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COMPUTER INVESTIGATION OF THE CAPACITOR-STORAGE IGNITION SYSTEMS

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

COMPUTER INVESTIGATION OF THE CAPACITOR-STORAGE IGNITION SYSTEMS

by

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1. INTRODUCTION

The purpose of this investigation was to find, by means of an analog computer, the voltages and currents in capacitor discharge and line discharge ignition systems. Various forms of these systems have been described in a previous Technical Report.[1] Of particular interest here are the peak value of the voltage $V_2(t)$ across the spark gap, the wave shape of this voltage, the time (t_m) to the voltage peak, and the primary current i_1. The effect of an early spark on the primary current was investigated. The values of network elements were varied, and the influence of those different parameters on voltages and currents were observed.

2. TABLE-TOP ANALOG COMPUTER

The computer investigation has been carried out using table-top analog computers in the Computer Laboratory of the Department of Aeronautical Engineering. The computers, designed and built in the Aeronautical Engineering Department, consist of six or ten high-gain DC amplifiers. Each amplifier consists of three stages and has an amplification factor of the order of 50,000.

The currents and voltages in the ignition system are represented by the voltages of the computer network. The basic operations carried out on those voltages in the

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[1] J. L. Stewart, "Theory and Design of Capacitor-Storage Ignition System," Engineering Research Institute, University of Michigan, Technical Report No. 1 on ERI Project No. 2370-2, December, 1955.

computer operation are change of sign, multiplication by a constant, summation, and integration. In an operational amplifier if a resistor is connected in the feedback the amplifier changes the sign, multiplies by a constant, and sums up the input voltages applied to the input resistor. If a capacitor is connected in the feedback the amplifier changes the sign, multiplies by a constant, and integrates the input voltages. These basic elements of the computer setup are shown schematically in Fig. 1.

The voltages that represent the primary current and the voltage across the spark gap are recorded on a Brush recorder. The expected accuracy of the solutions is ± 4%.

3. THE COMPUTER EQUIVALENT OF THE CAPACITOR DISCHARGE IGNITION SYSTEM

The basic ignition network is shown in Fig. 2. The actual transformer in the network can be replaced for the purpose of this investigation by a series (leakage) inductance L and a shunt (magnetizing) inductance L_m followed by an ideal transformer with the step-up ratio 1:n. The resistance R_a includes the resistance of the transformer windings. The resulting network is shown in Fig. 3. This network is further simplified for the computer investigation by eliminating the ideal transformer. The impedances of the secondary are transformed to the impedance level of the primary as shown in Fig. 4.

This network can be represented by the following equations in terms of the mesh currents I_1, I_2, I_3 (see Fig. 4):

$$I_1\left(\frac{1}{C_1 p} + R_a + pL + \frac{R}{n^2}\right) + I_2\,\frac{R}{n^2} + I_3\,\frac{R}{n^2} = 0 \qquad (1)$$

$$I_1\,\frac{R}{n^2} + I_2\left(pL_m + \frac{R}{n^2}\right) + I_3\,\frac{R}{n^2} = 0 \qquad (2)$$

$$I_1\,\frac{R}{n^2} + I_2\,\frac{R}{n^2} + I_3\left(\frac{1}{pn^2C} + \frac{R}{n^2}\right) = 0 \qquad (3)$$

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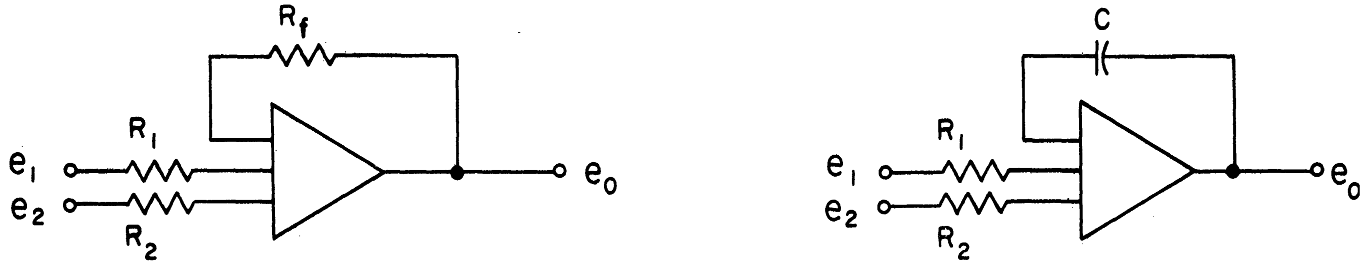

$$e_0 = -e_1 \frac{R_f}{R_1} - e_2 \frac{R_f}{R_2}$$

$$e_0 = -\frac{\int_0^t e_1 dt}{R_1 C} - \frac{\int_0^t e_2 dt}{R_2 C}$$

FIG I BASIC ELEMENTS OF AN ANALOG COMPUTER

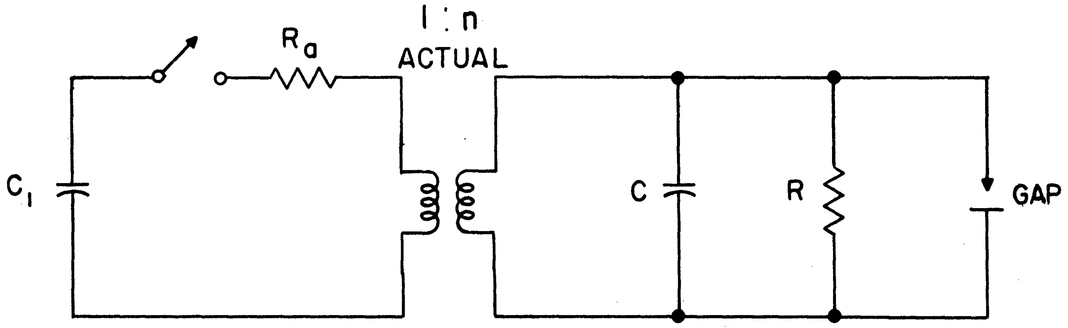

FIG 2 CAPACITOR STORAGE IGNITION SYSTEM

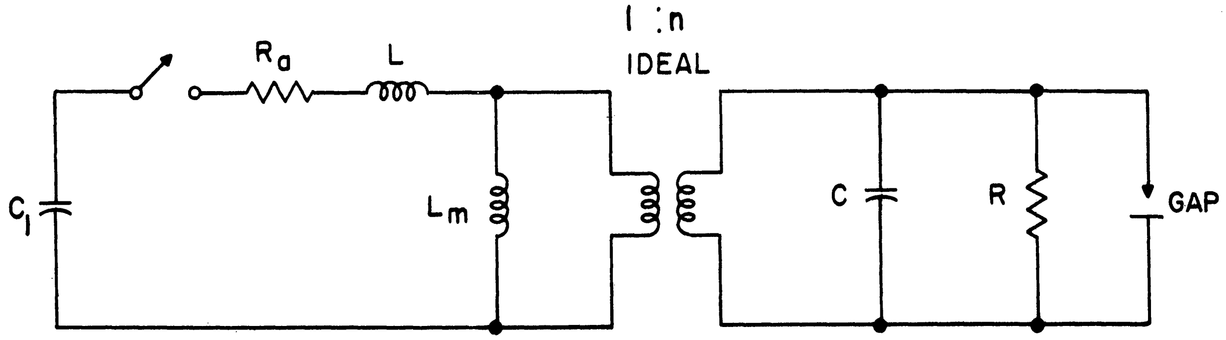

FIG 3 IDEAL-TRANSFORMER REPRESENTATION OF A CAPACITOR STORAGE IGNITION SYSTEM

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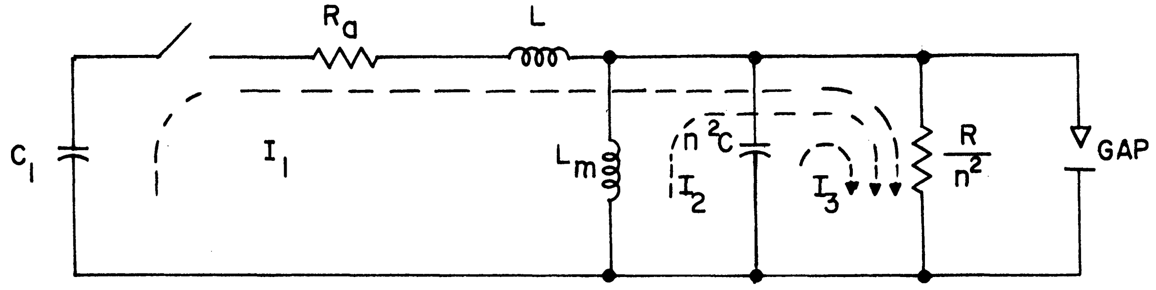

FIG 4 MESH CURRENTS IN A STORAGE CAPACITOR IGNITION SYSTEM NETWORK

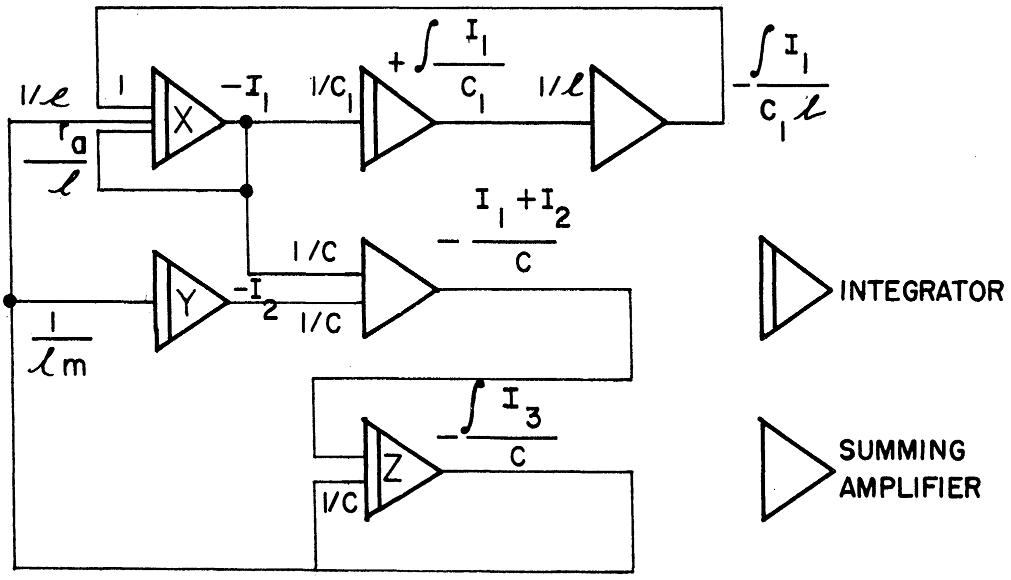

FIG. 5 COMPUTER BLOCK DIAGRAM FOR SIMULATING THE CAPACITOR STORAGE IGNITION SYSTEM

The mesh currents were traced in such a way as to minimize the number of terms containing 1/pC and pL. This minimum (four in this case) is equal to the number of independent energy-storage elements in the network. Such a procedure eliminates the possibility of having more integrators in the analog-computer setup than the number of independent L and C elements in the original network.

