MORPHOLOGY OF TJOARVEKRAJGGE,- THE LONGEST CAVE OF SCANDINAVIA

TORSTEIN FINNESAND[1] and RANE CURL[2]
[1]The Tjorve Project, Sølve Solfengsvei 4, N-0956 Oslo, Norway. finnesand@getmail.no
[2]Dept. Chemical Engng, Univ. of Michigan, Ann Arbor, MI 48109-2136. ranecurl@umich.edu

Tjoarvekrájgge (Tjorve), with a surveyed length of 21,814 meters, the longest cave of Scandinavia, is found in one of four marble bands of stripe karst in Bonå, some miles north of the Polar circle in Norway. The cave is a two-dimensional labyrinth system situated close to a shoulder of a "U" shaped valley. Morphometric and fractal analysis can be made with over 99 % of the passage dimensions.

Morphometric parameters of Tjorve yield a passage density of 47.5 km/km^2 and a cave porosity of 0.8 %, intermediate between the values of confined and unconfined settings, and an areal coverage of 21.8 %, close to the values for confined settings. Values for the uppermost part of the cave (cave porosity: 3.6 %, areal coverage: 32.6 %) are closer to or within the values for confined settings. The values might reflect a cyclic development of the cave over several glacial-interglacial cycles. Four levels in the cave can be discerned in vertical profile, possibly corresponding to ancient water tables that have been step-wise lowered in successive glacial periods. Tjorve may have developed over a long time-period, from perhaps the Tertiary.

The Linked Modular Element (LME) method (Curl 1986: http://tinyurl.com/6o53kd) is applied to Tjorve to determine the distribution of cave passage sizes. The distribution of LME sizes fit a power-law function from 1.8 to 5.9 m and exhibits a fractal dimension of 2.929 (s.d. 0.068), similar to Little Brush Creek Cave (LBC), Utah (fractal dimension 2.79). The proper modulus is near 1.1 m, compared to 0.6 m for LBC, indicating perhaps less complete exploration.

1. Introduction

Tjoarvekrájgge (Tjorve), with a surveyed length of 21 814 meters and a depth of c. 497 m, - the longest cave of Scandinavia - is found in one of four marble bands of stripe karst (Horn 1937) in Bonådalen, Nordland county, some kilometers north of the Polar circle in Norway.

Bonådalen is a north-south "U"-shaped valley, widened and deepened by the glaciers in the last 2.5 million years. Tjorve is situated close to the western shoulder of the valley. The marble band is 50 to 60 m thick, surrounded by insoluble mica schist. The marble dips 25 to 40 degrees to the south and southeast (following the local folding), adding depth to the cave system. The resurgence is at 84 masl, close to the valley bottom. Tjorve has five known proper entrances. They have no drainage area today. There is a short cave above Tjorve, Stoppenålen (496 m long and 190 m deep), leading straight toward Tjorve, but without obvious proper connections.

Tjorve has tubes, canyons, rock blocks, and clay. The tubes follow the "Tjorve plane" (Fig. 1) – horizontal in an east-west direction (the strike) and sloping in the dip direction. Canyons above the groundwater level also follow the dip. Large areas of the upper parts of Tjorve contain boulders, mostly from breakdown, but also injected during glaciations. The clay deposits are especially prominent in the phreatic tubes in the upper part of the cave, but can be found in most other places, including on breakdown.

The survey is done to BCRA grade 5, using Suunto compass and clinometer, tape and in recent years laser meters and digital clinometers. Due to many loops and side passages, stations are placed on bedrock, boulders and clay, and are normally properly marked. The survey includes 214 loops, with an average loop closure of 2.1 %. Survey data are downloaded into an Excel file developed for the Tjorve project. Export can be done to Compass, Therion and Excel workbooks for additional analysis. A total of 2 805 valid survey shots have been recorded. Over 99 % of the shots have passage dimensions, which allow morphometric and fractal analysis.

