumption that, however the players may be partitioned into coalitions during the negotiations, they will act as a Grand Coalition in order to obtain the maximum joint payoff.

The concept of the Bargaining Set, on the other hand, is applied to each possible coalition structure separately. Given a particular coalition structure, the concept of the Bargaining Set provides answers to the question of how the payoff accruing to each coalition will be apportioned among the members of the coalition. Here, too, the question of how the players will actually partition themselves is by-passed. However, some leverage with regard to this question is provided by the theory of the Bargaining Sets in the following way. Given a coalition structure and a *proposed* distribution of payoffs within each coalition, the theory answers the question of whether the two are "compatible." If they are, the coalitions can be expected to persist until the proposed apportionment of payoffs is effected. If not, then "something has got to give." Either another apportionment of payoffs has to be proposed, or the coalition that is incompatible with the proposed apportionment will dissolve. The Quota can be regarded as a solution only if we assume that pair coalitions will form.

It should be mentioned in passing that the concepts discussed here do not by any means exhaust the "solutions" offered in the theory of the n-person game. In particular, they do not include the very first class of solutions offered by John von Neumann and Oskar Morgenstern (1947) who laid the foundation of the theory. A discussion of the von Neumann-Morgenstern solution would take us too far into mathematics unfamiliar to most readers. Besides, the connection between this theoretical solution and actual behavior of players of n-person games is too tenuous to be enlightening.

So, returning to the three concepts discussed, we now ask a question that can be answered by empirical evidence. Are any of the solutions implied by these concepts actually realized when real players play games defined by the characteristic function? Some experiments designed to answer this question have been performed, and we shall examine the results. Before doing so, however, some preliminary remarks are in order.

In the context of game theory, payoffs associated with the outcomes of games are assumed to be in units of "utility." Roughly, the utility of a payoff is the "value" that a player places on it. The value, being a subjective concept, is not amenable to direct observation. It can only be inferred from the behavior of a player, and, moreover, the inference is beset by considerable difficulties. Suppose, for example, that the payoffs are in money. It is a simple matter to determine whether a player prefers more money to less money: one can simply offer him a choice between two unequal amounts of money. It is another matter to determine *by how much* he prefers a larger amount of money to a smaller one. Does he, for example, necessarily prefer $10 to $1 by a factor of 10? Better asked, what might be the meaning of such a question, if any? And what sort of evidence can provide an answer to it?

One way of comparing magnitudes of preference differences was proposed by von Neumann and Morgenstern (1947) and is still the foundation of so-called utility theory. Suppose we offer a person a choice between the following two alternatives: $1 outright or a "lottery ticket" that has one chance in ten of winning $10. If the person chooses the first alternative, we can then say that, in this instance, the utility of $10 is to him less than ten times the utility of $1. We shall then proceed to give him another chance to choose, this time increasing the chances of winning $10. If he again chooses the $1 outright, we can increase still further the chances of winning $10. Eventually his preference might change. We can certainly expect it to change when the lottery ticket wins the $10 with probability 1, but it may change before then, namely, when the probability of winning \$10 is large enough. Suppose the change comes when the chances of winning \$10 are forty percent. More precisely, the person is indifferent between \$1 outright and the lottery ticket with a forty percent chance of winning \$10. We can now calculate the relative utilities (to him) of \$1 and \$10. The calculation is made on the basis of a definition of utility as the quantity whose statistical expectation is maximized in any choice involving numerical prizes, with probabilities of winning them attached (i.e., choices between risky alternatives). If $u(x)$ is the utility of x dollars, then the utility of the lottery ticket is $0.6 \times 0 + 0.4 \times u(10)$. If this is equal to $u(1)$, the utility of \$1, we conclude that the utility of \$10 is, for our subject, 2.5 times the utility of \$1.

As has been said, the payoffs of games considered in game theory are supposed to have been already translated into utility units. For this reason, it can be anticipated that the rational player will try to maximize his statistically expected payoff. In actual experiments, however, payoffs are usually given in money or some such commodity that can easily be manipulated and reckoned with. To take for granted that the assumptions of game theory apply to monetary payoffs is tantamount to supposing that the utility of money is proportional to the amount of money. The supposition is questionable. Nevertheless, for want of more specific information about experimental players' evaluative modes, we have little choice but to make it. We shall make that supposition in the cases that follow.

