Acoustic emission from plastic deformation of a pure single crystal

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Acoustic emission (AE) during plastic deformation is analyzed for a pure single crystal neglecting the effects of grain boundaries, impurities, and second-phase particles. Acceleration of a moving dislocation is considered to be the principal AE source. There are two major mechanisms of dislocation motion related to acceleration, initial, and continuous oscillatory motion. Initial motion induced by the creation of mobile dislocations is modeled as a step function of velocity. Continuous oscillatory motion produced by interactions with neighboring dislocations is modeled as a harmonic function. These mechanisms vary with strain and strain rate due to dislocation multiplication. AE can thus be described in terms of strain and strain rate. Annihilation at a free surface is also regarded as an AE source in addition to the initial and oscillatory motions. The kinetic and strain energies stored around a moving dislocation are dissipated during annihilation, and can be related to AE. The frequency spectrum of AE is also determined. A shift of the spectrum to higher frequencies with increasing strain is explained by an increase in the interaction force between dislocations.

I. INTRODUCTION

Acoustic emission (AE) is regarded as an elastic wave generated by the release of energy. There are two main forms of acoustic emission: continuous and burst emission. The continuous emission consists of a large number of low-amplitude emissions produced by a series of dislocation motions during plastic deformation without obstacle interruptions. When a crystal is deformed, even though the macroscopic deformation is uniform and continuous, the microscopic strain mechanism is discrete and discontinuous resulting in the radiation of wave energy that propagates through the medium.

It has been proposed that acoustic emission from dislocation motion is produced by radiation of wave energy from accelerating dislocations.[1,2] Another approach to analyzing AE during dislocation motion was undertaken by Scruby[3] using the elastic Green's function to obtain the resulting surface displacement. The former analysis determines the amount of AE energy from the sources regardless of the directionality of the source mechanism. However, the latter approach is good for analyzing the orientation of a source mechanism since the measured AE varies differently depending on the transducer location. Since the orientation of a moving dislocation is not readily determinable, AE from dislocation motion during plastic deformation can be analyzed by quantifying the total radiation of wave energy from dislocation motion.

Dislocation motion is composed of initial motion and steady oscillatory motion. For the initial motion, the Peierls stress, which is the maximum from a potential trough to a crest, is required to surmount the potential barrier due to lattice periodicity. After a dislocation passes over the crest, the potential energy is converted to kinetic energy and, as a result, it is not necessary to have an equally large Peierls force to surmount the next crest which enables sudden acceleration. And, due to the high initial static Peierls stress,[4] there is an abrupt initial motion

of a dislocation with a step change in velocity as shown in Fig. 1(a). Following the initial motion, the dislocation undergoes a variation of potential energy due to the strain fields of other dislocations, which causes a variation in the dislocation velocity. Both the initial and steady oscillatory motions are sources of wave energy radiated during the plastic deformation of a pure single crystal. However, the variation of the Peierls potential can be neglected in analyzing acoustic emission, since the wave energy radiated from that has relatively high frequency and small amplitude compared to the energy associated with dislocation interactions. The stress and strain fields of an accelerating dislocation as well as the exact acceleration of dislocations during plastic deformation are difficult to determine. Only the average velocity is obtainable during plastic deformation.

Thus, the acceleration of a dislocation is modeled approximately as a step function for initial motion, and as a harmonic function for continuous motion, where both motions can be related to the average velocity. The initial motion can be represented by a Heaviside step function, [Fig. 1(a)] and the subsequent steady oscillatory motion due to the interaction with neighboring dislocations by a periodic function [Fig. 1(b)]. Thus, the dislocation motion will be described as a superposition of the two mechanisms, and AE energy from the two different motion of dislocations during plastic deformation will be obtained in terms of strain and strain rate. In addition, the AE generated during annihilation of dislocations will be analyzed.

II. DISLOCATION FLOW AND PLASTIC DEFORMATION: KINEMATICS

During plastic deformation, moving dislocations are continuously annihilated at the free surface after gliding through the slip plane, and new mobile dislocations are generated to maintain the deformation. The flow of dislocations may be represented as
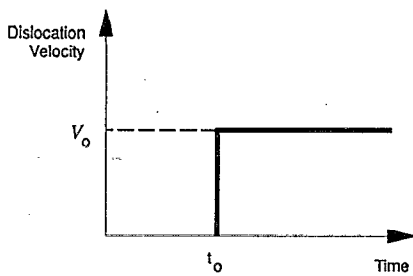
(a) Step Motion at time, t_o
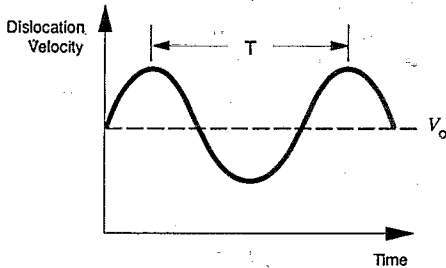
(b) Periodic Motion with period, T

FIG. 1. Velocity function of a moving dislocation.

$$\Delta\rho_m = \Delta\rho_{mc} - \Delta\rho_{out}, \qquad (1)$$

where ρ_{mc} is the density of newly created moving dislocations, ρ_m is the density of moving dislocations, and ρ_{out} is the density of dislocations annihilated at a free surface. Considering Eq. (1) in terms of strain ϵ,

$$N_{mc} = N_m + N_{out}, \qquad (2)$$

where $N = \partial\rho/\partial\epsilon$ is the rate of change of dislocation density with respect to strain. And, for the time variation, we have

$$\dot{\rho}_{mc} = \dot{\rho}_m + \dot{\rho}_{out}, \qquad (3)$$

where $\dot{\rho} = \partial\rho/\partial t$ is the rate of change of dislocation density with respect to time, and is given by

$$\dot{\rho} = \frac{\partial\rho}{\partial t} = \frac{\partial\rho}{\partial\epsilon}\frac{\partial\epsilon}{\partial t} = N\dot{\epsilon}. \qquad (4)$$

