This article shows how methods for the analysis of moment structures based on latent variates can be used to investigate theories hypothesizing interactive effects of exogenous constructs on endogenous constructs. Building on the work of Kenny and Judd, the authors discuss the specification of structural equation models with latent interaction terms, and they show that the structural coefficient relating an endogenous construct to the interaction of two exogenous constructs does not depend on the origin of the components of the multiplicative exogenous construct. The authors report evidence from simulations that the χ^2 *statistics and standard errors provided by maximum likelihood and generalized least squares may not be trustworthy, but that model testing through robust and asymptotically distribution-free procedures leads to appropriate assessments of model fit and the significance of parameter estimates. They conclude the article with an application of the procedure to the context of expectancy-value attitude models.*

Specification, Estimation, and Testing of Moment Structure Models Based on Latent Variates Involving Interactions Among the Exogenous Constructs

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Until recently, a serious shortcoming of structural equation modeling with latent variates has been the fact that conventional models for moment structure analysis are only applicable if the structural relationships among the variables in the model are linear. One implication of this is that theoretical frameworks that posit interaction effects at the unobservable level (e.g., expectancy-value attitude models, cf. Fishbein and Ajzen 1975) are not amenable to testing by these procedures. This is unfortunate, because the damaging

AUTHORS' NOTE: *We would like to thank Seenu Srinivasan for stimulating our thinking on the problem considered here and Jan-Benedict Steenkamp for helpful comments on an earlier version of this article.*

SOCIOLOGICAL METHODS & RESEARCH, Vol. 24 No. 2, November 1995 187-213

effects of measurement error may be exacerbated when products of unreliable measures are used as explanatory variables (see Bohrnstedt and Marwell 1978; Busemeyer and Jones 1983), and one of the advantages of moment structure analysis is that estimates of structural relationships can be corrected for attenuation if multiple indicators of endogenous and/or exogenous constructs are available.

Several years ago, Kenny and Judd (1984) proposed a procedure for handling interactions at the unobservable level that explicitly models the complexities introduced by using products of observed variables as indicators of multiplicative exogenous constructs.[1] Kenny and Judd (1984) discussed their procedure in the context of a model with two exogenous constructs and their interaction. In specifying their model, they assumed that the means of the two simple exogenous constructs were equal to zero. They did not show, however, that this assumption could be made without loss of generality, in the sense that the coefficient of the latent interaction term is consistently estimated even if the means of the two simple exogenous constructs are not equal to zero. Furthermore, Kenny and Judd (1984) generated only one data set from an assumed underlying model and demonstrated that the latent structure could be recovered very well in this particular instance by generalized least squares estimation. The authors noted that maximum likelihood estimation and testing were inappropriate for the model considered because of known a priori violations of assumptions, and they used generalized least squares estimation because of its weaker distributional assumptions. However, even generalized least squares were apparently expected to provide faulty χ^2 goodness-of-fit statistics and standard errors because model testing was not considered by the authors. Finally, because their model requires the imposition of nonlinear constraints on certain parameters, Kenny and Judd (1984) used the COSAN computer program (Fraser 1980) for model estimation, which is not as widely available as other programs but which at the time was the only program with the explicit capability to estimate models with nonlinearly constrained parameters.

The present article builds on the work of Kenny and Judd (1984) but gets around the aforementioned problems. First, we show that, similar to regression analysis, the structural coefficient expressing the effect of a latent interaction term on an endogenous construct does not depend on the origin of the components of the multiplicative exogenous construct. Second, we discuss how models with interaction effects can be estimated using commonly available computer software, particularly programs with the capability to impose nonlinear constraints on model parameters. Third, we report evidence from simulations that the χ^2 statistics and standard errors provided by maximum likelihood and generalized least squares may not be trustworthy, but that model testing through robust and asymptotically distribution-free procedures leads to appropriate assessments of model fit and the significance of parameter estimates. We conclude the article with an application of the procedure to the context of expectancy-value attitude theory, which points to the potential of the method for investigations of theoretical frameworks that contain interaction effects among exogenous constructs.

SPECIFICATION

The purpose of this section is to briefly review how exogenous constructs that are products of other exogenous constructs can be accommodated in structural equation models with latent variates (Kenny and Judd 1984). For simplicity, we will deal with the case in which one endogenous construct is influenced by two exogenous constructs and their interaction. Extensions to more complex cases are straightforward, although the models get rather complex quickly.

The latent variable model, which specifies the structural relations among the endogenous and exogenous constructs, is given by

$$\eta = \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3\xi_3 + \zeta, \tag{1}$$

where η is an endogenous construct, ξ_1 and ξ_2 are two simple (i.e., nonmultiplicative) exogenous constructs, $\xi_3 = \xi_1\xi_2$ is a multiplicative exogenous construct, γ is a vector of structural coefficients to be estimated, and ζ represents the error in the functional relationship. As usual, we assume that the expected values of all latent variates are equal to zero (in the case of η, ξ_1, ξ_2, and ξ_3 this means that they are written in mean-deviation form) and that ξ and ζ are independent. We will show later that the assumption of zero expected values for ξ_1 and ξ_2 can be made without loss of generality, in the sense that the coefficient of the multiplicative exogenous construct, γ_3, is not affected by changes in the means of the simple exogenous constructs. The final assumption is that ξ_1, ξ_2, and ζ are normally distributed; ξ_3 is not normally distributed because products of normal variates cannot be normally distributed (cf. Kendall and Stuart 1963).

For the measurement model, which links the endogenous and exogenous constructs to their indicators, we assume, again for purposes of simplicity, that η is measured without error by a single indicator and that the two simple exogenous constructs ξ_1 and ξ_2 are each measured by two indicators. We then have

$$y = \eta \tag{2}$$

$$x_1 = \xi_1 + \delta_1 \tag{3}$$

$$x_2 = \lambda_{21}\xi_1 + \delta_2 \tag{4}$$

$$x_3 = \xi_2 + \delta_3 \tag{5}$$

$$x_4 = \lambda_{42}\xi_2 + \delta_4, \tag{6}$$

where ξ_1 and ξ_2 are assumed to be independent of δ, and δ is assumed to be independently normally distributed with a mean of zero.