First we examine experimental results in constant-sum games.

Subjects (players) are informed of what each of the possible coalitions will get; that is, they are told the characteristic function of the game. The "play" will consist of the negotiations among the players in the process of forming coalitions and dividing among the members the payoffs accruing to the coalition. The players may make any arrangements they wish. The result of the negotiations (that is, the outcome of the game) is the agreed-upon apportionment of the total payoff among the individual players. The payoffs to the several coalitions in four-person constant-sum games are shown in Table 1.

Note that in Game 1 the total payoff to be apportioned is 80 points. For instance, if players 1 and 2 join in one coalition and players 3 and 4 join in another, 1 and 2 will get 60 while 3 and 4 will get 20. On the other hand, if 1 joins with 3, and 2 with 4, each coalition will get 40. If any three join, they can get all of 80, while the player left out will get nothing. In a way, this game resembles the divide-the-dollar game; as before, a majority can appropriate the whole amount. However, in this game we also have so-called "blocking coalitions" (pairs), which, although they cannot themselves appropriate the whole amount, can prevent others from doing so. Moreover, the "power" of the pairs is different. Pairs (12) and (23) get 60; pairs (13) and (24) get only 40, while pairs (14) and (34) can get only 20. One feels

TABLE 1[a]

	v(S)			
Coalition (S)	Game 1	Game 2	Game 3	Game 4
(1)	0	-40	-20	-20
(2)	0	10	-20	-40
(3)	0	0	-20	-40
(4)	0	-50	-20	-20
(12)	60	10	0	30
(13)	40	0	0	0
(14)	20	-50	0	-10
(23)	60	50	0	10
(24)	40	0	0	0
(34)	20	-10	0	-30
(123)	80	50	20	20
(124)	80	0	20	40
(134)	80	-10	20	40
(234)	80	40	20	20
(1234)	80	0	0	0

a. See KALISCH et al. (1954).

that player 2 is in the strongest position, since he must be a member of the "strongest" coalitions. Similarly, player 4 is the "weakest."

This disparity of power is reflected in the Shapley Value of Game 1, as shown in Table 2.

Each of the players played this game eight times, the composition of the players in each quadruple being changed from play to play in order to prevent the fixation of outcomes. We can now compare the average payoffs received by the players—or rather, by the roles assumed by the players as players 1, 2, 3, and 4. The comparison of average payoffs with both the Shapley Value and the Quota solutions is shown in Table 2.

The average payoffs of the four players are somewhere between the Shapley Value and the Quota.

Consider now Game 4. Its characteristic function looks quite unlike the characteristic function of Game 1. Yet in a certain sense the two games are equivalent. If we multiply the value accruing to each coalition by 3/2 and collect an "entrance fee" of 20 units from player 1 and player 4 and of 40 units from players 2 and 3, we shall obtain the characteristic function of Game 4. For instance, in Game 1, v(12) = 60; 60 x 3/2 = 90. Taking 20 from player 1 and 40 from player 2 gives this coalition 30, which is v(12) in Game 4. The same holds for all the other values of the characteristic function. Games 1 and 4 are considered equivalent for the

TABLE 2
COMPARISON OF SHAPLEY VALUE, QUOTA, AND AVERAGE OBSERVED PAYOFF IN GAME 1

	Players			
	1	2	3	4
Shapley Value	20	26.7	20	13.3
Quota	20	40	20	0
Average payoff (observed)	15	34.4	20.3	9.6

following reasons: multiplying each value by a constant amounts to no more than changing the units of payoffs. This is like changing dollars to dimes or any other money unit, and should not make any difference in the way the game is played by rational players. Nor should charging each player some fixed amount (or paying each player some fixed amount) make any difference in the way the game is played, because these charges and fees are independent of what the players do in the game.

The Shapley Value and the average payoffs obtained by the players (player roles) in Game 4 are shown in Table 3.

Here the average payoffs are not too far from the Shapley Value. But the Quota fails as a prediction of the average apportionment.