2. Morphometric Analysis

Morphometric parameters (Klimchouk, 2003) of Tjorve

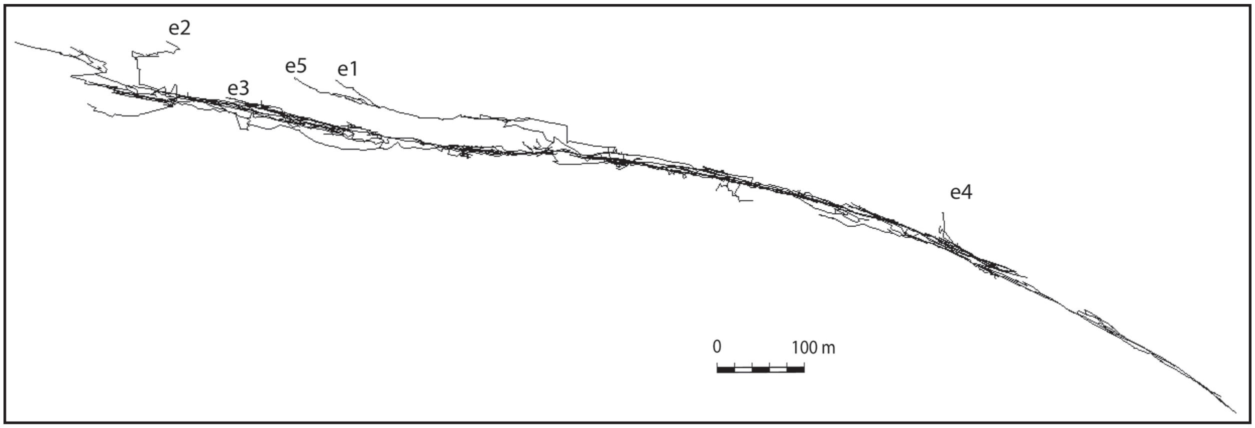

Figure 1: Tjorve looking northeast (40o) and looking up a slope (+24o). From this view the Tjorve plane is a thin line. The invasion passages can be identified. (from Finnesand et al. 2007). Proper entrances are numbered.

and parts of Tjorve (Table 1) can be calculated in different ways. Our definitions are:

1 Surveyed passages (column 2, 3, 4 and 5 in Table 1) use survey data: Length is given as surveyed (3D) and projected horizontally (2D). Surveyed area is the plan area of the passages (seen from above, i.e. passages situated above others (seldom found in Tjorve) are not included). Volume is the horizontal length multiplied with an elliptical cross-sectional area (from left-right and up-down (LRUD) measurements at stations).

2 Cave extent (column 6, 7 and 8 in Table 1) is the two- and three-dimensional area that the cave occupies. The two dimensional extent is a horizontal area (cave field) calculated from a polygon surrounding the cave. The height is calculated at the location where the vertical distance between the lowest and highest point is largest (minimum value would be the highest passage). Volume is the part of the rock volume (cave field x height) in which the cave is developed.

The passage density (47.5 km/km^2) and the cave porosity (1.0 %) are intermediate between the values of confined and unconfined (after Klimchouk, 2003) settings, and areal coverage (22.1 %) is close to the values for confined settings. Almost all (93 % of the length) of Tjorve lies in the Tjorve plane (Fig. 1), which has a higher porosity (2,2 %) than Tjorve. The missing 7 % is mainly the first few hundred meters of passages from three entrances, which appear to be invasion systems (Fig. 1).

Values for the uppermost and old part of the cave such as Galleries (cave porosity: 3.6 %, areal coverage: 32.6 %) are closer to or within the values for confined settings. These intermediate values might also reflect a cyclic development of the cave over several glacial-interglacial cycles. In a vertical profile (Fig. 2) one can possibly discern four levels in the cave, corresponding to ancient water tables that have been step-wise lowered by glacial erosion during the glacial periods. If this model holds true, Tjorve must have developed over a long time-period, perhaps originating in the Tertiary.