Because there are two simple indicators for each simple exogenous construct, there are four possible ways of combining an indicator of ξ_1 with an indicator of ξ_2. That is,

$$x_1x_3 = \xi_1\xi_2 + \xi_1\delta_3 + \xi_2\delta_1 + \delta_1\delta_3 \tag{7}$$

$$x_1x_4 = \lambda_{42}\xi_1\xi_2 + \xi_1\delta_4 + \lambda_{42}\xi_2\delta_1 + \delta_1\delta_4 \tag{8}$$

$$x_2x_3 = \lambda_{21}\xi_1\xi_2 + \lambda_{21}\xi_1\delta_3 + \xi_2\delta_2 + \delta_2\delta_3 \tag{9}$$

$$x_2x_4 = \lambda_{21}\lambda_{42}\xi_1\xi_2 + \lambda_{21}\xi_1\delta_4 + \lambda_{42}\xi_2\delta_2 + \delta_2\delta_4 \tag{10}$$

In equations (7) to (10), x_1x_3, x_1x_4, x_2x_3, and x_2x_4 are multiplicative observed variables, and $\xi_1\xi_2$, $\xi_1\delta_3$, $\xi_1\delta_4$, $\xi_2\delta_1$, $\xi_2\delta_2$, $\delta_1\delta_3$, $\delta_1\delta_4$, $\delta_2\delta_3$, $\delta_2\delta_4$ are multiplicative latent variates.[2] Of these, the only multiplicative latent variate of theoretical interest is $\xi_3 = \xi_1\xi_2$; the remaining multiplicative latent variates are needed solely to avoid misspecification of the measurement model. It should be noted that the specification of the measurement equation for x_2x_4 in (10) requires the imposition of a nonlinear constraint on the matrix of factor loadings such that the coefficient of $\xi_1\xi_2$ be equal to $\lambda_{21}\lambda_{42}$.

From equations (7) to (10), it can be seen that the multiplicative observed variables (e.g., x_1x_3) are weighted functions of several multiplicative latent variates. Thus it would be a misspecification to assume, for example, that

$$x_ix_j = \lambda_{ij,\,st}\xi_s\xi_t + \delta_{ij}. \tag{11}$$

However, it is exactly in this manner that multiplicative observed variables have generally been related to latent multiplicative terms in past research investigating interactive theoretical frameworks through analysis of moment structures (cf. Busemeyer and Jones 1983; Evans 1991).

The most important changes arising from the inclusion of multiplicative terms concern the variance-covariance matrices of ξ and δ, denoted by Φ and Θ^δ. Only the variances of, and covariances between, simple latent variates are freely estimated. Variances of, and covariances between, multiplicative latent variates are functions of variances of, and covariances between, simple latent variates. They can be obtained using the formulas derived by Bohrnstedt and Goldberger (1969):

$$C[ab,c] = E[a]C[b,c] + E[b]C[a,c] \tag{12}$$

$$\begin{aligned} C[ab,cd] = {} & E[a]E[c]C[b,d] + E[a]E[d]C[b,c] + E[b]E[c]C[a,d] \\ & + E[b]E[d]C[a,c] + C[a,c]C[b,d] + C[a,d]C[b,c], \end{aligned} \tag{13}$$

where C is the covariance operator; E is the expectation operator; and a, b, c, and d are variables having a multivariate normal distribution.

In the present case, all covariances between a multiplicative exogenous latent variate and any other exogenous latent variate (simple or multiplicative) are zero. The variances of the various multiplicative latent variates in equations (7) to (10) are given by:

$$C[\xi_1\xi_2, \xi_1\xi_2] = C[\xi_1, \xi_1]C[\xi_2, \xi_2] + (C[\xi_1, \xi_2])^2 \tag{14}$$

$$C[\xi_i\delta_j, \xi_i\delta_j] = C[\xi_i, \xi_i]C[\delta_j, \delta_j] \tag{15}$$

$$C[\delta_i\delta_j, \delta_i\delta_j] = C[\delta_i, \delta_i]C[\delta_j, \delta_j] \tag{16}$$

It should be noted that the derivation of the covariance structure implied by the model specified in equations (1) to (10) depends critically on the assumption that ξ_1 and ξ_2 as well as δ be normally distributed, otherwise formulas (12) and (13) are not applicable. This is more restrictive than in the usual case, where only the existence of second moments has to be assumed to derive the covariance structure implied by a model. In practical applications, it is important that the normality of the simple exogenous latent variates be ascertained via an examination of the approximate normality of their observed measures (see Bollen 1989 for details).[3]

By expressing the observed variances and covariances as functions of the model parameters and solving for each parameter in terms of the observed variables, it is readily shown that all parameters are identified. The model involves 13 free parameters to be estimated (λ_{21}, λ_{42}, γ_1, γ_2, γ_3, ϕ_{11}, ϕ_{21}, ϕ_{22}, θ_{11}, θ_{22}, θ_{33}, θ_{44}, and ψ, where the ϕ_{ij} and θ_{ij} are specific elements of Φ and Θ^δ and ψ is the variance of ζ). Ten parameters are functions of these 13 parameters (9 parameters in Φ and Θ^δ plus $\lambda_{21}\lambda_{42}$). Because the variance-covariance matrix of the observed variables contains 45 distinct elements, there are 32 degrees of freedom available for estimation. A graphical representation of the complete specification of the model is shown in Figure 1.

The specification of the model shown in equations (1) through (10) was first suggested by Kenny and Judd (1984). In deriving the covariance structure for this model, they assumed that the means of ξ_1 and ξ_2 were equal to zero. Some authors (e.g., Schoenberg 1984) have expressed concern that, because variances and covariances involving

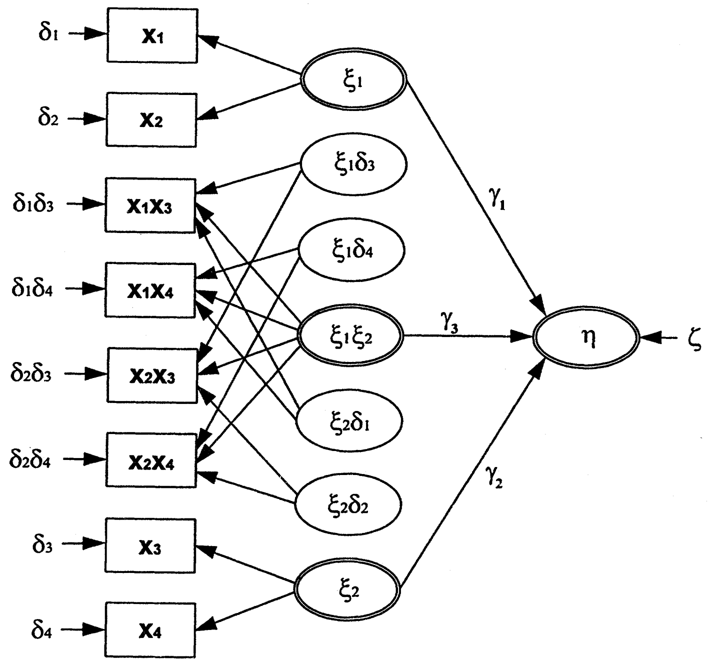

Figure 1: Graphical Representation of the Model

NOTE: Constructs enclosed by double ellipses are of theoretical interest; all other constructs are needed solely to avoid model misspecifications. Correlations among some of the exogenous constructs have been omitted for simplicity.

multiplicative variates depend on the means of the variables involved (cf. equations [12] and [13]), this assumption cannot be made without loss of generality. We will now address this issue in more detail.