Actually, the payoffs to the several players ranged widely in the individual plays of the game when different individuals occupied the various roles. For example, in Game 1 the payoff to player 1 ranged from 0 to 35.5, and in Game 4, from -20 to +25. The payoff to player 4 ranged from 0 to 24 in Game 1, and from -10 to +25 in Game 4. Over the different plays, and hence players, however, these differences averaged out to values approximately in accord with the theoretical values given by the Shapley Value.

A similar picture emerges in Games 2 and 3. Game 3 is clearly symmetric in that every player playing alone gets the same payoff (-20); every pair gets the same payoff (0); every

TABLE 3
COMPARISON OF SHAPLEY VALUE, QUOTA, AND AVERAGE OBSERVED PAYOFF IN GAME 4

	Players			
	1	2	3	4
Shapley Value	10	0	-10	0
Quota	10	20	-10	-20
Average payoff (observed)	13.0	- 3.9	-11.2	2.1

triple gets +20. Consequently, the Shapley Value awards the same amount to each player, and this must be zero, since the game is zerosum. Comparison with average payoffs is shown in Table 4.

Again the average payoffs are close to the Shapley Value, although the payoffs varied widely from play to play. In this case, the Quota solution coincides with the Shapley Value solution.

Game 2 is equivalent to Game 3 in the same way that Game 4 is equivalent to Game 1. Game 2 is obtained from Game 3 by giving players 1 and 4 a fixed bonus of 20 and 30 points respectively, and by charging players 2 and 3 a fixed fee of 30 and 20 respectively. Comparison with Shapley Value and the Quota is shown in Table 5. Here again the Shapley Value and the Quota coincide.

The agreement is not bad, but worse than before. Player 3 gets more than expected, and player 4 less than expected.

TABLE 4
COMPARISON OF SHAPLEY VALUE, QUOTA, AND AVERAGE OBSERVED PAYOFF IN GAME 3

	Players			
	1	2	3	4
Shapley Value	0	0	0	0
Quota	0	0	0	0
Average payoff (observed)	-2.9	3.1	-0.2	0

TABLE 5
COMPARISON OF SHAPLEY VALUE, QUOTA, AND AVERAGE OBSERVED PAYOFF IN GAME 2

	Players			
	1	2	3	4
Shapley Value	-20	30	20	-30
Quota	-20	30	20	-30
Average payoff (observed)	-21.5	29.0	30.1	-37.6

Still, considering the fact that the symmetry of this game was camouflaged, the payoffs are to a certain extent in accord with the Shapley Value. In one case where the Shapley Value and Quota differed, the average payoffs were intermediate between the two. In another case where the two differed, Shapley Value was definitely a better prediction. Possibly, if the payoffs of very many players in very many plays were averaged, the distribution of payoffs would be quite close to the Shapley Value. If so, then this theoretical construct may well be an accurate predictor of how people behave "on the average" in n-person games, as defined by their characteristic functions. Recall that this construct embodies certain considerations of power relations and also certain considerations of equity.

We shall now examine the results of an experiment designed to test the Bargaining Set as a predictor of behavior in three-person games. M. Maschler examined several three-person games, each played once by a triple of Israeli high school students (see Maschler, 1962). The results fall into three categories.

(1) Ten games in which the Grand Coalition was formed and the apportionment of the payoffs was in accord with the corresponding Bargaining Set apportionment. All the games in this category had cores (recall that a game has a core if it is possible to apportion the payoffs in such a way that no subset of players stands to gain by breaking away from the Grand Coalition to form a coalition of its own). In that case, all the imputations of the core are in the Bargaining Set. An example of such a game is one with the following characteristic function:

$$v(12) = 60; v(13) = 80; v(23) = 100; v(123) = 150. \qquad [8]$$

In this case, the actually observed apportionment was 46, 50, 54 to players 1, 2, and 3 respectively. This imputation is in the core, since 1 and 2 jointly get 96, more than they could have gotten in coalition (i.e., 60). Similarly, 1 and 3 get $100 > 80$; 2 and 3 get $104 > 100$. The expectation is, of course, that when the game has a core the apportionment of payoffs will be an imputation in the core. There is no motivation for any player to leave the Grand Coalition.