We will define i_1 as the current carried by the switch. In this case $i_1 = I_1$. The voltage across the spark gap is given by $V_2 = n\left(\int_0^t I_3\,dt\right)/\,n^2C$.

The mesh equations are rewritten in view of setting the computer equivalent. The highest derivatives of I_1, I_2, and I_3 are explicitly stated.

$$\begin{aligned} \dot{I}_1 &= \frac{1}{L}\left[-(I_1 + I_2 + I_3)\frac{R}{n^2} - I_1R_a - \frac{\int I_1 dt}{C_1}\right] \\ &= \frac{1}{L}\left[\frac{I_3 dt}{n^2C} - I_1R_a - \frac{\int I_1 dt}{C}\right] \end{aligned} \tag{4}$$

$$\dot{I}_2 = \frac{1}{L_m}\left[-(I_1 + I_2 + I_3)\frac{R}{n^2}\right] = \frac{1}{L_m}\,\frac{\int I_3 dt}{n^2C} \tag{5}$$

$$I_3 = \frac{1}{R}\left[-\frac{\int I_3 dt}{n^2C} - (I_1 + I_2)\frac{R}{n^2}\right] \tag{6}$$

The impedances are normalized by setting $R/n^2 = 1$. Moreover, the frequency is decreased by a factor of 10^6. The values of L and C are increased in this ratio. The circuit constants used in the first computer runs are given in Table I. Column i gives the actual values, Column ii gives the values transformed from secondary to the primary (in correspondence with Fig. 4), Column iii gives the values scaled down in frequency by the factor of 10^6. The last column gives the values of Column iii normalized to make the value of R/n^2 equal to unity. The lower-case letters denote those normalized values which are actually used in the computer setup.

TABLE I

i	ii	iii	iv
$R = 200{,}000\Omega$	$R/n^2 = 10\ \Omega$	$10\ \Omega$	$r = 1$
$C_1 = 1\ \mu F$	$1\ \mu F$	$1F$	$c_1 = 10$
$C = 50\ \mu\mu F$	$Cn^2 = 1\ \mu F$	$1F$	$c = 10$
$L = 25\ \mu H$	$25\ \mu H$	$25\ H$	$\ell = 2.5$
$L_m = 250\ \mu H$	$250\ \mu H$	$250\ H$	$\ell_m = 25$
$R_a = 2.5\ \Omega$	$2.5\ \Omega$	$2.5\ \Omega$	$r_a = .25$
$n = 141$			

Using these normalized values, the computer equations are:

$$\dot{I}_1 = \frac{1}{\ell}\left[\frac{\int I_3 dt}{c} - I_1 r_a - \frac{\int I_1 dt}{c_1}\right] \tag{7}$$

$$\dot{I}_2 = \frac{1}{\ell_m}\ \frac{\int I_3 dt}{c} \tag{8}$$

$$\frac{I_3}{c} = -\frac{\int I_3 dt}{c^2} - \frac{I_1 + I_2}{c} \tag{9}$$

The computer block diagram corresponding to these equations is presented in Fig. 5. The equations are implemented by summing the terms on the right side of Eqs 7, 8, and 9 at the integrators X, Y, and Z, respectively. At the outputs of these integrators we obtain the expressions appearing on the left side of the equations, integrated once and changed in sign: $-I_1$, $-I_2$, and $-\int I_3 dt/C$. The detailed computer setup using the values of Table I is shown in Fig. 6. The numbers refer to resistor values in megohms and capacitor values in microfarads.

The initial conditions of the ignition system (i.e., the voltage V_o across the capacitor C_1) is implemented by an initial condition of about 40 volts, which is set up on the integrator W. The Brush recorder is calibrated in terms of this voltage; therefore, no direct voltage measurement is necessary.

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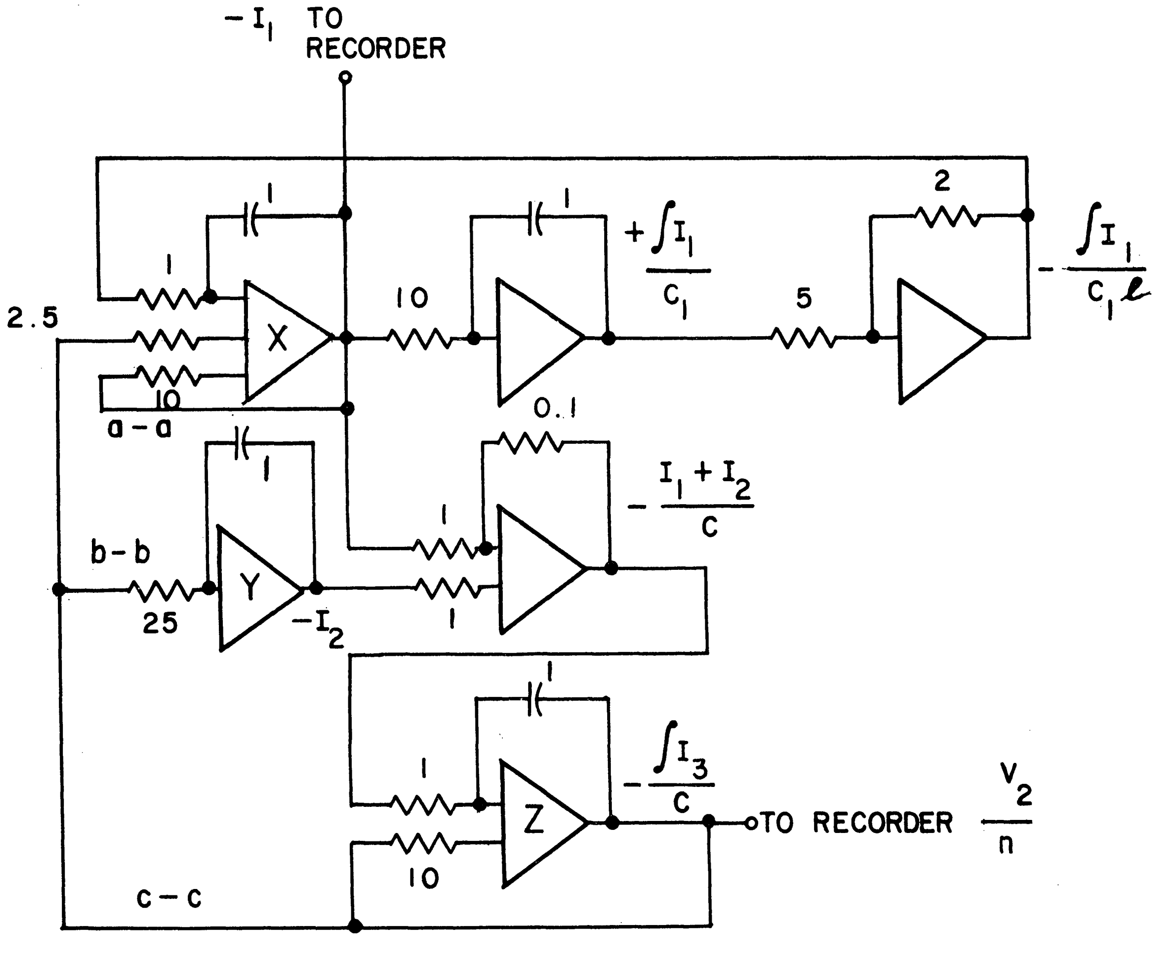

FIG 6 COMPUTER SET-UP FOR SIMULATING THE CAPACITOR STORAGE IGNITION SYSTEM

To make $r_a = 0$, the connection a-a is removed. To make $\ell_m \to \infty$, the connection b-b is removed. The sparking of the gap ($V_2 = 0$) is simulated at different gap voltages by shorting the connection c-c to ground. This is done manually by the operator when the recording of $V_2(t)$ on the Brush recorder reaches the assumed breakdown value.

4. VOLTAGES AND CURRENTS IN THE CAPACITOR-STORAGE IGNITION SYSTEMS

The data on the primary currents and spark gap voltages were read off the computer solutions recorded on the Brush recorder. A few representative solutions are shown in Figs. 7 and 8.

Preliminary investigation was concerned with the effects of the magnetizing inductance L_m and the series resistance R_a on the voltages and currents. It was noted that within the range of values that appear reasonable for an ignition system, the magnetizing inductance has very little effect. When the magnetizing inductance was changed from $L_m = \infty$ to $L_m = 10L$, the peak secondary voltage decreased about 5%, while the peak primary current increased about 1%.