Since the plastic displacement provided by the flow-out of a dislocation at a free surface is equivalent to the Burgers vector b, N_{out} is constant. Thus, the total length of the annihilated dislocations ρ_{out} during strain ϵ of a unit volume is

$$\rho_{out} = N_{out}\epsilon, \qquad (5)$$

and the rate of change of annihilated dislocation density $\dot{\rho}_{out}$ is given by

$$\dot{\rho}_{out} = N_{out}\dot{\epsilon}. \qquad (6)$$

Since N_{out} is constant, $\dot{\rho}_{out}$ is also constant for plastic deformation at a constant strain rate.

III. RADIATION OF WAVE ENERGY BY DISLOCATION MOTION

A. Elastic wave energy from initial motion

Abrupt motion of a dislocation is induced by breakaway from a large static frictional stress, which causes a sudden change in dislocation velocity. The strain field of a dislocation moving from rest to a constant velocity has been obtained by the Laplace transformation[5,6] and by use of the Green's function.[7] For a dislocation with a uniform velocity v starting at $t=0$, only the time-dependent terms will be related to the radiation wave energy. Time-independent terms are associated with steady-state motion of dislocations, and generate no radiation energy during motion inside a material, but do generate radiation energy at a free surface.

For screw dislocations, there is only displacement u_z in the z direction, with the displacements, u_x and u_y, in the x and y directions, respectively, being zero. The elementary motion of mass vector $\mathbf{d}$ is in the z direction, and the propagation vector of the dislocation $\mathbf{p}$ in the x direction. Only a shear wave is emitted from a moving screw dislocation because $\mathbf{d}_z \cdot \mathbf{p} = 0$. For an edge dislocation parallel to the z direction, a force in the x direction causes motion of mass element in the x and y directions. The elementary mass motion vector $\mathbf{d}$ is in the x-y plane, and the propagation vector $\mathbf{p}$ is in the x direction, so that, $\mathbf{d}_y \cdot \mathbf{p} = 0$ and $\mathbf{d}_x = \pm\mathbf{p}$. Thus, an edge dislocation generates a shear wave by the element motion in the y direction $\mathbf{d}_y$ and a longitudinal wave by motion in the x direction $\mathbf{d}_x$.

The stress field around a screw dislocation when it starts moving with constant velocity v in the x direction starting from a static condition at time $t=0$ is[5]

$$\sigma_{yz} = \mu\frac{\partial u_z}{\partial y} = \frac{\mu b}{2\pi}\left(\frac{\beta(x-vt)}{(x-vt)^2 + \beta^2 y^2} + \frac{1}{vt}(1-\beta^{-1})\right), \qquad (7)$$

where $\beta = [1 - (v^2/c_t^2)]^{1/2}$, μ is shear modulus, b is Burgers vector, v is velocity, and c_t is shear wave velocity. The first term of Eq. (7) represents the stress field of a dislocation moving with velocity v, and the second term is the stress imposed on a moving dislocation due to the external stress σ'_{yz}, which is the source of the acceleration that results in a step increase in velocity.

The rate of radiation of energy $\dot{E}_t$ per unit length of screw dislocation due to transient motion can be written as

$$\dot{E}_t = -F_x v = -\sigma'_{yz} bv,$$

where F_x is the force component in the x direction applied to a unit length of dislocation, and σ'_{yz} is the dislocation stress resulting from an external force applied to the dislocation core to initiate and sustain motion of the dislocation in the x direction with velocity v. Substituting for σ'_{yz}, $\dot{E}_t$ becomes

$$\dot{E}_t = \frac{\mu b^2}{2\pi}(\beta^{-1}-1)\frac{1}{t},$$

with the total energy radiated during the interval t being

$$E_t = \int_{t_0}^{t} \dot{E}_t \, dt = \frac{\mu b^2}{2\pi} (\beta^{-1} - 1) \ln \frac{t}{t_0}.$$

The radiation is assumed to start at a distance b, because the core region inside $r = b$ is not analytic for a Volterra model. Thus, it is considered valid only for the region $c_t t \gg b$, giving $c_t t_0 = b$. For a relatively low dislocation velocity,

$$E_t \cong K_{is} v^2 \ln(t/t_0), \tag{8}$$

where $K_{is} = \mu b^2 / 4\pi c_t^2$.

For an edge dislocation, the stress field is[6]

$$\sigma_{xy} = \mu \frac{\partial u_x}{\partial y} = \frac{\mu b}{2\pi} \left(\frac{2\gamma(x - vt)}{(x - vt)^2 + \gamma^2 y^2} \right.$$

$$- \frac{(\alpha^2 \beta + \alpha^2 \beta^{-1})(x - vt)}{(x - vt)^2 + \beta^2 y^2}$$

$$\left. + \frac{(1 + 2\gamma^2)}{\gamma vt} - \frac{(1 + 5\beta^2)}{2\beta vt} \right),$$

where $\alpha = [1 - (v^2/2c_t^2)]^{1/2}$, $\gamma = [1 - (v^2/c_t^2)]^{1/2}$, $\beta = [1 - (v^2/c_l^2)]^{1/2}$, and c_l is the longitudinal wave velocity. Following the same approach as for the screw dislocation, $\dot{E}_t$ is given by

$$\dot{E}_t = -F_x v = -\sigma'_{xy} b v.$$

For an edge dislocation, σ'_{xy} constitutes the last two terms of σ_{xy}. The resulting radiation energy rate $\dot{E}_t$, is then