(2) Eight games in which the Grand Coalition formed but the apportionment of payoffs deviated from the corresponding Bargaining Set apportionments. Of these eight games four had no cores, two had cores with just one imputation, and two had larger cores.

(3) Twelve games in which the Grand Coalition did not form; that is, two players joined against the third (who got nothing). The payoff accruing to the two was divided in accordance with the prescription of the corresponding Bargaining Set.

(4) Ten games in which the Grand Coalition did not form, and the apportionment between the two players in coalitions deviated from the prescription of the Bargaining Set.

Several observations are of interest. First, the formation of the Grand Coalition seems to depend strongly on the presence of a core. In all cases where the game had a core, the Grand Coalition was formed; where the game had no core, for the most part the Grand Coalition did not form.[3] Second, where deviations from the Bargaining Set apportionments occurred, they were overwhelmingly in the direction of a more egalitarian apportionment. In all the games examined, player 3 is at least as strong as player 2 (generally stronger) and stronger than player 1. Player 2 is at least as strong (generally stronger) than player 1, who is the weakest. This is reflected in the characteristic functions of all of the games examined: $v(23) \geqslant v(13) \geqslant v(12)$. This means that player 3 is a member of the two coalitions who get the largest payoffs; player 1 is a member of the two worse coalitions. This inequality is reflected in the Bargaining Set apportionments. As has been said, wherever deviations from these apportionments occur, as in games grouped under category 2 and category 4, predominantly the deviations are in the direction of more equal division. Finally, it is interesting to note which pair-coalitions formed when the Grand Coalition did not form (in categories 3 and 4). In games of category 3, the strongest coalition (23) formed in ten cases out of twelve. In one case the middle coalition formed (13); and in one case, the weakest coalition (12). In games of category 4 (where deviations were observed) the strongest coalition (23) formed in four cases out of ten; the middle coalition (13) also formed in four cases; the weakest coalition, in two cases. The conjecture is that the more straightforward the game is (as reflected in the realization of the Bargaining Set apportionments), the more likely is the strongest coalition to form. This conjecture is supported by the observation that, of the four cases where the strongest coalition formed in games of category 4, three were associated with the smallest deviations. The finding is interesting, because it seems to refute the widespread conviction that the weak are likely to unite against the strong instead of the other way around. However, no conclusion is warranted before a more careful definition of coalition strength is made. We shall return to this question.

It is also instructive to compare the apportionments of payoffs in games of categories 1 and 2 (where the Grand Coalition formed) with those prescribed by the Shapley Value. The comparisons are shown in Table 6.

Here the Shapley Value seems to be an even better predictor of average apportionment of payoffs than in the constant-sum game experiments cited above.

The theory of games with more than two players is an excellent example of serendipity: in the process of searching for one thing, one stumbles on another that proves in the long run to be more valuable than the original object of search. Columbus' discovery of the New World is a classical example. Originally game theory was inspired by a search for rational strategies in conflict situations. Once the general problem of the two-person constant-sum game was solved in principle, attention turned to nonconstant-sum games and to games with more than two players. In the latter case, the problem of coalitions immediately came to the forefront, and

TABLE 6

Games Where the Grand Coalition Formed and No Deviations From the Bargaining Set Were Observed

	Players		
	1	2	3
Average payoff	27.8	36.7	48.8
Shapley Value	28.9	38.0	50.9

Games When the Grand Coalition Formed and Deviations From the Bargaining Set Were Observed

	Players		
	1	2	3
Average payoff	23.6	31.9	45.1
Shapley Value	23.7	28.3	48.5

the original problem—that of finding rational strategies—was essentially abandoned. It seemed rational for players in n-person games to join in coalitions, since players in a coalition can in general accomplish more than by playing every man for himself. But then entirely new problems arose. It may be more rational for some of the n players to join in a coalition than to play independently. But a player can join any of several coalitions. How much he will get in each depends on what his coalition partners are willing to promise him. But, if preplay bargaining is free, every player, in bargaining for his share of the payoff accruing to the coalition he contemplates joining, must contend with the fact that other players may want to enter the same coalition and that his prospective coalition partners are at the same time considering offers from others to leave the coalition and to join another.

