A self-consistent approach to spectral hole burning in quantum wire lasers

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In a semiconductor laser above threshold, carriers are extracted at the lasing energy at a high rate due to stimulated emission and are injected at higher energies. This creates a "hole burning" phenomenon resulting in gain compression. This effect is studied in a quantum wire laser by solving the Boltzmann equation with sink and source terms by a novel Monte Carlo technique. The results for various values of the characteristic injection times are given. A formalism is also proposed for the fully self-consistent determination of the laser operating parameters from the rate equations with the inclusion of nonlinear gain effects by substituting the correct form of the distribution function in presence of hole burning into the standard expressions for the laser material gain. The nonlinear gain effect is then described completely starting from the wire band structure and scattering rates. The generality of the proposed technique and its possible extensions and applications to the problem of nonlinear gain in quantum-well lasers are discussed.

In a semiconductor laser biased below threshold, the electron-hole recombination times are ≈ 1 ns, and the electron and hole carrier distributions are well described by quasi-Fermi statistics. However, for a laser biased well above threshold, the stimulated emission rate increases dramatically resulting in a modification of the statistical properties of electrons and holes in the active medium. The carrier distribution is no longer given by the Fermi–Dirac function, but it is determined instead by the interplay of the depletion of carriers at the lasing wavelength and the relaxation of injected carriers into the depleted region. At high injections the relaxation processes are not sufficiently fast to fill the spectral hole formed by virtue of optical transitions.[1,2] This phenomenon is observed experimentally as the decline in the rate of the increase of the light output power at high operating currents. Although a number of other effects such as carrier heating[3,4] have a similar impact on the light-current characteristic of the laser diode, spectral hole burning is believed to be the major source of the observed nonlinearity.

A number of models have been proposed to gain a quantitative understanding of the hole burning effect.[1–8] In theoretical models the density matrix formalism is used to find the nonlinear gain coefficient by applying the perturbation theory analysis. However, all models assume a phenomenological intraband relaxation time, usually of the order of a few picoseconds, as an adequate description of the relaxation processes. Since recent studies of electron capture and relaxation in quantum wells and wires have suggested the nonlinear gain effects are very sensitive to the rate of energy loss by the injected electrons, it is our view that the formulation of a self-consistent method of accounting for the effect of spectral hole burning on the carrier distribution is desirable.

In this communication, we present a method that has been developed to yield the steady-state carrier distribution function for arbitrary injection and extraction characteristic times based on a numerical solution of the Boltzmann equation by a Monte Carlo technique and the results obtained by the application of our approach to hole burning in quantum wire lasers. To reduce the complexity of the problem the following assumptions are made: (i) The hole distribution is well described by the quasi-Fermi statistics since the equilibration of holes occurs much faster than that of electrons owing to weaker hole confinement and higher density of states in the valence band; (ii) extraction and injection times are taken to be equal in steady state in order to prevent buildup or depletion of charge in the active region.

The primary energy-loss mechanism in semiconductors is emission of polar optical phonons (POP). It has been found that quantum confinement increases the time necessary for carriers to equilibrate owing to the suppression of the energy-loss rate by phonon emission caused by the reduction in momentum space[9] and the absence of randomizing intrasubband electron-electron (EE) scattering. For example, in the 100×100 Å^2 quantum wire the relaxation time has been estimated to be ≈ 100 ps.[10] These results suggest that nonlinear gain effects will become more pronounced as the carriers are quantum mechanically confined in a greater number of dimensions.

The deformation potential coupling between electrons and acoustic phonons (AP) is much weaker than the Fröhlich POP-electron interactions. The former cannot be neglected, however, since it is believed to be responsible for overcoming the bottlenecks in the carrier relaxation process formed owing to the nonergodicity of the one-dimensional system.[9,10] The time evolution of the system is described by the Boltzmann equation for a spatially homogeneous system with no applied fields:

$$\begin{aligned}\frac{\partial f(E)}{\partial t} =& f(E+\hbar\omega)[1-f(E)]W_{\rm ems}(E+\hbar\omega,E)\\ &+f(E-\hbar\omega)[1-f(E)]W_{\rm abs}(E-\hbar\omega,E)\\ &-f(E)[1-f(E+\hbar\omega)]W_{\rm abs}(E,E+\hbar\omega)\\ &-f(E)[1-f(E-\hbar\omega)]W_{\rm ems}(E,E-\hbar\omega), \qquad (1)\end{aligned}$$

where W_{abs} and W_{ems} represent the scattering rates for emission and absorption of phonons for k states at corresponding energies. The time derivative is set to zero in order to solve for the steady-state distribution function. It is easy to verify that any function satisfying the condition of detailed balance