The influence of the resistance R_a, within the range of its expected values, was more noticeable. For the set of circuit parameters presented in Table I ($R_a = 2.5\ \Omega$; $R/n^2 = 10\ \Omega$), the reduction in the peak voltage was 23%. Thus, only slightly more than straight-potentiometer reduction $\left[R_a/(R_a + R/n^2) = 2.5/(2.5 + 10) = 20\%\right]$ was observed. The decrease in the peak primary current was 24%.

The investigation is fairly complicated because there are several varying parameters in the network. It was decided to concentrate the investigation on the effect of changing the inductance L and the turns ratio n. In the numerous circuits with different values of L and n that were simulated, the magnetizing inductance L_m was taken as infinite on the assumption that its effect can be neglected. The series resistance R_a was taken as zero on the assumption that the effect of a series resistance can be ascertained to a first approximation by a straight potentiometer relationship. The spark-gap leakage

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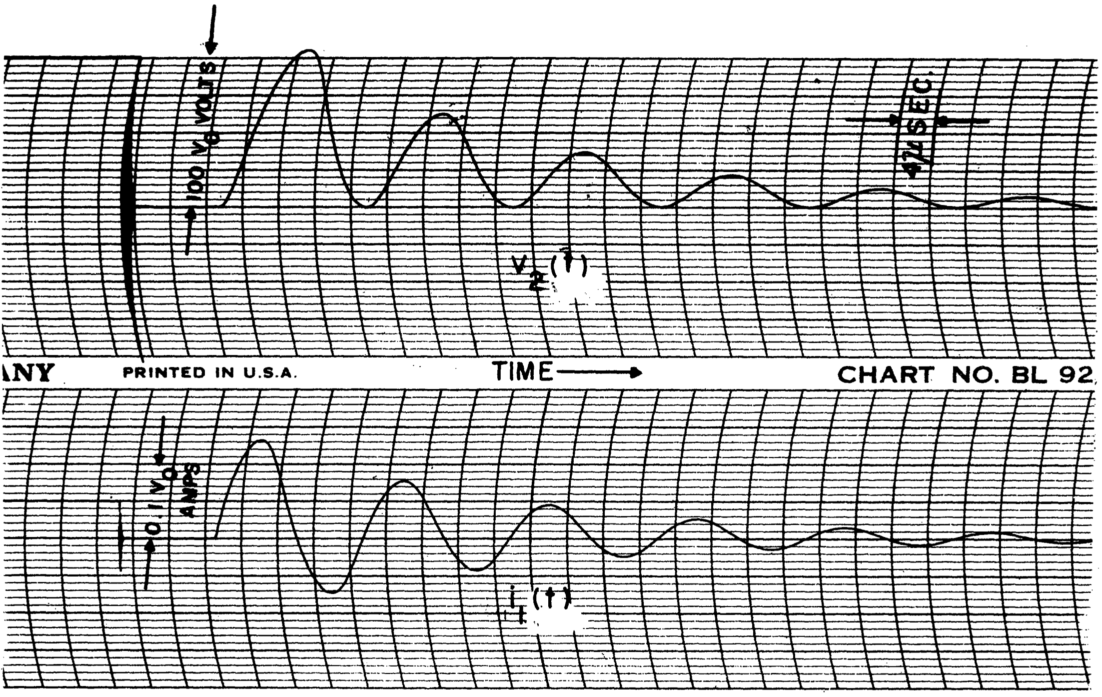
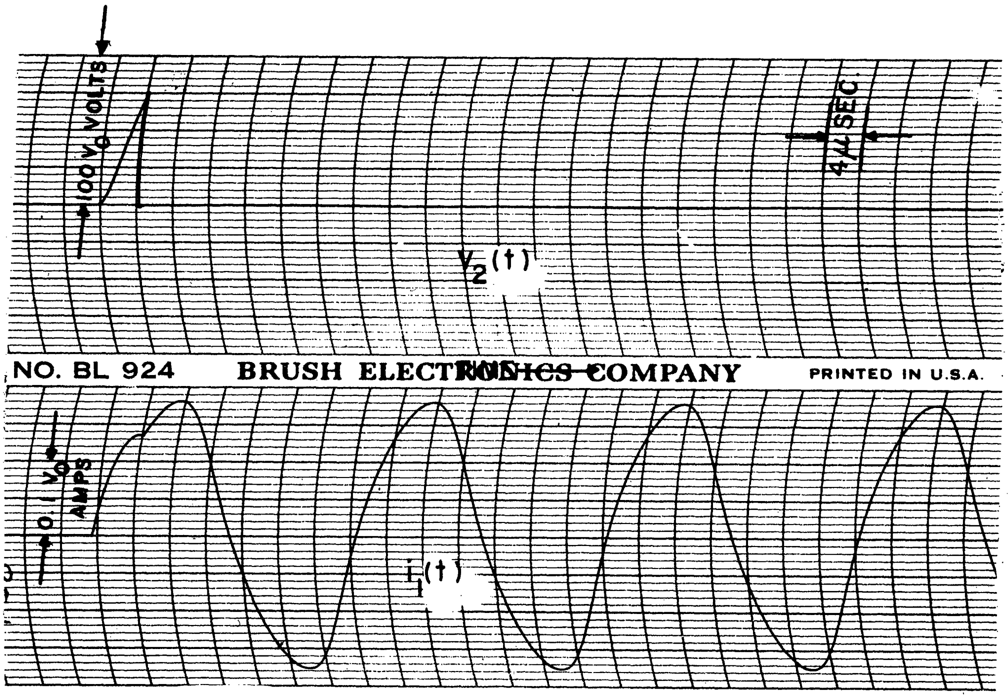


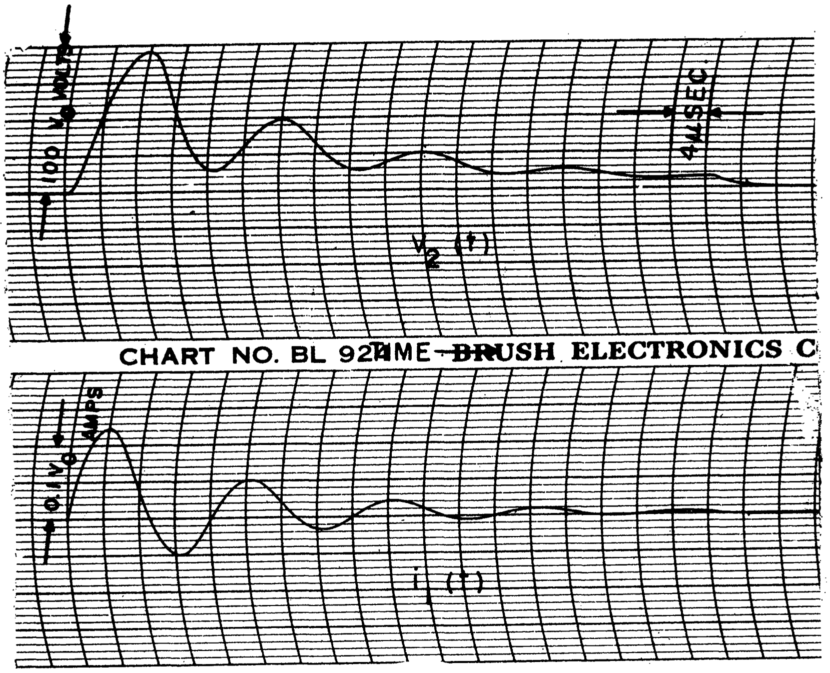
(a) NO SPARK SOLUTION



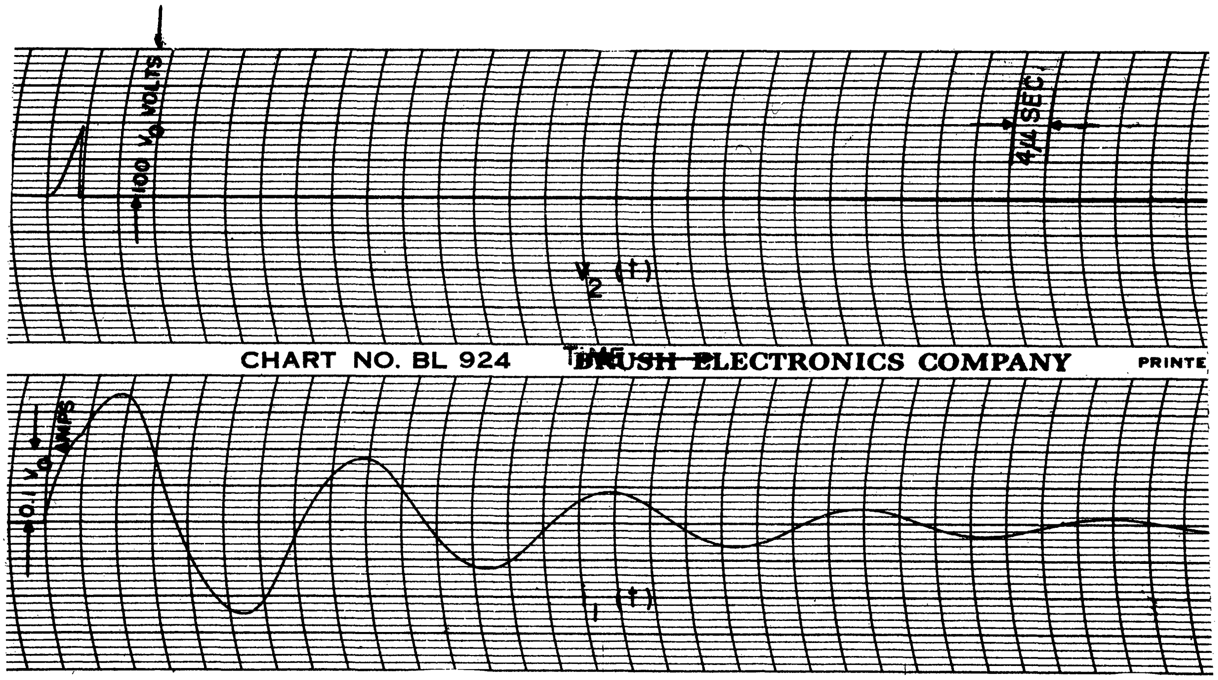
(b) THE EFFECT OF A SPARK AT $V_2 = 0.75\ V_{2(t_m)}$