$$\dot{E}_t = \frac{\mu b^2}{2\pi} \left(\frac{(1 + 2\gamma^2)}{\gamma} - \frac{(1 + 5\beta^2)}{2\beta} \right) \frac{1}{t},$$

and the total energy E_t in a finite time t is

$$E_t = \int_{t_0}^{t} \dot{E}_t \, dt = \frac{\mu b^2}{2\pi} \left(\frac{(1 + 2\gamma^2)}{\gamma} - \frac{(1 + 5\beta^2)}{2\beta} \right) \ln \frac{t}{t_0},$$

where $t_0 = b/c_l$. Again, with a relatively low dislocation velocity, we have

$$E_t \cong K_{ie} v^2 \ln(t/t_0), \tag{9}$$

where

$$K_{ie} = \frac{\mu b^2}{2\pi} \left(\frac{1}{c_t^2} - \frac{1}{2c_l^2} \right).$$

Consequently, the wave energy radiated from a dislocation initially at rest at $t = 0$ and suddenly accelerated to a velocity v depends on the duration of dislocation motion t and is proportional to the square of the dislocation velocity v.

B. Elastic wave energy from harmonic motion

The motion of a dislocation is affected by the stress fields of neighboring dislocations which result in a variation in potential energy of the moving dislocation and, consequently, a periodicity of motion (Fig. 2). When a dislocation accelerates or decelerates as it passes through
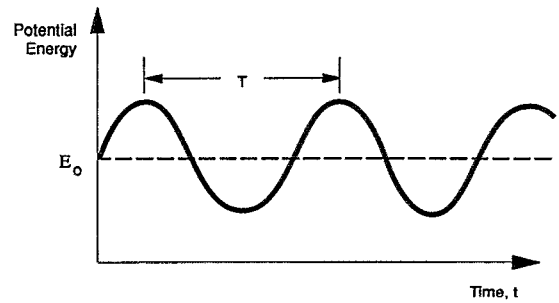

FIG. 2. A moving dislocation undergoing variation of potential energy.

the varying stress fields, energy is dissipated which propagates as an elastic wave.

The oscillatory motion is superposed on the uniform motion. When $x'(t)$ is the displacement of the dislocation in the reference frame moving with average velocity v, the absolute displacement $x(t)$ is

$$x(t) = x'(t) + vt.$$

At relatively low velocities, the motion of a dislocation is determined not by inertia but by the periodic potential (see Appendix). Thus, the amplitude is determined by half the periodicity of the potential,

$$x'(t) = (d/2)\sin \omega t,$$

where d is the distance between dislocations and ω is angular velocity. The velocity of the dislocation v_d is then

$$v_d(t) = (\omega d/2)\cos \omega t + v. \tag{10}$$

The radiation wave energy is only related to the first term of Eq. (10) since there is no radiation energy from a uniformly moving dislocation. The radiation of shear wave energy from harmonic motion of a unit length of screw dislocation was determined by Eshelby[8] as follows:

$$\dot{E}_h = \frac{\mu b^2 \omega}{8c_t^2} \left(\frac{\omega d}{2} \cos \omega t \right)^2,$$

where $\dot{E}_h$ is the rate of radiation of energy from harmonic motion of a dislocation. The energy radiated per oscillation can be obtained by integration over the period T,

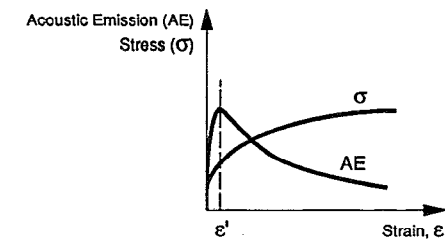$$E_h = \int_{0}^{2\pi/\omega} \dot{E}_h \, dt = K_{hs} v^2, \tag{11}$$

where $K_{hs} = \mu b^2 \pi^3 / 8c_t^2$.

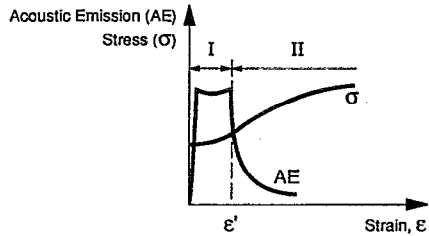The kinetic energy E_k of a uniformly moving edge dislocation[9] is given by

$$E_k = 2E_0 \frac{c_t^2}{v^2} (\gamma + \gamma^{-1} + \alpha^4 \beta^{-1} + \alpha^4 \beta^{-3} - 4\alpha^2 \beta^{-1}),$$

where E_0 is the strain energy of a dislocation and is given by $(\mu b^2/4\pi)\ln(d/2b)$. Expanding α, β, and γ at a relatively low dislocation velocity, it can be shown that

$$E_k = \frac{1}{2} E_0 c_t^2 \left(\frac{1}{c_l^4} + \frac{1}{c_t^4} \right) v^2.$$

(a) Homogeneous deformation (pure Aluminum)

(b) Stretched yield deformation (Aluminum alloy)

FIG. 3. Stress and acoustic emission from two different deformation models.

