The effect of nonlinear gain on the stability of evanescently coupled semiconductor laser arrays

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We show that nonlinear gain saturation can enhance the stability of evanescently coupled semiconductor laser arrays.

Arrays of semiconductor lasers are widely used as compact sources of intense radiation, with potential for generating watts of power in diffraction limited beams.[1] Coherent arrays rely on the principle of phase locking, and in previous articles, we have analyzed the stability of the phase-locked state by solving coupled rate equations for the laser array.[2,3] A key result of this analysis was that the phase-locked state is unstable over a wide range of coupling strength between the lasers. In the unstable regime, the coupled lasers are predicted to exhibit sustained self-pulsations at gigahertz frequencies. Evidence of such self-pulsations has been reported in several experiments.[4] One factor that may help extend the stable range of operation of coupled lasers is the presence of nonlinear gain saturation in which the optical gain depends on the photon density. In this article, we include the effect of nonlinear gain on the dynamics of coupled lasers and find that it can reduce the domain of instability for the phase-locked state.

The coupled mode equations for the carrier density, N_j, and the complex electric field, $\widetilde{E}_j$, in each laser of the array may be written[3]

$$\frac{d\widetilde{E}_j}{dt}=\frac{1}{2}\left(\Gamma G(N_j)-\frac{1}{2}\right)(1-i\alpha)\widetilde{E}_j+iK(\widetilde{E}_{j-1}+\widetilde{E}_{j+1}),\tag{1}$$

$$\frac{dN_j}{dt}=P-\frac{N_j}{\tau_s}-\Gamma G(N_j)\,|\widetilde{E}_j|^2,\quad j=1,2,...,\tag{2}$$

where τ_p is the photon lifetime, τ_s is the carrier lifetime, P is the pump rate, K is the coupling strength between adjacent lasers, α is the linewidth enhancement factor, G is the gain, and Γ is the mode confinement factor. The gain function $G(N_j)$ may be linearized about its value at the carrier density N_0 required to achieve transparency

$$G(N_j)=g(N_j-N_0),\tag{3}$$

where g is the differential gain. In our previous analyses, the differential gain was taken to be independent of intensity. At high intensity, however, the gain does saturate in the manner

$$g=\frac{g_0}{(1+\epsilon|E|^2)^\beta},\tag{4}$$

where $\beta=1$ for a phenomenological two-level model or $\beta=\frac{1}{2}$ according to a recent nonperturbative treatment.[5] The nonlinear gain parameter ϵ has a value of order 1×10^{-17} cm^3 for InGaAsP lasers and the electric field is normalized so that $|E|^2$ gives the photon density.

For two coupled lasers, the rate equations are

$$\frac{dE_1}{dt}=\frac{1}{2}\left(\frac{g_0'(N_1-N_0)}{1+\epsilon E_1^2}-\frac{1}{\tau_p}\right)E_1-KE_2\sin\theta,\tag{5a}$$

$$\frac{dE_2}{dt}=\frac{1}{2}\left(\frac{g_0'(N_2-N_0)}{1+\epsilon E_2^2}-\frac{1}{\tau_p}\right)E_2+KE_1\sin\theta,\tag{5b}$$

$$\frac{d\theta}{dt}=-\frac{1}{2}\alpha g_0'(N_2-N_1)+K\left(\frac{E_1}{E_2}-\frac{E_2}{E_1}\right)\cos\theta,\tag{5c}$$

$$\frac{dN_1}{dt}=P-\frac{N_1}{\tau_s}-g_0'\frac{(N_1-N_0)}{1+\epsilon E_2^2}E_1^2,\tag{5d}$$

$$\frac{dN_2}{dt}=P-\frac{N_2}{\tau_s}-g_0'\frac{(N_2-N_0)}{1+\epsilon E_2^2}E_2^2.\tag{5e}$$

Here, $E_{1,2}=|\widetilde{E}_{1,2}|$ and $\theta=\phi_2-\phi_1$ is the phase difference between the fields in the two lasers. We also define the unsaturated modal gain as $g_0'=\Gamma g_0$. In these equations, we have used the form $(1+\epsilon E^2)^{-1}$ to represent gain saturation. From the linear stability analysis that follows, we find that the results for gain saturation of the form $(1+\epsilon E^2)^{-1/2}$ can be obtained simply by letting $\epsilon\to\epsilon/2$ in the expressions for the stability boundaries. This is clear from the linearized form of the two models for gain saturation.