Consider again the latent variable model specified in equation (1) but assume now that the means of ξ_1 and ξ_2 are actually not equal to zero. We then have

$$\eta^* = \gamma_1\xi_1^* + \gamma_2\xi_2^* + \gamma_3\xi_1^*\xi_2^* + \zeta, \tag{17}$$

with $\xi_1^* = \mu_1 + \xi_1$ and $\xi_2^* = \mu_2 + \xi_2$. What would happen if we proceeded as if the means were equal to zero? To investigate this possibility, rewrite (17) in terms of ξ_1 and ξ_2 using the fact that $\xi_1^* = \mu_1 + \xi_1$ and $\xi_2^* = \mu_2 + \xi_2$. After simplifying we get

$$\eta^* = \gamma_0^* + \gamma_1^*\xi_1 + \gamma_2^*\xi_2 + \gamma_3\xi_1\xi_2 + \zeta \tag{18}$$

or equivalently

$$\eta^{**} = \gamma_1^*\xi_1 + \gamma_2^*\xi_2 + \gamma_3\xi_1\xi_2 + \zeta, \tag{19}$$

where $\gamma_0^* = \gamma_1\mu_1 + \gamma_2\mu_2 + \gamma_3\mu_1\mu_2$, $\gamma_1^* = \gamma_1 + \gamma_3\mu_2$, $\gamma_2^* = \gamma_2 + \gamma_3\mu_1$, and $\eta^{**} = \eta^* - \gamma_0^* - \gamma_3E[\xi_1\xi_2]$ so that $E[\eta^{**}] = 0$. Comparing equation (1) with equation (19), it is readily seen that the coefficient of the multiplicative construct, γ_3, is the same in both cases, whereas the coefficients of ξ_1 and ξ_2 differ (i.e., $\gamma_1 \neq \gamma_1^*$ and $\gamma_2 \neq \gamma_2^*$). This result is analogous to the well-known fact in multiple regression analysis that the coefficient of a multiplicative variable is invariant under changes in the means of the constituent simple variables (cf. Allison 1977; Bagozzi 1984; Cohen 1968, 1978; Evans 1991; Schmidt 1973).

In contrast to regression analysis, ξ_1 and ξ_2 are not directly observable. The measurement model corresponding to (1) is given by equations (2) to (10), as before. The measurement model corresponding to (19) is the same except that the indicator of η^{**} is now given by y^*, say. Note that the assumption of zero means for ξ_1 and ξ_2 implies that the simple indicators x_i have to be mean centered before multiplicative indicators x_ix_j are formed. Given mean centering, the variance-covariance matrix of **x** does not depend on the means of the latent variates in ξ. It can also be shown that $C[y, x_ix_j] = C[y^*, x_ix_j]$. Note that equation (12) is not applicable in evaluating this expression because y and y^* are not normally distributed. On the other hand, the covariances between y or y^* and a simple observed measure of an exogenous construct are not equal, that is, $C[y, x_i] \neq C[y^*, x_i]$. Because $\gamma_3 = (\phi_{33})^{-1}C[\eta, \xi_3] = (\phi_{33})^{-1}C[\eta^{**}, \xi_3]$ and because both the numerator and denominator in this expression are functions of terms in the variance-covariance matrix of the observed variables that are invariant under changes in the means of ξ_1 and ξ_2, γ_3 itself is unaffected by changes in the means of ξ_1 and ξ_2. Thus, with respect to the estimation of γ_3, it can safely be assumed that the means of ξ_1 and ξ_2 are zero. In contrast, the coefficients of ξ_1 and ξ_2 depend on the means of ξ_1 and ξ_2, and it is not possible to recover these coefficients when the means of the simple exogenous constructs are not actually equal to zero and the observed variables have been mean centered.

The foregoing derivations are based on population variances and covariances of the observed variables. In practice, only sample estimates of the population values are available. However, if the model is specified correctly and if the population variance-covariance matrix is consistently estimated by the sample variance-covariance matrix, the value obtained for γ_3 based on sample statistics will be a consistent estimate of the true parameter value.

The ability to assume that the means of exogenous constructs are zero constitutes a simplification of the analysis but, more important, makes the interpretation of interaction terms meaningful even when the means of simple exogenous constructs cannot be estimated meaningfully. It is possible to specify a model that explicitly incorporates the means of exogenous constructs into the analysis, but the procedure requires that these latent means be estimated. This can be done by equating the latent mean of an exogenous construct to the observed mean of one of its indicators (cf. Schoenberg 1984). However, this is only meaningful if the indicator is measured on a ratio scale, otherwise the estimate of the latent mean is completely arbitrary. Thus in many cases the procedure breaks down because the available measures have at most interval quality. The fact that the coefficient of the interaction term does not depend on the means of the variables involved therefore represents a significant gain in the applicability of moment structure analysis with multiplicative exogenous constructs.[4] It should be acknowledged, however, that mean centering makes the interpretation of the coefficients of the linear terms problematic, and if there is any substantive interest in the linear effects, observed variables will have to be measured on a ratio scale and their means incorporated into the analysis.

ESTIMATION AND TESTING

Two problems arise with respect to model estimation. One is that various nonlinear constraints have to be observed during estimation (equations 10 and 14-16). Until recently, specialized computer programs such as COSAN (Fraser 1980, based on the work of McDonald 1978, 1980), which were not widely available, or trick procedures such as those proposed by Hayduk (1987; see also Rindskopf 1984 and

Wong and Long 1987), which made the estimation of even fairly simple models rather cumbersome, had to be used to impose the necessary nonlinear constraints on model parameters. However, the latest versions of several widely available computer programs allow the user to impose nonlinear constraints in a rather straightforward way so that the availability of suitable software is no longer a major concern.