SPARK PLUG VOLTAGE V_2 AND PRIMARY CURRENT i_1 IN A CAPACITOR STORAGE IGNITION SYSTEM. $C_1 = 1\mu f$, $L = 20\mu h$ $R_a = 0$, $n = 100$, $R = 200\,K$, $C_2 = 50\mu\mu f$

FIG 7



(a) NO SPARK SOLUTION



(b) THE EFFECT OF A SPARK AT $V_2 = 0.5\ V_{2(tm)}$

SPARK PLUG VOLTAGE V_2 AND PRIMARY CURRENT i_1 IN A CAPACITOR STORAGE IGNITION SYSTEM $C_1 = 1\mu f$, $L = 20\mu h$, $R_a = 1\,\Omega$, $n = 100$, $R = 200K$, $C_2 = 50\mu\mu f$

FIG 8

resistance $R = 200{,}000\ \Omega$ was not varied, since it is the design goal of the ignition system to assure sparking even at this low resistance value. The secondary capacitance C was kept constant at 50 $\mu\mu F$. This is thoughtto be a fairly representative value. The storage capacitor C_1 was chosen at 1 μF, except for a few cases where the effect of a larger capacitor was investigated.

The results are presented in the form of graphs: The peak voltage, $V_2(t_m)$, the time t_m to the voltage peak, and the peak primary current are all plotted as functions of the turns ratio n with different values of inductance L as a parameter. This is given in Fig. 9. In Fig. 10, the same variables are plotted as functions of L for two values of parameter n.

It can be seen from these graphs that the peak voltage increases with decreasing L, the only result to be expected. However, beyond $L = 50\ \mu H$, the decrease in L by a factor of 2 to 25 μH results in less than a 10% increase in peak voltage. For any specific inductance, there is an optimum turns ratio for which the peak voltage is a maximum. For the range of inductances considered, the values of n for which the ratio $V_2(t_m)/V_0$ is a maximum range from $n = 80$ to $n = 120$, where V_0 is the initial voltage of the storage capacitor. The maxima are fairly broad, falling off more sharply in the direction of decreasing n.

Thus, from the point of view of maximum peak voltage alone, it is desired to have the smallest possible leakage inductance and a certain optimum turns ratio. However, other considerations show that this might not be the most desirable solution.

The primary current increses with increasing n. Thus if carrying large currents through the switching device presents a problem, it is desirable to choose n slightly below the value for maximum secondary voltage. From the same point of view, the reduction in L cannot be carried too far, since the currents increase rapidly. Decreasing L from 50 μH to 25 μH increases the current by as much as 30% (for only a 10% increase in the peak voltage, as stated previously).