Equations (5) have two symmetric steady-state solutions with $E_1=E_2\equiv E$, $N_1=N_2\equiv N$, and $\theta=0$ (in-phase), or $\theta=\pi$ (out-of-phase). The steady-state carrier density is

$$N=\frac{N_{\rm th}+\epsilon P/g_0'}{1+\epsilon/g_0'\tau_s},\tag{6}$$

where the threshold carrier density is given by $N_{\rm th}=N_0+1/g_0'\tau_p$. For the steady-state field intensity, we have

$$E^2=\frac{(P-P_{\rm th})\tau_p}{(1+\epsilon/g_0'\tau_p)},\tag{7}$$

with the threshold pump rate defined as $P_{\rm th}=N_{\rm th}/\tau_p$.

To proceed with the stability analysis, we linearize Eqs. (5) about the steady-state solution. Taking $E_{1,2}=E+e_{1,2}(t)$, $N_{1,2}=N+n_{1,2}(t)$, and $\theta=\theta_0+\delta(t)$, we obtain the following linear differential equations for the perturbation δ, $e=e_2-e_1$, and $n=n_2-n_1$:

 0021-8979/93/010459-03$06.00

$$\begin{bmatrix} \dot{e} \\ \dot{n} \\ E\dot{\delta} \end{bmatrix} = \begin{bmatrix} \frac{-\epsilon E^2}{\tau_p(1+\epsilon E^2)} & \frac{g_0' E}{2(1+\epsilon E_2)} & 2KE\cos\theta_0 \\ \frac{-2E}{\tau_p(1+\epsilon E^2)} & -\left(\frac{1}{\tau_s}+\frac{g_0' E^2}{1+\epsilon E^2}\right) & 0 \\ -2K\cos\theta_0 & -\frac{1}{2}\alpha g_0' E & 0 \end{bmatrix} \times \begin{bmatrix} e \\ n \\ E\delta \end{bmatrix}. \quad (8)$$

The stability of the steady state is governed by the above matrix of coefficients. Application of the Routh–Hurwitz criterion permits us to determine the regions of parameter space in which the steady-state solution is stable. We find that the out of phase solution is stable if

$$K > \frac{\alpha g_0' \tau_s E^2}{2\tau_p[1+(\epsilon+g_0'\tau_s)E^2]}, \quad (9)$$

while the in-phase solution is stable if

$$4K^2\epsilon + \frac{(g_0'\tau_s+\epsilon)}{\tau_s^2}\left(1+\frac{\tau_s(g_0'\tau_p+\epsilon)E^2}{\tau_p(1+\epsilon E^2)}\right) > 2K\alpha g_0'. \quad (10)$$

To make contact with our previous analyses,[3] we introduce the dimensionless quantities $\eta = K\tau_p$, $T = \tau_s/\tau_p$, $\gamma = \epsilon/g_0'\tau_s$, and $p = \frac{1}{2}(g_0'\tau_s+\epsilon)E^2$. Here, p represents the normalized excess pump current beyond the threshold value. The parameter γ is generally a small quantity ($\sim 10^{-3}$), hence, we can obtain the following approximate expressions for the stability boundaries from Eqs. (9) and (10):

$$\eta > \frac{\alpha p}{1+2p} \quad \text{(in-phase)}, \quad (11)$$

$$p > \frac{2\eta T(\alpha - 2\eta T\gamma) - 1}{2(1+T\gamma)} \quad \text{(out-of-phase)}. \quad (12)$$

Figure 1(a) shows the stability domain in the plane of the variables η and p for different values of the nonlinear gain parameter γ. From these results, it is clear that nonlinear gain can significantly increase the amount of phase space under which the stable operation of evanescently coupled arrays may occur. One striking feature of these results is the fact that the in-phase mode of operation is substantially unaffected by the presence of the nonlinear gain, whereas, the out-of-phase mode is clearly very strongly affected. This is indicative of the fact that the stability of the in-phase mode is dependent chiefly upon the extent to which the photons coupled in from the adjacent emitter represent a significant fraction of the photons generated within the device, i.e., the extent to which the array acts as one large laser. The stability of this mode is thus more dependent upon what goes on between the lasers than what goes on within them.