The second, more serious problem is that commonly used estimation methods such as maximum likelihood (ML) and generalized least squares (GLS) require that the observed variables be distributed multivariate normally (for ML estimation) or at least that fourth-order cumulants of the multivariate observations be zero (for GLS estimation). The assumption underlying GLS is slightly less restrictive than the assumption of multivariate normality. The multiplicative observed indicators and the indicator(s) of the endogenous construct(s) in the models considered here are functions of products of normally distributed variates and thus cannot be normally distributed (cf. Kendall and Stuart 1963). Although the estimates obtained through ML and GLS will be consistent, χ^2 goodness-of-fit statistics and standard errors may be misleading. That this is indeed the case will be demonstrated below.

One way to get around this problem is to use an asymptotically distribution-free (ADF) method of estimation. The primary advantage of ADF procedures is that, under mild assumptions, asymptotically efficient estimates and asymptotically valid χ^2 goodness-of-fit statistics and standard errors can be obtained regardless of the form of the distribution underlying the observed variables (cf. Browne 1982, 1984). However, a weight matrix has to be estimated that is a quartic function of the number of variables in the model (Arminger and Schoenberg 1989), and this requirement may make ADF estimation infeasible in practical situations.

A second way to address the problem of nonnormality is to base goodness-of-fit assessments and tests of significance of individual parameters on corrected χ^2 statistics and standard errors that take into account violations of normality. Bentler (1989) referred to these statistics as robust χ^2 values and standard errors. With robust estimation and testing, the parameter estimates are the same as those for ML and GLS, but the χ^2 values and standard errors are adjusted to correct for the lack of normality in the data. The primary advantage of this approach is that it eliminates the need to estimate the weight matrix necessary for the application of ADF procedures. Furthermore, recent simulation evidence by Hu, Bentler, and Kano (1992) indicates that robust estimation performs well when the data are nonnormal and may outperform ADF methods at sample sizes commonly encountered in practice.

To investigate the applicability of ML, GLS, ML robust, GLS robust, and weighted least squares (WLS) to the estimation and testing of structural equation models with latent variable interactions, we performed the following simulation study using the model specified in equations (1) through (10) and depicted graphically in Figure 1. One hundred different random samples of 500 observations each were generated from a multivariate normal distribution for the 7 latent variates ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, and ζ using the GGNSM subroutine in the IMSL software package (IMSL 1980). In the population, the 7 variates had means of zero and were mutually uncorrelated, with the exception of ξ_1 and ξ_2, which were assumed to have a correlation of .2. The population variances assumed for the 7 latent variates are given in the first column of Table 1. From the simulated data, observations on y, x_1, x_2, x_3, and x_4 were obtained using equations (2) to (6) and the coefficient values given in Table 1. Product indicators were formed by multiplying simple indicators, as shown in equations (7) to (10). All variables were mean centered before multiplicative indicators were obtained.[5]

We used LISREL 8 (Jöreskog and Sörbom 1993a) to get ML, GLS, and ADF estimates of all model parameters for the 100 different variance-covariance matrices of the observed measures. The weight matrix necessary for ADF estimation (which is referred to as WLS estimation in LISREL) was obtained through PRELIS 2 (Jöreskog and Sörbom 1993b).[6] To get ML and GLS robust estimates, we used EQS (Bentler 1989). In LISREL—as well as in several other programs such as the PROC CALIS procedure in SAS (SAS Institute 1989), LINCS (Schoenberg 1989), and COSAN (Fraser 1980)—the nonlinear constraints can be specified in a straightforward manner.[7] The method suggested by Hayduk (1987) was used to estimate the model in EQS.[8]

To assess how well the underlying model could be recovered, we computed the bias and mean squared error (MSE) associated with each parameter and estimation procedure. Bias indicates the deviation of

TABLE 1: **Means of Parameter Estimates and Percentage Bias for Different Estimation Procedures**

		ML[a]		*GLS*[b]		*WLS*[c]	
Parameter	*True Value*	*Means of Parameter Estimates*	*Percentage Bias*	*Means of Parameter Estimates*	*Percentage Bias*	*Means of Parameter Estimates*	*Percentage Bias*
λ_{21}	0.600	0.601	0.0024	0.601	0.0010	0.600	0.0004
λ_{42}	0.700	0.699	–.0011	0.700	0.0002	0.695	–0.0064
$\lambda_{21}\lambda_{42}$	0.420	0.420	0.0015	0.421	0.0015	0.418	–0.0056
θ_{11}	0.810	0.812	0.0022	0.788***	–0.0271	0.755*	–0.0674
θ_{22}	0.490	0.485	–0.0110	0.473*	–0.0356	0.449*	–0.0829
θ_{33}	0.640	0.624	–0.0248	0.609*	–0.0484	0.574*	–0.1024
θ_{44}	1.000	1.004	0.0042	0.976*	–0.0241	0.931*	–0.0687
$\theta_{11}\theta_{33}$	0.518	0.507	–0.0219	0.480*	–0.0738	0.434*	–0.1638
$\theta_{11}\theta_{44}$	0.810	0.816	0.0076	0.770*	–0.0496	0.703*	–0.1315
$\theta_{22}\theta_{33}$	0.314	0.303***	–0.0352	0.288*	–0.0824	0.258*	–0.1773
$\theta_{22}\theta_{44}$	0.490	0.487	–0.0065	0.461*	–0.0584	0.419*	–0.1458
γ_1	–0.150	–0.160**	0.0692	–0.160**	0.0671	–0.157	0.0490
γ_2	0.350	0.339	–0.0301	0.339	–0.0312	0.339	–0.0302
γ_3	0.700	0.695	–0.0073	0.693	–0.0104	0.686**	–0.0193
ϕ_{11}	2.560	2.562	0.0012	2.511**	–0.0308	2.376*	–0.1151
ϕ_{21}	0.384	0.393	0.0329	0.382	–0.0080	0.337*	–0.1692
ϕ_{22}	1.440	1.440	–0.0003	1.409**	–0.0262	1.357*	–0.0694
ϕ_{33}	3.834	3.859	0.0065	3.695**	–0.0361	3.343*	–0.1280
ψ_{11}	0.160	0.146	–0.0849	0.170	0.0654	0.195*	0.2195

NOTE: Parameter estimates marked with an asterisk are significantly different from the population value, with *p <. 01; **p < .05; ***p < .10.
a. ML = maximum likelihood.
b. GLS = generalized least squares.
c. WLS = weighted least squares.