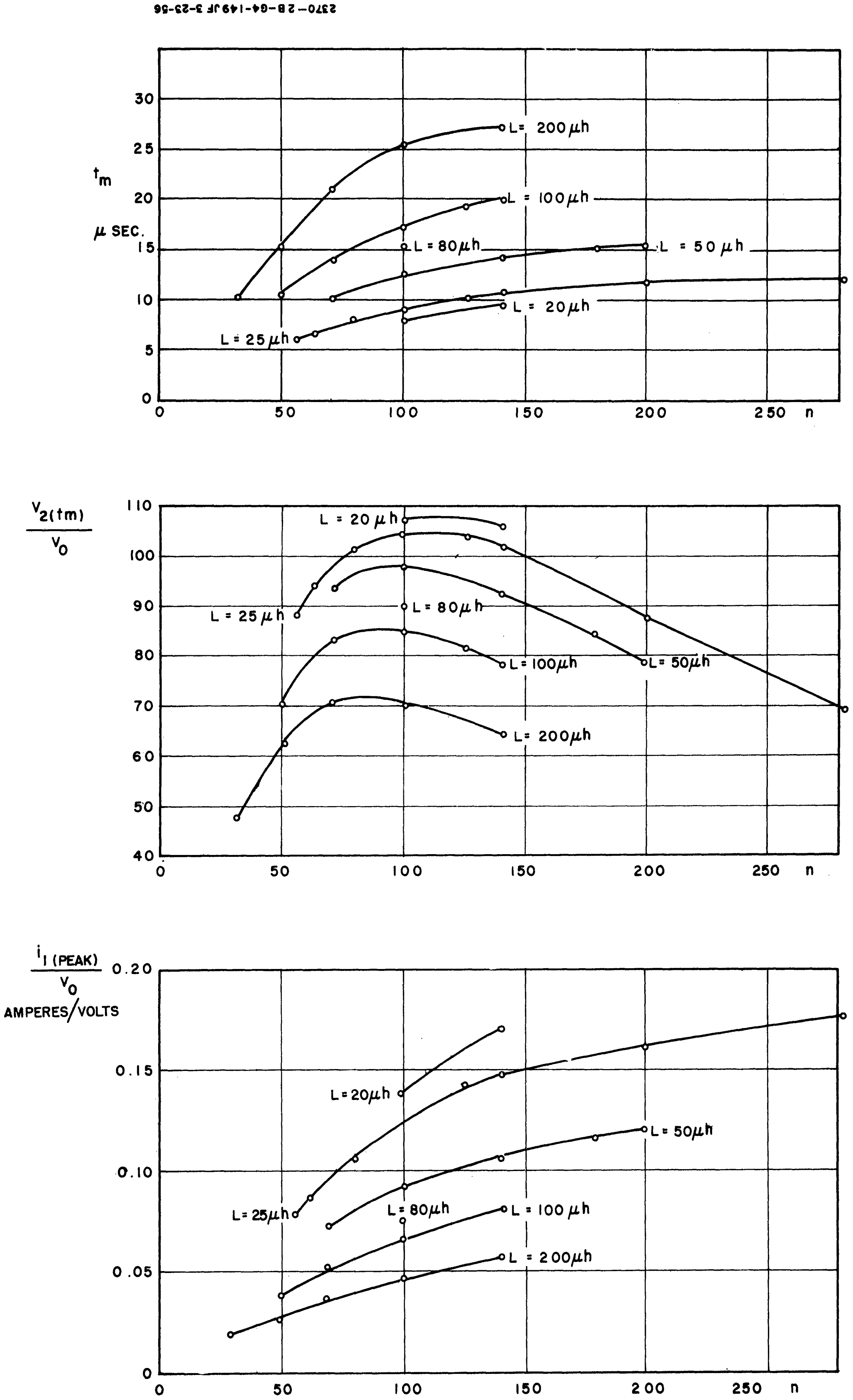


FIG 9 PEAK SPARK PLUG VOLTAGE, PEAK PRIMARY CURRENT AND TIME TO VOLTAGE PEAK VS n

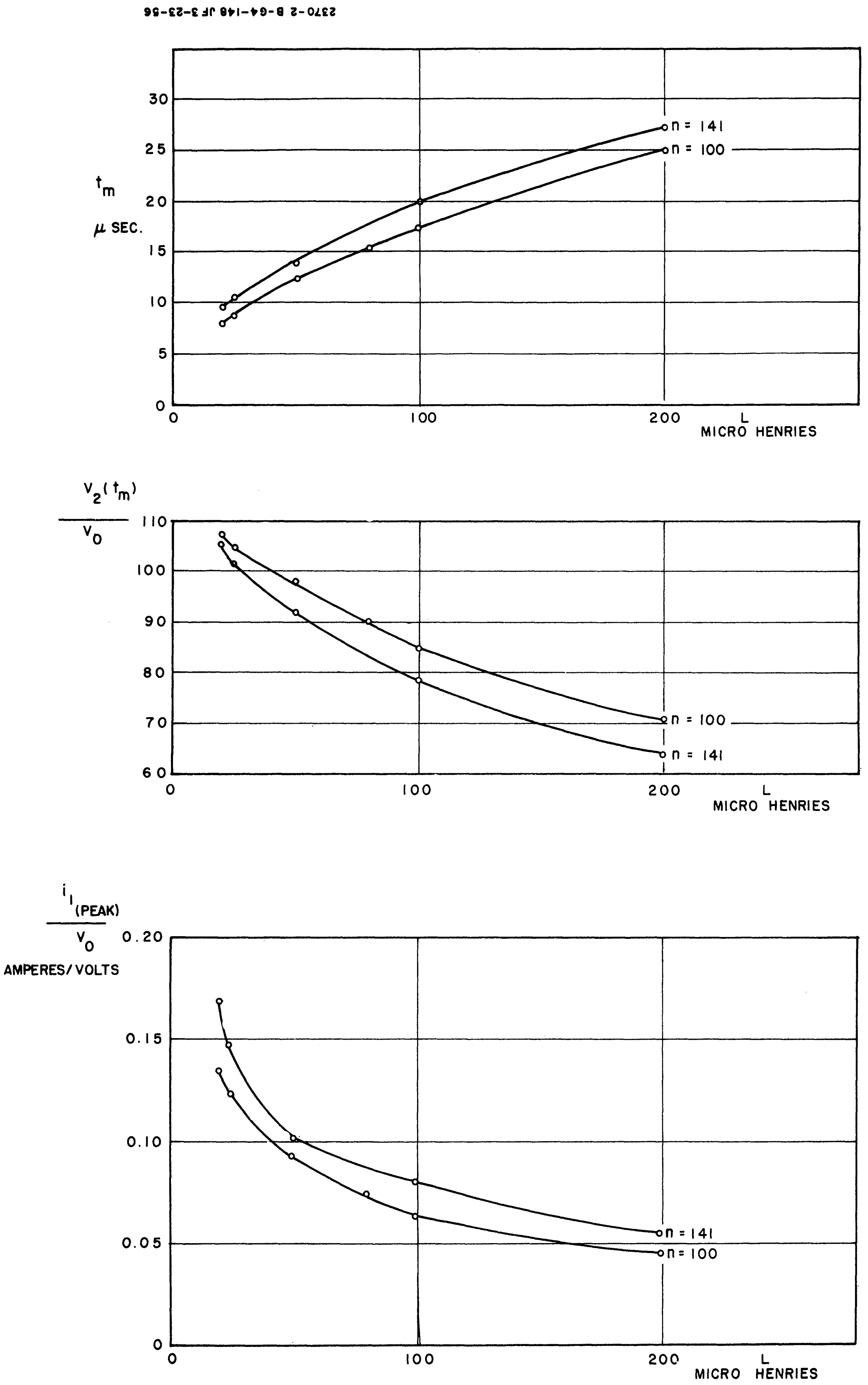


FIG 10 PEAK SPARK PLUG VOLTAGE, PEAK PRIMARY CURRENT
AND TIME TO VOLTAGE PEAK VS L

Similar directives as far as L is concerned, and quite opposite as far as n is concerned, result if one desires to extend the duration of high secondary voltage. This consideration arises because the ionization of the spark gap requires a certain time. The voltage requirements for the gap breakdown are higher if the time that the voltage applied is very short. Moreover, the spark duration is longer for the case where the no-spark solution has high voltage for a longer time. This dwell time of voltage is closely related to t_m and it can be seen that for longer t_m, a higher turns ratio or a higher leakage inductance must be chosen.

The merits of different network parameters in satisfying the requirements of high peak voltage, small primary current, and extended voltage dwell time can be presented in a somewhat different form. An arbitrary pair of criteria[1] have been chosen for optimizing the system design:

(a) The peak secondary voltage in the absence of a spark must be at least 25 kv.

(b) The secondary voltage in the absence of a spark must exceed 20 kv for at least 10 microseconds.

Optimum design is the solution to the following problem. What is the lowest initial capacitor voltage V_0 required to satisfy the above two criteria? Under these conditions the no-spark peak primary current, and the increase in primary current with sparking are quantities of interest.

Plots of V_0 and i_1 (max) to satisfy criterion (a) and a plot of V_0 to satisfy criterion (b) as functions of n are given in Figs. 11, 12, and 13. In Fig. 13, the points for L = 100 μH and 80 μH were omitted, since they coincide in the region of interest with the curve for L = 50 μH.

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[1] Quantitative results in this section are derived on the basis of these two rather arbitrary criteria, and should be taken with this in mind. It is known that the ionization time is generally much less than 10 microseconds, but this rather conservative value was chosen for criterion (b) partly to give longer duration of spark current.

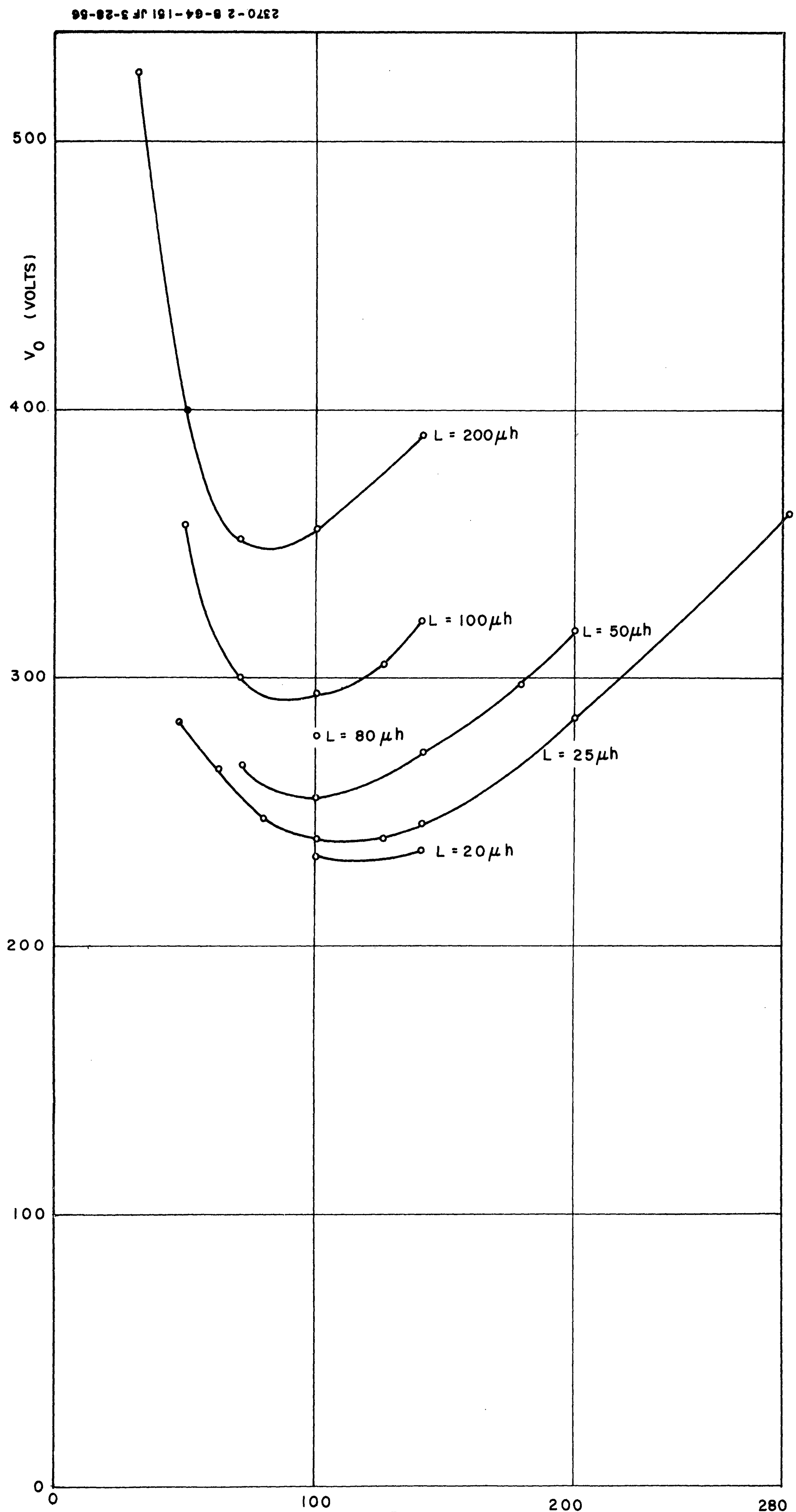

FIG 11 INITIAL STORAGE CAPACITOR VOLTAGE V REQUIRED TO PRODUCE 25 KV SPARK PLUG VOLTAGE PEAK VS. n

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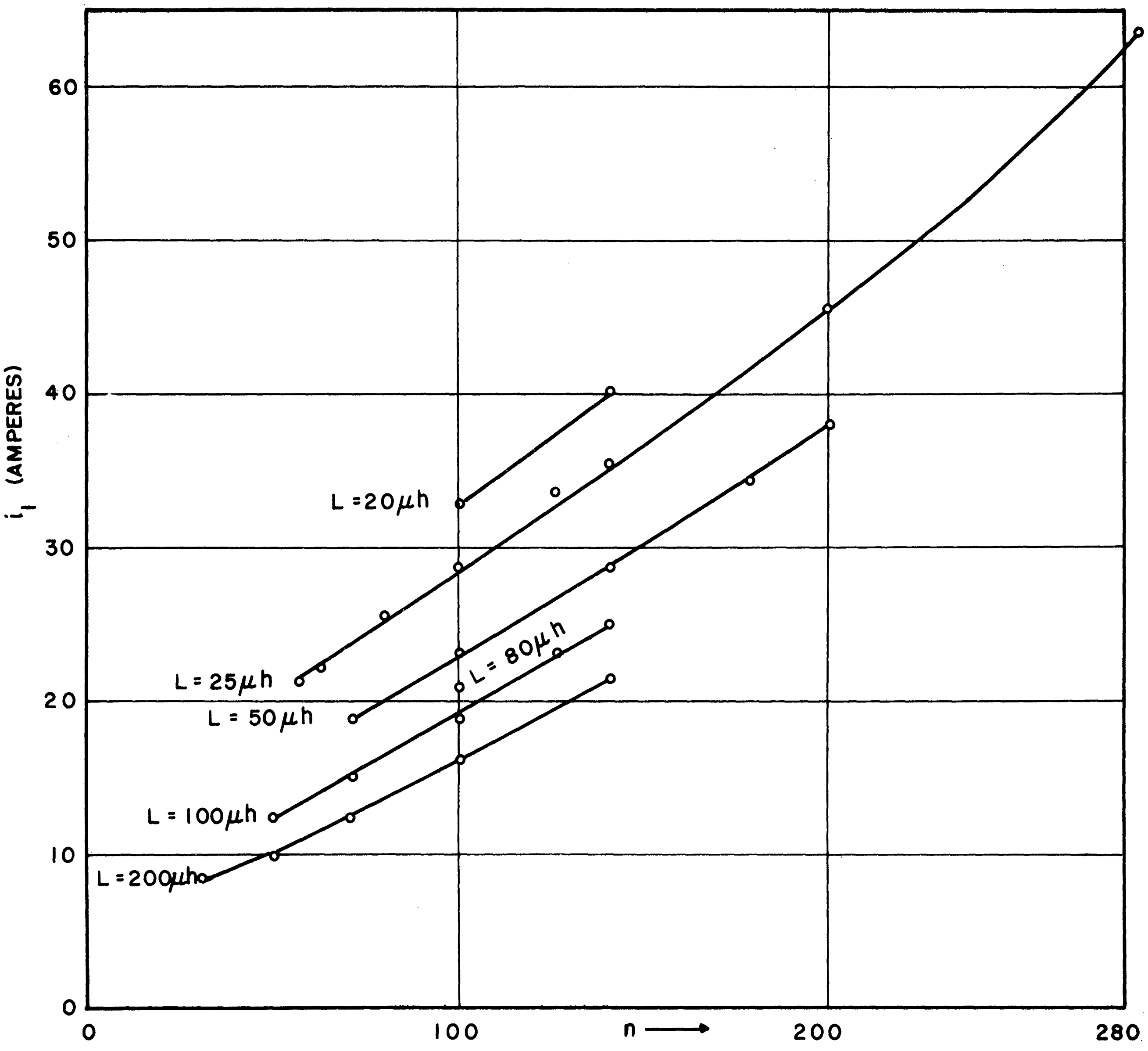