	Surveyed passages				Cave extent			Specific	Passage	Areal	Cave
Cave, or part of the cave	length (3D) km	length (2D) km	area km^2	volume m^3*10^6	area km^2	height m	volume m^3*10^6	volume m^3/m	density km/km^2	coverage %	porosity %
Tjoarvekrajgge	21.814	19.658	0.090	0.224	0.414	64.0	26.5	10.3	47.5	21.8	0.8
- Tjorve plane	20.279	18.409	0.087	0.199	0.414	22.0	9.1	9.8	44.5	20.9	2.2
- Galleries	8.807	8.037	0.051	0.120	0.157	21.0	3.3	13.6	51.2	32.6	3.6
- Down below	2.239	1.975	0.007	0.009	0.064	6.5	0.4	4.0	30.9	11.6	2.2
Average values from 4 unconfined caves (Klimchouk 2003)									16.6	6.4	0.4
Average values from 21 confined caves (Klimchouk 2003)									167.3	29.7	5.0

Table 1: Morphometry, derived from surveyed (proper) passages. Specific volume is the sum of passage volume divided by the sum of passage length (3D). Passage density is the sum of horizontal passage length (2D) divided by the cave field. Areal coverage is the ratio of the sum of the horizontal passage area seen from above to the cave field. Cave porosity is the ratio of the sum of the passage volume to the rock volume. Data 1993-2008.

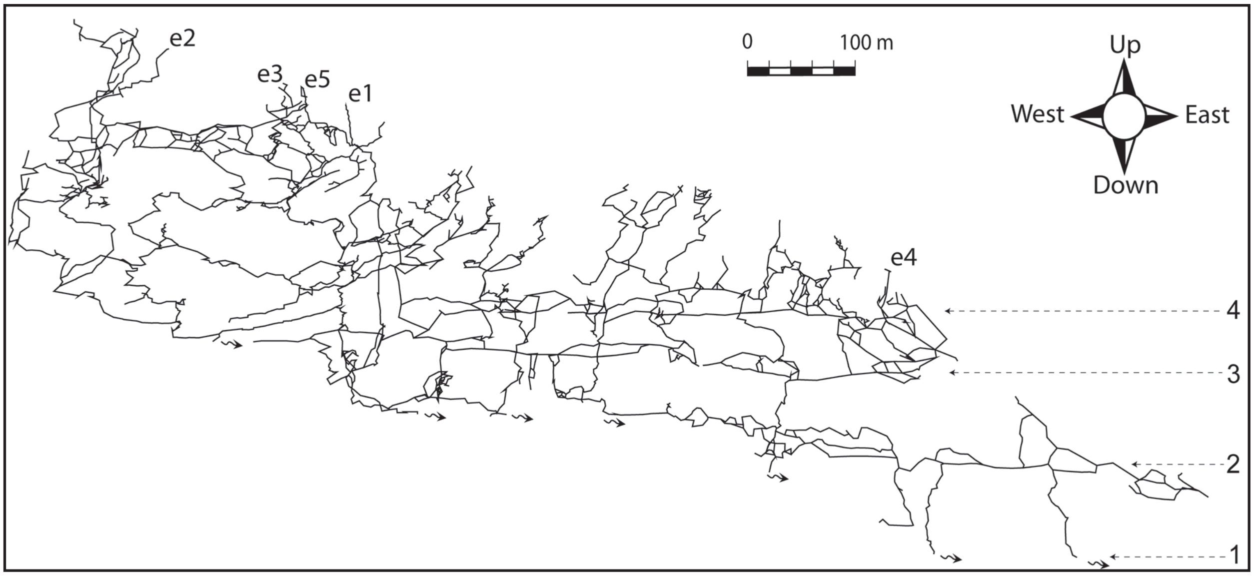

Figure 2: Tjorve profile, looking north. The bottom of the cave is closer to the observer and the top is further away. Level 1 is the river of today. Level 2 is the tubes in "Down below". Level 3 is the lower part of the "Galleries". Level 4 is in the main part of the Galleries (the Galleries have passages that extend towards the surface). Canyons are often found between the levels, in particular between level 1 and 2, and level 2 and 3. Since the cave dips 25o-40o to the south, the plan map of Tjorve is quite similar to the profile map. 19 line segments are needed to identify a polygon that includes the centerline.