The increase in external stress $\Delta\sigma_{xy}$ required for a velocity increase Δv is

$$\Delta\sigma_{xy} = \frac{1}{bv}\frac{\partial(\Delta E_k)}{\partial t} = \frac{\mu b c_t^2 \Delta v}{4\pi t}\left(\frac{1}{c_l^4}+\frac{1}{c_t^4}\right).$$

Using the same procedure as for screw dislocations,[8] the radiation energy from a unit length of edge dislocation is then

$$\dot{E}_h = \frac{1}{8}\mu b^2 c_t^2\omega\left(\frac{1}{c_l^4}+\frac{1}{c_t^4}\right)\left(\frac{\omega d}{2}\cos\omega t\right)^2.$$

The energy radiated per oscillation of a unit edge dislocation length then becomes

$$E_h = \int_0^{2\pi/\omega}\dot{E}_h\,dt = K_{he}v^2, \tag{12}$$

where

$$K_{he} = \frac{\mu\pi^3 b^2 c_t^2}{8}\left(\frac{1}{c_l^4}+\frac{1}{c_t^4}\right).$$

IV. AE AND PLASTIC DEFORMATION UNDER CONSTANT STRAIN RATE

Typical stress-strain and AE-strain relations of pure aluminum and aluminum alloy specimen are shown in Fig. 3.[10] Even though the results are for polycrystalline materials while our analysis is for single crystals, we use them for illustration since they have basic similarities to results that would be obtained from single crystals. Work hardening begins from the strain ϵ'. The initial portion of the curve, involving strain from 0 to ϵ', is relatively small and

represents a transition from elastic to full work hardening. This stage is more evident with alloys[11] such as occurs in Lüders band formation, where dislocation motion in the stretched yield region is mainly determined by interaction with impurities, generating high AE activity, while for a pure material with uniform deformation most of the AE is generated after ϵ'. A relatively high dislocation velocity in the stretched yield region due to impurity effects compared to that in uniform deformation of a pure material produces high-amplitude acoustic emission signals. After the initial stage, work hardening starts and dislocation velocity is reduced, resulting in a notable decrease in AE signals after ϵ'. Considering only homogeneous deformation arising after ϵ', where the forest dislocation density correlates well with the strain hardening, if the function f_{AE} defines AE energy per unit strain and unit volume, the AE energy E_{AE} during deformation from ϵ_1 to ϵ_2 of a unit volume is given by

$$E_{AE} = \int_{\epsilon_1}^{\epsilon_2} f_{AE}\,d\epsilon.$$

Plastic deformation and radiation energy are produced only by the mobile dislocations ρ_m, but the average dislocation distance d and interaction force are determined by the overall dislocation density ρ. The ratio of mobile dislocation density to total dislocation density γ_m can be assumed constant during deformation under the assumption that temperature change is negligible during deformation,[12]

$$\rho_m/\rho = \gamma_m = \text{const}.$$

As strain increases, both the total and mobile dislocation densities are raised by dislocation multiplication.

Using the Taylor equation, which is based on the dislocation interaction force, the relationship between flow stress and dislocation density in stage II can be written as

$$\sigma = k_m\rho^{0.5}, \tag{13}$$

where $k_m = \mu b/2\pi\alpha_s$, and α_s is 1 for a screw dislocation, $1-\nu$ for an edge dislocation, and ν is Poisson's ratio. A macroscopic stress-strain relation is

$$\sigma = k\epsilon^n \quad (\epsilon > \epsilon'), \tag{14}$$

where n is the work hardening exponent and k is a constant. ρ can then be obtained in terms of strain ϵ using Eqs. (13) and (14),

$$\rho = C_n\epsilon^{2n}, \tag{15}$$

where $C_n = (k/k_m)^2$. The initial dislocation density ρ_i is given by $C_n\epsilon'^{2n}$. The moving dislocation density can also be expressed in terms of ϵ and n as

$$\rho_m = \gamma_m C_n\epsilon^{2n}. \tag{16}$$

Considering plastic deformation with a given constant strain rate $\dot{\epsilon}_c$,

$$v = \dot{\epsilon}_c/\rho_m b.$$

The average velocity v is then obtained in terms of strain by substituting for ρ_m from Eq. (16),

$$v = c_v \epsilon^{-2n}, \tag{17}$$

where $c_v = \dot{\epsilon}_c / b \gamma_m C_n$.

A. AE from step motion of a dislocation

From Eq. (16), we have

$$N_m = \frac{\partial \rho_m}{\partial \epsilon} = 2n \gamma_m C_n \epsilon^{2n-1}. \tag{18}$$

Substituting Eq. (18) into Eq. (2) gives

$$N_{mc} = 2n \gamma_m C_n \epsilon^{2n-1} + N_{out}, \tag{19}$$

and further from Eqs. (3), (4), and (16), we have

$$\dot{\rho}_{mc} = \frac{\partial \rho_m}{\partial \epsilon} \frac{\partial \epsilon}{\partial t} + \dot{\rho}_{out} = 2n \dot{\epsilon} \gamma_m C_n \epsilon^{2n-1} + \dot{\rho}_{out}.$$

For a given length of glide plane h, the average duration of dislocation glide t is given by

$$t = h/v. \tag{20}$$

For a single crystal, h depends on the dimensions of the workpiece. E_t for screw dislocations can thus be obtained by substituting Eqs. (17) and (20) into Eq. (8),

$$E_t = \beta_d K_{ts} c_v^2 \epsilon^{-4n},$$

where β_d is given by substituting Eq. (20) in the logarithmic factor in Eq. (8). Acoustic emission energy is obtained as the sum of the individual contributions of radiation wave energy E_t from transient motion resulting from the creation of mobile dislocations. Only the term $\partial \rho_m / \partial \epsilon$ in Eq. (19) determines the variation of N_{mc} in terms of strain, since N_{out} is constant for a unit strain. Considering the variation of AE with respect to strain, the AE energy per unit strain and unit volume f_{AE}, from step motion of screw dislocations is then obtained as

$$f_{AE} = N_{mc} E_t = E_{t0} \epsilon^{-2n-1}, \tag{21}$$

where $E_{t0} = 2n K_{ts} \gamma_m C_n c_v^2 \beta_d$. AE from the step motion of edge dislocations with strain is found in a similar manner using Eq. (9).