$$N_q[1-f(E+\hbar\omega)]f(E) = (N_q+1)f(E+\hbar\omega)[1-f(E)], \quad (2)$$

where N_q is the phonon occupation number, is indeed the solution of the Boltzmann equation. Although the Fermi–Dirac distribution function is not a unique solution of the Boltzmann equation, it is the solution the electron gas will be driven to in the long run under the joint influence of intersubband EE and AP scattering. The effect of stimulated emission at the lasing wavelength and carrier injection at the edge of the potential well formed by the barrier region is now taken into account by introducing the source $([1-f(E)]/\tau)$ and sink $[-f(E)/\tau]$ terms. The energy spectrum of interest is then subdivided into small intervals (in this work, 200 intervals 1 meV wide), and the Boltzmann equation can be converted into a system of coupled nonlinear equations. The equations are greatly simplified if solely POP scattering is included because only points separated by multiples of the POP energy are coupled. The AP scattering is then treated as contributing a broadening of the solution of the simplified equation with a linewidth proportional to the AP scattering rate. This is an adequate approximation since POP scattering rates are greater than AP scattering rates in quantum wires by at least an order of magnitude. The scattering rates are computed using the well-known formalism for bulk phonon modes.[9,11,12] The structure considered in this communication is the 100×100 Å^2 GaAs/$Al_{0.3}Ga_{0.7}As$ wire at room temperature.

The direct numerical solution of the resulting system of nonlinear coupled equations is cumbersome for a fine mesh on the energy spectrum. Instead we solve the problem by a Monte Carlo technique shown in Fig. 1. Excellent convergence is obtained for injection times greater than 1 ps. The steady-state distribution function for a characteristic time of 10 ps is shown in Fig. 2. This injection-extraction time roughly corresponds to a current density 100 times the threshold current density calculated for the 100×100 Å^2 wire. The effect of injection and extraction terms on the distribution function is seen to be twofold: a spectral hole is burned around the lasing wavelength and the electron gas effective temperature is raised. The magnitude of gain compression can be estimated by calculating the material gain from the Fermi Golden Rule (in Gaussian units):

$$g(\hbar\omega) = \frac{4\pi^2 e^2 \hbar}{n_0 c m_0^2 \hbar\omega} \frac{1}{A} \frac{2}{2\pi} \int dk_z \sum_{n,m} |\hat{\epsilon} \cdot P_{nm}(k_z)|^2 \times \delta[E_n^e(k_z) - E_m^h(k_z) - \hbar\omega][f^e(E_n^e) - f^h(E_m^h)], \quad (3)$$

where $\hat{\epsilon}$ is the polarization of light, P_{nm} is the momentum matrix element between the electron and hole states, and A is the cross-sectional area of the wire. We substitute the electron distribution function obtained with the inclusion of spectral hole burning effects. The result is shown in Fig. 3. Mimicking the singularity in the joint density of states, the gain spectrum is sharply peaked around the lasing fre-

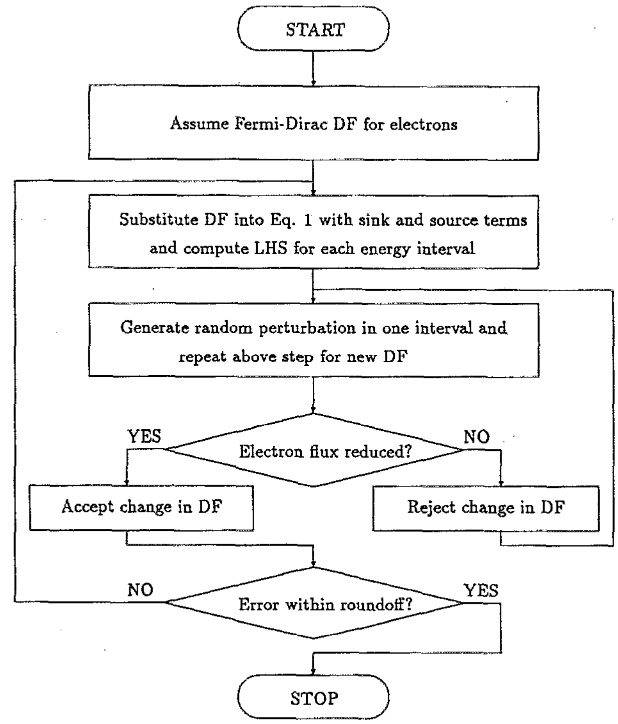

FIG. 1. The flowchart illustrating the Monte Carlo technique used to solve Eq. (1) numerically.