The method by which the cave field is calculated is of great importance to the passage density, areal coverage and cave porosity (Klimchouk 2003). We calculated the "Minimum Horizontal Polygon Field" – defined such that any line segment of the polygon 1) is as long as possible and 2) does not cross a shot (Finnesand et al. 2007). This polygon in Tjorve has 19 line segments. Perhaps more common (and easier) is to calculate the area from the smallest rectangle that includes the cave (the cave field of Tjorve would then increase from 0.41 km^2 to 0.63 km^2). Klimchouk (2003) identified polygons by using a lot of line segments, - the goal is to have a polygon that reasonably closely embraces the plan array of a cave. The polygon can be drawn in many ways, but in practice the cave field would have values within 10-15 % (Klimchouk, 2003). Doing that in Tjorve, the cave field become 0,34 km^2 (75 stations), which would increase the three affected parameters by 23 %. In theory, one can increase the number of line segments in the polygon until the cave field will be the sum of the passage area and the area within the "loops" which would occur in plan view.

There are still some passages to be surveyed in Tjorve, although the extent of the cave would probably not change. Average size of remaining passages are probably small, which will reduce the specific volume. The passage density, areal coverage and cave porosity would increase somewhat, in particular in the Galleries.

3. Fractal Analysis

A fractal analysis of Tjorve was done to determine whether the cave exhibited self-similar fractal structure and, if so, to estimate the unsurveyable (non-proper) length and volume of the cave and possibly that of the entire karst terrain. The method used was that of Curl (1986 – hereafter cited as **RC**) and discussed further in Curl (1999). This is done by placing virtual spherical *linked modular elements* (LME) of diameter η (cm) at survey stations and interpolating additional LMEs linearly between stations, as explained in **RC** (pp 776-777, Fig 7). The diameter of LME at stations is chosen as the lesser of the measured or estimated LRUD distances because this is what limits exploration and hence defines the limiting scale of the proper cave. Counts of LMEs were sorted into a histogram using equal logarithmic interval binning corresponding to 2 % differences in η, with q_i LME per bin centered at η_i. These are shown in log-log coordinates in Figure 3.

Tjorve yielded 15 768 LME between 10 and 1 305 cm. The q_i fall off rapidly at η smaller than at the data peak (at approximately 110 cm) because of the physical difficulty or impossibility of surveying in smaller passages, and are also truncated at large η because of such factors as limited strata thickness, rock strength, and extent of solution. The nearly linear slope in a range of η larger than at the data peak suggests a power-law model of the form