B. AE from harmonic motion of a dislocation

A moving dislocation should keep gliding under its inertial force, until it encounters obstacles or is annihilated at a free surface. It may not radiate wave energy while gliding with constant velocity. However, moving dislocations, even in a pure crystal, will undergo variation of potential energy as a result of interactions with neighboring dislocations.

Harmonic motion of a dislocation can be determined by considering the periodic glide distance d which is a function of the dislocation density ρ,

$$d = \rho^{-0.5}. \tag{22}$$

The average glide distance d_m that results in a unit strain due to a mobile dislocation density ρ_m is given by
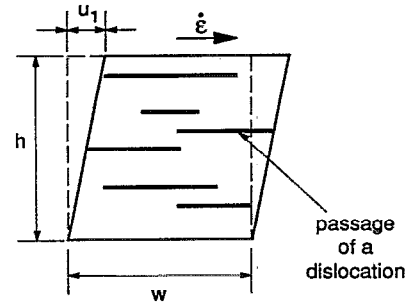
$$d_m = 1/\rho_m b.$$

FIG. 4. Dislocation flow for a specimen of width x, height y, and thickness z.

Thus, the number of oscillations f experienced by each dislocation per unit strain is

$$f = \frac{d_m}{d}.$$

And then from Eqs. (11), (16), and (22), the acoustic emission energy is obtained in terms of strain as

$$f_{AE} = \rho_m E_h f = E_{h0} \epsilon^{-3n}, \tag{23}$$

where $E_{h0} = (K_{hs} c_v^2 \sqrt{C_n})/b$.

V. FREE-SURFACE EFFECTS ON AE

The emergence of a dislocation at the free surface of a crystal is accompanied by the emission of elastic energy which propagates through the medium. Likewise, the annihilation of dislocations within the crystal also generates elastic wave energy. However, for simplicity, we focus our analysis on energy generated due to annihilation at a free surface.

The total glide area L_a for shear deformation of a material with dimensions of width W, height H, and thickness Z as shown in Fig. 4 is represented by

$$L_a = \rho_m WHZ d_m,$$

where d_m is the average glide distance of a moving dislocation. However, the glide distance is limited to the length of glide plane given by the workpiece geometry. Hence a dislocation reaches the free surface after gliding a distance D, where the average of D for moving dislocations is considered to be half of the glide plane, i.e.,

$$D = W/2.$$

The total length of dislocations Q_{out} that break out of the surface after a glide area L_a is then given by

$$Q_{out} = L_a/D = 2 \rho_m HZ d_m. \tag{24}$$

If we consider S as the free surface providing annihilation of dislocations, i.e., $S = 2HZ$, and V as the deformed volume, i.e., $V = WHZ$ (Fig. 4), then the surface-to-volume ratio γ_s is

$$\gamma_s = S/V = 2/W. \tag{25}$$

Substituting Eq. (25) into Eq. (24) gives

$$Q_{out} = \gamma_s \rho_m WHZ d_m .$$

Considering Q_{out} per unit volume,

$$\rho_{out} = Q_{out}/WHZ = \gamma_s \rho_m d_m .$$

By substituting $\rho_m d_m = 1/b$, ρ_{out} per unit strain can be obtained as

$$N_{out} = \gamma_s/b. \tag{26}$$

The AE energy per unit strain and unit volume f_{AE}, from the annihilation of dislocations at a surface is thus determined as

$$f_{AE} = N_{out} E_a , \tag{27}$$

where E_a is the radiation energy due to the emergence of a dislocation at a free surface. For a moving dislocation, E_a is the sum of the strain and kinetic energies, and for a uniformly moving screw dislocation with a relatively slow velocity, E_a is obtained[9] as

$$E_{a\ screw} = E_0 + (E_0 v^2/2c_t^2), \tag{28}$$

where $E_0 = (\mu b^2/4\pi)\ln(d/2b)$ is the strain energy per unit dislocation length when it is stationary. From Eqs. (26), (27), and (28), we obtain

$$f_{AE} = \frac{E_0 \gamma_s}{b}\left(1 + \frac{v^2}{2c_t^2}\right),$$

where only v is a function of strain ϵ. Substituting for v from Eq. (17), we obtain

$$f_{AE} = \frac{E_0 \gamma_s}{b}\left(1 + \frac{c_v^2}{2c_t^2}\epsilon^{-4n}\right). \tag{29}$$

The intensity of AE from the release of kinetic energy thus decreases with strain ϵ to the power of $-4n$ because the kinetic energy diminishes as a result of a reduction in dislocation velocity during work hardening.

For an edge dislocation moving with a relatively slow velocity, AE can be obtained by substituting in the following equation:

$$E_{a\ edge} = \frac{E_0}{1-v} + \frac{E_0 v^2}{2c_t^2}\left(1 + \frac{c_t^4}{c_l^4}\right).$$

VI. ACOUSTIC EMISSION AND STRAIN RATE

Up to this point, acoustic emission has been considered at a constant strain rate during deformation. The flow stress depends not only on the strain but also on the strain rate, and can be expressed as

$$\sigma = C\epsilon^n \dot\epsilon^m, \tag{30}$$

where C is a constant depending on temperature and material, and m is the strain rate sensitivity. From Orowan's equation, $\dot\epsilon = \rho b v$, an increase in strain rate raises either the mobile dislocation density or the velocity, or both:

$$\Delta\dot\epsilon = \Delta(\rho_m v)b = \Delta\rho_m vb + \rho_m \Delta vb.$$

Acoustic emission produced by dislocation dynamics depends on whether $\Delta\dot\epsilon$ is influenced by $\Delta\rho_m$ or Δv.