The gist of the matter is that games of this sort cannot be solved by a straightforward "balance of power" principle as can the two-person constant-sum game. If any final apportionment of payoffs is to have "stability," other principles often must be invoked. These may have something to do with "equity," a recognition on the part of the players that the situation awards to each player something that he "deserves" in some definable sense. In this way, n-person game theory becomes more a theory of conflict resolution than a theory of strategic conflict, which supposedly prescribes to each player a course of action that will ensure for him the largest possible payoff under the constraints of the situation. The point is that these constraints are obvious only in the case of the two-person constant-sum game, where each of the players has a clear opponent with diametrically opposed interests. In an n-person game, the largest possible gain to each individual player (the value of the game to the coalition consisting of him alone) in general falls far short of what he can get in a coalition with others. Indeed, if the n-person game is nonconstant-sum, the Grand Coalition can always get more than the several coalitions jointly. But the apportionment of the Grand Coalition payoff, if it is to be *the* apportionment, must, in general, be based on a principle other than a well-defined balance of power. To be sure, in constant-sum n-person games, the coalition of n - 1 players can always get as much as a coalition of n players. The question, then, is who is to be left out. But the original question of how the coalition of n - 1 players will divide the spoils still remains.

One way of getting a feeling for this situation is to participate in an n-person coalition game. The following game was specially invented for this purpose. Whether it is "fun" to play depends on the temperaments and the sensitivities of the players. Many who have played it report a painful experience. Therefore a recommendation to play this game must be coupled with a warning. The game is called "So Long, Sucker." As players find out, it well lives up to its name (compare Shubik, 1964).

There are four players. Each takes seven playing cards of one suit and is thereby designated by the suit. The values of the cards do not matter. The player to move first is decided by chance. He moves by placing one of his cards on the table.

Thereupon he calls on any of the other players to make the next move. He may also call on himself. The successive order of play is determined in a similar manner. When there are cards on the table, a card may be placed separately or on another card. In this way, piles of cards may be built up. If a player chooses to put his card on a pile, he may give the next move to any player whose suit is *not* represented in that pile. Or, if all suits are represented, he must give the move to the player whose last card played is furthest down in the pile.

If a card is placed on a pile where the top card is of the same suit, the pile is captured. The capture, however, is effected by the player designated by the suit of the card played. The player effecting the capture must then "kill" a card of his choice. The killed card is out of the game. The captor keeps the remaining cards.

Eventually players will hold cards of a suit different from their own. These cards are called prisoners. A player may at any time kill any prisoners in his possession. He may also transfer a prisoner to another player. Such transfers are made unconditionally. For example, a player cannot *provisionally* transfer a prisoner, pending some move that he wants another player to make, assuming that if the move is not made he can retract the transfer. In fact, although promises and deals can be made freely (but openly) in this game, no promise needs to be kept. A player makes a deal entirely at his own risk. A player may not transfer cards of his own suit or kill them except in a captured pile.

When a player is called upon to move and cannot move, not having any cards, he is thereby defeated and drops out of the game. However, his defeat is not final until all the other players have refused to come to his rescue. He may be rescued by a transfer of a prisoner card, which he can then play.

When a player is defeated, the move rebounds to the player who gave him the move. If that player is thereby defeated, the move rebounds to the player who gave *him* the move, and so on. The cards of a defeated player not yet killed are in the possession of other players as prisoners. These cards remain in the game but are ignored in determining the order of play.

The winner is the player who has survived all others.

Instructive as it may be to participate in a cut-throat game like "So Long, Sucker," it can at best help form impressions of what it feels like to be in a dog-eat-dog world, where cooperation is only a matter of expedience and where allies can suddenly become enemies. We know that such real-life situations are common, especially where conflict and competition are constantly a principal theme; e.g., in business, diplomacy, and war. But such situations do not exhaust the range of human conflict. The social scientist naturally wants to gain some *systematic* knowledge about the dynamics of such situations, not merely impressions. For this reason, some social scientists have looked to the theory of games as a conceptual framework in which controlled conflict situations might be described or experimentally arranged.