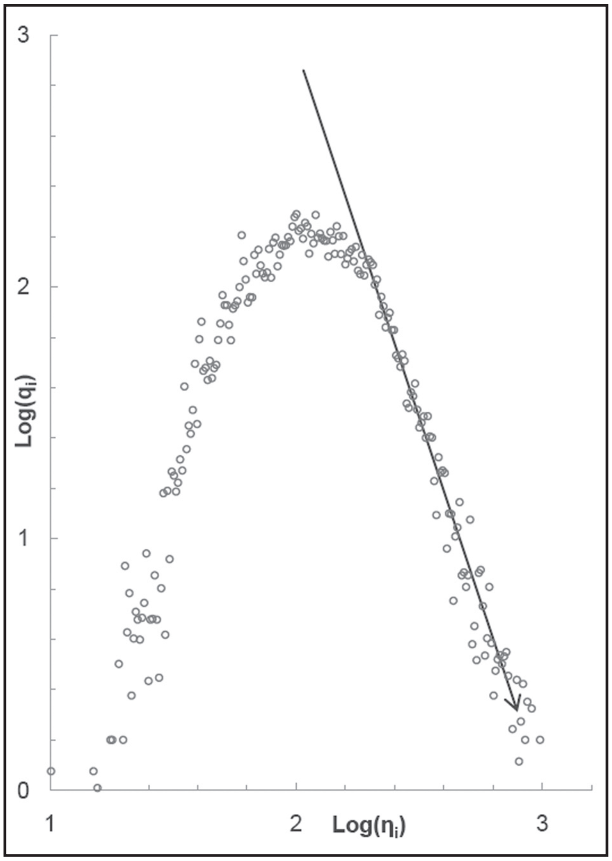

Figure 3: Histogram of LME number distribution with logarithmic interval binning of 2 % of size (cm). The apparently linear power-law range was chosen to be 184 to 591 cm for the estimation of the fractal dimension of the cave. The peak is at the nominal proper modulus (1.1 m) of the cave.

$$q_i = c\eta_i^{-D} \qquad (1)$$

where D is the *fractal dimension*. This is characteristic of self-similar geometric fractals (Mandelbrot 1983).

The data may also be plotted as the more conventional cumulative, defined as the number Q(η) of LME larger than a value η, as shown in Figure 4. Here Q(η) appears to also exhibit a power-law range, as in Eqn. 1 (but with a different constant c). Newman (2005) gives numerous examples of similar power-law distribution behaviors of number versus size, known as Zipf's Law, a Pareto distribution, and 'rank-frequency' plots, for such phenomena as word frequencies, earthquake magnitudes, moon crater sizes and population data. The causes of power-law behavior have seldom been explained, but it is related to there being no unique defining scales for the phenomenon.

The least-square slope of the apparent power-law part of the cumulative plot was used in **RC** (p. 778, Fig. 8) to estimate D. There are, however, two problems with this: Q(η) data are not statistically independent or homoscedastic, and an upper cutoff at large values of η due to the factors noted previously. The first problem has been addressed by using a maximum likelihood (ML) estimator for D from the histogram data. Geyer (2007) details the general ML theory. Newman (2005) derived an ML estimator for power-law data not truncated above. We have derived the following estimators for D and its standard deviation σ_D for data truncated to the range $\eta_l < \eta < \eta_u$ where η_l and η_u are the lower and upper bounds of the chosen range of η. Derivations are included in Supporting Online Material:

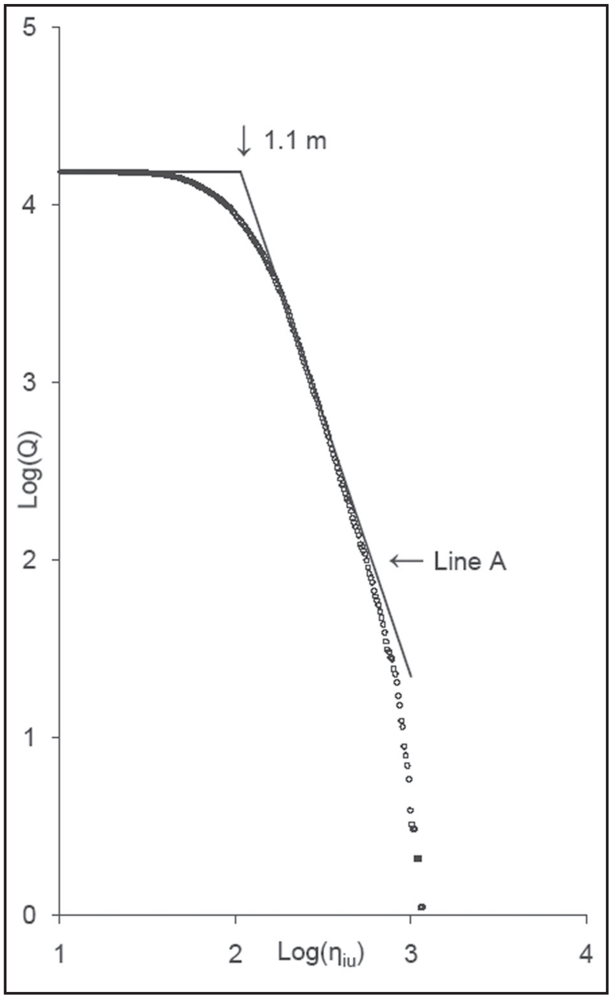

Figure 4: Number of LMEs, Q(η,) larger than η(cm). The slope in the apparently linear range of a log-log plot approximates D, but is too high due to the truncation of Q(η) at large η. A correction for truncation is shown by line A, which has slope D

$$\left[\frac{1}{\hat{D}} + \frac{r^{\hat{D}}\ln(r)}{\left(1-r^{\hat{D}}\right)}\right] = \frac{1}{n}\sum_{l}^{u} q_i \ln\left[\frac{\eta_i}{\eta_l}\right] \tag{2}$$

$$\sigma_D = \frac{\hat{D}}{\sqrt{n}}\left[1 - \frac{\hat{D}^2 r^{\hat{D}}\left(\ln(r)\right)^2}{\left(1-r^{\hat{D}}\right)^2}\right]^{-\frac{1}{2}} \tag{3}$$

where r = η_l/η_υ and n is the sum of q_i over the range. Eqn. (2) must be solved iteratively for the D estimator $\hat{D}$. An Excel spreadsheet and instructions for performing these calculations are in the Supporting Online Material.

The second problem with using a least-squares regression for the data in Figure 4 to estimate D is the necessity of correcting for the truncation of Q(η) data at large η: this correction is shown by line A, now with the lower slope – $\hat{D}$, calculated by adding a derived constant to all the Q(η) up to η_υ. D must be estimated from the histogram to apply this correction, so regression of Q(η) data themselves does not provide an unbiased estimate of D.

The value for $\hat{D}$ is somewhat sensitive to the range (η_l, η_u) chosen because of the inaccuracy with which LRUD are measured at survey stations, often only to the nearest integer meter in large passage. The values of L, R, U and D at each survey stations were randomized uniformly over a local interval of ± 5 % to reduce this effect. This adjustment is less than the precision of surveyed LRUD distances, but reduces the apparent scatter of the data in Figure 3. For η_l = 184 cm and η_u = 591 cm, with station LRUD randomization, $\hat{D}$ = 2.929 and σ_D = 0.068. Previous analyses have reported values for of 2.79 (Little Brush Creek Cave, (LBC; Colorado: **RC**), and 2.5 (Stagebarn Crystal Cave, South Dakota: Curl and Nepstad 1991). The earlier applications were less thorough than the current one.

The estimate of D permits estimating the total volume of non-proper cave (smaller than surveyable) if it is assumed that the known proper cave is fully connected. That is, that there exists no unknown connected cave passages larger than the proper modulus of the survey. This is true of the Menger Sponge (**RC**, p. 775, Fig. 6) but it is likely that if smaller passages could be explored, additional large proper passage would be found. Therefore an extrapolation of the data in Figure 3 to η = 0 will provide only a lower limit to the remaining volume in the karst terrain.

The volume of known proper cave can be estimated from the sum of $q_i\eta_\iota^3$ from η_l up. The estimated volume of cave below η_l from a theoretical extrapolation of η to 0, is given by

$$V(0,\eta_l) = \frac{n\hat{D}\eta_l^3}{(1-r^{\hat{D}})(3-\hat{D})} \tag{4}$$

which gives $V(0,\eta_l)$ = 831 000 m³ (19 600 m³ known), compared to 108 800 m³ for the cave above η = 1 m.