To simplify the analysis, we consider the two cases separately. First, assuming that an increase in strain rate is induced by Δv, we have

$$\Delta\dot\epsilon = \rho_m b\Delta v.$$

Δv is then proportional to an increase in strain rate and $\Delta\rho_m = 0$. Since the radiation energy from the initial and harmonic motion is proportional to the square of the velocity and the number of AE sources is constant, the acoustic emission due to $\Delta\dot\epsilon$ is then proportional to the square of the strain rate.

The other assumption is made that an increase in strain rate is related to an increase in ρ_m with constant dislocation velocity, i.e.,

$$\Delta\dot\epsilon = \Delta\rho_m bv.$$

Since E_h, E_i, and E_a are functions of v only, they are then independent of strain rate. And further, since $\Delta\rho_m$, $\Delta\rho_{mc}$, and $\Delta\rho_{out}$ are proportional to the change in strain rate, the acoustic emission energy is then proportional to the strain rate.

In practice, the two situations will arise during plastic deformation simultaneously. An increase in the strain rate can raise both the mobile dislocation density and the velocity. At a given strain and temperature, Eq. (30) is expressed as

$$\sigma\alpha\dot\epsilon^m. \tag{31}$$

A direct relationship between average velocity and applied stress is given by a well-known empirical relation,

$$v = v_0(\sigma/\sigma_0)^{m'}, \tag{32}$$

where v_0 is taken as a unit velocity, m' is dislocation velocity sensitivity, and σ_0 is the stress necessary to produce a unit velocity. The effect of strain rate on dislocation velocity is obtained by substituting Eq. (31) into Eq. (32), giving

$$v\alpha\dot\epsilon^\xi, \tag{33}$$

where $\xi = mm'$. At a given strain rate, substituting Eq. (33) into Orowan's equation gives the mobile dislocation density as

$$\rho_m \alpha\dot\epsilon^{1-\xi}.$$

The relative contributions of $\Delta\rho_m$ and Δv to the AE energy due to $\Delta\dot\epsilon$ are thus determined by ξ. Since the radiation energy from moving dislocations is proportional to the source number and to the square of the velocity, the acoustic emission power in terms of strain rate is

$$\dot E_{AE}\alpha\dot\epsilon^{1+\xi}.$$

For the assumption that a change in strain rate is due primarily to a change in mobile dislocation density, $\Delta v = 0$, and thus $m' = 0$, giving $\xi = 0$, and for the assumption that it is due primarily to mobile dislocation velocity, $\xi = 1$. And considering the AE energy per unit strain,

$$(\dot E_{AE}/\dot\epsilon)\alpha\dot\epsilon^\xi.$$

Thus, the acoustic emission energy during plastic deformation is determined by both the strain and strain rate,

$$f_{AE}(\epsilon, \dot{\epsilon}) \alpha f_{AE}(\epsilon) \dot{\epsilon}^{\xi}$$

or

$$\log f_{AE} = \xi \log \dot{\epsilon} + C,$$

where C is given by $f_{AE}(\epsilon)$ defined in Eq. (21) for initial motion and Eq. (23) for harmonic motion of dislocations in terms of strain. A linear relationship is thus obtained between the log of AE and the log of strain rate, at constant strain.

VII. AE SPECTRAL CHARACTERISTICS

Frequency domain analysis can be used to distinguish between various sources of acoustic emission. The three different source mechanisms discussed thus far are described in the time domain as follows: step increase in dislocation velocity; harmonic motion of dislocations; and annihilation of dislocations at a free surface. Here, only the AE from harmonic motion experiences a change in frequency as strain increases. The frequency of forced motion of a dislocation line $\bar{\omega}$ can be related to the strain,

$$\bar{\omega} = v/d.$$

Substituting for v from Eq. (17) and d from Eqs. (15) and (22), we see that

$$\bar{\omega} \alpha \epsilon^{-n}. \tag{34}$$

Consequently, during plastic deformation, the AE from harmonic motion shifts to lower frequencies as strain increases.

We now consider the spectral characteristics of AE from the free motion of dislocations. The elastic wave radiated from a source mechanism interacts with dislocations during propagation, and is scattered by the strain fields of the dislocations. A force is thus exerted on the dislocations by propagating waves, inducing oscillations of the impinged dislocations. Secondary elastic waves are emitted from the oscillation of the dislocations which can be considered as sinusoidal oscillations of small amplitude. The frequency characteristics are different between the primary and secondary waves. That of the latter depends on the free motion of dislocations, while that of the former depends on the forced motion. The interaction force between neighboring dislocations is inversely proportional to the distance between them. For a finite displacement x of a dislocation from an equilibrium position with neighboring distance d, the interaction force f_i per unit length is expressed as

$$f_i = \kappa_i \left(\frac{1}{d-x} - \frac{1}{d+x} \right),$$

where $\kappa_i = \mu b^2 / 2\pi \alpha_s$. The equation of motion of a free oscillating dislocation without damping is

$$m_e \ddot{x} = f_i,$$

where m_e is the effective mass of the dislocation. Assuming a solution of x gives $A \sin \bar{\omega}_n t$ for small displacements, we have

$$m_e \bar{\omega}_n^2 \cong 2\kappa_i / d^2$$

and

$$\bar{\omega}_n \cong \frac{1}{d} \left(\frac{2\kappa_i}{m_e} \right)^{0.5}.$$

Substituting for d from Eq. (22), we have

$$\bar{\omega}_n \alpha \rho^{0.5}. \tag{35}$$

Substituting Eq. (15) into Eq. (35), the natural frequency of a dislocation $\bar{\omega}_n$ is obtained in terms of strain as follows:

$$\bar{\omega}_n \alpha \epsilon^n. \tag{36}$$

The radiation energy of a secondary wave is relatively small compared to that of a primary wave. Even though it is difficult to describe the momentum transfer from a propagating primary wave to a secondary wave, the amount of secondary wave energy increases with an increase in dislocation density since the dislocation itself acts as the source of the secondary wave. Thus, in considering the free vibration of dislocations, a shift of the spectrum to higher frequencies is expected with an increase in ϵ.