We have already examined some experimental results on three- and four-person games. Note that the experiments cited were guided by fundamental concepts of game theory. The games were designed with a view of testing results derived from the theory; for instance, whether payoff apportionments are in accord with those prescribed by the Quota, the Bargaining Set, or the Shapley Value.

The social psychologist, however, approaches the subject from another point of view. Game-theoretical concepts on the technical level, to the extent that he is familiar with them, are not often central to his concern. His interest is focused, for the most part, on concepts expressed in nontechnical language. For example, given a group of actors of unequal "strengths," and an opportunity to form coalitions, it is natural to ask whether patterns of coalition formation can be observed reflecting the strengths of the actors. Will the weak combine against the strong, for example? This sounds like a question that can be formulated in game-theoretic terms, but all too often questions of this sort are vague. Without a more precise definition of the nature of the game, of strength, or of weakness, answers will be equally vague. In game theory, strength may be defined in terms of certain features of the characteristic function of the game. In real life, however, strength manifests itself in quite other ways, often in personal characteristics of the players. There is no necessary connection among the various conceptions of strength suggested by intuition.

THEORIES OF GROUP BEHAVIOR

In spite of this vagueness of concepts not strictly operationally defined, a number of theories put forward by some social psychologists, in an effort to systematize thinking and observations of group behavior, are worth examining.

The Minimum Resource Theory states that if, among the possible coalitions, some are winning, then the winning coalition that will form will be the one that commands just sufficient resources to win. As stated. the nature of winning is not specified. In the games examined above, winning might have been defined as appropriating the largest portion of the prize. But here, for the most part, the coalitions that *could* appropriate the biggest prize, because they were coalitions of the stronger players, did appropriate it in seeming contradiction to the Minimum Resource Theory. In another sense, however, the theory may reflect some observations. If all prizes are of *equal* value, then, perhaps, the prize-winning coalition that will form will be the one that commands the minimum resources necessary to win. In this interpretation, the problem remains of defining resources.

In an experiment conducted by W. E. Vinacke and A. Arkoff (1957), players 1, 2, and 3 were allotted numbers that were essentially proportional to their chances of winning, as shown in Table 7. The three games listed were played 90 times each.

The coalitions with the smallest numbers of these votes, sufficient to form a majority, are shown in the second column of the table. Observed results are shown in the last three columns. Note that, if coalitions are formed by chance, each coalition has a probability of .33 of being formed. In Game 1, the predicted coalition formed seventy percent of the time. In Game 2, either of two coalitions is minimal. Hence, by chance alone, the theory would be corroborated sixty-seven percent of the time. In fact, in this game, minimal coalitions were formed again seventy percent of the time; i.e., at about chance level. Still, the nonminimal coalition was formed only seventeen percent of the time, below its chance level of thirty-three percent. In Game 3, the minimal coalition was formed sixty-six percent of the time, again well above chance level.

These results seem to corroborate the Minimum Resource Theory. The significance of the result is strengthened by the fact that the three players all had equal voting *power,* since any two had a majority in all three games. Therefore, the Shapley Value of each of these games is equally distributed. Nevertheless, the frequencies of the various coalitions were different—statistically in accord with the Minimum Resource Theory.

TABLE 7[a]

	Distribution of Resources Player			Minimal Winning Coalition	Observed Coalitions (Number) of times out of 90 plays)		
	1	2	3		Minimal	Other	None
Game 1	3	2	2	(23)	64	25	1
Game 2	1	2	2	(12) or (13)	64	15	1
Game 3	4	3	2	(23)	59	29	2

a. See Vinacke and Arkoff.

In experiments conducted by William Gamson (1964), the resources played a more explicit role. The players were asked to imagine that they were chairmen of delegations at a political convention and that political patronage was to be the prize of victory (nominating the candidate). The resources were votes commanded by the respective chairmen, namely 17, 17, 17, 25, 25. Here the Minimum Resource Theory has a certain real-life plausibility. The costs of recruiting a majority involve the number of votes possessed by the wooed parties and also the amount of patronage the allies will demand after victory. The coalition of minimal resources minimizes its costs.