FIG 12 PEAK PRIMARY CURRENT REQUIRED TO PRODUCE 25KV SPARK PLUG VOLTAGE PEAK VS n

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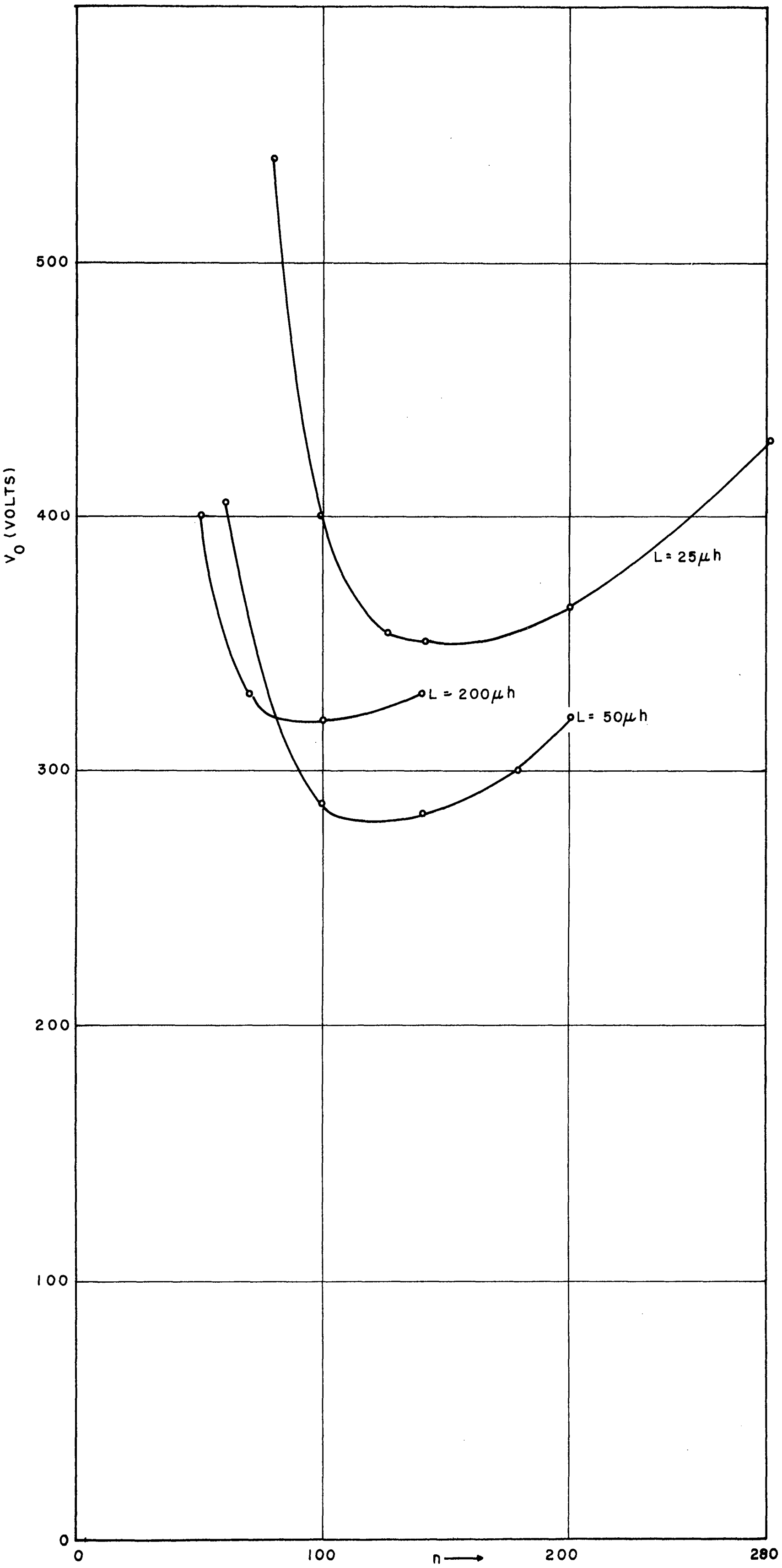

FIG 13 INITIAL STORAGE CAPACITOR VOLTAGE V REQUIRED
FOR SPARK PLUG VOLTAGE TO EXCEED 20 KV FOR
10 SEC. VS n

It can be seen from the curves in Figs. 11 and 13 that for the network parameters $n = 100$ and $L = 80\ \mu H$, the system satisfies both of the design criteria of 25 kv peak and 20 kv 10 μ sec dwell time with the smallest initial voltage. The initial storage-capacitor voltage required to satisfy both criteria is $V_o = 290$ volts. From Fig. 12 it can be seen that the peak primary current for this condition will be $0.075\ V_o$, or, for $V_o = 290$ volts, approximately 22 amps.

The investigation showed that the peak primary current is substantially increased by the occurrence of the spark. The amount of the increase depends very much on the timing of the spark. For the case when the spark occurs at the spark-plug voltage $V_2(t) = V_2(t_m)/2$, the increase in the peak value of primary current was as much as 50%.

The above data apply to circuits with $C_1 = 1\ \mu F$. For some cases the value of C_1 was varied to 2 μF, and it was noted that this brought about an increase of the order of 25% in the peak secondary-voltage peak. The duration of both the high voltage dwell and the spark current under sparking became considerably longer. It should be observed that this improvement is obtained at the cost of increasing the input energy of the system by the factor of 2.

5. THE COMPUTER EQUIVALENT OF THE LINE DISCHARGE SYSTEM

The desirability of obtaining an approximately rectangular wave shape for the spark plug voltage (in the absence of the spark) led to the idea of replacing the discharge capacitor by a delay line. It was thought that a pi-section consisting of two capacitors and a connecting inductance would be a practical approximation to the delay line. If the discharge capacitor of the ignition network (Fig. 3) is replaced with such a line, keeping the same total capacitance, the network of Fig. 14 is obtained. After eliminating the transformer, the equivalent circuit shown in Fig. 15 is obtained.

The mesh equations in terms of the three loop currents can be stated as follows:

$$\frac{\int I_1 dt}{C_2} + L_1 \dot{I}_1 + \frac{\int I_1 dt}{C_1} = 0 \tag{10}$$

$$- \frac{\int I_1 dt}{C_1} + \frac{\int I_2 dt}{C_1} + (R_a + R)\, I_2 - R I_3 = 0 \tag{11}$$

$$- R I_2 + R I_3 + \frac{\int I_3 dt}{C} = 0 \tag{12}$$

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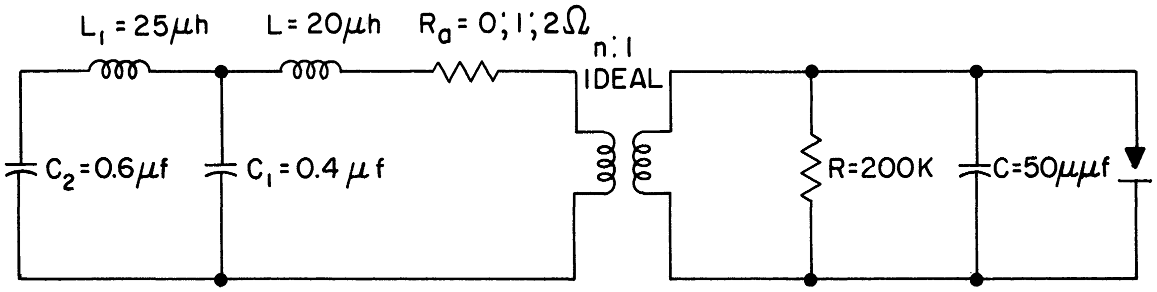


FIG 14 IDEAL TRANSFORMER REPRESENTATION OF A LINE DISCHARGE IGNITION SYSTEM

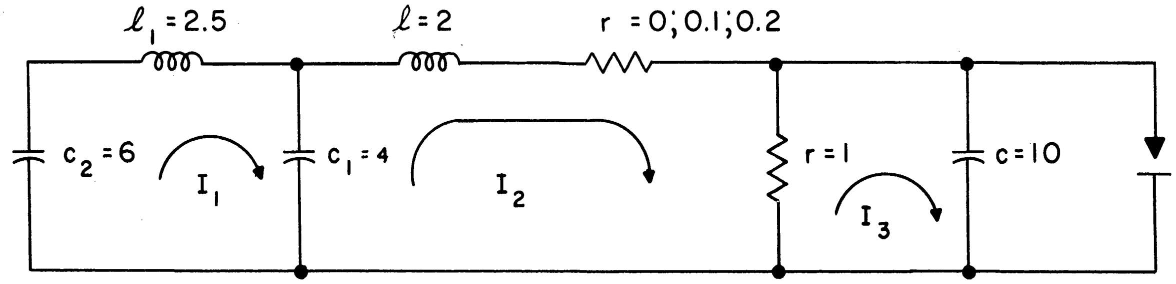


FIG 15 MESH CURRENTS IN A LINE DISCHARGE IGNITION SYSTEM NETWORK

These equations can be rewritten in a form convenient for setting the equations on the computer. Again, the lower case leters denote the coefficients actually used in the computer set-up; that is, the impedances normalized at the desired frequency and impedance level.

$$-\dot{I}_1 = \frac{1}{\ell_1}\left\{\frac{\int I_1 dt}{C_2} - \frac{\int (I_2 - I_1)dt}{C_1}\right\} \tag{13}$$

$$\dot{I}_2 = \frac{1}{\ell}\left\{-\frac{\int (I_2 - I_1)dt}{C_1} - rI_2 - r_a I_2 + rI_3\right\} \tag{14}$$

$$-I_3 = I_2 - \frac{\int I_3 dt}{rC} \tag{15}$$

The computer set-up for solving these equations is shown in Fig. 16. The numbers represent resistor values in megohms and capacitor values in micro-farads. The values in the set-up correspond to the circuit parameters in Fig. 14 (n = 141).