TABLE 2: Mean Squared Error by Method of Estimation

Parameter	*ML*[a]	*GLS*[b]	*WLS*[c]
λ_{21}	0.0010	0.0010	0.0010
λ_{42}	0.0033	0.0033	0.0038
$\lambda_{21}\lambda_{42}$	0.0018	0.0018	0.0021
θ_{11}	0.0143	0.0140	0.0164
θ_{22}	0.0024[d]	0.0026[d]	0.0037[e]
θ_{33}	0.0120[d]	0.0121[d, e]	0.0150[e]
θ_{44}	0.0079[d]	0.0084[d]	0.0122[e]
$\theta_{11}\theta_{33}$	0.0139	0.0136	0.0172
$\theta_{11}\theta_{44}$	0.0205	0.0198	0.0269
$\theta_{22}\theta_{33}$	0.0038	0.0040	0.0059
$\theta_{22}\theta_{44}$	0.0044	0.0050	0.0085
γ_1	0.0023	0.0023	0.0023
γ_2	0.0051	0.0051	0.0057
γ_3	0.0039	0.0039	0.0041
ϕ_{11}	0.0533[d]	0.0541[d]	0.0863[e]
ϕ_{21}	0.0144	0.0133	0.0146
ϕ_{22}	0.0234	0.0234	0.0291
ϕ_{33}	0.3435	0.3116	0.4494
ψ_{11}	0.0237[d]	0.0213[d, e]	0.0177[e]

NOTE: Estimates of mean squared error marked with different superscripts are significantly different from each other at $\alpha = .05$.
a. ML = maximum likelihood.
b. GLS = generalized least squares.
c. WLS = weighted least squares.

sample estimates from the true parameter value in the population, and MSE measures the spread of estimates around the true value, trading off unbiasedness with precision. Mean parameter estimates across the 100 samples and percentage bias for each estimation procedure are given in Table 1, and the results for MSE are contained in Table 2.

Table 1 shows that all ML estimates except one are within sampling fluctuation of the true value. In contrast, a substantial number of GLS and WLS estimates differ significantly from their true values (at $\alpha = .05$). However, in percentage terms bias is generally small, and in most cases significant biases lead to underestimation of the true value. Table 2 shows that, in spite of the findings for bias, all three estimation procedures performed about equally well in terms of MSE. The findings thus indicate that all three estimation procedures recovered

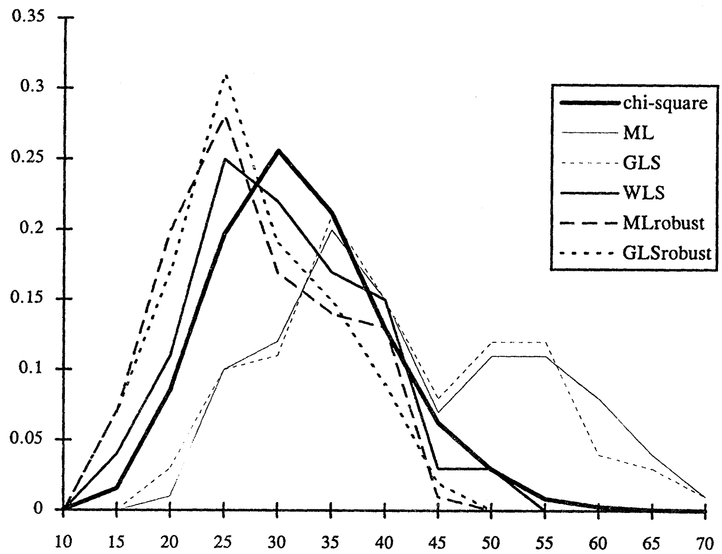

Figure 2: Empirical Distribution Functions of c2 Values Obtained From Different Estimation Methods

the underlying model well, with similar performance in terms of MSE even though GLS and WLS yielded a greater proportion of biased estimates than ML.

Once the parameters of the specified model have been estimated, the researcher would like to ascertain how well the model fits the data and whether the estimated parameters are significantly different from zero. Without these statistics, the estimation results are of limited value. To see how useful ML, GLS, ML robust, GLS robust, and WLS procedures are for this purpose, we looked at goodness-of-fit statistics and estimated standard errors across the 100 data sets.

Figure 2 shows the theoretical distribution function for a χ^2 variate with 32 degrees of freedom and the five empirical distribution functions for the χ^2 values obtained through ML, GLS, ML robust, GLS robust, and WLS estimation. It is apparent that the empirical distribution function for WLS estimation closely matches the theoretically expected distribution of χ^2 values, whereas the distribution functions for ML and GLS estimation have much too heavy right-hand tails. ML and GLS robust χ^2 values approximate the theoretically expected distribution function better than their nonadjusted counterparts, but as Figure 2 shows there is a tendency toward overcorrection.

These conjectures are confirmed by Kolmogorov-Smirnov tests, which indicate that, for ML, GLS, ML robust, and GLS robust, the hypothesis that the obtained fit values are distributed as χ^2 with 32 degrees of freedom is rejected ($p < .001$), whereas for WLS this hypothesis cannot be rejected ($p > .10$). The rejection rates for α levels of .1, .05, and .01 are 42%, 36%, and 21% for ML; 39%, 33%, and 16% for GLS; 1%, 0%, and 0% for ML robust; 2%, 0%, and 0% for GLS robust; and 6%, 3%, and 0% for WLS. It is apparent that too many models get rejected by ML and GLS and that too few models get rejected by ML and GLS robust. On the other hand, the WLS rejection rates are in rough agreement with theoretical expectations, although a slight tendency toward overcorrection is apparent.

As is to be expected from the foregoing results, a multivariate repeated measures analysis of variance shows that there are significant differences in the magnitude of χ^2 values obtained by the five estimation methods. ML and GLS χ^2 values are significantly larger than WLS χ^2 values ($p < .001$), whereas ML and GLS robust χ^2 values are significantly smaller than WLS χ^2 values ($p < .001$). In addition, both adjusted and unadjusted ML χ^2 values are significantly larger than their GLS counterparts ($p < .001$), although the absolute magnitude of the latter difference is small.