The three chairmen with 17 votes each united in coalition thirty-three percent of the time, whereas, by chance, this particular coalition among all the possible ones would have formed only ten percent of the time—another apparent corroboration of the Minimum Resource Theory.

The Minimum Power Theory is a modified form of Minimum Resource Theory suggested by the concept of Shapley Value, according to which the winning coalition that commands the minimum voting power (in the Shapley Value sense) will form most frequently. In neither Vinacke and Arkoff (1957) nor Gamson's (1964) experiments could this theory be pitted against the Minimum Resource Theory, because in each case each of the players had equal voting power despite the disparity of resources. To juxtapose the two theories, a game should be designed where the coalition with minimum resources has actually more voting power than some other coalition, or else the coalition with minimum voting power has more resources than another. An example is a four-person voting game where the players have 10, 5, 3, and 3 votes respectively, and where two-third's majority is required to win. The winning coalitions are (12), (123), (124), (134), and (1234). Player 1, who, incidentally, has veto power since he is a member of every winning coalition, has 7/12 of the voting power. Player 2 has 3/12, and players 3 and 4, 1/12 each. Coalition (134) is the winning coalition with minimal power, but coalition (12) is the winning coalition with minimal resources.

The Minimum Conflict Theory, another theory of coalition formation, posits the principle of "minimizing intra-coalition conflict." This theory is more difficult to state in terms of clearly observable features of conflict than the preceding theories. The theory may relate to the way people choose partners in coalitions. They may, of course, do so on personal grounds, choosing people they are attracted to or people most resembling themselves. If so, then the theory says little more than that birds of a feather flock together. Conceivably the theory may refer to the fact that the resources of people in a coalition should be as equal as possible to forestall conflicts about payoff apportionments. In this context, the theory can be verified. In fact, the frequency of coalitions of the three equal players in Gamson's games is in accord with the Minimum Conflict Theory as much as with the Minimum Resource Theory. Note that, in Vinacke and Arkoff's Game 2, the number of cases where *no* coalition was formed was largest. In that game also, the coalition with equal partners (23) was at the same time the only pair-coalition with *maximal* resources. Conceivably, in that situation, the tendency to form a coalition with minimal resources clashed with the tendency to form a coalition with equal partners, resulting in a large number of cases where no coalition formed. It stands to reason, if these proposed theories are taken seriously, a great deal more experimental work needs to be done to put them to significant tests. Moreover, the experiments should be designed so as to pit the competing theories against each other.

SUMMARY

In summary, game theory provides a rich conceptual framework for studying group behavior. It should be pointed out, however, that this framework is most helpful when it is used as a principal guide in designing experiments rather than as an afterthought for interpreting experiments suggested on other theoretical grounds. The concepts of n-person games suggest many lines of investigation that can be pursued in an entirely systematic manner: testing various normative hypotheses of game theory sequentially, pitting one against another, and so forth. The by-products of these observations will of themselves suggest conjectures relevant to questions of interest to social psychologists and sociologists, to be tested further in appropriate settings. In short, game theory becomes a most useful tool if the formal characteristics of the games are put at the *start* of an investigation instead of trying to fit them to situations observed or designed with other interests in mind.

NOTES

1. "A man for himself" does not appear as a coalition in the everyday meaning of the phrase, but is considered a "coalition of one" in game theory in the interest of uniform terminology.

2. In a constant-sum two-person game, each player can choose a strategy that will guarantee him a minimum expected payoff, whatever the other player does. Assuming rationality of both players, these strategies, called *Minimax* strategies, and their associated payoffs, constitute the "solution" of the game. The minimum guaranteed payoff is called the *value* of the game to the player in question.

3. It must be pointed out, however, that in games where the Grand Coalition failed to form, v(123) did not exceed v(23) and in many cases was actually smaller, violating the rule stated in equation [4]. In order to test the dependence of the formation of the Grand Coalition on the core, many more games should have been introduced where $v(123) > v(23)$, but where the core is nevertheless empty. Only two of Maschler's games (1962) had this property.

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