The initial conditions of the network are: voltage V_0 across the capacitors C_1 and C_2. The initial currents in the inductors and the initial voltage across the capacitor C are zero. These initial conditions are simulated on the computer by applying voltages of equal magnitude and opposite polarities as the initial conditions on the integrators W and X.

Again, the plug voltage V_2 and the switch current i_1 are recorded. In this case the switch current is given by $i_1 = I_2$.

6. VOLTAGES AND CURRENTS IN THE LINE DISCHARGE SYSTEM

The line-discharge system was compared with the capacitor discharge system of the same initial voltage V_0 and the same initial energy. Since the value of 1 μF was used for the capacitor discharge system,the sum of the capacitor values $C_1 + C_2$ was also made equal to 1 μF. It was found that the values $C_1 = 0.6$ μF and $C_2 = 0.4$ μF give a good approximation to the rectangular plug voltage pulse. Consequently those values were used in the circuits investigated.

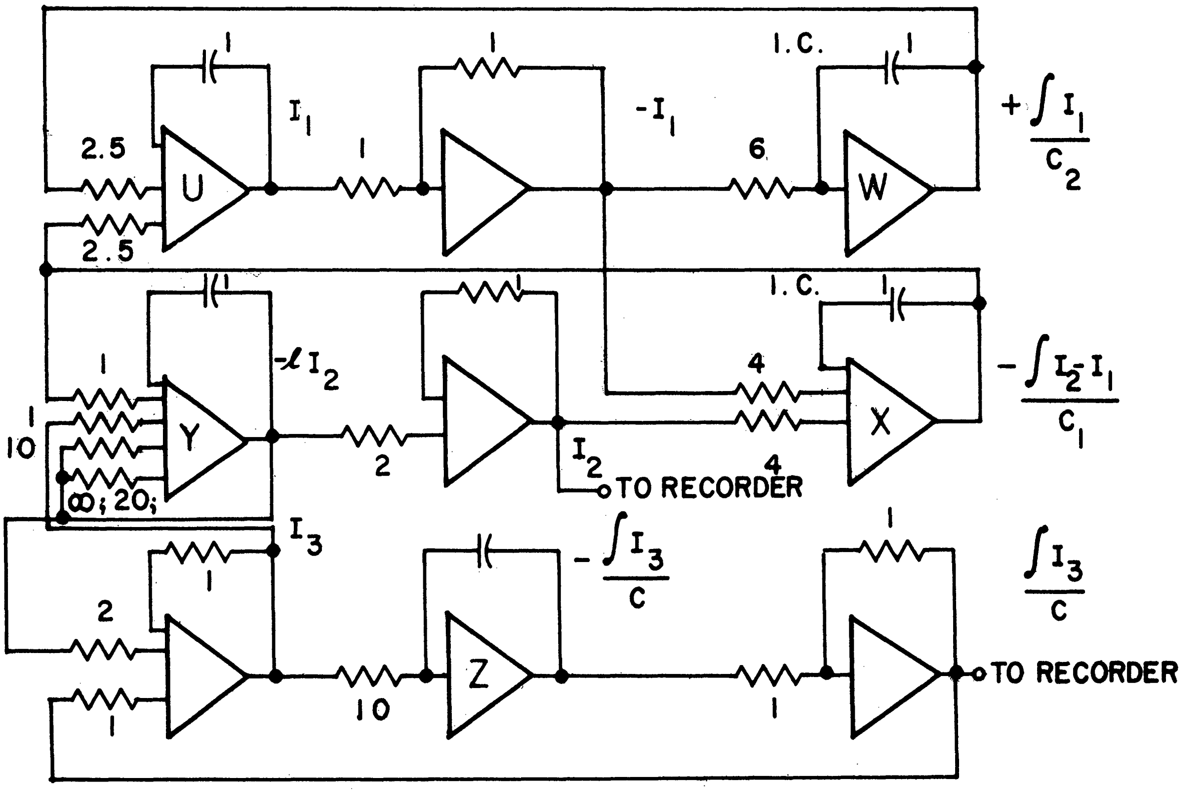


FIG 16 COMPUTER SET-UP FOR SIMULATING THE LINE IGNITION SYSTEM DISCHARGE

Two line discharge circuits were investigated, one with $L_1 = 20$ μH and $n = 100$ and the second with $L_1 = 25$ μH and $n = 141$. The effect of a series resistance R_a on voltages and currents was investigated. The spark plug voltage, V_2, and the switch current, i_1, of the first circuit are shown in Fig. 17 and Fig. 18 for $R_a = 0$ and $R_a = 1$ respectively.

7.--COMPARISON OF CAPACITOR AND LINE DISCHARGE SYSTEMS

The results of voltage and current measurements for those two line-discharge circuits and two comparable capacitor-discharge systems are presented in Table II.

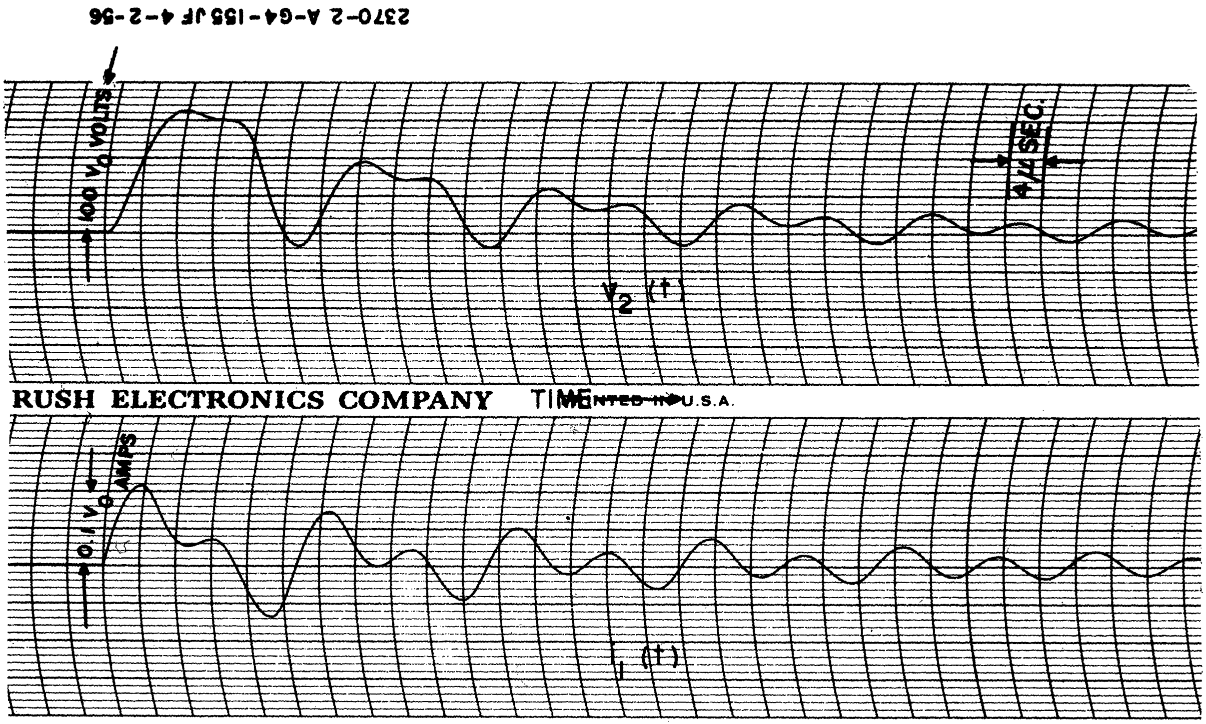
In comparing the line discharge systems with similar capacitor discharge systems, it can be seen that the peak plug voltage of the line system is approximately 25 percent lower than the capacitor system. Correspondingly, the requirement of 25 kv peak calls for a considerably higher initial voltage V_o than in the case of capacitor discharge. On the other hand the wave form in the line discharge is such that the second criterion for plug voltage of 20 kv or more during 10 μ sec is satisfied with a much lower initial voltage. Actually the line discharge system with $n = 100$ and $L_1 = 20$ μH meets both requirements when charged to approximately the same voltage V_o that is required in the optimized capacitor discharge system with $L = 80$ μH and $n = 100$. It should be pointed out, however, that in this later the peak primary current is 22 amperes as compared with 33.1 amperes in the line discharge case.

It is concluded, therefore, that the line discharge system shows its best advantage only in the case of small leakage inductance (i.e., $L = 20$ μH). In this case the desired shaping of the plug voltage waveform allows both criteria to be satisfied with a lower value of V_o than is possible with the capacitor discharge system.

