Electrohydrodynamic Solutions for Nematic Liquid Crystals

P. Andrew Penz*
Scientific Research Staff, Ford Motor Company, Dearborn, Michigan 48121

and

G.W. Ford
Department of Physics, University of Michigan, Ann Arbor, Michigan 48104
(Received 29 November 1971)

We have solved the boundary value problem associated with the Williams domain mode and the variable grating mode in nematic liquid crystals. Using linearized electrohydrodynamic equations of motion, standard constitutive relations, and experimentally observed material constants, we reproduce the significant experimental observations.

The electrohydrodynamic modes of nematic liquid crystals (NLC) have been studied intensively recently due to their electro-optic display potential. The first mode was discovered by Williams,[1] and the basic features of this experiment are shown in Fig. 1. The NLC is placed in a transparent electrode "sandwich" capacitor, the cross section of which is shown in Fig. 1. The electrodes are wiped to promote orientation of the nematic director in a unique direction, defined as the x direction. The electric field is applied across the capacitor in the z direction. Williams observed that upon the application of a sufficient electric field bright lines appeared, giving the NLC an appearance of domains running in the plane of the sample. These lines are indicated in cross section in Fig. 1 and are to be thought of as extending in both directions perpendicular to the figure (y direction). The wavelength λ associated with the domain pattern gives rise to a relative phase factor $\varphi = \pi d/\lambda$, where d is the sample thickness. Further investigation revealed five crucial experimental observations[2]: (i) The Williams domain mode (WDM) has a critical voltage for formation; (ii) the wavelength is inversely proportional to thickness, i.e., φ is independent of d; (iii) fluid motion in a vortex-type pattern is coincident with the domains; (iv) the visibility of the domains is associated with a focusing produced by a spatial variation of the director $\mathbf{n}$ and the anisotropic index of refraction associated with NLC; (v) the wavelength of the pattern is not a sensitive function of voltage above threshold for relatively thick ($d > 10$ μm) samples.

Greubel and Wolff[3] have recently observed a new mode which is similar to the WDM in the first four items, but which differs in an important manner in the fifth. They used very thin samples with high resistivity and found that the phase factor φ varied linearly with voltage above threshold. They produced a diffraction grating which operates in what might be called a variable grating mode (VGM).

We have recently made a rigorous calculation of the boundary value problem associated with these electrohydrodynamic effects and have found solutions that correspond to both the WDM and VGM.[4] We report here a brief summary of the theory and the results. The basic physical mechanisms involved in the problem were discussed by Helfrich, and this theory is an extension of his treatment.[5] Pikin[6] has also extended Helfrich's theory in a manner similar to our method and has demonstrated a critical voltage. The key assumption is that the NLC has an Ohmic conductivity which is larger along the director than perpendicular to it. Generally this means that the current and electric field need not be parallel and permits the formation of space charge in layers parallel to the yz plane. Because of the static electric field's interaction with these layers, a shear flow can be set up. The shear reorients the director, and more space charge is generated which further turns the director. The possibility for an instability is apparent.

The calculation begins by assuming the standard hydrodynamic differential equations associated with anisotropic liquids. The electric forces and torques are expressed as operations on the Maxwell stress tensor. This description allows a concise statement of the equations of motion. We employed Leslie's formulation of the viscous stress tensor. We assumed an anisotropic dielectric tensor and expressed the elastic torques as operations on the standard Frank free energy. The conditions at the boundary were assumed to be that the velocity vector had to be zero, the director had zero z component, and the electric field could have no x component.

The mathematical problem was linearized in the distortion amplitudes, isothermal and steady-state conditions

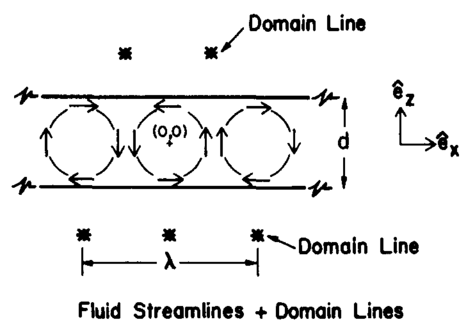

FIG. 1. Schematic drawing of the streamlines and domain lines is given. The figure shows a cross section of the experimental geometry: a capacitor filled with liquid crystal. The electric field is applied in the z direction ($\hat{e}_z$), and the electrodes are rubbed to promote orientation of the director in the x direction ($\hat{e}_x$). Above a critical voltage, vortex motion is observed, the vortices extending in the y direction and antiparallel in adjacent cells. The periodicity of the fluid motion is λ. When the sample is illuminated from below, an observer above the sample sees bright domain lines extending in the y direction. There are two sets of domain lines, one above the sample and one below. It can be shown that these lines result from focusing by the director spatial distribution in the NLC.