Recent research has indicated that, among the many alternative fit indexes that have been suggested in the literature, the Comparative Fit Index proposed by Bentler (1990)—which in most cases is equal to the Relative Noncentrality Index (RNI) suggested by McDonald and Marsh (1990)—and the Tucker and Lewis (1973) Index (TLI) have the most desirable properties (cf. Marsh, Balla, and McDonald 1988). Both of these indexes are relative fit indexes, and the baseline model in the present case is the one of complete independence among the observed measures. The CFI and TLI are very close to one for all estimation methods (means of .995, .999, and 1.000 for ML, GLS and WLS, respectively, for both CFI and TLI). Although the differences in CFI and TLI values obtained by the three estimation methods are small in absolute magnitude, they are highly reliable; WLS models fit better than GLS models, which in turn fit better than ML models (all

p values smaller than .001). We also computed Steiger's (1990) Root Mean Square Error of Approximation (RMSEA), which assesses model discrepancy per degree of freedom. The means for ML, GLS, and WLS were .021, .020, and .007, respectively, and the p values for the test that RMSEA is smaller than .05 indicated close fit in every instance (cf. Browne and Cudeck 1993). However, as in the case of CFI and TLI, WLS models fit reliably better than GLS models, which in turn fit better than ML models (p values smaller than .001).

To assess the validity of significance tests for individual parameters, we compared the means of the estimated standard errors across the 100 data sets and the standard deviations of parameter estimates across samples. The results are shown in Table 3. Three things are apparent in Table 3. First, the standard deviations of estimates are quite similar for all three estimation methods. Second, the mean standard errors are consistently lower for ML and GLS than for ML robust, GLS robust, or WLS. Multivariate repeated measures analyses of variance show that these differences are highly significant ($p < .001$). WLS standard errors are reliably lower than ML and GLS robust estimates in most cases, but the absolute magnitude of this difference is small. Third, standard errors provided by ML and GLS, which are based on normal distribution theory, are consistently lower than the standard deviations of parameter estimates across the 100 sets of data. This is not the case for ML robust, GLS robust, and WLS, where the agreement between the two values is generally much closer than for ML and GLS and where the estimated standard errors do not consistently exaggerate the statistical significance of the estimated parameters. Thus ML and GLS robust standard errors and WLS standard errors yield similar conclusions about the significance of individual parameter estimates, and these conclusions are in line with the inherent variability of parameter estimates at the specified sample size.

In summary, although ML and GLS performed well with respect to model estimation, our simulation suggests that the χ^2 statistics and standard errors based on normal distribution theory may not be trustworthy. On the other hand, if the sample size is large enough and if formulas (12) and (13) are applicable, ML robust, GLS robust, and WLS estimation appear to be useful methods for obtaining appropriate χ^2 goodness-of-fit statistics and standard errors.[9] Although there was a tendency for ML and GLS robust χ^2 values to overcorrect for lack of normality, the corresponding standard errors accurately reflected inherent sampling variability. If WLS is not applicable because of insufficient sample size, robust ML and GLS estimation and testing seem most appropriate. In cases where the sample size is large enough for the WLS approach to be feasible, we would recommend that both sets of procedures be applied so that their convergence can be assessed.

AN ILLUSTRATIVE EXAMPLE OF THE USE OF MULTIPLICATIVE STRUCTURAL EQUATION MODELS

To illustrate the potential of structural equation modeling with interactions for investigating substantive problems, we applied the procedure to the context of expectancy-value attitude models. Specifically, we tested whether, as hypothesized by expectancy-value theory (e.g., Fishbein and Ajzen 1975), beliefs and evaluations interact in determining overall attitude. The attitudinal act of interest was consumers' use of coupons for grocery shopping, and the two consequences of coupon use considered in our example were the beliefs that using coupons would lead to savings on the grocery bill and that using coupons would contribute to a feeling of being a thrifty shopper. Although a variety of other beliefs underlie consumers' attitudes toward using coupons, Shimp and Kavas (1984) have shown that the rewarding consequences of coupon use considered here are by far the most important determinants of consumers' overall attitudes.

A total of 253 female staff members at two large public universities responded to a written questionnaire that was sent to them via campus mail. Beliefs about the two rewarding consequences of coupon use were assessed in the usual way on 7-point likely-unlikely scales, and the associated evaluations were measured on 7-point good-bad scales. Overall attitude toward using coupons for grocery shopping in the upcoming week was assessed with three 7-point semantic differential scales (pleasant-unpleasant, good-bad, and favorable-unfavorable). The model shown in Figure 1 was estimated by ML, GLS, ML robust, GLS robust, and WLS methods (the only difference being that three indicators of η were available), and the results are shown in Table 4.[10]

(Text continues on page 208)

TABLE 3: Standard Deviations of Estimates Across Simulated Samples and Means of the Estimated Standard Errors for Different Estimation Procedures

	ML[a]		*GLS*[b]		*WLS*[c]	
Parameter	*Standard Deviation of Estimates*	*Mean Standard Error*	*Standard Deviation of Estimates*	*Mean Standard Error*	*Standard Deviation of Estimates*	*Mean Standard Error*
λ_{21}	0.031	0.023 (0.035)	0.032	0.024 (0.036)	0.032	0.032
λ_{42}	0.058	0.037 (0.060)	0.058	0.039 (0.061)	0.062	0.054
$\lambda_{21}\lambda_{42}$	0.042	0.028 n.a.	0.042	0.029 n.a.	0.046	0.040
θ_{11}	0.120	0.090 (0.117)	0.117	0.091 (0.116)	0.117	0.104
θ_{22}	0.049	0.036 (0.050)	0.049	0.037 (0.050)	0.046	0.044
θ_{33}	0.109	0.071 (0.100)	0.106	0.072 (0.101)	0.104	0.092
θ_{44}	0.089	0.054 (0.083)	0.089	0.054 (0.081)	0.087	0.069
$\theta_{11}\theta_{33}$	0.118	0.078 n.a.	0.111	0.077 n.a.	0.101	0.092
$\theta_{11}\theta_{44}$	0.144	0.095 n.a.	0.136	0.094 n.a.	0.126	0.110

$\theta_{22}\theta_{33}$	0.061	0.040	0.058	0.039	0.053	0.049
		n.a.		n.a.		
$\theta_{22}\theta_{44}$	0.067	0.041	0.065	0.040	0.058	0.051
		n.a.		n.a.		
γ_1	0.047	0.030	0.047	0.031	0.048	0.039
		(0.040)		(0.040)		
γ_2	0.071	0.044	0.071	0.045	0.075	0.052
		(0.053)		(0.054)		
γ_3	0.062	0.041	0.062	0.042	0.063	0.059
		(0.059)		(0.061)		
ϕ_{11}	0.232	0.156	0.228	0.156	0.230	0.207
		(0.262)		(0.253)		
ϕ_{21}	0.120	0.102	0.116	0.100	0.112	0.094
		(0.107)		(0.104)		
ϕ_{22}	0.154	0.102	0.150	0.103	0.150	0.139
		(0.168)		(0.165)		
ϕ_{33}	0.588	0.329	0.543	0.023	0.459	0.455
		n.a.		n.a.		
ψ_{11}	0.154	0.095	0.146	0.094	0.129	0.112
		(0.122)		(0.122)		

NOTE: Numbers in parentheses are robust standard errors. The notation n.a. means that robust standard errors are not available in these cases.
a. ML = maximum likelihood.
b. GLS = generalized least squares.
c. WLS = weighted least squares.