In order to gain some insight into the effect of the nonlinear gain on real devices, it is useful to examine the way in which an array of InGaAsP double heterostructure

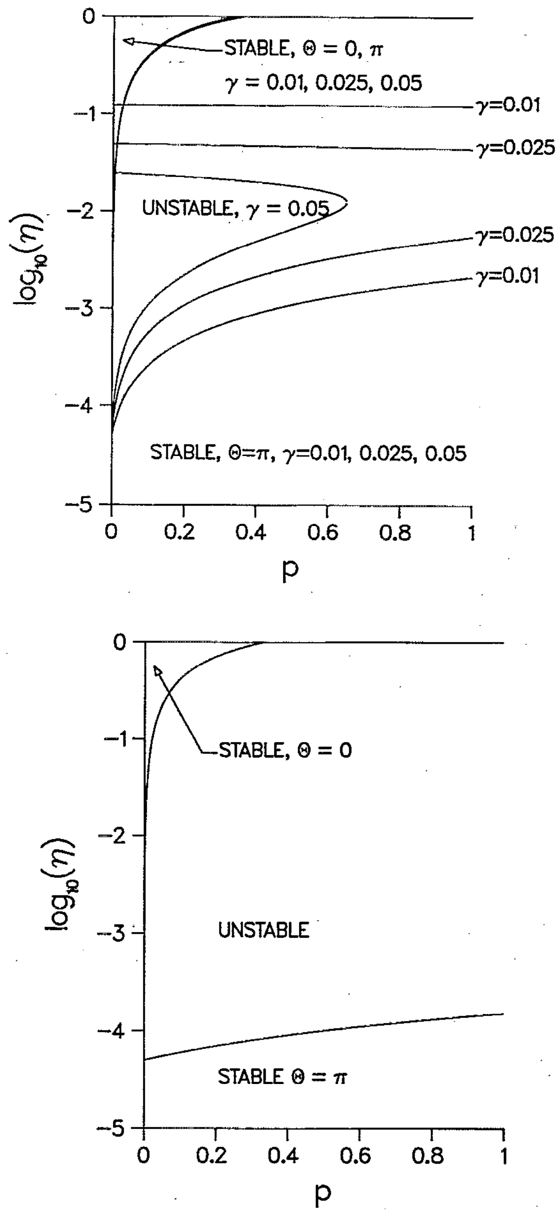

FIG. 1. Stability domain of the in-phase and out-of-phase solutions for (a) several values of nonlinear gain parameter $\gamma = \epsilon/g_0'\tau_s$ and (b) in the absence of nonlinear gain. Here, p is the excess pump current and η is the coupling strength.

lasers might be affected. If we take typical values of $g = 2\times10^{-6}$ cm^3 s^{-1}, $\epsilon = 1\times10^{-17}$ cm^3, $\tau_s = 2\times10^{-9}$ s, and $\Gamma = 0.2$, we find $\gamma = 1.25\times10^{-2}$. From Fig. 1(a), we see that this value of the nonlinear gain parameter significantly extends the stable domain of the out-of-phase mode. For longer wavelength strained multiple quantum well lasers, the damping effect is expected to be even stronger. For AlGaAs lasers, where the nonlinear gain parameter is about five times smaller,[6] we expect a much smaller effect on the stable domain of operation. For AlGaAs lasers, $\gamma \sim 10^{-3}$ and we can see, by comparing Fig. 1(a) to the stability boundaries in the absence of nonlinear gain, plotted in Fig. 1(b), that the AlGaAs array should be only weakly affected by nonlinear gain.

In summary, a stability analysis of twin-element evanescently coupled laser arrays has been extended to include the effects of nonlinear gain. The presence of the nonlinear gain has been shown to have a significant damping effect on the instability of evanescently coupled arrays.

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