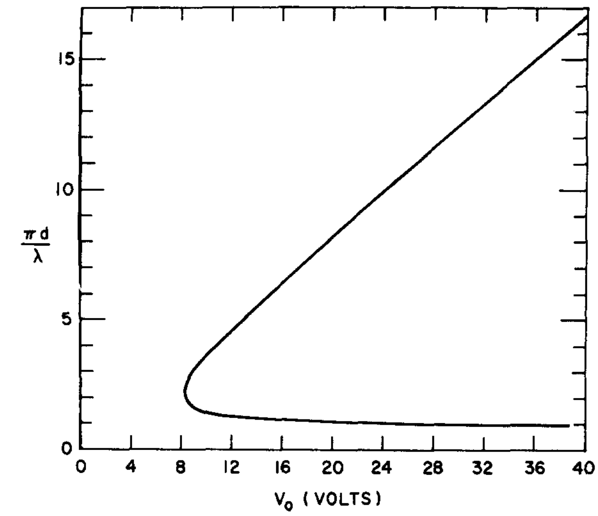

FIG. 2. Computer calculation of the $\pi d/\lambda$ and V values which solve the boundary value problem discussed in the text. The NLC is PAA at 125 °C. The dispersion relation is double valued for any given voltage above the threshold voltage of 8.2 V. This means that there are two normal modes possible for any V_0 above 8.2 V. The low-phase region corresponds to the WDM. The linear phase-voltage region at higher voltages corresponds to the VGM. The spatial distributions of field, velocity, and director have also been calculated and correspond exactly with Fig. 1.

were assumed, and sinusoidal trial functions were used. Two sets of linear homogeneous equations resulted. The simultaneous solution of both sets of equations was performed numerically and resulted in the dispersion relation shown in Fig. 2. This curve represents the normal modes of the system, i.e., those values of λ and V_0 which satisfy the boundary value problem. This dispersion relation was calculated using experimentally determined contants for p-azoxyanisole (PAA). It can be seen that the phase factor is a double-valued function of voltage. The lower branch shows φ roughly independent of voltage and can be identified with the WDM. The upper branch shows that φ varies in a roughly linear manner with voltage and can be identified with the VGM. The qualitative agreement between the theory and these two modes has been demonstrated by numerical calculations of velocity and director patterns. The quantitative agreement between the theory and the WDM for PAA is very satisfactory. The VGM experiments were not performed with PAA, and so the comparison can only be semiquantitative. The phase factor for the VGM experiment was on the order of 3 at 30 V, whereas PAA could be expected to have a phase factor of 12 at 30 V. Considering the difference in materials, we are satisfied with this order-of-magnitude agreement.

The linear steady-state nature of the calculation precludes a theoretical prediction of the conditions under which the upper or lower branch of the dispersion relation would be observed. The problem is further complicated because there are a family of theoretical dispersion relations, *each* curve of similar shape and nested inside the *previous* curve at roughly 7-V higher threshold voltage. These additional curves correspond to increasing numbers of vortex layers across the sample and have not been presented in Fig. 2 for reasons of clarity. A theoretical stability analysis would be guided by the experimental observation that the VDM is realized in thin samples ($d < 10$ μm) with high resistivity.[3]

In conclusion, we wish to report that two steady-state electrohydrodynamic modes in NLC can be understood on the basis of conduction-induced alignment.[5] A stability treatment of the theory is now necessary to uniquely predict the proper conditions of the WDM, VGM, and the dynamic scattering mode, which is probably expressible as a turbulent solution.

We wish to thank Dr. W. Weber for valuable assistance in the numerical computations associated with this paper.

*Present address: Central Research Laboratory, Texas Instruments, Dallas, Tex. 75222.

[1]R. Williams, J. Chem. Phys. 39, 384 (1963).
[2]Several workers have contributed to the experimental understanding of the WDM. For a review of this work, see P.A. Penz, Mol. Cryst. 15, 141 (1971).
[3]W. Greubel and U. Wolff, Appl. Phys. Letters 19, 213 (1971); see, also, L.K. Vistin, Kristallografiya 15, 594 (1970) [Sov. Phys. Cryst. 15, 514 (1970)].
[4]P.A. Penz and G.W. Ford, Phys. Rev. A (to be published).
[5]W. Helfrich, J. Chem. Phys. 51, 4092 (1969).
[6]S.A. Pikin, Zh. Éksperim. i Teor. Fiz. 60, 1185 (1971) [Sov. Phys. JETP 33, 641 (1971)].