TABLE 4: Results for Illustrative Example

	ML[a]			*GLS*[b]			*WLS*[c]		
Parameter	*Estimate*	*Standard Error*	T *Value*	*Estimate*	*Standard Error*	T *Value*	*Estimate*	*Standard Error*	T *Value*
λy_{21}	1.05	0.08	13.53	1.07	0.08	12.82	0.99	0.07	13.63
		(0.09)	(11.82)		(0.10)	11.14)			
λy_{31}	1.16	0.08	13.68	1.15	0.09	12.97	1.11	0.08	14.61
		(0.09)	(12.78)		(0.09)	(12.24)			
$\theta\varepsilon_{11}$	0.71	0.08	8.88	0.68	0.08	8.66	0.67	0.11	5.88
		(0.12)	(5.82)		(0.12)	(5.68)			
$\theta\varepsilon_{22}$	0.46	0.07	7.07	0.42	0.06	6.69	0.48	0.11	4.63
		(0.13)	(3.84)		(0.14)	(3.14)			
$\theta\varepsilon_{33}$	0.50	0.08	6.61	0.49	0.07	6.74	0.25	0.07	3.40
		(0.12)	(4.11)		(0.12)	(4.20)			
λx_{21}	1.14	0.13	8.78	1.09	0.10	11.01	0.91	0.06	15.72
		(0.17)	(6.88)		(0.13)	(8.16)			
λx_{42}	0.86	0.08	11.44	0.90	0.06	15.65	0.84	0.04	19.38
		(0.08)	(10.79)		(0.06)	(13.97)			
$\theta\delta_{11}$	0.84	0.10	8.54	0.69	0.08	8.39	0.47	0.08	6.01
		(0.12)	(7.29)		(0.12)	(5.93)			
$\theta\delta_{22}$	1.19	0.13	9.05	0.58	0.09	6.34	0.56	0.08	6.68
		(0.38)	(3.11)		(0.17)	(3.34)			
$\theta\delta_{33}$	0.30	0.04	6.77	0.18	0.03	5.46	0.11	0.02	4.75
		(0.08)	(3.70)		(0.03)	(5.17)			
$\theta\delta_{44}$	0.54	0.04	12.28	0.31	0.03	9.53	0.19	0.03	6.33
		(0.11)	(4.84)		(0.06)	(4.89)			

γ_1	−0.42	0.39 (0.76)	−1.08 (−0.55)	0.03	0.16 (0.15)	0.16 (0.17)	0.10	0.09	1.09
γ_2	1.58	0.50 (1.02)	3.14 (1.55)	0.95	0.21 (0.21)	4.63 (4.51)	0.85	0.15	5.57
γ_3	0.17	0.11 (0.11)	1.59 (1.56)	0.13	0.09 (0.08)	1.53 (1.61)	0.15	0.06	2.73
ϕ_{11}	0.68	0.10 (0.16)	6.87 (4.20)	0.75	0.09 (0.14)	8.08 (5.36)	0.95	0.11	8.79
ϕ_{21}	0.46	0.04 (0.07)	10.35 (7.03)	0.48	0.04 (0.04)	11.11 (11.12)	0.52	0.02	21.83
ϕ_{22}	0.45	0.05 (0.10)	8.69 (4.62)	0.53	0.05 (0.07)	10.65 (7.56)	0.54	0.04	12.95
ψ_{11}	0.40	0.12 (0.20)	3.26 (1.99)	0.49	0.09 (0.10)	5.51 (4.84)	0.57	0.09	6.46
Goodness of fit		$\chi^2(48) = 245.38$ (111.43) CFI[d] = 0.84, TLI[e] = 0.82, RMSEA[f] = .13 ($p < .0001$) $R^2 = .62$			$\chi^2(48) = 174.64$ (89.77) CFI = 0.98, TLI = 0.97 RMSEA = .10 ($p < .0001$) $R^2 = .51$			$\chi^2(48) = 112.81$ CFI = 0.99, TLI = 0.99 RMSEA = .07 ($p = .02$) $R^2 = .47$	

NOTE: Numbers in parentheses are robust standard errors, t values, and χ^2 statistics.
a. ML = maximum likelihood.
b. GLS = generalized least squares.
c. WLS = weighted least squares.
d. CFI = Comparative Fit Index.
e. TLI = Tucker and Lewis Index.
f. RMSEA = Root Mean Square Error of Approximation.

Table 4 indicates that the models estimated by ML robust, GLS robust, and WLS fit the data fairly well, yielding similar overall χ^2 goodness-of-fit tests. The two incremental fit indexes (CFI and TLI) also show that the GLS and WLS models are in agreement with the data. In contrast, the fit of the ML model is relatively poor, in terms of both the (uncorrected) χ^2 statistic and the two alternative fit indexes. The RMSEA index indicates a lack of fit for ML and GLS and a borderline fit for WLS. Beliefs, evaluations, and the interaction of beliefs and evaluations account for about 50% of the variance in overall attitude. All individual parameter estimates are reasonable, and most are similar in magnitude and level of statistical significance across estimation methods. As hypothesized by expectancy-value attitude theory, the data suggest that attitudes toward using coupons are an interactive function of consumers' beliefs that using coupons will have rewarding consequences and their evaluative reactions toward these consequences.[11] In the case of ML and GLS, the interaction of expectancies and values is only of borderline significance, but for WLS it is clearly significant. The latter finding is consistent with the results of a regression analysis in which the addition of an expectancy-value term (i.e., the product of the average belief rating for the two consequences and the average of their evaluations) to a model already containing the main effects of beliefs and evaluations accounts for a significant incremental portion of the variance in consumers' overall attitude toward using coupons ($t_{249} = 2.34, p < .05$).[12]

In summary, the procedures discussed in this article seem to lead to a valid structural equations specification of expectancy-value models, in contrast to early applications of latent variable modeling in this context (cf. Busemeyer and Jones 1983). The results based on ML robust, GLS robust, and WLS estimation are reasonable and suggest that the method may have potential for investigations of theoretical frameworks specifying interaction effects among latent exogenous constructs.