VIII. DISCUSSION

It is evident from the analysis, which is based on straight dislocation models, that the AE energy due to both the initial and harmonic motions of a dislocation is proportional to the source number and to the square of the velocity [Eqs. (8), (9), (11), and (12)]. However, harmonic motion generates a continuous acoustic emission signal, while initial motion does generate a burst-type AE signal which has a maximum in the initial stage of motion and decays rapidly with time.

The AE energy from the deformation of a crystalline material is represented in terms of strain by superposition of the release of elastic energy from initial motions, harmonic motions, and annihilation of dislocations which are obtained from Eqs. (21), (23), and (29), since the three source mechanisms occur simultaneously during strain as expressed in Eq. (1).

Equations (16) and (19) express the mobile dislocation and newly created mobile dislocation densities in terms of strain, and are shown in Fig. 5. It is also shown that the number of AE sources by initial motion is determined by the work hardening exponent. However, the density of annihilated dislocations per unit strain is constant during strain with a constant strain rate. It is also evident from Eqs. (16) and (19) that the number of source mechanisms increases in harmonic motion, and decreases in initial motion when the work hardening index n is smaller than 0.5. However, as the strain increases, the overall acoustic emission energy decreases due to the falloff of dislocation velocity shown in Eq. (17), since the velocity effect on the AE signal is more dominant than that of the number of sources.

A comparison between the measured and calculated AE is shown in Fig. 6, where the two solid lines are measured data from deformation of a pure aluminum specimen[10] and the broken lines are calculated from Eq. (21)
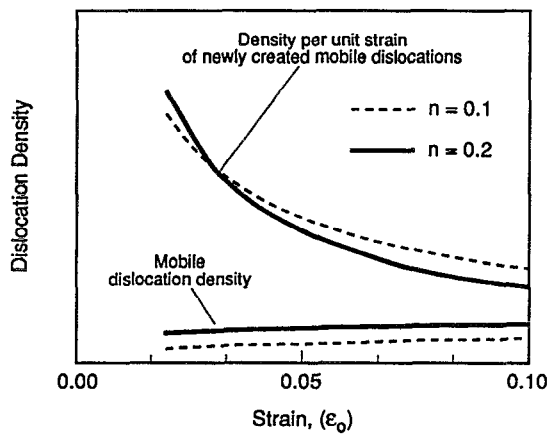
FIG. 5. Density of newly created and mobile dislocations in terms of strain.

for initial motion and Eq. (23) for harmonic motion. As shown in Fig. 6, the measured AE energy can be obtained by a linear interpolation of AE from harmonic motion and AE from initial motion, since the measured AE is between them.

Energy is also radiated during the dissipation of a strain field at a free surface. In this case, the effect of dislocation velocity is negligible since the strain energy is much greater than the kinetic energy at a relatively low dislocation velocity. Thus, AE energy per unit strain from annihilation of dislocations at a surface is relatively constant during deformation, regardless of dislocation multiplications. In deforming a finite volume, the surface-to-volume ratio is found to be one of the parameters that determines the free-surface effect on AE as shown in Eq. (29).

Experimental data is available that shows the strain rate effect on AE for aluminum.[1,3,10] All results show that the strain rate has a linear relationship with the AE power and that the AE energy from a unit strain is constant regardless of the strain rate. The m value for pure aluminum is 0.02 at room temperature. Hence ξ becomes very
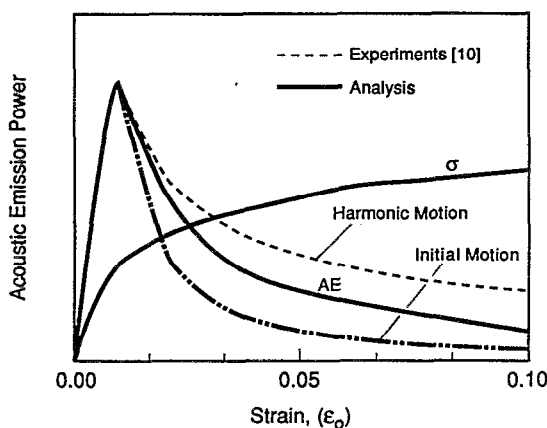
small, with m' being of the order of 1, which agrees with the experimental results. An increase in strain rate does not change the AE spectrum,[13] which means that increasing the strain rate raises the density of mobile dislocations rather than the dislocation velocity. That means ξ approaches zero. Carpenter and Chen[14] have shown experimentally for zinc that the value of ξ is from 0 to 0.4 depending on the annealing time, which results in a linear relationship between log of AE and log of strain rate. More experimental results for other materials are needed to enable more general conclusions to be drawn. However, for most metals at room temperature, the strain rate sensitivity m is considered to be extremely low, of the order of 10^{-2} at room temperature,[15] which would result in the dislocation velocity being insensitive to strain rate, and ξ approaching 0. Consequently, at a given strain, the strain rate would be more dominantly influenced by ρ_m and ρ_{mc} than v. An increase in strain rate then results in a linear increase in the number of AE sources without a change of velocity. Thus, for the lower strain rate sensitivity condition, ξ approaches 0 and the rate of emission of AE energy approaches a linear relationship with strain rate $\dot{\epsilon}$.