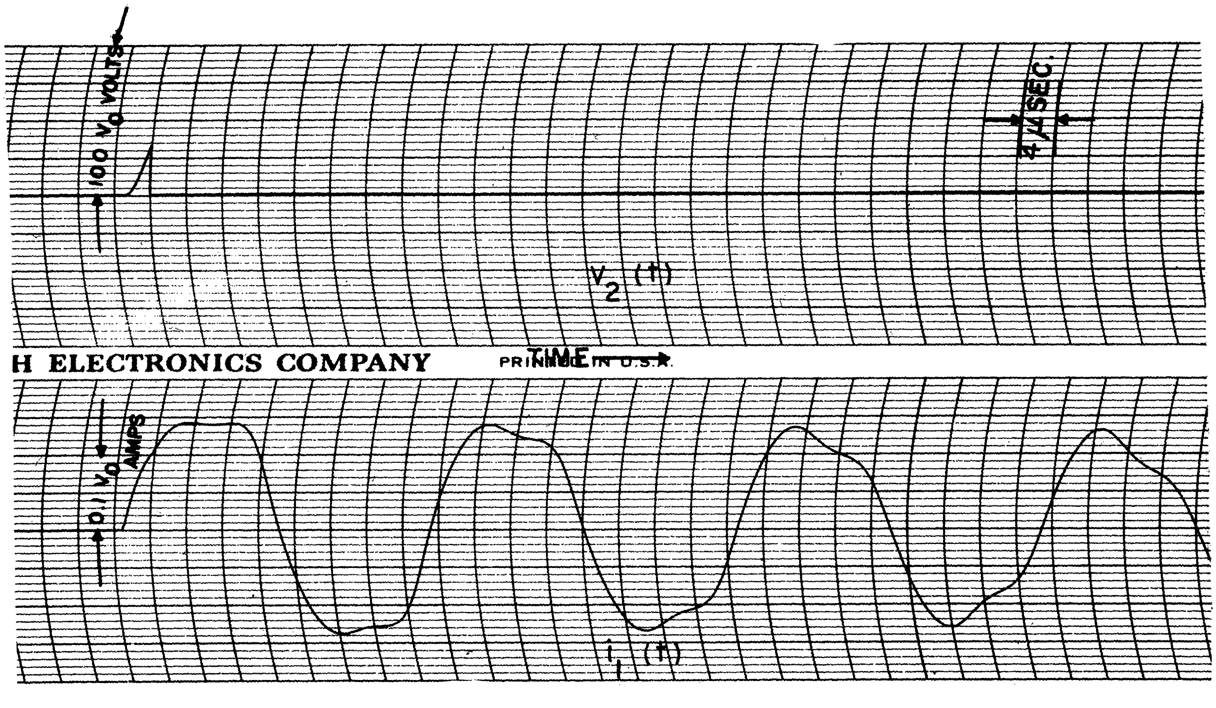
Table II

	$n = 100$		$n = 141$	
	Line Discharge $L_1 = 20\ \mu H$	Capacitor Discharge	Line Discharge $L_1 = 25\ \mu H$	Capacitor Discharge
Ratio $V_2(t_m)/V_o$ of Peak Plug Voltage to Initial Primary Voltage	83	105	79	107
Initial Primary Voltage V_o Required to Produce 25 kv Plug Voltage Peak (Volts)	301	238	316	283
Initial Primary Voltage V_o Required to Make the Plug Voltage Exceed 20 kv for 10 μ sec (Volts)	295	503	234	405
Ratio of Peak Switch Current i_1 to Initial Primary Voltage (Amperes/Volts)	0.11	0.132	0.129	0.17
Peak Switch Current i_1 Required to Make The Plug Voltage Exceed 20 kv for 10 μ sec and Exceed 25 kv Peak (Amperes)	33.1	66.4	41.0	66.9

Voltages and Currents in Line Discharge and Capacitor Discharge Systems



(a) NO SPARK SOLUTION

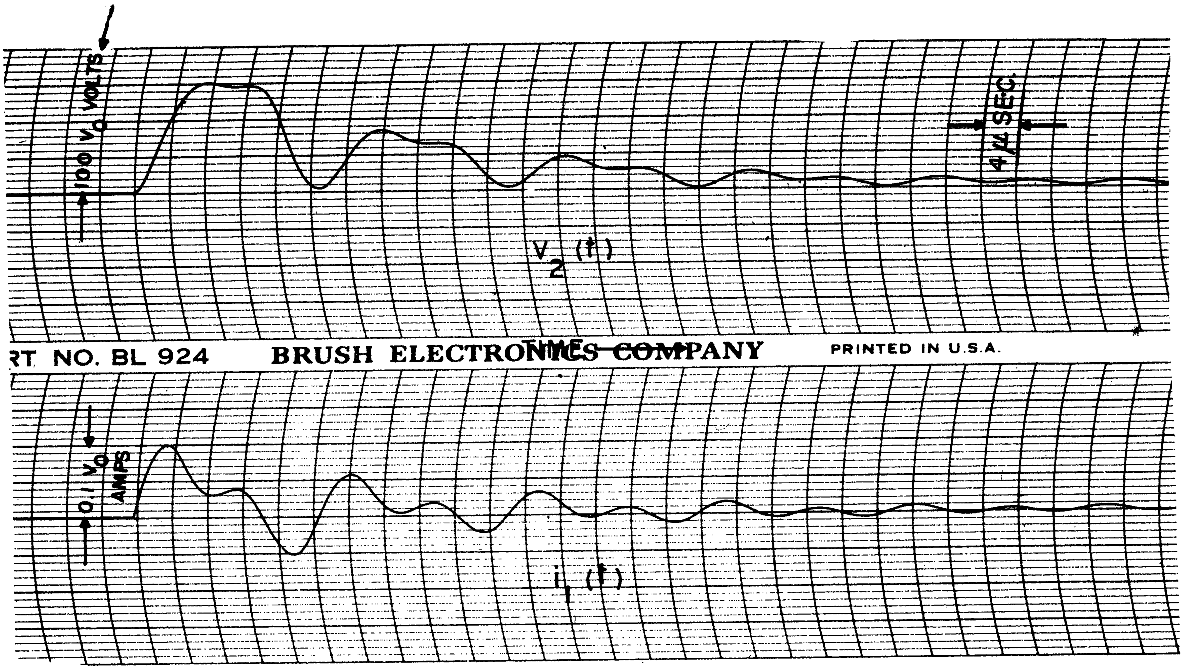


(b) THE EFFECT OF AN EARLY SPARK

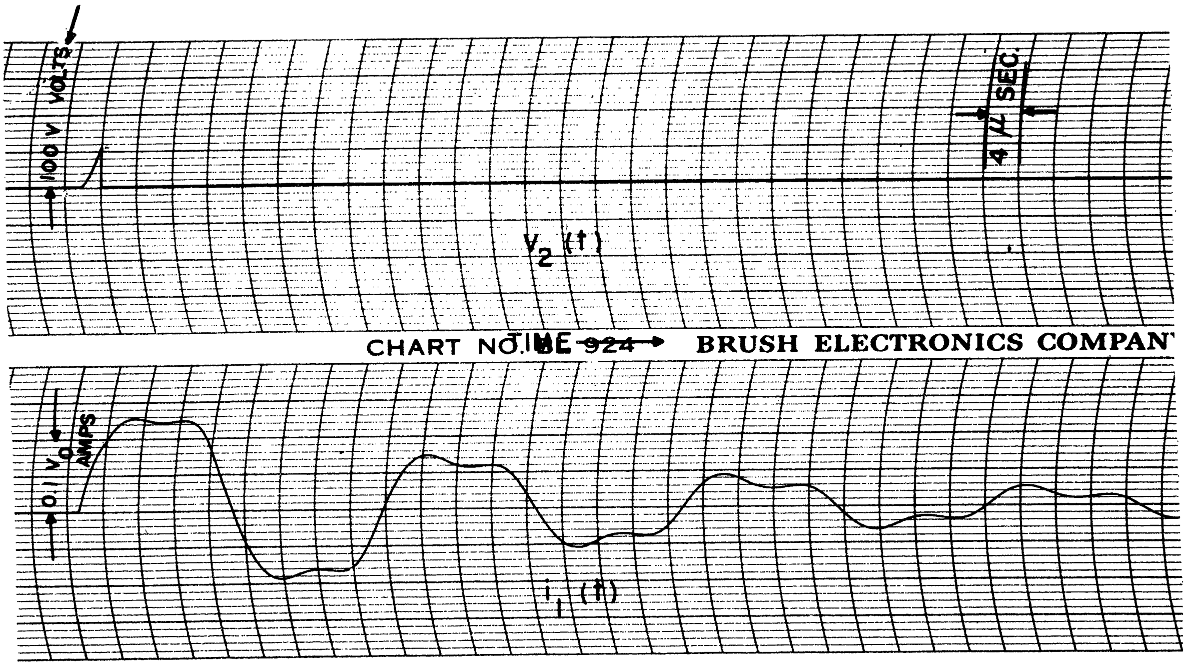
SPARK PLUG VOLTAGE V_2 AND PRIMARY CURRENT i_1 IN
A LINE DISCHARGE IGNITION SYSTEM
$C_1 = 0.4 \mu F$, $C_2 = 0.6 \mu F$, $L_1 = 20 \mu H$, $L = 20 \mu H$, $R_a = 0$
$n = 100$, $R = 200 K\Omega$, $C = 50 \mu\mu F$

FIG. 17

2370-2 A-64-156 JF 4-2-56



(a) NO SPARK SOLUTION



(b) THE EFFECT OF AN EARLY SPARK

SPARK PLUG VOLTAGE V_2 AND PRIMARY CURRENT i_1
IN A LINE DISCHARGE IGNITION SYSTEM
$C_1 = 0.4\,\mu F$, $C_2 = 0.6\,\mu F$, $L_1 = 20\,\mu H$, $L = 20\,\mu H$, $R_a = 1\,\Omega$,
$n = 100$, $R = 200\,K\Omega$, $C = 50\,\mu\mu F$

FIG 18

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