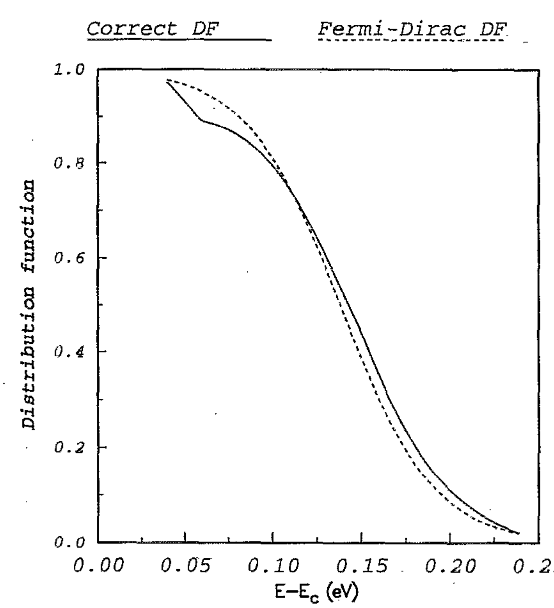

FIG. 2. The distribution function in presence of spectral hole burning is shown by the solid line. For comparison, the equilibrium Fermi-Dirac distribution function is given by the dashed line. The characteristic injection and extraction time is 10 ps, approximately corresponding to a current density 100 times that at threshold.

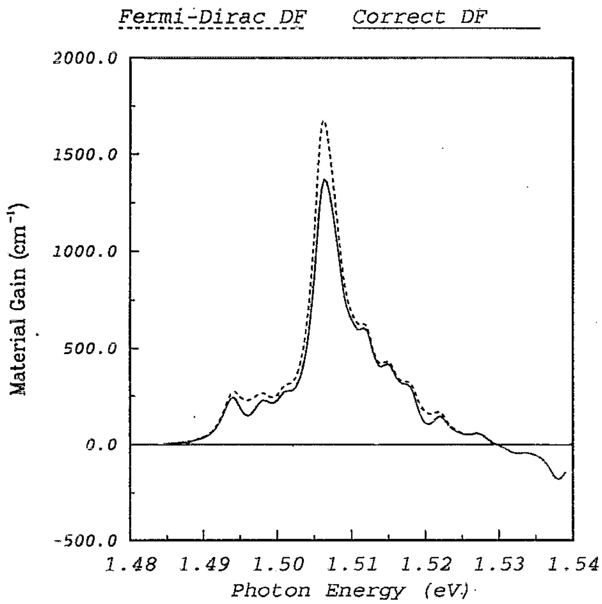

FIG. 3. Gain compression resulting from the inclusion of spectral hole burning. The solid line represents the material gain spectrum obtained by using the correct distribution, while the dashed line represents that obtained using the Fermi-Dirac distribution. The characteristic and extraction injection time is 10 ps, approximately corresponding to a current density 100 times that at threshold.

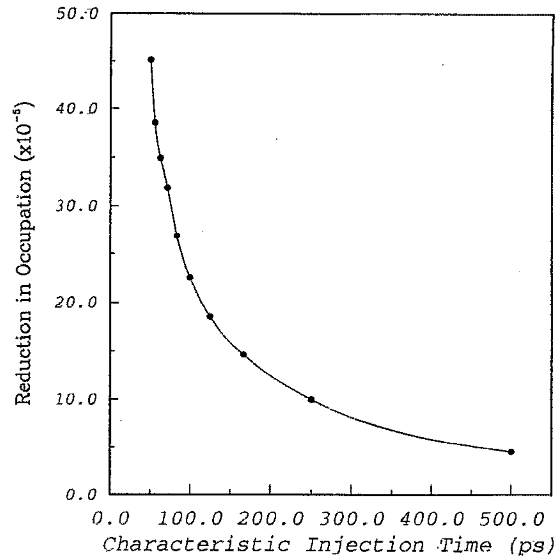

FIG. 4. The peak reduction in the occupation probability at the lasing wavelength as a function of the characteristic injection and extraction time.

quency, and the nonlinear gain effects are clearly seen to amount to the suppression of the peak gain. Note that in the context of this communication, we refer to the spectral hole formed in the distribution function rather than in the gain spectrum as the "hole burning" effect. No hole in the gain spectrum is observed because of the assumption of single mode lasing and the sharpness of the gain curve. We illustrate the reduction in the occupation probability at the lasing energy as a function of the characteristic extraction-injection time in Fig. 4.

Although the presented method does not require extraction at a single energy, we have made the assumption of single-mode operation above threshold which is excellent considering the sharpness of the gain peak in quantum wire lasers. However, our method is not limited by this assumption in general. The results for the distribution function obtained by the method described in this communication can then be used in a fully self-consistent solution of the laser rate equations, in which the injection characteristic time is consistent with the injected current density.

In brief summary, we have proposed a novel method for deriving the correct form of the carrier distribution from a solution of the scattering rate equation for the distribution function. The form of the distribution function obtained in this manner is consistent with the exact description of the relaxation processes in quantum wire laser structures. The correct form of the distribution function is then substituted into the perturbation theory expression for the laser material gain and the rate equations. This approach allows one to compute all parameters necessary to describe gain compression due to spectral hole burning in quantum wire lasers from the wire band structure and scattering processes. We would like to emphasize particularly the generality of the technique presented in this communication in that it can be applied with equal success to calculation of the nonlinear gain effects in quantum-well lasers. A detailed analysis of the effect of spectral hole burning on the quantum wire laser operating characteristics is forthcoming.

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