It has been shown experimentally that the AE signal spectrum from plastic deformation of a single crystal moves to higher frequencies as strain increases,[13,16] and this was explained on the basis that the duration of dislocation motion could decrease as a result of a decrease in mean glide distance under an assumption of constant velocity. However, dislocation velocity is not constant during work hardening because an increase in interaction force reduces velocity. Both the velocity and glide distance decrease with an increase in strain, and the frequency of harmonic motion thus decreases with an increase in strain as obtained in Eq. (34). Hence, a spectral change in AE from the source as observed experimentally may only be explained by a change in the free vibration of dislocations. Both mobile and immobile dislocation lines vibrate due to the radiation energy from moving dislocations. The natural frequency is raised as dislocation density increases with strain as shown in Eq. (36).

IX. CONCLUSIONS

Initial and harmonic motion of dislocations during plastic deformation of a crystal is considered as a source of acoustic emission. AE from initial motion of the newly created mobile dislocations decreases with strain to the power of $-2n-1$, and that from harmonic motion decreases with strain to the power of $-3n$.

The presence of a free surface provides an AE source since the strain and kinetic energies of dislocations are dissipated during annihilation at the surface. The resulting AE energy is less dependent on strain ϵ but linearly increases with the surface-to-volume ratio.

The rate of AE energy is proportional to strain rate to the power of $\xi + 1$, and AE energy per unit strain is proportional to strain rate to the power of ξ. For a low strain rate sensitivity condition, ξ approaches 0 and AE energy per unit strain is then independent of strain rate.

FIG. 6. AE variation with time. A comparison between calculated values and experimental data from the literature (see Ref. 10).

The natural frequency of a dislocation line increases with strain to the power of n, and that results in a shift of the AE signal spectrum to higher frequencies during plastic deformation.

APPENDIX

Oscillation of a dislocation moving through a lattice was obtained by Hart[17] considering the Peierls potential. The interaction of neighboring dislocations also provides a periodic potential, and induces oscillation of moving dislocations. The latter oscillation can be analyzed in a manner similar to that due to lattice potential variance.

The effective mass m_e per unit length of a dislocation is given by

$$m_e = E_0/c^2,$$

where c is wave velocity, and the self-strain energy of a unit dislocation $E_0 = (\mu b^2/4\pi)\ln(R/b)$, where R is the distance to a stress-free boundary such as a free surface. For neighboring dislocations of same sign, R is $0.5d$. c is c_t for a screw dislocation and c_l for an edge dislocation. The potential energy E due to the presence of other dislocations can be written as

$$E = \lambda E_i \sin(2\pi x/d),$$

where E_i is the interaction energy between two dislocations, x is the displacement of a dislocation, and λ is less than 1 and depends on the orientation between the glide plane and neighboring dislocations. E_i is equal to $2E_0$. A dislocation moving with average velocity v through the varying potential experiences an oscillatory force F given by

$$F = -\frac{dE}{dx} = -\frac{2\pi\lambda E_i}{d}\cos\omega t,$$

where $\omega = 2\pi v/d$.

The equation of motion for oscillation from its mean position can be written as

$$m_e \frac{d^2 x'}{dt^2} = \frac{2\pi\lambda E_i}{d}\cos\omega t, \tag{A1}$$

where x' is the displacement from the origin of a coordinate system moving with uniform velocity v. The solution for x' in Eq. (A1) is

$$x' = A\cos\omega t,$$

with

$$A = \frac{4\pi\lambda c^2}{d\omega^2} = \frac{\lambda d}{\pi}\left(\frac{c}{v}\right)^2. \tag{A2}$$

The amplitude of oscillation is thus inversely proportional to the square of the mean velocity. The maximum limit on amplitude is half the periodic potential; the critical velocity can be found by replacing A with $0.5d$ in Eq. (A2), giving

$$v_c = (2\lambda/\pi)^{0.5}c.$$

Thus, when $v > v_c$, the motion of dislocations is determined by their inertia, and the amplitude of motion is determined by Eq. (A2). For a relatively slow velocity of dislocation with $v < v_c$, it is controlled by the potential periodicity due to neighboring dislocations, and the amplitude becomes $0.5d$.

[1] N. Kiesewetter and P. Schiller, Phys. Status Solidi A 38, 569 (1976).
[2] K. Kuribayashi and T. Kishi, Mater. Sci. Eng. 33, 159 (1978).
[3] C. Scruby, H. Wadley, and J. E. Sinclair, Philos Mag. 44, 249 (1981).
[4] F. R. N. Nabarro, Theory of Crystal Dislocations (Oxford University Press, London, 1967).
[5] R. J. Clifton and X. Markenscoff, J. Mech. Phys. Solids 29, 227 (1981).
[6] X. Markenscoff and R. J. Clifton, J. Mech. Phys. Solids 29, 253 (1981).
[7] J. Kiusalaas and T. Mura, Recent Adv. Eng. Sci. 1, 543 (1964).
[8] J. P. Hirth and J. Lothe, Theory of Dislocations (Wiley-Interscience, New York, 1982).
[9] J. Weertman, Response of Metals to High Velocity Deformation, edited by P. G. Shewman and V. F. Zackay (Wiley-Interscience, New York, 1961), pp. 205–247.
[10] H. Hatano, J. Appl. Phys. 49, 3873 (1976).
[11] F. R. N. Nabarro, Z. S. Basinski, and D. B. Holt, Adv. Phys. 13, 193 (1964).
[12] W. C. Leslie, The Physical Metallurgy of Steels (McGraw-Hill, New York, 1982).
[13] H. Hatano, J. Acoust. Soc. Am. 57, 639 (1975).
[14] S. H. Carpenter and C. Chen, J. Acoust. Emis. 7, 161 (1988).
[15] W. F. Hosford and R. M. Caddell, Metal Forming Mechanics and Metallurgy (Prentice-Hall, Englewood Cliffs, NJ, 1983).
[16] D. Rouby and P. Fleischmann, Phys. Status Solidi A 48, 439 (1978).
[17] E. W. Hart, Phys. Rev. 98, 1775 (1955).