CONCLUSION

We have shown how methods for the analysis of moment structures based on latent variates can be properly used to test theories hypothesizing interactive effects of exogenous constructs on endogenous constructs. The procedure discussed in this article involves an explicit consideration for product indicators and the resulting nonlinearities in the variance-covariance matrix of the exogenous latent variates and the matrix of factor loadings. In an extension of the original work of Kenny and Judd (1984), we also demonstrated that the structural coefficient relating an endogenous construct to the interaction of exogenous constructs is invariant under changes in the origin of the components of the multiplicative exogenous construct.

With respect to model estimation, we found that, within the specific context of the interaction of two exogenous constructs, each measured by two indicators, mean squared errors of parameter estimates were comparable across estimation methods, even though GLS and WLS led to a greater proportion of biased estimates. However, in percentage terms, bias was generally slight. With respect to model testing, we found that χ^2 values based on normal distribution theory led to the rejection of too many models, but that assessment of overall fit based on ADF estimation procedures yielded appropriate results. Although ML and GLS robust χ^2 values tended to overcorrect for the violation of the normality assumption, they approximated the theoretically expected distribution better than their nonadjusted counterparts. Standard errors of parameters based on normal distribution theory were also found to be misleading because of exaggerated significance levels, whereas the standard errors reported by ML robust, GLS robust, and WLS usually were in line with the standard deviation of parameter estimates across samples. An empirical illustration of the procedure in the context of expectancy-value attitude models confirmed that multiplicative structural equation models may have potential in investigating theories that specify interactions among exogenous constructs.

In summary, this article shows how multiplicative structural equation models can be specified properly so that coefficients relating endogenous constructs to latent interaction terms are invariant under changes in the origin of the components of the multiplicative construct. Furthermore, these models can now be easily estimated and tested using programs that allow the researcher to impose nonlinear constraints on parameter estimates. The use of ADF procedures yielded promising results, but the problem is that fairly large sample sizes are required for their applicability. A more practical alternative may be the use of robust ML or GLS procedures, which led to similar results as WLS estimation but which can be used even when the sample size is smaller. Future research will have to investigate what the minimum sample size requirements for the application of structural equation models with interactions are. We would also recommend that both the straightforward imposition of nonlinear constraints on model parameters and the provision of robust test statistics be implemented in commonly used computer programs so that the models discussed in this article can be estimated and tested in a routine fashion.

NOTES

1. Kenny and Judd (1984) also considered the specification and estimation of models with quadratic exogenous constructs. We will only deal with interactions among exogenous constructs, but our discussion of estimation and testing should readily transfer to models with quadratic terms. Furthermore, we will not deal with interactions in the measurement model (cf. Etezadi-Amoli and McDonald 1983; Mooijaart and Bentler 1986).

2. We use the general expression *exogenous latent variate* for terms such as ξ_1, δ_1, $\xi_1\xi_2$, $\xi_1\delta_3$, and $\delta_1\delta_3$. The expression *exogenous construct* is reserved for latent variates that are of theoretical interest (i.e., ξ_1, ξ_2, $\xi_1\xi_2$).

3. If the normality assumption does not hold, equations (12) and (13) may be used as asymptotic approximations. The usual asymptotic approximation procedure for the covariance of two product variables (see Kendall and Stuart 1963, p. 232; cf. Bohrnstedt and Goldberger 1969) would yield a value of zero when the expected values of all simple variables are assumed to be zero. If normality cannot be assumed but a, b, c, and d have expected values of zero, the exact expressions corresponding to (12) and (13) are given by C[ab,c] = E[abc] and C[ab,cd] = E[abcd] – C[a,b]C[c,d].

4. After we had completed work on this article, we received a working paper by Jöreskog and Yang (1995) in which they argue that the means of the observed variables have to be incorporated into the specification of the model and that estimation should be based on weighted least squares based on the augmented moment matrix. The issues involved are complex and will have to be investigated in more detail in future research.

5. The specific parameter values assumed for the underlying structure are similar but not identical to the ones used by Kenny and Judd (1984). Note that we followed Kenny and Judd (1984) in generating simulated data for the 7 latent variates ξ_1, ξ_2, δ_1, δ_2, δ_3, δ_4, and ζ because we were only interested in the effects of nonnormality resulting from the multiplication of normal variates.

6. PROC CALIS does at present not offer asymptotically distribution-free methods of estimation.

7. Tests of multivariate normality based on skewness and kurtosis indicated that the normality assumption was violated in each of the 100 samples (all p's smaller than .0001). Tests of univariate normality showed that this was caused by the product indicators and the indicator of the endogenous construct (which is a function of nonnormal variables), which showed marked

deviations from normality. In contrast, the distribution of the nonproduct indicators generally conformed to normality.

8. LINCS (Schoenberg 1989) allows the user to impose nonlinear constraints on the parameters and also provides corrected standard errors that take into account nonnormality. Although this makes LINCS easier to use than EQS in the present application, we did not have access to LINCS and therefore used EQS. Note that, for certain model parameters, the Hayduk procedure actually estimates functions of the parameters of interest (e.g., the square root of θ_{11}, rather than θ_{11}). In these cases, the delta method was used to transform the robust standard errors (cf. Miller 1986).

9. Our results thus provide a more promising assessment of the utility of asymptotically distribution-free methods than the recent article by Hu, Bentler, and Kano (1992).

10. Although statistical tests indicated that the distributions of x_1, x_2, x_3, and x_4 deviated from normality, these violations were less severe than for the product indicators. For example, the univariate kurtoses were .48, 1.86, –.67, and –.59 for the simple indicators versus 2.13, 8.03, 9.53, and 8.94 for the product indicators. However, the lack of normality and the relatively small sample size imply that the results have to be interpreted with caution.

11. Note that the interpretation of the main effects of beliefs and evaluations on attitude is problematic because they are not invariant under changes in the means of ξ_1 and ξ_2.

12. The three indicators of η were also averaged to obtain a single dependent variable in the regression analysis.

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