4. Discussion and Conclusions

One of the major problems when surveying in Tjorve (as in most caves) is the often ill-defined walls due to the sloping marble, and usually the width and height of difficult passages have been very roughly estimated. The labyrinthine nature of Tjorve, with numerous side passages, also add to the challenge. There are still probably some more kilometers of minor passages unsurveyed and unexplored in Tjorve.

Likewise, unresolved problems in the fractal analysis are the inaccuracy of passage profile measurements and the more general question of how a passage profile should be used as a local measure of passage size. The cave defined by the current LME mehod of analysis does not represent the complexity of cave passages, but rather some measure of *proper* size – that is, an anthropomorphic size. Nevertheless the fractal analysis provides estimates of cave morphology that can otherwise not be measured.

The relations between the parameters of the above morphometric and fractal analyses are unclear. A related question was asked some time ago (Curl 1963) when the modulus of a cave was first defined: are there defining scales for speleogenesis that imprint themselves upon caves? If there were, one would expect to find multiple peaks in the histogram of the distribution of passage size. From this and previous analyses, there appear to only be two: the size of explorers (which has nothing to do with speleogenesis) and the upper LME cutoff above about 5 meters, probably due to limits in strata thickness, rock strength, and extent of solution.

The proper modulus is near 1.1 m, compared to 0.6 m for LBC, indicating perhaps less complete exploration. In the power-law range there appear to be no defining scales, or so many that they overlap in a way to produce a power-law dependence of LME size. How this works has not as yet been determined.

The total volume of the karst terrain including Tjorve and nearby caves, for any given modulus, cannot yet be estimated by the method of **RC** (p. 774, Eqn. 20) because the karst terrain has not as yet been analyzed to estimate the number and lengths of all caves in the terrain, with and without entrances.

Acknowledgements

Since Tjorve was discovered by Torbjörn Doj and Johannes Lundberg 29 July 1993, surveying has been done while exploring. Acknowledgements are due to the 75 cavers from Norway, Sweden, and four other countries who surveyed the cave.

References

Curl, R. (1963) One the Defintion of a Cave. *National Speleological Society Bulletin (USA)* **26** (1), 1-6.

Curl, R. (1986) Fractal dimensions and geometries of caves. *Mathematical Geology* **18**, 765-783. (http://deepblue.lib.umich.edu/handle/2027.42/43195)

Curl, R., and J.A. Nepstad (1991) Fractal analysis of Stagebarn Crystal Cave, South Dakota (abstract). *Proceedings of 1991 annual meeting, National Speleological Society*, in NSS Bulletin, **53** (1), p. 44.

Curl, R. (1999) Entranceless and Fractal Caves Revisited. *Proceeding of the Karst Modeling Symposium* (Karst Waters Institute), Charlottesville, p. 183-185.

Finnesand, T., K. Davidsen, S. Grundstrɯm, B. E. Johansen, J. Lundberg and R. Solbakk (2007). Tjoarvekrajgge 1993-2007. *Norsk Grotteblad* **49**, 28-71.

Geyer, C.J. (2007) Stat 5102 Notes: Fisher Information and Confidence Intervals Using Maximum Likelihood. (http://www.stat.umn.edu/geyer/old03/5102/notes/fish.pdf)

Klimchouk, A.B (2003) Unconfined versus confined speleogenetic settings: variations of solution porosity. *Speleogenesis and Evolution of Karst Aquifers* 1 (2), www.speleogenesis.info.

Mandelbrot, B.B., (1983) *The fractal geometry of nature*. W.H. Freeman and company, New York,.468 p.

Newman, M.E.J. (2005) Power laws, Pareto distributions and Zipf's law. **46**, 323-351. (http://arxiv.org/abs/cond-mat/0412004)

Supporting Online Material

http://www-personal.umich.edu/~ranecurl/2009ICS/SOM_09-0338.pdf

Derivations

Excel Spreadsheet